SAT "Final Push" – Day 2

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1 Today's Events

- Vocabulary and roots quiz.
- Review of Math section 4 from yesterday.
- Reading Strategies lecture.
- Reading section 1 practice exam.
- Review of Reading section 1.
- Lunch.
- Math section 3 practice exam.
- Review of Math section 3 practice exam.
- Writing section 2 practice exam.
- Rhetorical Strategies for the SAT Essay lecture.
- Basic essay mechanics lecture and sample essay discussion.

1.1 New Words from Reading Section

- bellow (v) - to yell very loudly; to shout.
- clamber (v) - to climb awkwardly.
- multitude (n) - a large number.
- contort (v) - to twist or bend out of shape.
- exultation (n) - triumphant joy.
- agitation (n) - a state of anxiety or nervous excitement.
- quell (v) - to subdue, put down forcibly.
- extirpate (v) - to tear up by the roots; to destroy totally.
- rapine (n) - the violent seizure of someone's property.
- blemish (v) - spoil the appearance of (something) that is otherwise aesthetically perfect.
- inviolable (adj) - sacred; of such a character that it must not be broken, injured, or profaned.
- clemency (n) - mercy; lenience.
- internecine (adj) - mutually destructive.
- benevolent (adj) - kindly, charitable.
- altruistic (adj) - unselfish, concerned with the welfare of others.
- fecundity (n) - fertility; fruitfulness.
- augment (v) - to make larger, increase.

1.2 New Words from Writing Section

- ephemeral (adj) - short-lived.
- orthodox (adj) - adhering to the traditional and established.
- expenditure (n) - an expense; the amount needed to be paid out.

1.3 Lecture Notes

1.3.1 Math

Some common factorizations that you should be intimately familiar with:

1. $(x+y)^2 = x^2 + 2xy + y^2$.
2. $(x-y)^2 = x^2 - 2xy + y^2$.
3. (Difference of Squares) $(x-y)(x+y) = x^2 - y^2$.

Problem 2.1. *If $8x + 8y = 18$ and $x^2 - y^2 = -\frac{3}{8}$, what is the value of $2x - 2y$?*

Proof. Note that

$$8x + 8y = 18 \implies 8(x+y) = 18 \implies x + y = \frac{9}{4}. \tag{1}$$

Furthermore, note that we can factor the second equation using difference of squares to get

$$x^2 - y^2 = -\frac{3}{8} \implies (x-y)(x+y) = -\frac{3}{8}. \tag{2}$$

Substituting the value for $x+y$ in (1) for $x+y$ in (2), we get

$$(x-y)\left(\frac{9}{4}\right) = -\frac{3}{8} \implies x - y = -\frac{1}{6}. \tag{3}$$

So, $2(x-y) = 2 \cdot -\frac{1}{6} \implies 2x - 2y = \boxed{-\frac{1}{3}}$. ☐

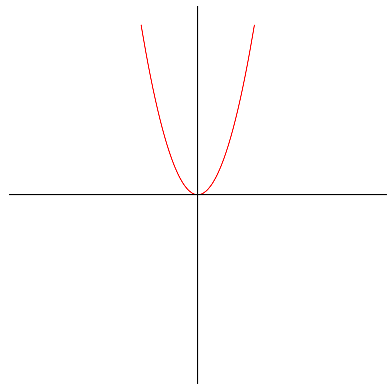
Problem 2.2. *A professional baseball team wishes to average 45500 ticket purchases per game for the entire 162-game season. Through the first 60 games of the season, the team has averaged 43000 ticket purchases per game. To the nearest 10, how many ticket purchases per game must the team average for the remainder of the season in order to hit its overall goal of an average of 45500 ticket purchases per game for the season?*

Proof. Note that, if the team wants to sell an average of 45500 tickets every game for 162 games, they want to sell a total of $45500 \cdot 162 = 7371000$ tickets over the entire season. They have sold an average of 43000 tickets every game for the first 60 games, so in total, they have sold $60 \cdot 43000 = 2580000$ tickets so far. Thus, in the remaining $162 - 60 = 102$ games, they want to sell $7371000 - 2580000 = 4791000$ tickets. This is an average of

$$\frac{4791000}{102} \approx \boxed{46970 \text{ tickets}}.$$

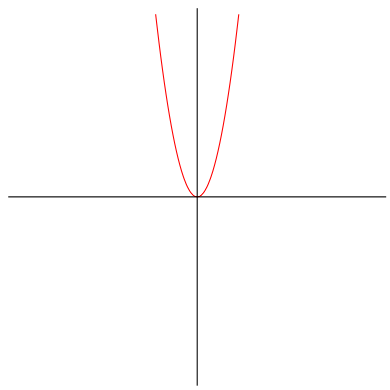
☐

Now, let's turn to a brief discussion of parabolas, and specifically what the coefficient in front of the x^2 does. Recall that the general form of a parabola is $y = a(x-h)^2 + k$, which expanded out is $y = ax^2 - 2ahx + (ah^2 + k)$, so the coefficient in front of the x^2 is a. First, consider the basic graph of $y = x^2$ as a reference point:

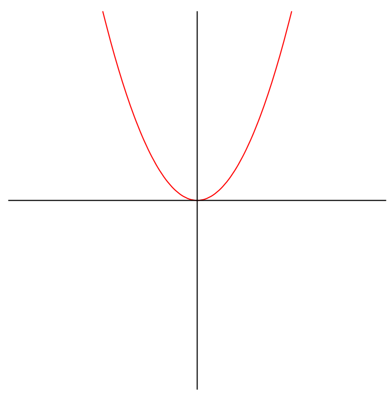


There are four intervals for a of which you need to be aware:

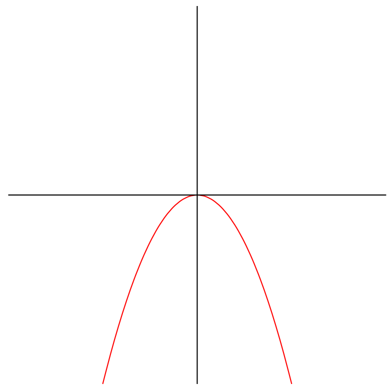
1. $a > 1$. This causes the parabola to stretch vertically, making it grow faster:



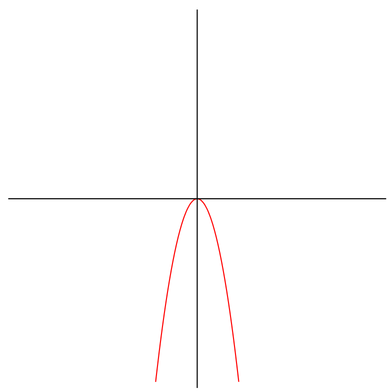
2. $0 < a < 1$. This causes the parabola to stretch horizontally, making it grow slower:



3. $-1 < a < 0$. This causes the parabola to reflect across the x-axis and stretches it horizontally:



4. $a < -1$. This causes the parabola to reflect across the x-axis and stretches it vertically:



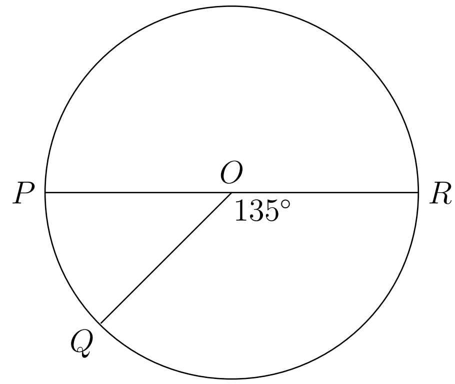
So, if $|a| < 1$, the parabola gets stretched vertically, and if $|a| > 1$ then it gets stretched horizontally. Furthermore, if $a > 0$ then the parabola is upwards-opening, and if $a < 0$ then the parabola is downwards-opening.

Problem 2.3. *Circle O is divided into three sectors. Points P, Q, and R are on the circumference of the circle. Sector POR has an area of 8π, and sector ROQ has an area of 6π. If the radius of circle O is 4, what is the measure of the central angle of sector QOP, in degrees?*

Proof. Recall that, for a sector S:

$$\frac{\text{area of sector } S}{\text{area of the circle}} = \frac{\text{central angle of sector } S}{360} \tag{4}$$

Since circle O has radius 4, its area is 16π. Applying equation (4), since sector POR has area 8π, its central angle $\angle POR$ is $\dfrac{8\pi}{16\pi} = \dfrac{\angle POR}{360} \implies \angle POR = 180°$. Similarly, $\angle ROQ = 135°$. Thus, $\angle QOP = 360 - \angle POR - \angle ROQ = 360 - 180 - 135 = \boxed{45°}$. The circle, then, looks like this, where PR is a diameter of circle O:



∎

Problem 2.4. Twelve Smooth-Glide pens and eight Easy-Write pencils cost exactly \$16.00 at Office World. Six Smooth-Glide pens and ten Easy-Write pencils cost \$11.00 at the same location. How much will nine Smooth-Glide pens and nine Easy-Write pencils cost at Office World?

Proof. Let p be the cost of one pen, and let c be the cost of one pencil. Then, we get the following system of equations:

$$\begin{aligned} 12p + 8c &= 16 \\ 6p + 10c &= 11 \end{aligned}$$

Adding the two equations, we get $18p + 18c = 27$, and dividing by two gives $9p + 9c = \boxed{\$13.50}$. ∎

Problem 2.5. *The chart below shows the population distribution for the 2400 occupants of the city of Centre Hill.*

	Adult Male	**Adult Female**	**Child**
% Living in Uptown	9	8	6
% Living in Midtown	22	20	15
% Living in Downtown	21	22	12
% Living in Suburbs	48	50	67

a) *If there are an equal number of adults and children, and adult females outnumber adult males by 200, what is the sum of the women living uptown and the children living in the suburbs of Centre Hill?*

b) *Centre Hill plans to annex the area around a nearby lake. This new part of Centre Hill will be called, appropriately, The Annex. The Annex will add to the current population of Centre Hill. The percent of adult males living in Uptown will decrease to 6% after incorporating The Annex into Centre Hill. How many adult males live in The Annex?*

Proof. a) Let the number of adult men be m. Then, since adult females outnumber adult men by 200, there are a total of $m + 200$ adult females. Thus, there are $m + (m + 200) = 2m + 200$ adults. Furthermore, we know that the number of adults is equal to the number of children, so the total population of Centre Hill, in terms of m, is $(2m + 200) + (2m + 200) = 4m + 400$.

Additionally, we know that the total population is equal to 2400, so $4m + 400 = 2400 \implies m = 500$. That tells us that:

Adult Men	**Adult Women**	**Children**
500	700	1200

The number of women living Uptown, then, is $8\% \cdot 700 = 56$, and the number of children living in the Suburbs is $67\% \cdot 1200 = 804$, so this sum is $\boxed{860}$.

b) First, let's find out how many men live in Uptown. This is $9\% \cdot 500 = 45$. Now, these 45 men only make up 6% of the total male population once new men move into The Annex. So, $6\% \cdot x = 45$, and solving for x gives $x = 750$. Thus, $750 - 500 = \boxed{250}$ men moved into The Annex.

$\square$

2 Homework

2.1 Vocabulary

Know the words in the New Words sections, as well as their definitions, their parts of speech, and how to use them in a sentence.

2.2 Latin and Greek Roots

- *aqu-*: From the Latin *aqua*, means **water**. Examples: aquamarine, aquarium, aqueduct, aquifer.
- *astr-:* From the Greek *astron* $(\alpha\gamma\tau\rho o\nu)$, means **star**. Examples: asterisk, astrology, astronomy, disaster.
- *bell-*: From the Latin *bellum*, means **war**. Examples: antebellum, belligerent, bellicose.
- *bio-*: From the Greek *bios* $(\beta\iota o\gamma)$, means **life**. Examples: biography, biology, bioluminescence, biosphere.
- *cand-*: From the Latin *candere*, means **glowing**. Examples: candela, candid, candle, candor, incandescent.