# Newton Raphson in N dimensions

→ Taylor series for a real valued function of a vector variable

$$f(x) = f(x_{n-1}) + \vec{\nabla} f(x_{n-1})(x - x_{n-1})$$

$$= f(x_{n-1}) + \begin{bmatrix} \frac{\partial f(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

→ Suppose we have n functions, so for each we have:

$$f_k(x) \approx f_k(x_{n-1}) + \begin{bmatrix} \frac{\partial f_k(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_k(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

$\therefore$ we have

$$f_1(x) \approx f(x_{n-1}) + \begin{bmatrix} \frac{\partial f_1(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_1(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

$$f_2(x) \approx f(x_{n-1}) + \begin{bmatrix} \frac{\partial f_2(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_2(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

$$\vdots$$

$$f_n(x) \approx f_n(x_{n-1}) + \begin{bmatrix} \frac{\partial f_n(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_n(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

$\therefore$ we have

$$\begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix} \approx \begin{pmatrix} f_1(x_{n-1}) \\ \vdots \\ f_n(x_{n-1}) \end{pmatrix} + \begin{bmatrix} \frac{\partial f_1(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_1(x_{n-1})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(x_{n-1})}{\partial x_1} & \cdots & \frac{\partial f_n(x_{n-1})}{\partial x_n} \end{bmatrix} (x - x_{n-1})$$

Jacobian evaluated at $x_{n-1}$

$\therefore$ we have n tangent $(n-1)$-dimensional hyperplanes at $x_{n-1}$:

$$f(x) \approx f(x_{n-1}) + J(f(x_{n-1})(x - x_{n-1})$$

A root of these hyperplanes may be found by equation this to the zero vector

$$0 = f(x_{n-1}) + J(f)(x_{n-1})(x - x_{n-1})$$

$$\Rightarrow \vec{x}_n = \vec{x}_{n-1} - J(\vec{x}_{n-1})^{-1} \vec{F}(\vec{x}_{n-1})$$

where $\vec{x}_n = (x_1, x_2, \ldots, x_n)^T$, $\vec{F}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \ldots, f_n(\vec{x}))^T$

$J(\vec{x})$ = Jacobian matrix of $\vec{F}(\vec{x})$