SAT Intensive Workshop - Day 21

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Please address any questions, comments, concerns, or errors to tomas.matzner@utexas.edu.

Don't forget to bring your calculators every time! Also, don't forget to bring a laptop tomorrow, Tuesday (and every Tuesday) if you want to type your essay. Otherwise, handwritten is also fine.

1 Policies

1.1 Class Homepage

All class materials will be posted on my Github, https://github.com/TomasMatzner/SAT-Intensive-2019. I will post the day's materials no later than 6:00 PM (except on Friday nights), unless I say otherwise.

1.2 Class Conduct

Like I said, I plan to keep it fairly casual in the classroom. You are not in middle school anymore, and college is not far off. As such, I plan to give you more privileges than you might get at school. For example, just tell me you're going to the restroom – it's not like I'm going to say no anyway. However, these privileges are just that – privileges. They can be taken away if you abuse them.

I am more than happy to allow class-related discussion while I am presenting, as I believe that learning happens best within peer groups and with laughter. However, I do ask that we pay attention when a fellow student is talking or presenting – getting up to talk is no small feat, and we want to make sure that everyone feels supported in their academic endeavors.

1.3 Electronics

Cell phones should be used sparingly, if necessary, throughout the day. Checking notifications every once in a while for critical messages is fine with me, as long as it doesn't turn into an extended Snapchat story browsing session, etc. If you feel that one of your classmates is using their phone too much, let me know and we'll hold a collective vote on whether the offender's phone privileges should be revoked for the day.

If you finish a practice exam, have double-checked your work, and feel confident about your answers, using your cell phone under the table is fine, but the screen must be out of view of the other students in the room. I do not anticipate this being a problem, but if I feel that a student is using their phone too much, I reserve the right to ask them to put it away.

Regarding laptops – some of you have expressed the desire to type some essays to save your hands. I agree. I probably won't be able to read some of your handwriting, so typed essays sound good to me too. However, we will do some handwritten essays to simulate the actual test-taking experience. This is a privilege, not a right – if I see anyone using their laptops for non-academic work, laptop privileges will be revoked for everyone in the classroom.

Of course, I reserve the right to change these rules should electronic use interfere with anyone's daily learning.

2 Today's Events

- Diagnostic PSAT Exam.
- Lunch.
- Review of Reading section 1 from the PSAT.
- Review of Math section 3 from the PSAT.
- Math section 4 practice exam.

2.1 Review of Reading section 1 from the PSAT

2.1.1 Strategies

When reading the passages, especially if you're struggling to finish in time, do a quick speed read of the passage to get a sense for what it's talking about. Furthermore, since all of the passages are well-written literature, reading the beginning and the end of each passage will also give you a good sense for what the main points in the passage will be.

Also, try to find concrete evidence (e.g. a line number) for each question you answer if you have time, even if the test doesn't ask you for it. If the test *does* ask you for evidence, be sure to consider questions in appropriate pairs.

We also discussed the trickiness of the Reading section. Collegeboard claims that there is one answer that is 100% correct, and three answers that are 100% wrong, but I personally disagree. There will often be cases where all four answer choices are technically correct, but only one is the "most" correct, whatever that means. Conversely, there are cases where none of the four answer choices really fit the question, but one of them fits it better than the other three. When doing the Reading section, try to be as specific as you possibly can.

2.1.2 New words

- abscond (v) - to leave hurriedly and secretly, generally to avoid arrest or detection.
- foyer (n) - an entrance hall in a house or apartment.
- waft (v) - to pass easily or gently through, as if through the air.
- murky (adj) - dark and gloomy, not clear.
- stride (v) - to walk with long, decisive steps in a specified direction.
- cavernous (adj) - roomy; big; literally, like a cavern.
- somber (adj) - dark; dull in color; gloomy; solemn.
- murmur (v) - to say something in a low or indistinct voice.

- reek (v) - to smell strongly and unpleasantly.
- faux (adj) - fake.
- temperate (adj) - showing moderation or self-restraint.
- aloof (adj) - cool and distant; not friendly; uninvolved and uninterested/
- intrinsic (adj) - occurring naturally.
- appreciable (adj) - large or important enough to be noticed.
- nudge (v) - to push slightly, sometimes to draw their attention.
- disposition (n) - a person's inherent (intrinsic) qualities of mind and character.
- labor (v) - to work hard at something without succeeding much.
- endeavor (v) - to attempt to achieve something.
- trudge (v) - to walk slowly and wearily.
- tantalizing (adj) - tormenting or teasing with the promise of something that cannot be gotten.
- cogitation (n) - the action of thinking deeply about something; contemplation.

2.2 Review of Math section 3 from the PSAT

2.2.1 Vieta's Formulas

Recall that if a quadratic x^2+bx+c has roots r_1 and r_2, then that quadratic can be written as $(x-r_1)(x-r_2)$. In other words, $x^2 + bx + c = (x - r_1)(x - r_2)$. Expanding the right-hand-side, we get $x^2 + bx + c = x^2-(r_1+r_2)x+r_1r_2$. Since the only way for two polynomials to be equal is for their coefficients to be equal, we conclude that $r_1 + r_2 = -b$, and $r_1r_2 = c$. This sort of neat fact has a name:

Theorem 3.3 [Vieta's Formulas for Quadratics]. Given a generic quadratic $ax^2+bx+c = 0$ with roots r_1 and r_2, then

$$r_1 + r_2 = -\frac{b}{a}$$
$$r_1r_2 = \frac{c}{a}$$

In particular, if the polynomial is monic, meaning that $a = 1$, then

$$r_1 + r_2 = -b$$
$$r_1r_2 = c$$

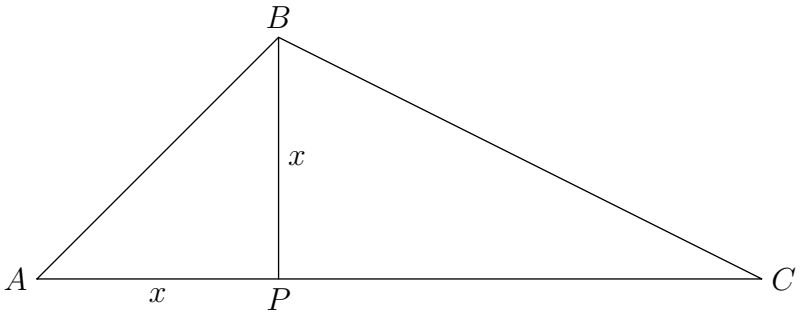
Example 3.4. Write a polynomial whose roots are 5 and 3.

Proof. Using Vieta's formulas, we can set $a = 1$, and $b = -r_1 - r_2 = -5 - 3 = -8$, while $c = 5 \cdot 3 = 15$. So, a possible polynomial is $\boxed{x^2 - 8x + 15}$. □

2.2.2 Other Problem

Notation 21.1. The area of a triangle $\triangle ABC$ is generally denoted by $[\triangle ABC]$.

Problem 21.2. Given the triangle on the following page such that $\angle APB = 90^\circ$ and that the area of $\triangle ABC$ is equal to $2x^2$. Find the cosine of $\angle BCP$.



Proof. Note that the area of $\triangle APB$ is $\frac{1}{2}x^2$, since it has both a base and a height of x. Also note that $[\triangle ABC] = [\triangle APB] + [\triangle BPC]$. So, we know that $2x^2 = \frac{1}{2}x^2 + [\triangle BPC] \implies [\triangle BPC] = \frac{3}{2}x^2$.

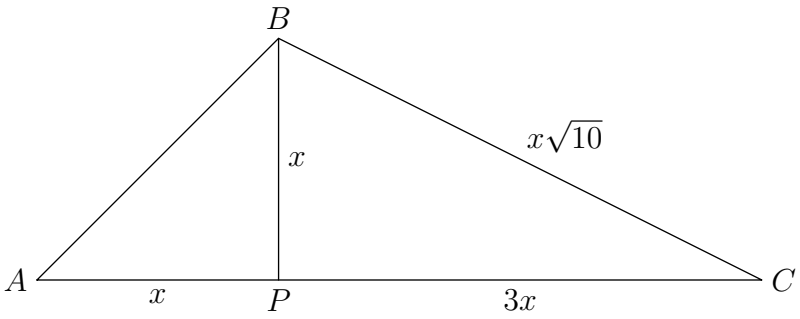
Now, let's try to find the other lengths of the legs of $\triangle BPC$. Since $[\triangle BPC] = \frac{3}{2}x^2$, the length of the base is $\overline{PC}$, and the height is x, we get the equation

$$\frac{3}{2}x^2 = \frac{1}{2}(\overline{PC})(x).$$

This just comes from the area $= \frac{1}{2}$(base)(height) equation for the area of a triangle. Solving for $\overline{PC}$, we get that $\overline{PC} = 3x$. Then, applying the Pythagorean Theorem to $\triangle BPC$ tells us that

$$\overline{BC} = \sqrt{x^2 + (3x)^2} = x\sqrt{10}.$$

So, our triangle has the following lengths:



Now, to find $\cos \angle BCP$, recall that cosine $= \dfrac{\text{adjacent}}{\text{hypotenuse}}$. The adjacent side is PC, and the hypotenuse is BC, so we get:

$$\cos \angle BCP = \frac{\overline{PC}}{\overline{BC}} = \frac{3x}{x\sqrt{10}} = \boxed{\frac{3}{\sqrt{10}}}.$$

□

3 Homework

No homework due tomorrow, but you should know the words in the "New words" section, as well as their definitions, parts of speech, and how to use them in a sentence – there will be a quiz on them on Wednesday.