SAT Intensive Workshop - Day 22

Tomas Matzner

9 July 2019

1 Today's Events

- Finished Review of Math section 3 from 8 July.
- Review of Writing section 2 from 8 July.
- Review of Math section 4 from 8 July PSAT.
- Review of Math section 4 from 8 July.
- Reading section 1 practice exam.
- Lunch.
- Writing section 2 practice exam.
- Math section 3 practice exam.
- Practice essay.
- Vocabulary practice.

1.1 Review of Math section 3 from 8 July

Problem 22.1. If $h(x) = x^a$ for some positive constant a, and $h(27x) = 3h(x)$ for all positive values of x, what is the value of a?

Proof. Note that $h(27x) = (27x)^a$, and $3h(x) = 3(x^a)$, so setting these equal, we get $(27x)^a = 3x^a$. Distributing the exponent through on the left-hand-side gives us $27^a \cdot x^a = 3x^a$ – dividing both sides by x^a gives $27^a = 3$.

But then, recall that $27 = 3^3$, so making this substitution, we get $(3^3)^a = 3 \implies 3^{3a} = 3^1$. For this equality to hold, and since the base is the same on both sides, it must be that the exponents on both sides are the same, yielding $3a = 1 \implies a = \boxed{\frac{1}{3}}$. □

Problem 22.2. A snack bar sells scoops of strawberry, chocolate, and vanilla ice cream. On Monday, the snack bar sold 100 scoops in total of these flavors of ice cream. The snack bar sold 3 times as many scoops of chocolate as it did strawberry and 2 times as many scoops of vanilla as it did chocolate. How many scoops of chocolate ice cream did the snack bar sell on Monday?

Proof. Let s be the number of strawberry scoops sold, let c be the number of chocolate scoops sold, and let v be the number of vanilla scoops sold. Since they sold a total of 100 scoops, we know that

$$s + c + v = 100.$$

Since they sold three times as many scoops of chocolate as they did of strawberry, we get

$$3s = c \implies s = \frac{c}{3}.$$

Finally, because they sold twice as much vanilla as chocolate,

$$2c = v.$$

Then, plugging the last two equations into the first one, we get:

$$\frac{c}{3} + c + 2c = 100 \implies c = \boxed{30}.$$

□

1.2 Review of Writing section 2 from 8 July

1.2.1 Redundancy

This section brings up the real slogan for the writing section: BE AS SPECIFIC AS POSSIBLE IN AS FEW WORDS AS POSSIBLE. There is no reason to say two things when one thing suffices.

Example 2.4. Fix the following sentence: *In addition, researchers at the Department of Horticulture at LSU found that students' science achievement scores increased notably and considerably after a semester of weekly gardening classes.*

There is no reason to say notably and considerably, since they both communicate the exact same information. Therefore, we can just write "... scores increased notably after ...", or "... scores increased considerable after ..." □

Example 22.3. Which of the answer choices fills in the following blank the best: *Physical therapists must earn a graduate degree from one of* [insert answer here].

A) more than 200 accredited programs

B) the 200 accredited programs, at minimum, that are accredited and award the graduate degree necessary to practice physical therapy

C) the programs, of which there are more than 200, that are accredited to award physical therapy degrees

D) many accredited educational programs that offer a physical therapy graduate degree

$\boxed{\text{A)}}$ is the correct answer, since it is the most concise and isn't redundant. B) is incorrect for many reasons, one of which is that it says accredited twice. C) is incorrect because it introduces an unnecessary clause, which makes the sentence murkier than it has to be. Finally, D) is incorrect because it mentions graduate degrees, even though earlier in the sentence, it was already mentioned that graduate degrees must be earned. □

1.2.2 Parallelism

Everyone knows, you can't compare apples to oranges. The same rule applies in grammar. When two phrases are being compared, the subject in each of the phrases must be the same. This can be a rather abstract concept, so maybe an example will help elucidate (if you don't know what elucidate means, look it up):

Example 2.5. Which of the answer choices fixes the following sentence: *Researchers in Texas, for example, found that elementary school student gardeners scored 5.6 points higher on a science achievement than those of students not gardening at school.*

A) Leave it as it is

B) Replace "those of students not gardening" with "the scores of students who did not garden"

C) Replace "those of students not gardening" with "did students who did not garden"

D) Replace "those of students not gardening" with "nongardening students' scores"

The presence of the word "than" gives us a clue that we are comparing two phrases. These are "elementary school student gardeners scored 5.6 points higher on a science achievement" and "those of students not gardening". The subject in the first phrase is clearly gardeners, and the subject in the second phrase is the scores (why? What is "those" referring to?). We're comparing gardeners and scores, apples and oranges, so that's not allowed. Similarly for B) and D) – the subject in each of those phrases is also the scores. The only answer choice whose subject is also gardeners is $\boxed{\text{C)}}$, comparing apples to apples. □

1.2.3 Restrictive and Non-Restrictive Clauses

Definition 22.4. A *restrictive clause* is a group of words in a sentence that if removed from the original sentence, the modified sentence loses its primary meaning.

Definition 22.5. A *non-restrictive clause* is a group of words in a sentence that if removed from the original sentence, the modified sentence still retains its primary meaning.

Example 22.6. Restrictive and non-restrictive clauses.

i) *Tomas, a UT student, has a summer job at ACES.* "A UT student" is a non-restrictive clause, because if it were removed from the sentence, the modified sentence retains its primary purpose – to notify the reader that my summer job is at ACES.

ii) *While I brought lunch, it was not enough and I was still hungry.* "While I brought lunch" is a restrictive clause. If we remove it, the modified sentence reads *It was not enough and I was still hungry.* We no longer know what "it" is referring to, so the modified sentence loses its primary meaning.

There is a type of non-restrictive clause that comes up very frequently on the SAT.

Definition 22.7. An *appositive phrase* is a non-restrictive clause that gives more information about the noun that directly precedes it.

Example 22.8. Appositive phrases.

i) See Example 22.6(i). The appositive phrase is "a UT student", because it provides more information about me, the noun directly preceding the phrase.

ii) *This devastating natural disaster buried the ancient city of Pompeii, near Naples, Italy, under millions of tons of hot ash and rocks.* The appositive phrase is "near Naples, Italy", because it gives more information about Pompeii, the noun directly preceding the appositive phrase.

A word of warning: appositive phrases *must* refer to the noun directly preceding it. Often, the SAT will include a fake appositive phrase that, in fact, refers to a previous noun. Be on the lookout for this common error.

1.3 Review of Math section 4 from 8 July PSAT

1.3.1 Statistics

Be sure you are completely familiar with the following statistics terms, as well as those in the Day 1 notes and line of best fit.

Definition 4.1. Given a set of data, the *mean* of the data is the average. So, given $\{x_1, x_2, x_3, \ldots, x_n\}$, the mean of this set of data is

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}.$$

Definition 4.2. Given a set of data, the *median* of the data is the middle point. So, consider a set of data $\{x_1, x_2, x_3, \ldots, x_{2n-1}\}$, so that there's an odd number of data points. Then, the median is

$$\text{median} = x_n.$$

Now consider a set of data that has an even number of data points, $\{x_1, x_2, x_3, \ldots, x_{2n}\}$. Then, the median is

$$\text{median} = \frac{x_n + x_{n+1}}{2}.$$

Definition 4.3. Given a set of data, the value (or values) that appears most often is called the *mode*.

Definition 4.4. Given a set of data, the *range* of the set is the largest element minus the smallest element.

1.3.2 Completing the Square

Occasionally, you'll want to factor a quadratic as the perfect square of a binomial. Let's say you have a quadratic equation $rx^2 + sx = 0$, and you want to write it in the form $a(x + b)^2 = c$. There are a number of steps through which you must go:

1. Turn the quadratic into a monic polynomial (a polynomial whose leading coefficient is 1). To do so, factor out an r from both terms on the left hand side to get

$$rx^2 + sx = 0 \implies r\left(x^2 + \frac{s}{r}x\right) = 0.$$

   Note that if the quadratic is already monic, it makes our lives much, much easier.

2. Now, we want to turn the expression inside the parentheses into the perfect square of a binomial. So, we add $\frac{s^2}{4r}$ to both sides to get
$$r\left(x^2 + \frac{s}{r}x\right) + \frac{s^2}{4r} = \frac{s^2}{4r}.$$
We now artificially factor out an r from the $\frac{s^2}{4r}$ term on the left hand side to push it into the parentheses, getting
$$r\left(x^2 + \frac{s}{r}x + \frac{s^2}{4r^2}\right) = \frac{s^2}{4r}.$$
Finally, we can factor the term inside the parentheses as a perfect square, getting the final answer of
$$r\left(x + \frac{s}{2r}\right)^2 = \frac{s^2}{4r}.$$

This is rather abstract, so let's look at the problem we did in class to try to illuminate us.

Example 8.1. Find the radius of the circle with equation $x^2 + y^2 + 4x - 2y = -1$.

Proof. Recall that the general form of a circle, $(x-h)^2 + (y-k)^2 = r^2$, tells us what the radius of a circle is. So, we want to shoehorn the equation we were given into the format of the general form of a circle, so that we can simply read off the radius. But, in order to do that, we must complete the square in both x and y.

Consider the x terms for a second, and pretend that the y terms don't exist. Our current equation is $x^2 + 4x = -1$. We want to convert $x^2 + 4x$ into something of the form $(x+b)^2$. So, after FOILing and comparing them, we see that $x^2+4x \leftrightarrow x^2+2bx+b^2$. We want these to be equal, so we set $2b = 4 \implies b = 2$. So, we need to add 4 to $x^2 + 4x$ in order to have it factor as the square of a binomial. Doing so, we get:

$x^2 + 4x = -1 \implies x^2 + 4x + 4 = -1 + 4 \implies (x+2)^2 = 3$. Now, our equation is $(x+2)^2 + y^2 - 2y = 3$. Doing similarly for y (work it out yourself), we get
$$(x+2)^2 + (y-1)^2 = 4 = 2^2.$$
It is clear, then, that the radius of the circle is $\boxed{2}$. □

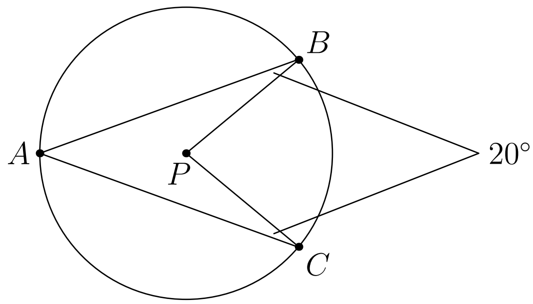
We can also complete the square to find the minimum value of a quadratic.

Problem 22.9. Find the minimum value of the function $f(x) = 10x^2 - 25x - 60$.

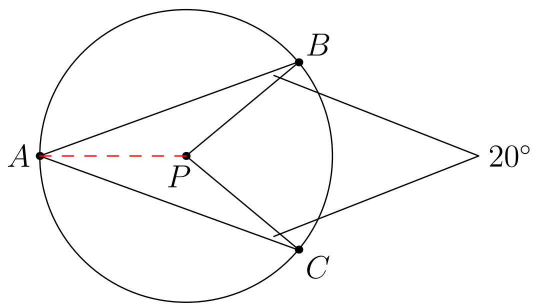
Proof. First, we factor a 10 out of each term, so that we get to work with a monic polynomial. This gives us
$$10\left(x^2 - \frac{25}{10}x - \frac{60}{10}\right) = 10\left(x^2 - \frac{5}{2}x - 6\right).$$
Then, to complete the square on the inside, the closest we can get is $\left(x - \frac{5}{4}\right)^2 = x^2 - \frac{5}{2}x + \frac{25}{16}$. So, we add and subtract $10\left(6 + \frac{25}{16}\right)$ to get:
$$10\left(x^2 - \frac{5}{2}x - 6\right) + 10\left(6 + \frac{25}{16}\right) - 10\left(6 + \frac{25}{16}\right) = 10\left(x - \frac{5}{4}\right)^2 - \frac{605}{8}.$$
Now, the smallest value that the square of a real number can take is 0, so the smallest value our expression above can take is $10(0) - \frac{605}{8} = -\frac{605}{8}$. So, the minimum value of $f(x) = 10x^2 - 25x - 60$ is $\boxed{-\frac{605}{8}}$. □

1.4 Review of Math section 4 from 8 July

Problem 22.10. Given that P is the center of the circle and that $\angle PBA = \angle PCA = 20°$, find $\angle BPC$.



Proof. The critical line to draw here is line AP:



Consider triangles $\triangle APB$ and $\triangle APC$. These are both isosceles triangles since $AP = PB = PC$ because they are all radii of the circle. So, $\angle PAB = \angle PBA = \angle PAC = \angle PCA = 20°$. Since the sum of the angles in a triangle is $180°$, we know that $\angle APC = \angle APB = 180° - 20° - 20° = 140°$. So, $\angle BPC = 360° - 140° - 140° = \boxed{80°}$. □

2 Homework

For tomorrow, you should know the words from yesterday, as well as their definitions, parts of speech, and how to use them in a sentence. Do the same for the following words:

- tout (v) - to praise something because you want others to think it is good or important.
- acrophobia (n) - extreme fear of heights.
- bourgeois (n) - of the middle class, often tied to the materialistic values they hold.
- debility (n) - weakness, especially as a result of illness.
- epistle (n) - a letter.
- hiatus (n) - a pause in a sequence or process.
- languid (adj) - slow and relaxed.
- obscured (adj) - concealed; kept from being seen.
- poised (adj) - having a composed and self-assured manner.
- resplendent (adj) - impressive by being richly colorful or brilliant.
- therapeutic (adj) - relating to the healing of disease.