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Soustavy lineárních rovnic a Gaussova eliminace

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Úkol 1. Vyřešte soustavu lineárních rovnic s parametrem $a \in \mathbb{R}$:

$$\begin{matrix}(1)\\(2)\\(3)\end{matrix}\left(\begin{array}{ccc|c} 2a+1 & a & 2a & 1 \\ a & 1 & a+1 & 0 \\ 2a & 0 & 2a & 0 \end{array}\right)\begin{matrix}-(3)\\ \sim \\ :2\end{matrix}$$

[10 b]

$$\sim \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ a & 1 & a+1 & 0 \\ a & 0 & a & 0 \end{array}\right) \sim \overset{-a\cdot(1)}{\searrow} \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ 0 & 1-a^2 & a+1 & -a \\ a & 0 & a & 0 \end{array}\right) \overset{-a\cdot(1)}{\searrow} \sim \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ 0 & 1-a^2 & a+1 & -a \\ 0 & -a^2 & a & -a \end{array}\right) \cdot$$

$$\sim \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ 0 & 1-a^2 & a+1 & -a \\ 0 & a^2 & -a & +a \end{array}\right) \overset{-(3)}{\nearrow} \sim \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & a^2 & -a & +a \end{array}\right) \sim \overset{-a^2\cdot(2)}{\searrow} \sim \left(\begin{array}{ccc|c} 1 & a & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -a^2-a & a \end{array}\right)$$

$x_3 \cdot (-a^2-a) = a$

$x_3 = \frac{a}{-a^2-a} = \frac{a}{-a(a-1)} = \frac{1}{-a-1} = -\frac{1}{a+1}$

$x_2 + 1 \cdot \left(-\frac{1}{a+1}\right) = 0$

$x_2 = \frac{1}{a+1}$

$x_1 + a \cdot \frac{1}{a+1} + 0 = 1$

$x_1 = 1 - \frac{a}{a+1} = \frac{a+1}{a+1} - \frac{a}{a+1} = \frac{1}{a+1}$

Pro $a \in \mathbb{R} \setminus \{0, 1\}$:

$x_1 = \frac{1}{a+1}$

$x_2 = \frac{1}{a+1}$

$x_3 = -\frac{1}{a+1}$

Pro $a = 0$:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = 1$

$x_2 = -x_3$

$x_3 \in \mathbb{R}$

Pro $a = -1$

$$\left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ -1 & 1 & 0 & 0 \\ -2 & 0 & -2 & 0 \end{array}\right) \overset{\cdot(-1)}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ 0 & 2 & 2 & -1 \\ -2 & 0 & -2 & 0 \end{array}\right) \overset{+2\cdot}{\sim}$$

$$\sim \left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 0 & 2 & 2 & -1 \\ 0 & -2 & -2 & -2 \end{array}\right) \overset{+1\times}{\sim} \left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 0 & 2 & 2 & -1 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$\text{rank}(A) < \text{rank}(A|b)$

$\Rightarrow$ žádné řešení