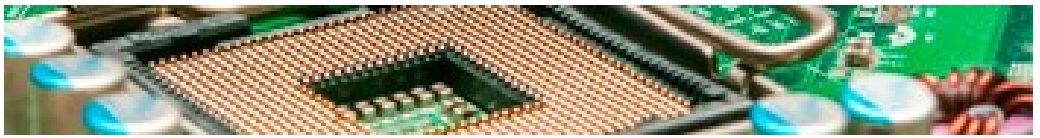
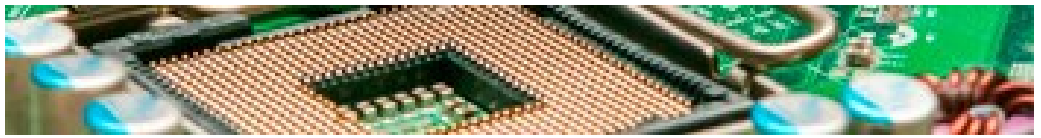
Content

- **Unit 3**: Data analysis
  - Predictive and descriptive tasks
  - Supervised techniques
  - Unsupervised techniques
  - Model evaluation



Evaluation: task-dependent

- Evaluation depends on the task:
  - Supervised learning: The problem is presented with example inputs and their desired outputs and the goal is to learn a general rule that maps inputs to outputs.
    - Classification: Output is categorical
    - Regression: Output is numerical
  - Unsupervised learning: No labels are given to the learning algorithm, leaving it on its own to find structure in its input.
    - Clustering, association rules…
  - Reinforcement learning: A computer program interacts with a dynamic environment in which it must perform a certain goal: Driving a vehicle or videogames.



Metrics for classification

- Given a set *S* of *n* instances, we define classification error:

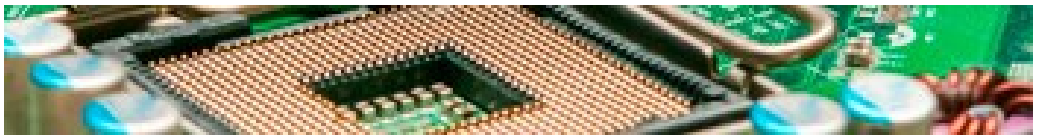
$$error_S(h) = \frac{1}{n} \sum_{x \in S} \partial(f(x), h(x))$$

Where $\delta(a,b)=0$ if a=b and 1otherwise.

Predicted class(h(x))	Actual class (f(x))	**Error**
Buy	Buy	**No**
No Buy	Buy	**Yes**
Buy	No Buy	**Yes**
Buy	Buy	**No**
No Buy	No Buy	**No**
No Buy	Buy	**Yes**
No Buy	No Buy	**No**
Buy	Buy	**No**
Buy	Buy	**No**
No Buy	No Buy	**No**

Mistakes/ Total

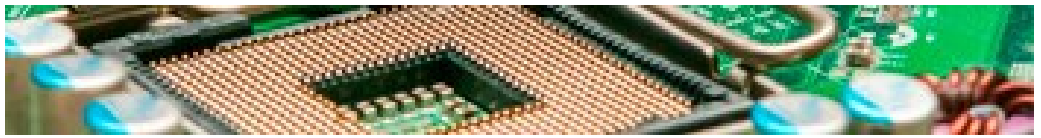
Error = 3/10 = 0.3



Metrics for classification

- Common measures in IR: Precision and Recall
  - Are defined in terms of a set of retrieved documents and a set of relevant documents.
    - Precision: Fraction of retrieved documents that are relevant to the query. (TP / Predicted Positives)
    - Recall: Percent of all relevant documents that is returned by the search. (TP / Actual Positives)
- Both measures are usually combined in one (harmonic mean):

$$\text{F-measure} = 2\frac{Precision \times Recall}{Precision + Recall} = \frac{2TP}{2TP + FP + FN}$$



Metrics for Regression

- Given a set *S* of *n* instances,
  - Mean Absolute Error:

$$MAE_S(h) = \frac{1}{n}\sum_{x \in S} |f(x) - h(x)|$$

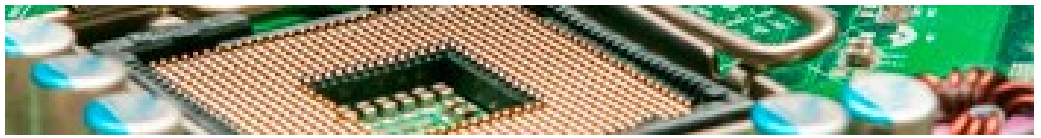
  - Mean Squared Error:

$$MSE_S(h) = \frac{1}{n}\sum_{x \in S} (f(x) - h(x))^2$$

  - Root Mean Squared Error:

$$RMSE_S(h) = \sqrt{\frac{1}{n}\sum_{x \in S} (f(x) - h(x))^2}$$

    - MSE is more sensitive to extreme values
    - RMSE and MAE are in the same magnitude of the actual values



Metrics for Regression

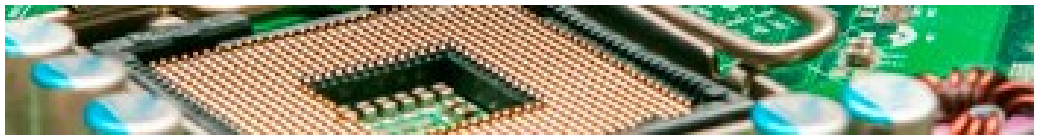
- Example:

Predicted Value(h(x))	Actual Value (f(x))	**Error**	**Error^2**
100 mill. €	102 mill. €	**2**	**4**
102 mill. €	110 mill. €	**8**	**64**
105 mill. €	95 mill. €	**10**	**100**
95 mill. €	75 mill. €	**20**	**400**
101 mill. €	103 mill. €	**2**	**4**
105 mill. €	110 mill. €	**5**	**25**
105 mill. €	98 mill. €	**7**	**49**
40 mill. €	32 mill. €	**8**	**64**
220 mill. €	215 mill. €	**5**	**25**
100 mill. €	103 mill. €	**3**	**9**

MAE= 60/10 =6

MSE= 744/10 = 74,4

RMSE= sqrt(744/10) = 8.63



Metrics for Regression

- Sometimes relative error values are more appropiate:
  - 10% for an error of 50 when predicting 500

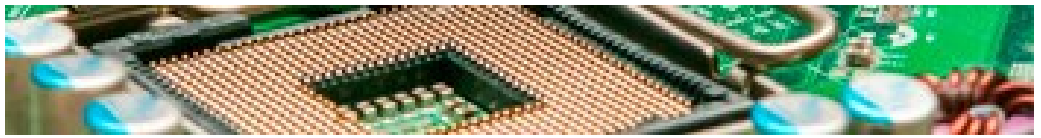
- How much does the scheme improve on simply predicting the average:
  - Relative Mean Squared Error

$$RSE_S(h) = \frac{\sum_{x \in S} \left(f(x) - h(x)\right)^2}{\sum_{x \in S} \left(\bar{f} - f(x)\right)^2}$$

  - Relative Mean Absolute Error

$$RAE_S(h) = \frac{\sum_{x \in S} \left|f(x) - h(x)\right|}{\sum_{x \in S} \left|\bar{f} - h(x)\right|}$$

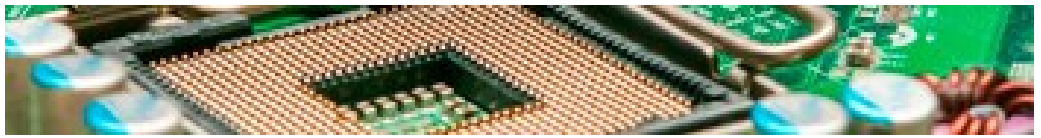
- Pearson/Spearman correlations also useful



Metrics for Unsupervised Learning

- Association Rules: finding interesting relations between variables in databases.
- Common metrics:
  - Support: Estimates the popularity of a rule
  - Confidence: Estimates the reliability of a rule

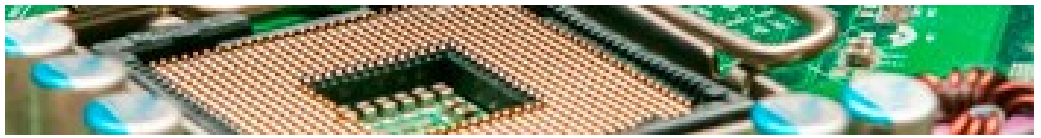
- Rules are ordered according to measures that combine both values



Metrics for Unsupervised Learning

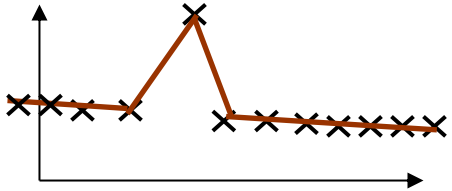
- Clustering: grouping a set of objects in such a way that objects in the same group (cluster) are more similar to each other than to those in other clusters.
  - Task difficult to evaluate

- Some evaluation measures based on distance:
  - Distance between borders of clusters
  - Distance between centres (centroids) of clusters
  - Radius and density of clusters

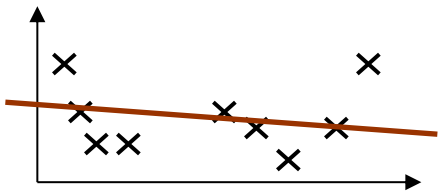


Overfitting?

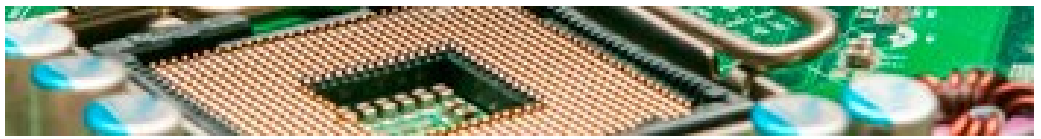
- What dataset do we use to estimate all previous metrics?
  - If we use all data to train the models and evaluate them, we get overoptimistic models:
    - Over-fitting:



  - If we try to compensate by generalising the model (e.g., pruning a tree), we may get:
    - Under-fitting:



  - How can we find a trade-off?

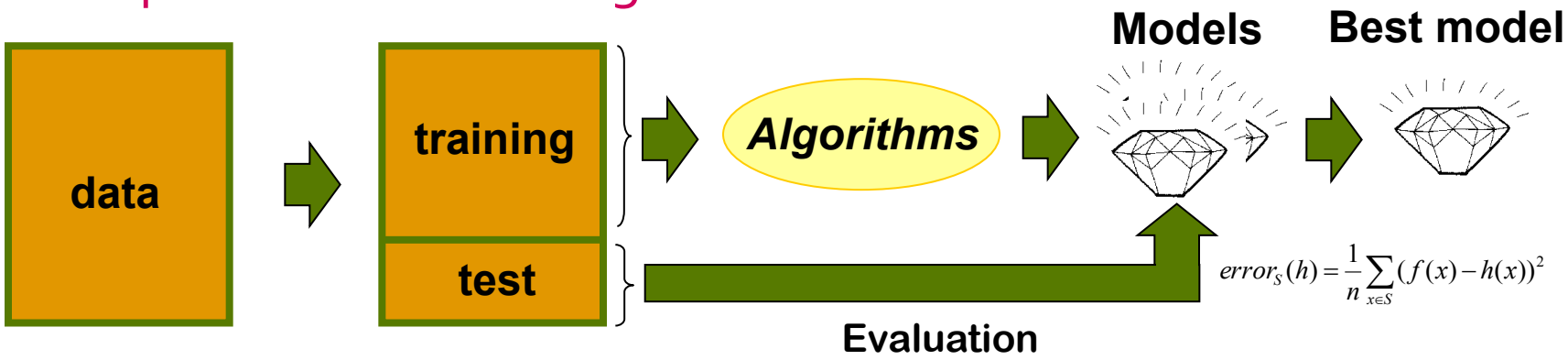


Splitting Train/Test

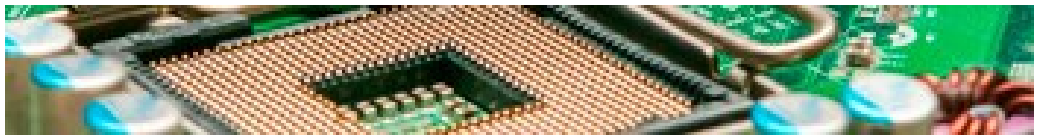
- Common solution (especially in supervised learning):

GOLDEN RULE: Never use the same example for training the model and evaluating it!!

  - Split between training and test data

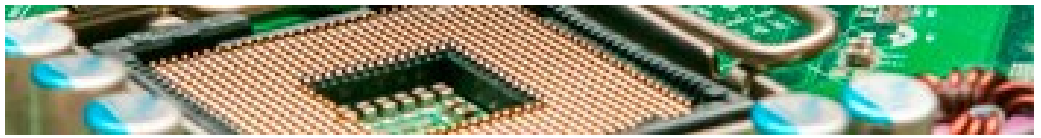


What if there is not much data available?



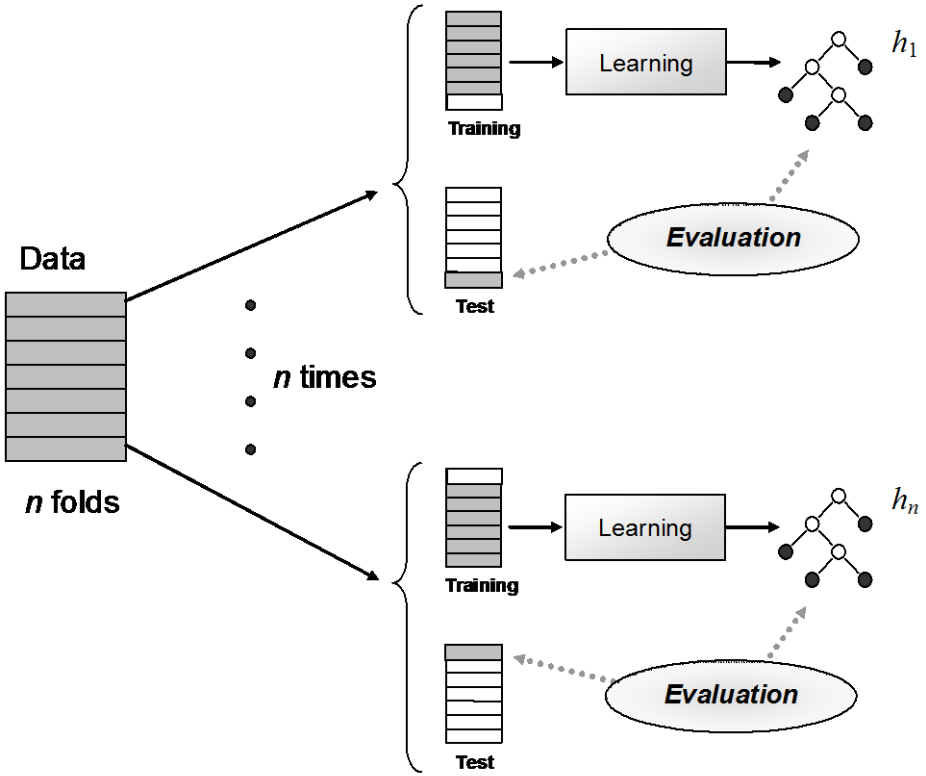
Taking the most of the data

- Too much training data: poor evaluation
- Too much test data: poor training
- Can we have more training data and more test data without breaking the golden rule?
  - Repeat the experiment!
    - Bootstrap: we perform *n* samples (with repetition) and test with the rest.
    - Cross validation: Data is split in *n* folds of equal size.
      - Hold-out: special case with as many folds as examples.

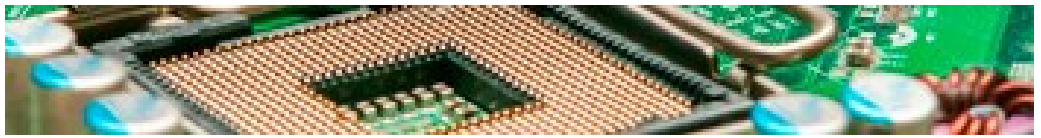


Cross validation

- We can train and test with all the data!



- We take all possible combinations with *n*–1 for training and the remaining fold for test.
- The error (or any other metric) is calculated *n* times and then averaged.
- A final model is trained with all the data.



Where Does my Model Fail?

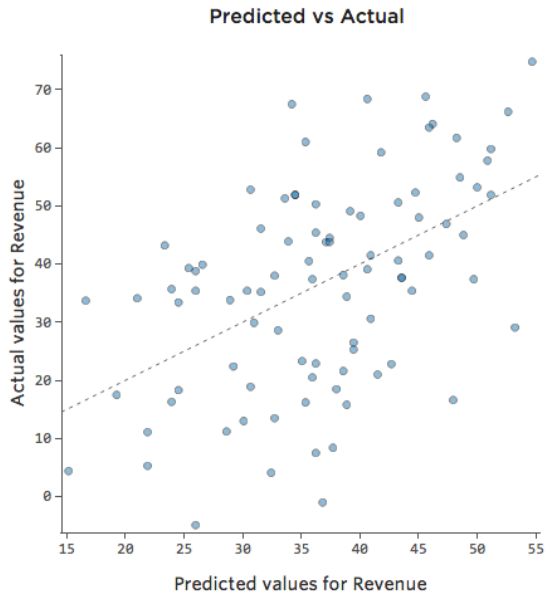
- See the contingency
  - Classification: a matrix

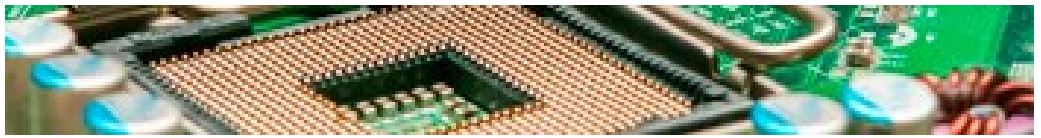
Actual

c	Buy	No Buy
Buy	4	1
No Buy	2	3

Pred.

  - Regression: a plot

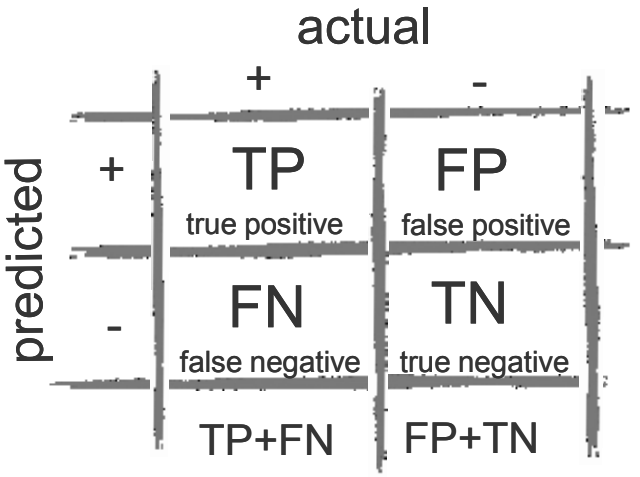




Binary Classification: Confusion Matrix

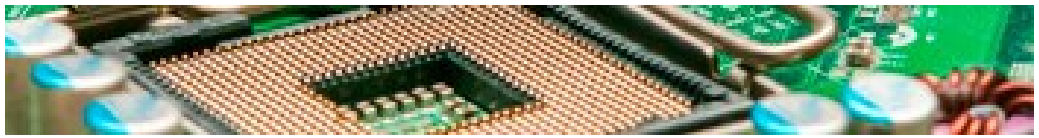
- Confusion matrix for two classes:

Pred. \ Actual	Buy	No Buy
Buy	4	1
No Buy	2	3



$$\text{Accuracy} = \frac{TP+TN}{n}$$

$$\text{Error=1-Accuracy} = \frac{FP+FN}{n}$$



Binary Classification: Confusion Matrix

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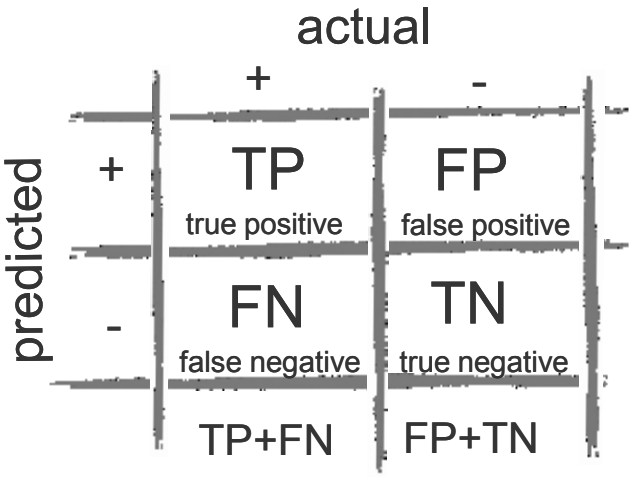
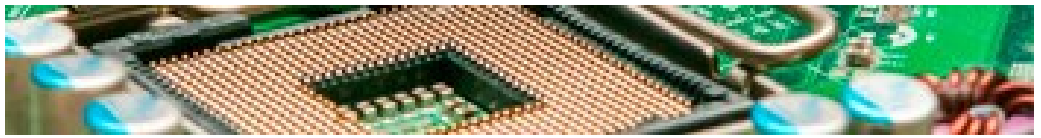


$$\text{TPRate, Sensitivity} = \frac{TP}{TP+FN}$$

$$\text{TNRate, Specificity} = \frac{TN}{FP+TN}$$



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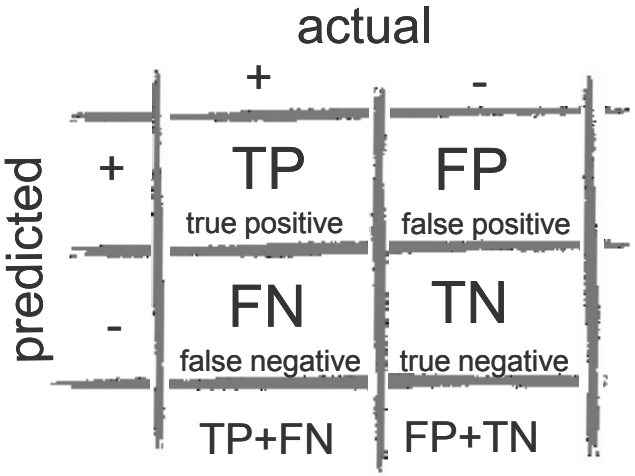
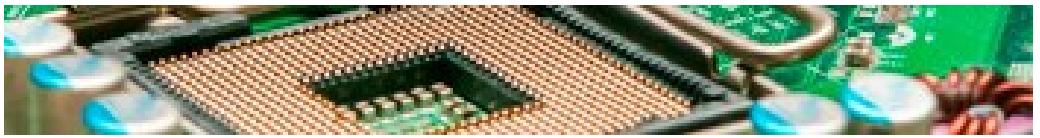


$$\text{TPRate, Sensitivity, Recall} = \frac{TP}{TP+FN}$$

$$\text{Positive predictive value (PPV), Precision} = \frac{TP}{TP+FP}$$

$$\text{F-measure} = 2\frac{Precision \times Recall}{Precision+Recall} = \frac{2TP}{2TP+FP+FN}$$

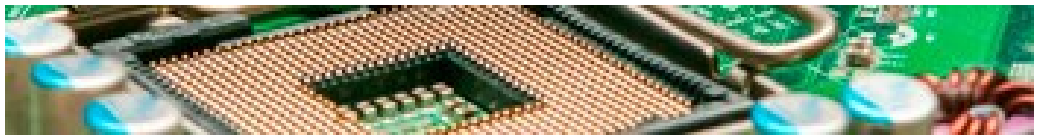


Multiclass problems

- Confusion (contingency) tables can be multiclass

ERROR		*actual*		
		low	medium	high
predicted	low	20	0	13
	medium	5	15	4
	high	4	7	60

- Measures based on 2-class matrices are computed
  - 1 vs all (average N partial measures)
  - 1 vs 1 (average N*(N-1)/2 partial measures)
    - Weighted average?



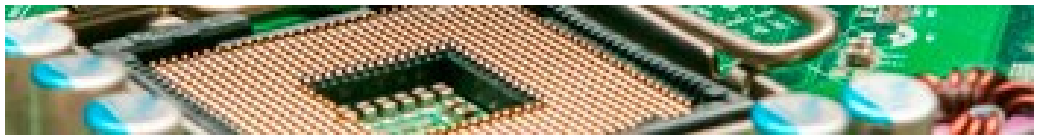
Imbalanced Datasets

- In some cases we can find important differences in the proportion of classes
  - A naïve classifier that always predictive majority class (ignoring minority classes) obtains good performance
    - In a binary problem (+/-) with 1% of negative instances, the naïve model "always positive" gets an accuracy of 99%.

- Macro-accuracy: Average of accuracy per class

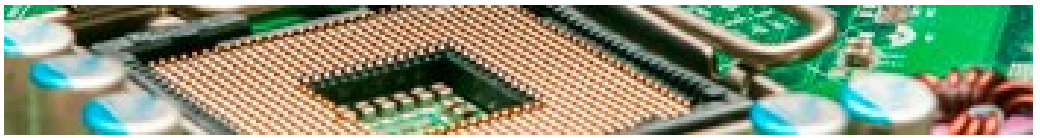
$$macroacc(h) = \frac{\frac{Hits_{\text{class 1}}}{total_{\text{class 1}}} + \frac{hits_{\text{class 2}}}{total_{\text{class 2}}} + \ldots + \frac{hits_{\text{class m}}}{total_{\text{class m}}}}{m}$$

  - The naïve classifier gets a macro-accuracy=0.5



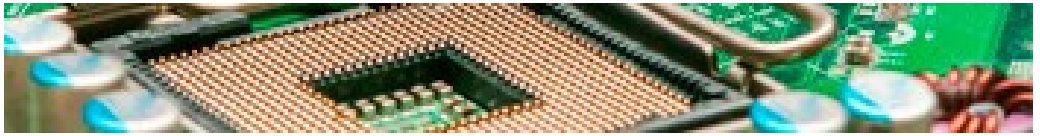
Soft classifiers

- Crisp and Soft Classifiers:
  - A "hard" or "crisp" classifier predicts a class between a set of possible classes.
  - A "soft" or "scoring" classifier (probabilistic) predicts a class, but accompanies each prediction with an estimation of the reliability (confidence) of each prediction.
    - Most learning methods can be adapted to generate this confidence value.



Soft classifiers: probability estimator

- A special kind of soft classifier is a class probability estimator.
  - Instead of predicting "a", "b" or "c", it gives a probability estimation for "a", "b" or "c", i.e., "p_a", "p_b" and "p_c".
    - Example:
      - Classifier 1: $p_a = 0.2$, $p_b = 0.5$ and $p_c = 0.3$.
      - Classifier 2: $p_a = 0.3$, $p_b = 0.4$ and $p_c = 0.3$.
  - Both predict "b", but classifier 1 is more confident.



Evaluating probabilistic classifiers

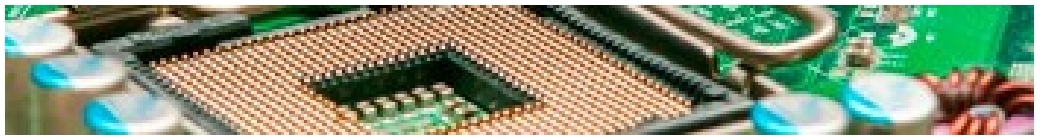
- Probabilistic classifiers: Classifiers that are able to predict a probability distribution over a set of classes
  - Provide classification with a degree of certainty:
    - Combining classifiers
    - Cost sensitive contexts
- Mean Squared Error (Brier Score)

$$MSE = \frac{1}{n}\sum_{i \in S}\sum_{j \in C}[f(i,j) - p(i,j)]^2$$

f(i,j)=1 if instance i is of class j, 0 otherwise.
p(i,j) returns de estimate of i in class j

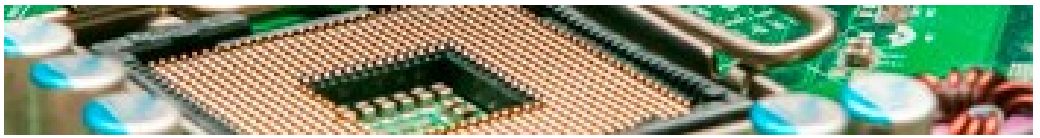
- Log Loss

$$Logloss = -\frac{1}{n}\sum_{i \in S}\sum_{j \in C}(f(i,j) * \log_2 p(i,j))$$



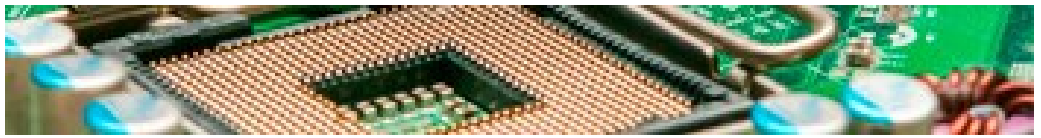
Evaluating probabilistic classifiers

- MSE or Brier Score can be decomposed into two factors:
  - BS=CAL+REF
    - Calibration: Measures the quality of classifier scores wrt class membership probabilities
    - Refinement: it is an aggregation of resolution and uncertainty, and is related to the area under the ROC Curve.
- Calibration Methods:
  - Try to transform classifier scores into class membership probabilities
    - Platt scaling, Isotonic Regression, PAVcal.



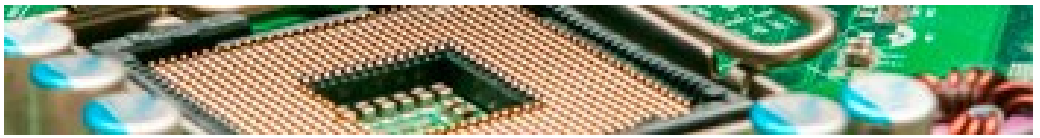
Soft classifiers: rankers

- "Rankers":
  - Whenever we have a probability estimator for a two-class problem:
    - $p_a = x$, then $p_b = 1 - x$.
  - Let's call one class "0" (neg) and the other class "1" (pos).
  - A ranker is a soft classifier that gives a value (score) monotonically related to the probability of class "1".
- We can rank instances according to estimated probability
  - CRM: You are interested in the top % of potential customers
    - Examples:
      - Rank customers according to how likely they can buy a product.
      - Rank spam messages from more likely to less likely.



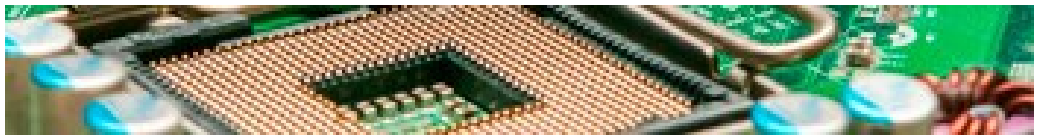
Evaluating Rankers

- Measures for ranking
  - AUC: Area Under the ROC Curve
    - Equivalent to the Wilcoxon-Mann-Whitney statistic, defined as:
      - "Given a positive example and a negative example, the probability that the model ranks the positive example above of the negative example".
  - Other: Distances between the estimated ranking and the perfect ranking (if there is a notion order in the true values).



Cost-sensitive Evaluation

- In classification, the model with highest accuracy is not necessarily the best model.
  - Some errors (e.g., false negatives) may be much more expensive than others.
    - This is usually (but not always) associated with imbalanced datasets.
    - A cost matrix is a simple way to account for this.
- In regression, the model with lowest error is not necessarily the best model.
  - Some errors (e.g., overpredictions) may be much more expensive than others (e.g., underpredictions).
    - A cost function is a simple way to account for this.



Cost-sensitive Evaluation

- Classification. Example: 100,000 instances (only 500 pos)
  - High imbalance (π_1=Pos/(Pos+Neg)=0.005, 1- π_1=π_0= Neg/(Pos+Neg)=0.995).

		Actual	
Pred.	c_1	open	close
	OPEN	300	500
	CLOSE	200	99000

ERROR: 0,7%

TPR= 300 / 500 = 60%
FNR= 200 / 500 = 40%
TNR= 99000 / 99500 = 99,5%
FPR= 500 / 99500 = 0.5%
PPV= 300 / 800 = 37.5%
NPV= 99000 / 99200 = 99.8%

Macroavg= (60 + 99.5) / 2 = 79.75%

		Actual	
	c_2	open	close
	OPEN	0	0
	CLOSE	500	99500

ERROR: 0,5%

TPR= 0 / 500 = 0%
FNR= 500 / 500 = 100%
TNR= 99500 / 99500 = 100%
FPR= 0 / 99500 = 0%
PPV= 0 / 0 = UNDEFINED
NPV= 99500 / 10000 = 99.5%

Macroavg= (0 + 100) / 2 = 50%

		Actual	
	c_3	open	close
	OPEN	400	5400
	CLOSE	100	94100

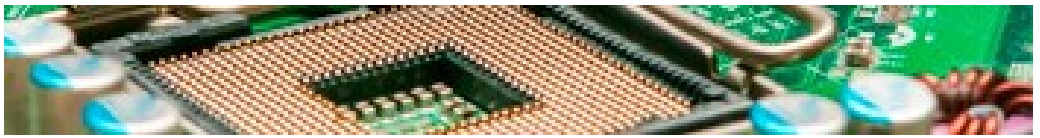
ERROR: 5,5%

TPR= 400 / 500 = 80%
FNR= 100 / 500 = 20%
TNR= 94100 / 99500 = 94.6%
FPR= 5400 / 99500 = 5.4%
PPV= 400 / 5800 = 6.9%
NPV= 94100 / 94200 = 99.9%

Macroavg= (80 + 94.6) / 2 = 87.3%

Sensitivity → Recall
Specificity → Precision

Which classifier is best?

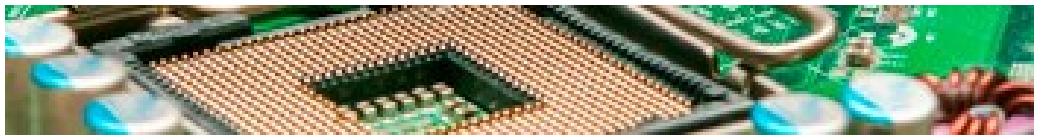


Cost-sensitive Evaluation

- Not all errors are equal.
  - Example: keeping a valve closed in a nuclear plant when it should be open can provoke an explosion, while opening a valve when it should be closed can provoke a stop.
  - Cost matrix:

		Actual: open	Actual: close
Predicted	OPEN	0	100€
	CLOSE	2000€	0

  - The best classifier is the one with lowest cost



Cost-sensitive Evaluation

- Easy to calculate (Hadamard product):

Confusion Matrices

Actual

c_1	open	close
OPEN	300	500
CLOSE	200	99000

Pred

Actual

c_2	open	close
OPEN	0	0
CLOSE	500	99500

Actual

c_3	open	close
OPEN	400	5400
CLOSE	100	94100

Actual

	open	close
OPEN	0	100€
CLOSE	2000€	0

Predicted

Cost Matrix

Resulting Matrices

c_1	open	close
OPEN	0€	50,000€
CLOSE	400,000€	0€

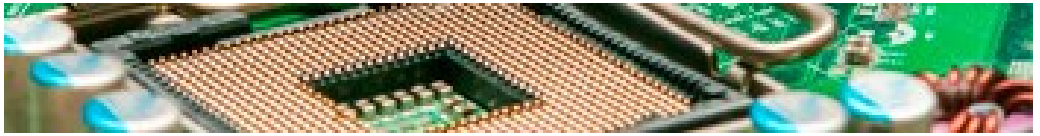
TOTAL COST: 450,000€

c_2	open	close
OPEN	0€	0€
CLOSE	1,000,000€	0€

TOTAL COST: 1,000,000€

c_3	open	close
OPEN	0€	540,000€
CLOSE	200,000€	0€

TOTAL COST: 740,000€



Cost-sensitive Evaluation

- What affects the final cost?
  - Cost per unit = FNcost*π_1*FNR + FPcost*(1-π_1)*FPR
  - If we divide by FNcost*π_1 we get M:
  - M = 1*FNR + FPcost*(1-π_1)/(FNcost*π_1) *FPR =
    = 1*FNR + **slope** *FPR

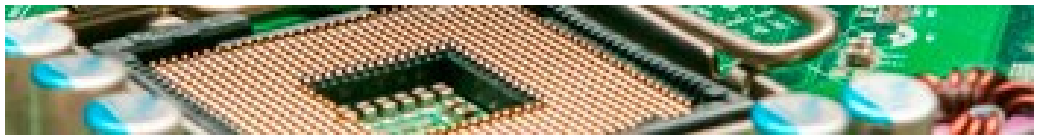
$$\frac{FPcost}{FNcost} = \frac{100}{2000} = \frac{1}{20} \qquad \frac{Neg}{Pos} = \frac{99500}{500} = 199 \qquad slope = \frac{1}{20} \times 199 = 9.95$$

Classif. 1: FNR= 40%, FPR= 0.5%
M1= 1 x 0.40 + 9.95 x 0.005 = 0.45
Cost per unit =
M1*(FNCost*π_0)=4.5

Classif. 2: FNR= 100%, FPR= 0%
M2= 1 x 1 + 9.95 x 0 = 1
Cost per unit =
M2*(FNCost*π_0)=10

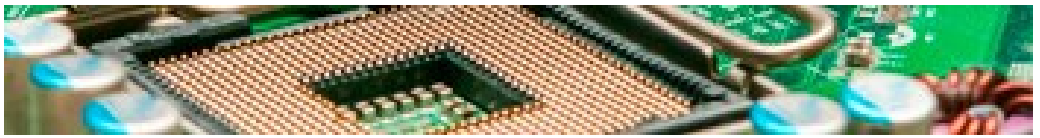
Classif. 3: FNR= 20%, FPR= 5.4%
M3= 1 x 0.20 + 9.95 x 0.054 = 0.74
Cost per unit =
M3*(FNCost*π_0)=7.4

For two classes, the value "**slope**" (with FNR and FPR) is sufficient to tell which classifier is best.
This is the **operating condition, context or skew**.



"ROC Analysis" (crisp)

- The context or skew (the class distribution and the costs of each error) determines the goodness of a set of classifiers.
  - PROBLEM:
    - In many circumstances, until the application time, we do not know the class distribution and/or it is difficult to estimate the cost matrix. E.g. a spam filter.
    - But models are usually learned before.
  - SOLUTION:
    - ROC (Receiver Operating Characteristic) Analysis.



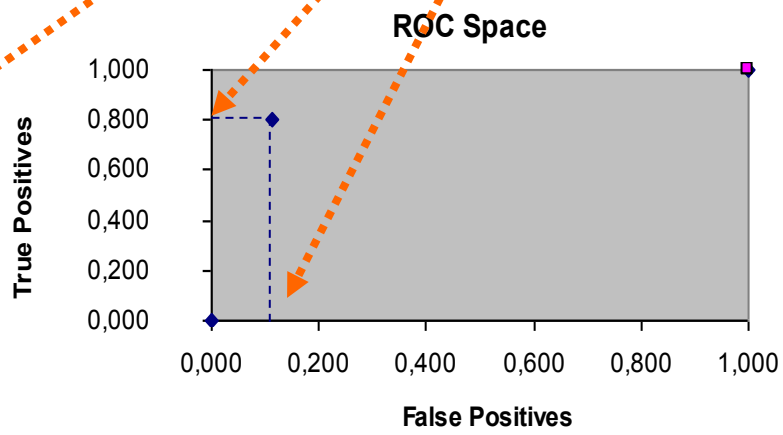
"ROC Analysis" (crisp)

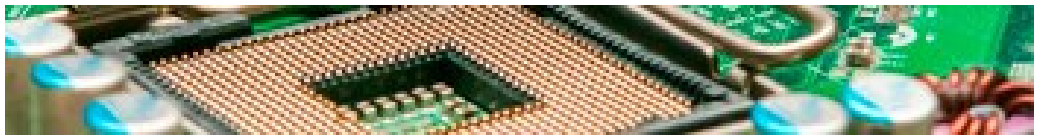
- The ROC Space
  - Using the normalised terms of the confusion matrix:
    - TPR, FNR, TNR, FPR:

	Actual: open	Actual: close
Pred: OPEN	400	12000
Pred: CLOSE	100	87500

	Actual: open	Actual: close
Pred: OPEN	0.8	0.121
Pred: CLOSE	0.2	0.879

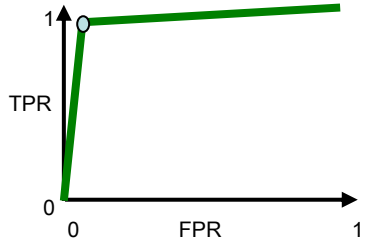
TPR= 400 / 500 = 80%
FNR= 100 / 500 = 20%
TNR= 87500 / 99500 = 87.9%
FPR= 12000 / 99500 = 12.1%



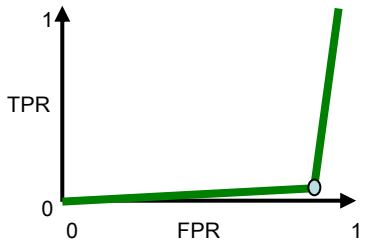


"ROC Analysis" (crisp)

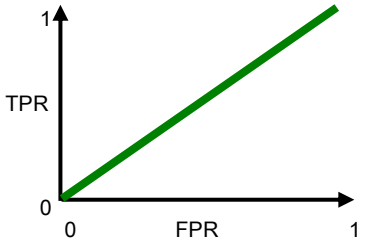
- ROC space: good and bad classifiers.



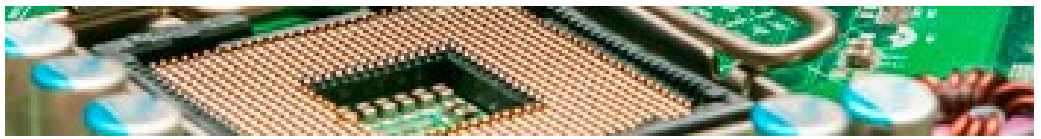
- Good classifier.
  - High TPR.
  - Low FPR.



- Bad classifier.
  - Low TPR.
  - High FPR.

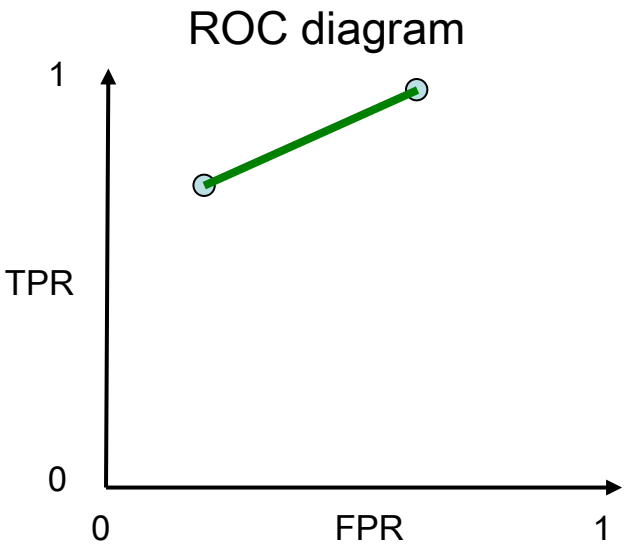


- Bad classifier (more realistic).

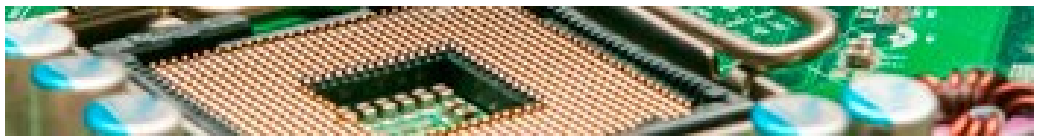


"ROC Analysis" (crisp)

- The ROC "Curve": "Continuity".

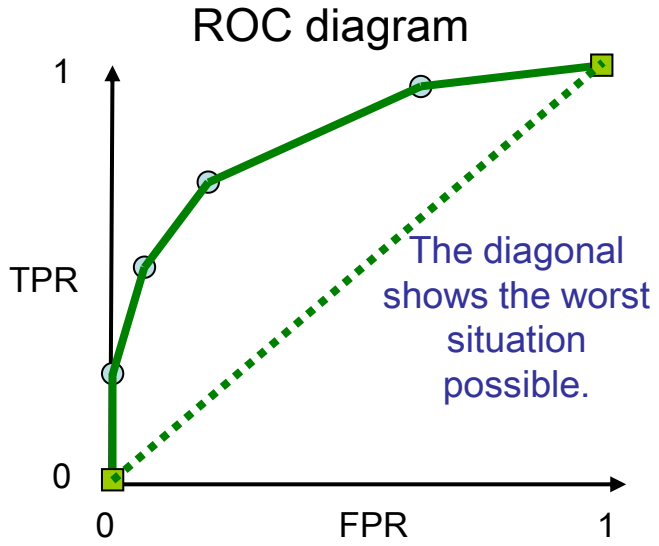


- Given two classifiers:
  - ✓ We can construct any "intermediate" classifier just randomly weighting both classifiers (giving more or less weight to one or the other).
  - ✓ This creates a "continuum" of classifiers between any two classifiers.



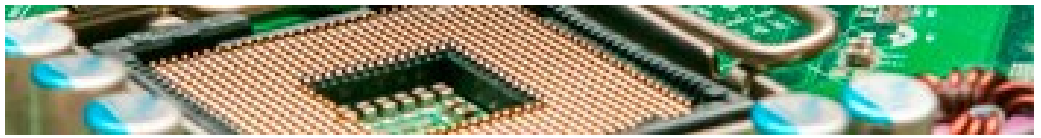
"ROC Analysis" (crisp)

- ROC Curve. Construction.



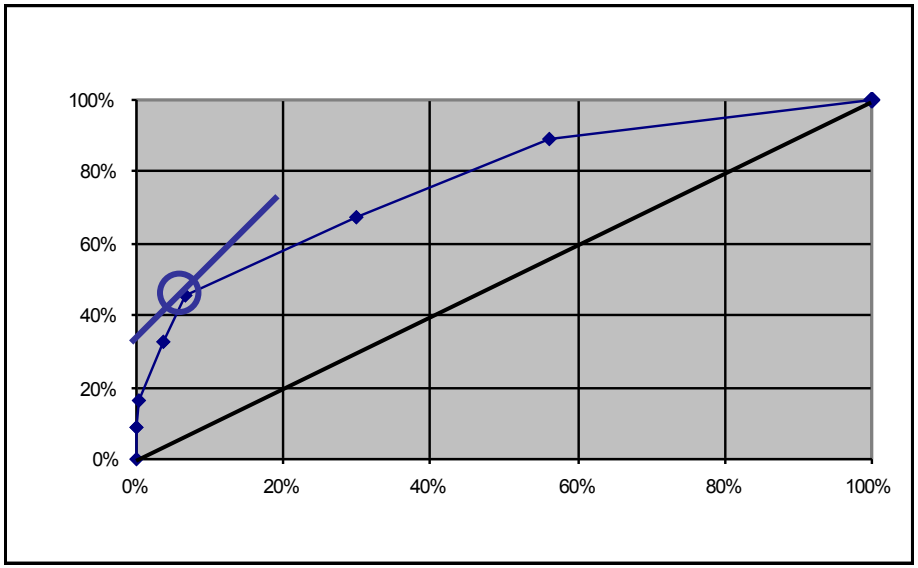
- Given several classifiers:
  - ✓ We construct the convex hull of their points (FPR,TPR) as well as the two trivial classifiers (0,0) and (1,1).
  - ✓ The classifiers below the ROC curve are discarded.
  - ✓ The best classifier (from those remaining) will be selected in application time…

We can discard those which are below because there is no combination of class distribution / cost matrix for which they could be optimal.



"ROC Analysis" (crisp)

- In the context of application, we choose the optimal classifier from those kept. Example 1:

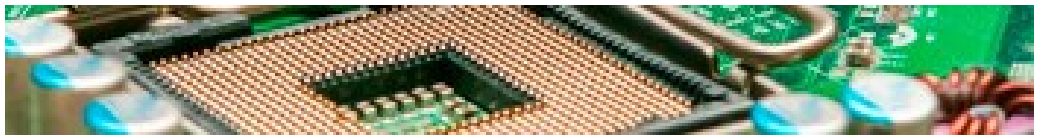


Context (skew):

$$\frac{FPcost}{FNcost} = 1/2$$

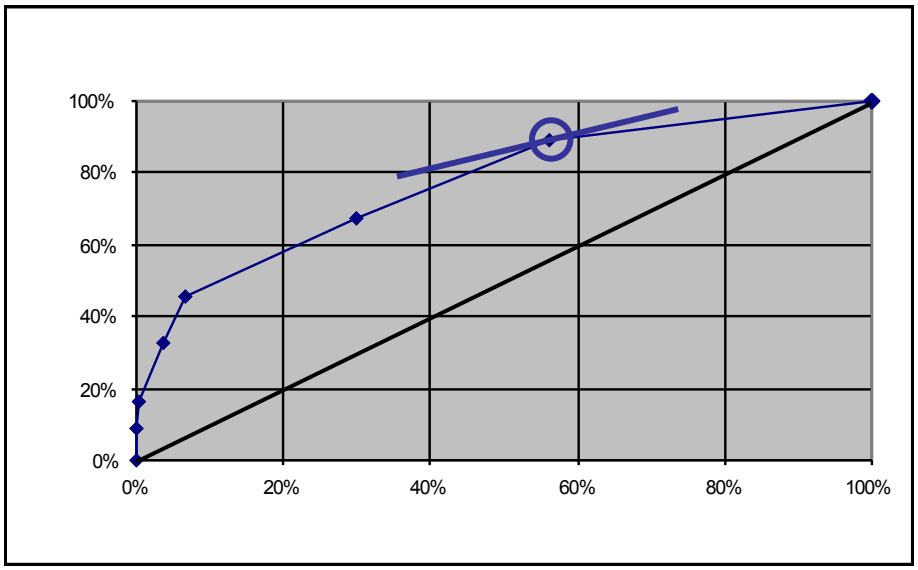
$$\frac{Neg}{Pos} = 4$$

$$slope = 4/2 = 2$$



"ROC Analysis" (crisp)

- In the context of application, we choose the optimal classifier from those kept. Example 2:

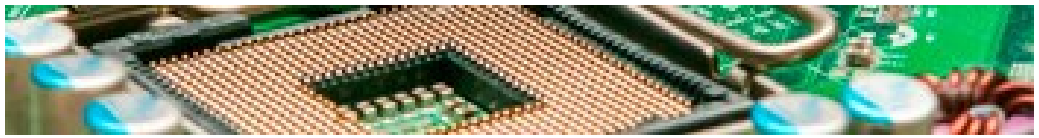


Context (skew):

$$\frac{FPcost}{FNcost} = 1/8$$

$$\frac{Neg}{Pos} = 4$$

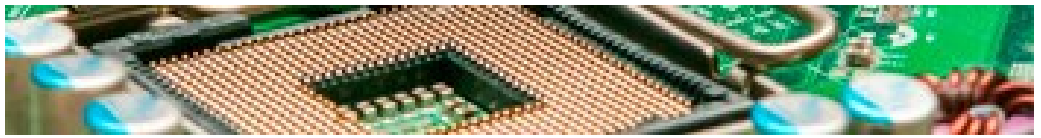
$$slope = 4/8 = .5$$



"ROC Analysis" (crisp)

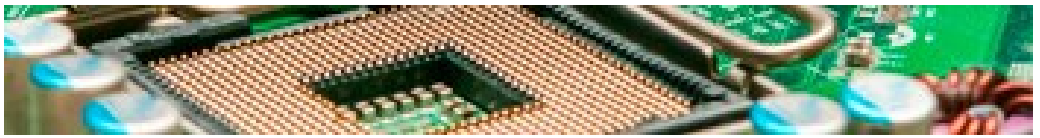
- What have we learned from this?
  - The optimality of a classifier depends on the class distribution and the error costs.
  - From this **context / skew** we can obtain the "slope", which characterises this context.
    - If we know this context, we can select the best classifier, multiplying the confusion matrix and the cost matrix.
    - If we don't know this context in the learning stage, by using ROC analysis we can choose a subset of classifiers, from which the optimal classifier will be selected when the context is known.

Can we go further than this?



True ROC Analysis (soft)

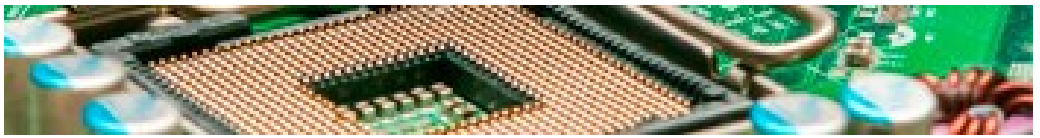
- Crisp and Soft Classifiers:
  - A "hard" or "crisp" classifier predicts a class between a set of possible classes.
  - A "soft" or "scoring" classifier (probabilistically) predicts a class, but accompanies each prediction with an estimation of the reliability (confidence or class probability) of each prediction.
    - Most learning methods can be adapted to generate this confidence value.



True ROC Analysis (soft)

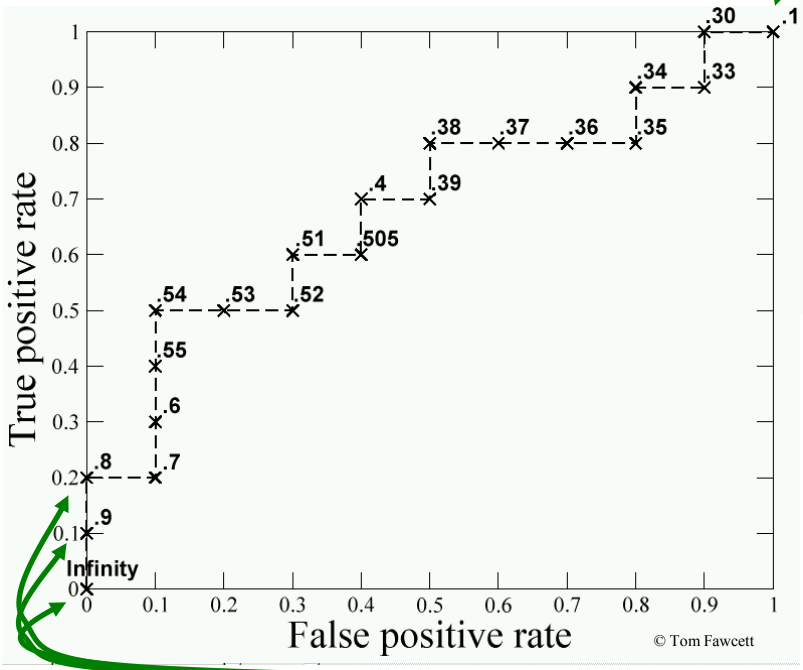
- ROC Curve of a Soft Classifier:
  - A soft classifier can be converted into a crisp classifier using a threshold.
    - Example: "if score > 0.7 then class A, otherwise class B".
    - With different thresholds, we have different classifiers, giving more or less relevance to each of the classes
  - We can consider each threshold as a different classifier and draw them in the ROC space. This generates a curve...

We have a "curve" for just one soft classifier



True ROC Analysis (soft)

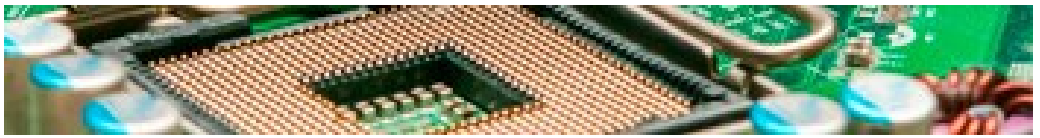
- ROC Curve of a soft classifier.



Actual Class

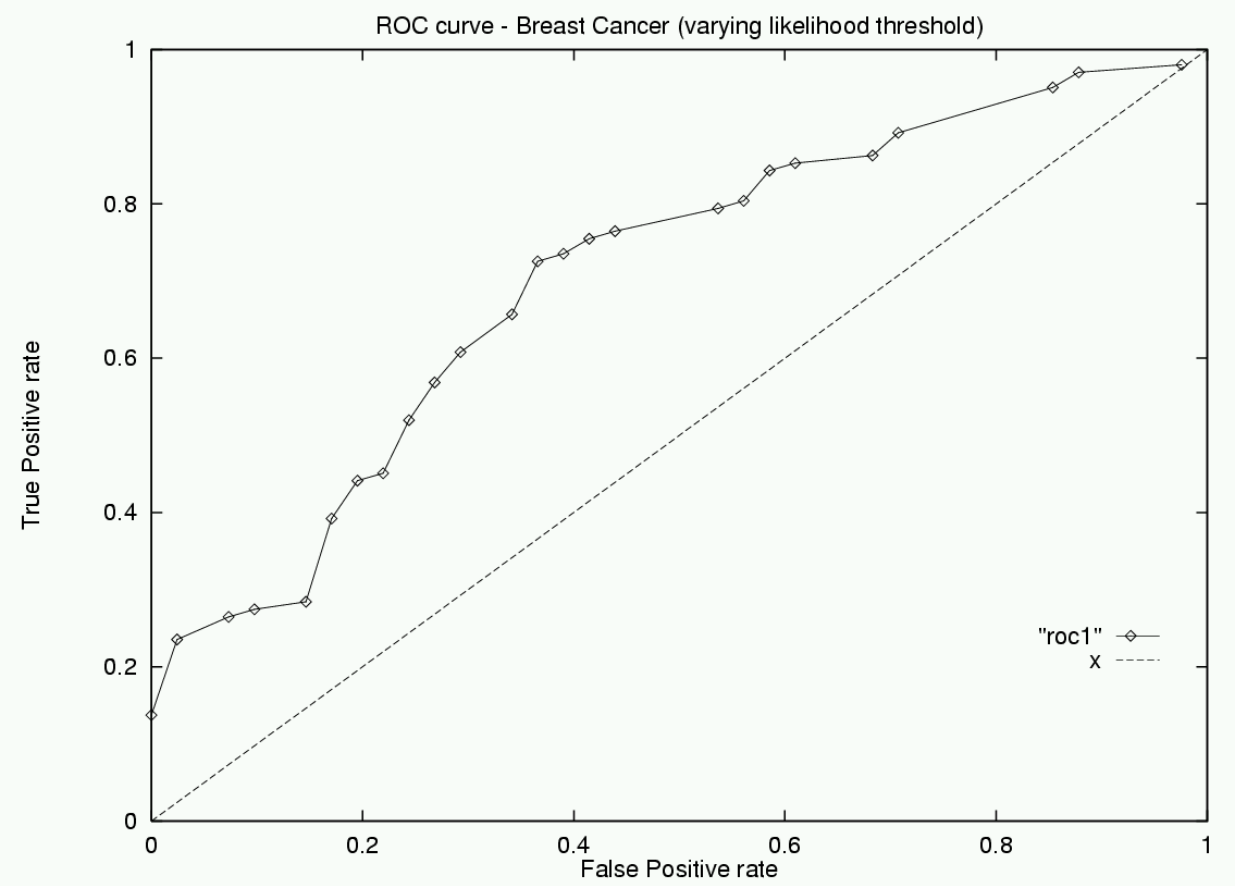
Predicted Class

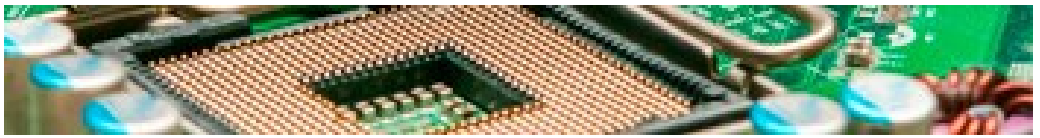
Inst#	Class	Score				
1	p	.9	n	p	p	p
2	p	.8	n	n	p	p
3	n	.7	n	n	n	p
4	p	.6	n	n	n	p
5	p	.55	n	n	n	p
6	p	.54	n	n	n	p
7	n	.53	n	n	n	p
8	n	.52	n	n	n	p
9	p	.51	n	n	n	p
10	n	.505	n	n	n	... p
11	p	.4	n	n	n	p
12	n	.39	n	n	n	p
13	p	.38	n	n	n	p
14	n	.37	n	n	n	p
15	n	.36	n	n	n	p
16	n	.35	n	n	n	p
17	p	.34	n	n	n	p
18	n	.33	n	n	n	p
19	p	.30	n	n	n	p
20	n	.1	n	n	n	p



True ROC Analysis (soft)

- ROC Curve of a soft classifier.



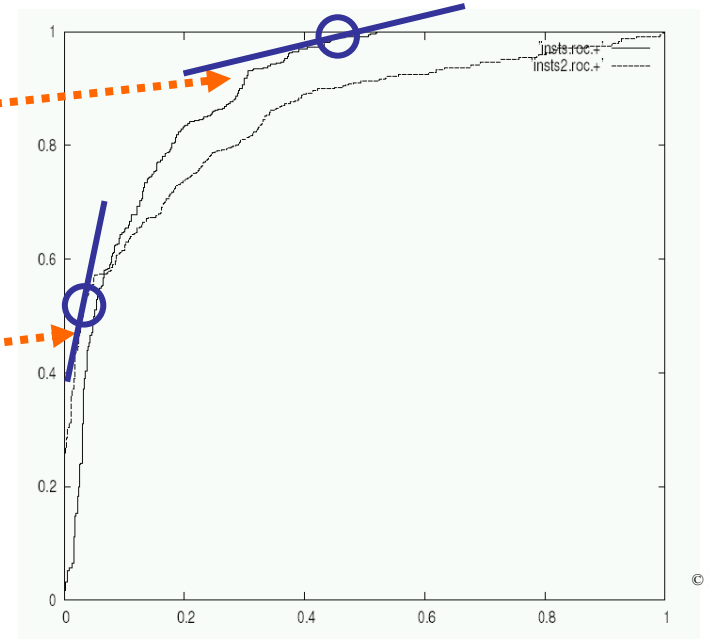


True ROC Analysis (soft)

- ROC Curve of a soft classifier.

In this zone the best classifier is “insts”

In this zone the best classifier is“insts2”



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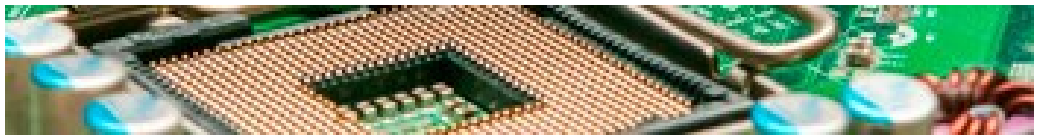
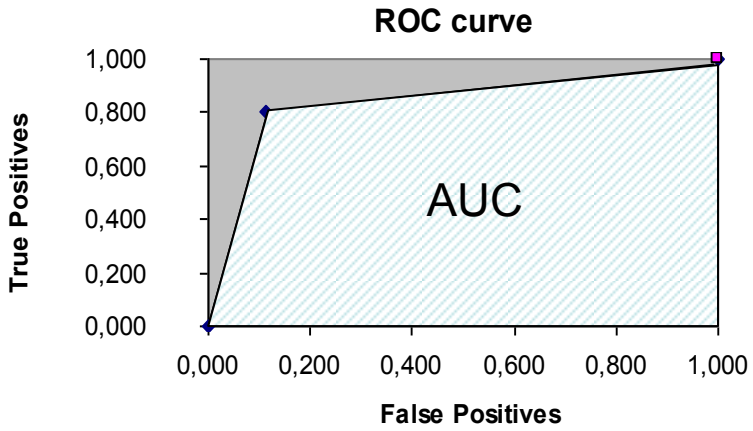
We must preserve the classifiers that have at least one “best zone” (dominance) and then behave in the same way as we did for crisp classifiers.

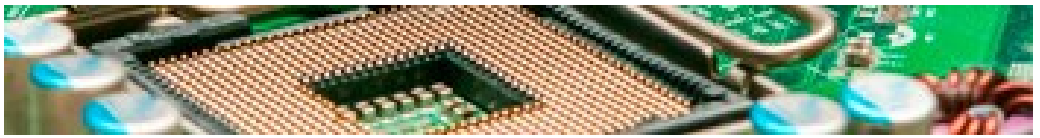


The AUC metric

- What if we want to select just one crisp classifier?
  - The classifier with greatest Area Under the ROC Curve (AUC) is chosen.

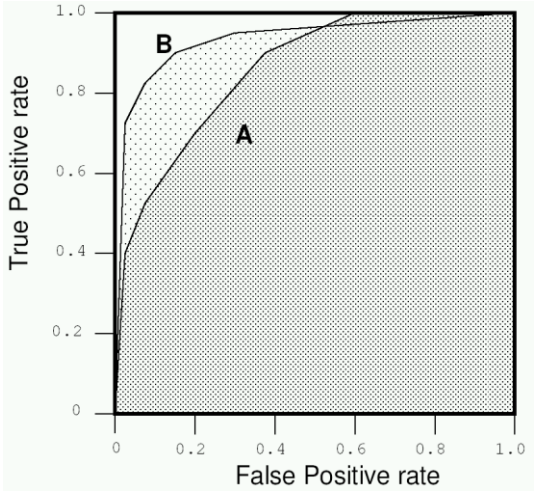


For crisp classifiers AUC is equivalent to the macroaveraged accuracy.



The AUC metric

- What if we want to select just one soft classifier?
  - The classifier with greatest Area Under the ROC Curve (AUC) is chosen.

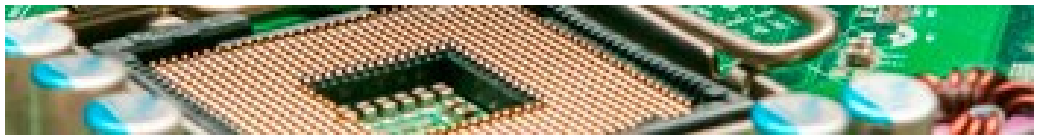


In this case

we select B.

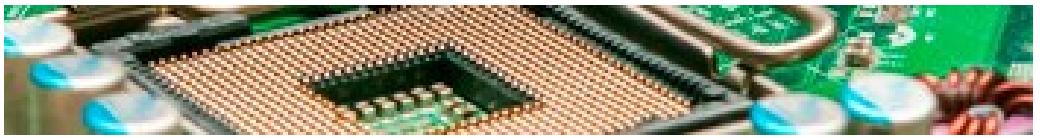
Remember the “Wilcoxon-Mann-Whitney statistic”:

The AUC really estimates the probability that, if we choose an example of class 1 and an example of class 0, the classifier will give a higher score to the first one than to the second one.



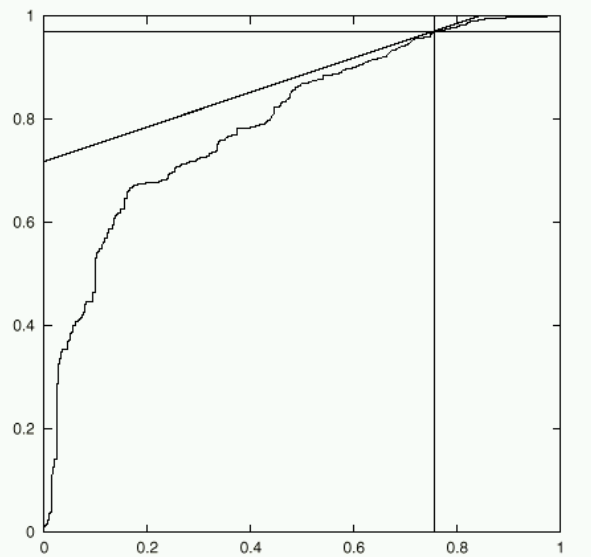
The AUC metric

- AUC is for classifiers and rankers:
  - A classifier with high AUC is a good ranker.
  - It is also good for a (uniform) range of operating conditions.
    - A model with very good AUC will have good accuracy for all operating conditions.
    - A model with very good accuracy for one operating condition can have very bad accuracy for another operating condition.
  - A classifier with high AUC can have poor calibration (probability estimation).

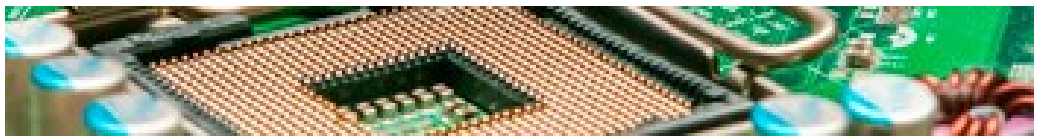


AUC is not ROC

- AUC is useful but it is always better to draw the curves and choose depending on the operating condition.



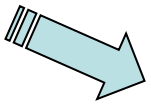
- Curves can be estimated with a validation dataset or using cross-validation.



Beyond binary classification

- Cost-sensitive evaluation is perfectly extensible for classification with more than two classes.

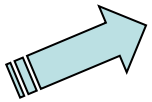
COST		*actual*		
		low	medium	high
predicted	low	0€	5€	2€
	medium	200€	-2000€	10€
	high	10€	1€	-15€



Total cost:

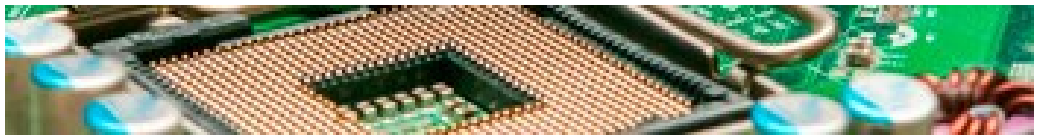
-29787€

ERROR		*actual*		
		low	medium	high
predicted	low	20	0	13
	medium	5	15	4
	high	4	7	60



- For regression, we only need a cost function
  - For instance, asymmetric absolute error:

$$\ell_{\alpha}^{A}(\hat{y},y) \triangleq \begin{cases} 2\alpha(y-\hat{y}) & \text{if } \hat{y} < y \\ 2(1-\alpha)(\hat{y}-y) & \text{otherwise} \end{cases}$$

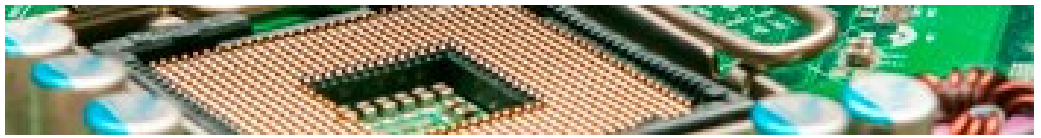


Beyond binary classification

- ROC analysis for multiclass problems is troublesome.
  - Given *n* classes, there is a $n \times (n-1)$ dimensional space.
  - Calculating the convex hull impractical.
- The AUC measure has been extended:
  - All-pair extension (Hand & Till 2001).

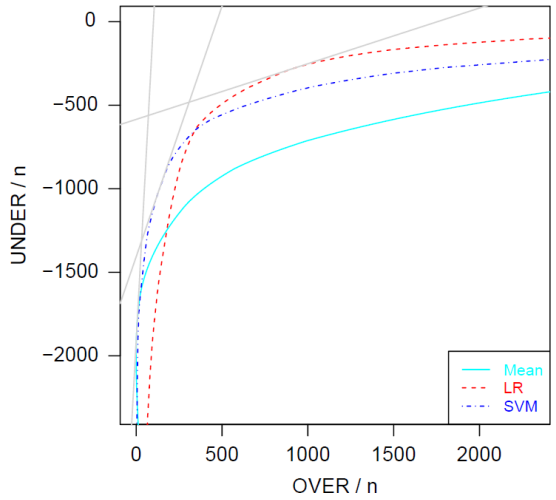
$$AUC_{HT} = \frac{1}{c(c-1)} \sum_{i=1}^{c} \sum_{j=1, j<i}^{c} AUC(i,j)$$

  - There are other extensions.



Beyond binary classification

- ROC analysis for regression (using shifts).
  - The operating condition is the asymmetry factor α. For instance if α=2/3 means that underpredictions are twice as expensive than overpredictions.



$$\ell_{\alpha}^{A}(\hat{y},y) \triangleq \begin{cases} 2\alpha(y-\hat{y}) & \text{if } \hat{y} < y \\ 2(1-\alpha)(\hat{y}-y) & \text{otherwise} \end{cases}$$

  - The area over the curve (AOC) is ½ the error variance. If the model is unbiased, then it is ½ MSE.