# Problema de la Mochila 0/1

Ejemplo

# El problema

Hay un ladrón con una mochila que puede llevar 4 kilos en cosas.

Hay tres cosas que puede robarse:

**Estereo**
$ 200K
3 Kg

**Laptop**
$ 300K
3 Kg

**Guitarra**
$ 150K
1 Kg

¿Qué items debería robar para maximizar el valor robado?

# Solución *naïve* (Fuerza Bruta / Combinatoria)

| **Estereo ($ 200 / 3)** | **Laptop ($ 300 / 3)** | **Guitarra ($ 150 / 1)** | **Ganancia / Peso restante** |
|---|---|---|---|
| NO | NO | NO | $ 0 / 4 |
| NO | NO | SI | $ 150 / 3 |
| NO | SI | NO | $ 300 / 1 |
| NO | SI | SI | $ 450 / 0 |
| SI | NO | NO | $ 200 / 1 |
| SI | NO | SI | $ 350 / 0 |
| SI | SI | NO | $ 500 / -2 |
| SI | SI | SI | $ 650 / -3 |

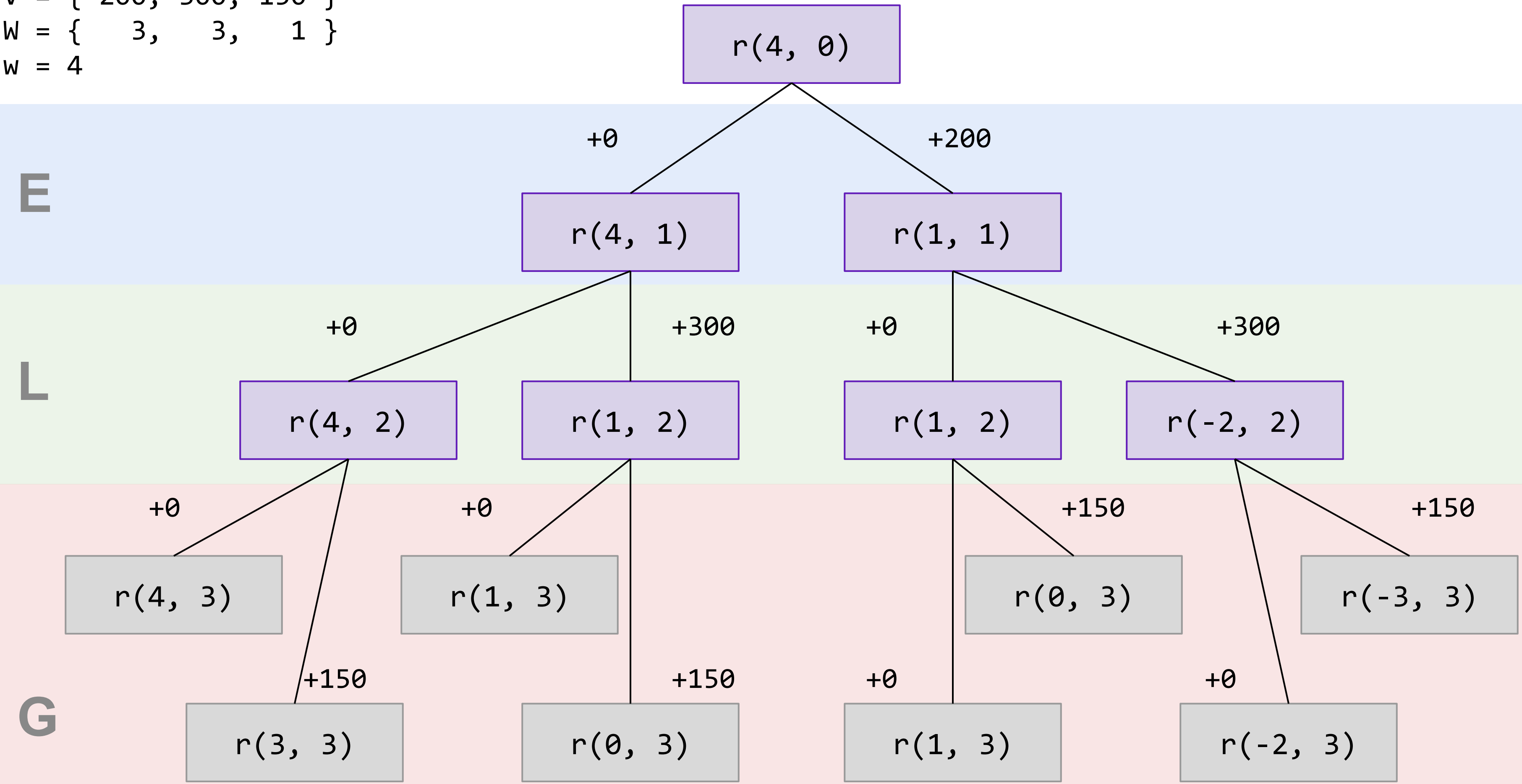
# Expresión recursiva → Enfoque Top-Down

$$r(w,i) = \begin{cases} 0 & \text{if } i = |V| \\ \max \begin{cases} 0 + r(w, i+1) \\ V_i + r(w - W_i, i+1) \end{cases} & \text{otherwise} \end{cases}$$

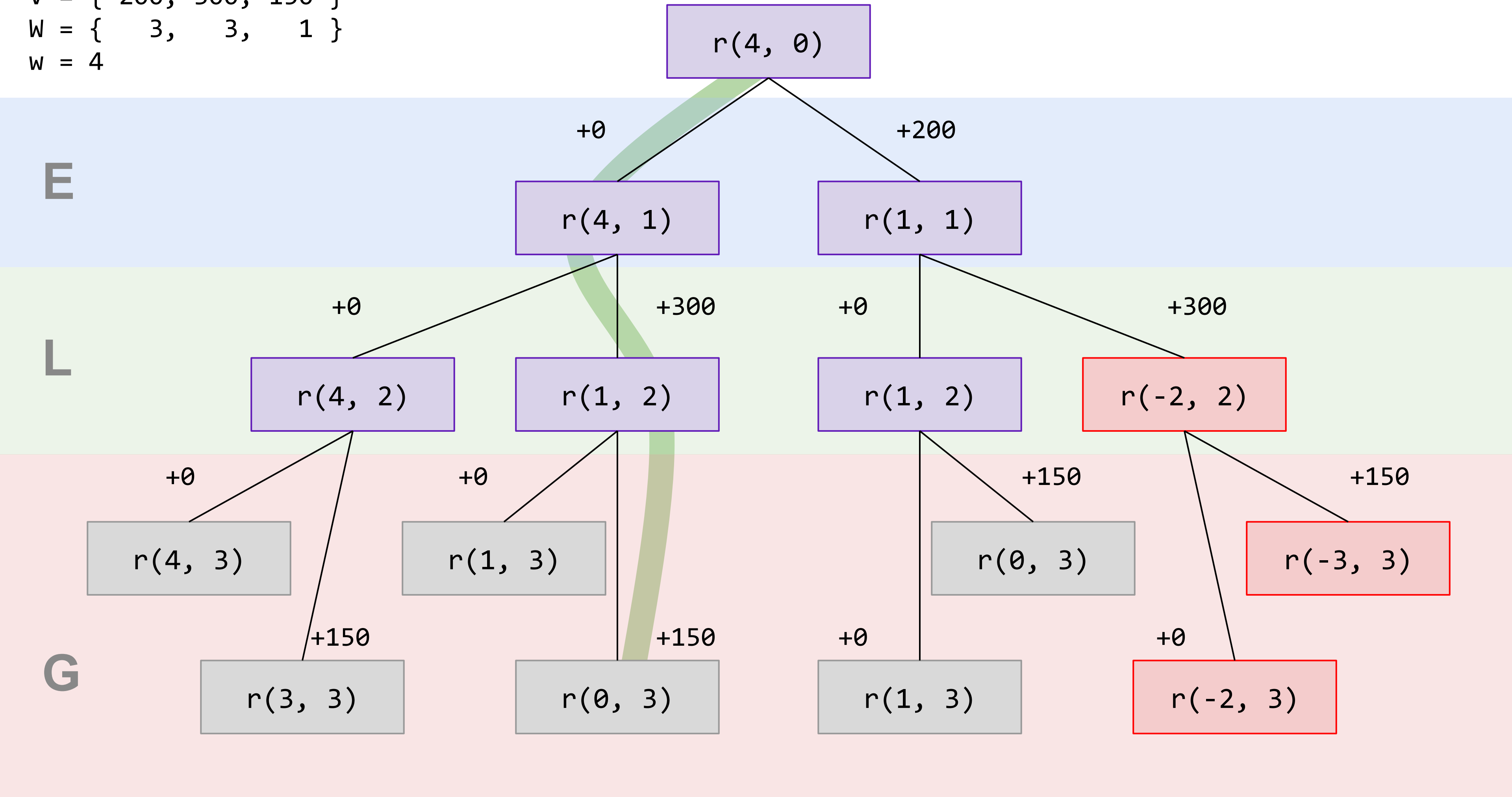
Siendo:

```
V = { 200, 300, 150 }
W = {   3,   3,   1 }
w = 4
i = 0
```

```
V = { 200, 300, 150 }
W = {   3,   3,   1 }
w = 4
```

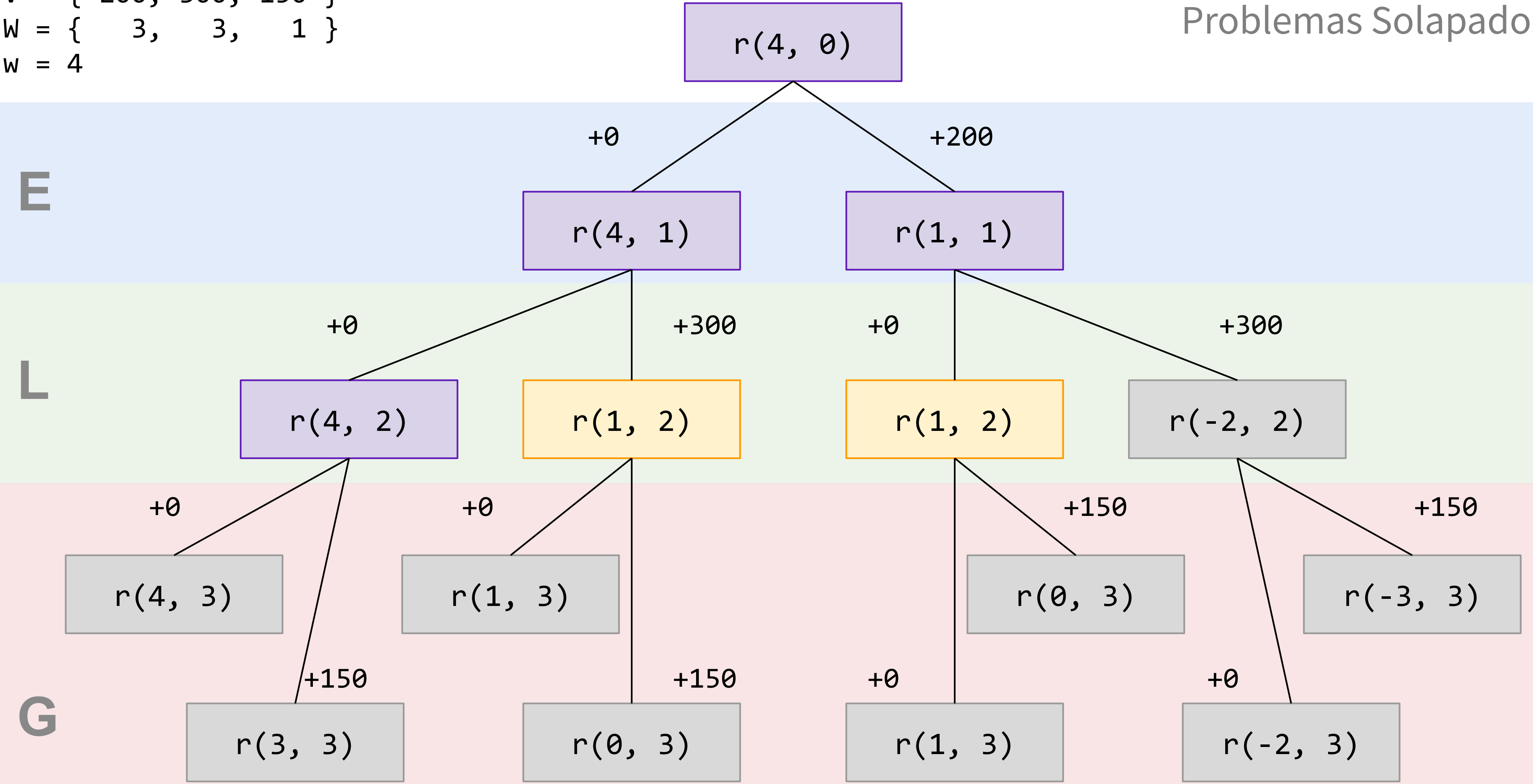


V = { 200, 300, 150 }
W = {   3,   3,   1 }
w = 4



# Problemas Solapados

```
V = { 200, 300, 150 }
W = {   3,   3,   1 }
w = 4
```



$2^3 = 8$

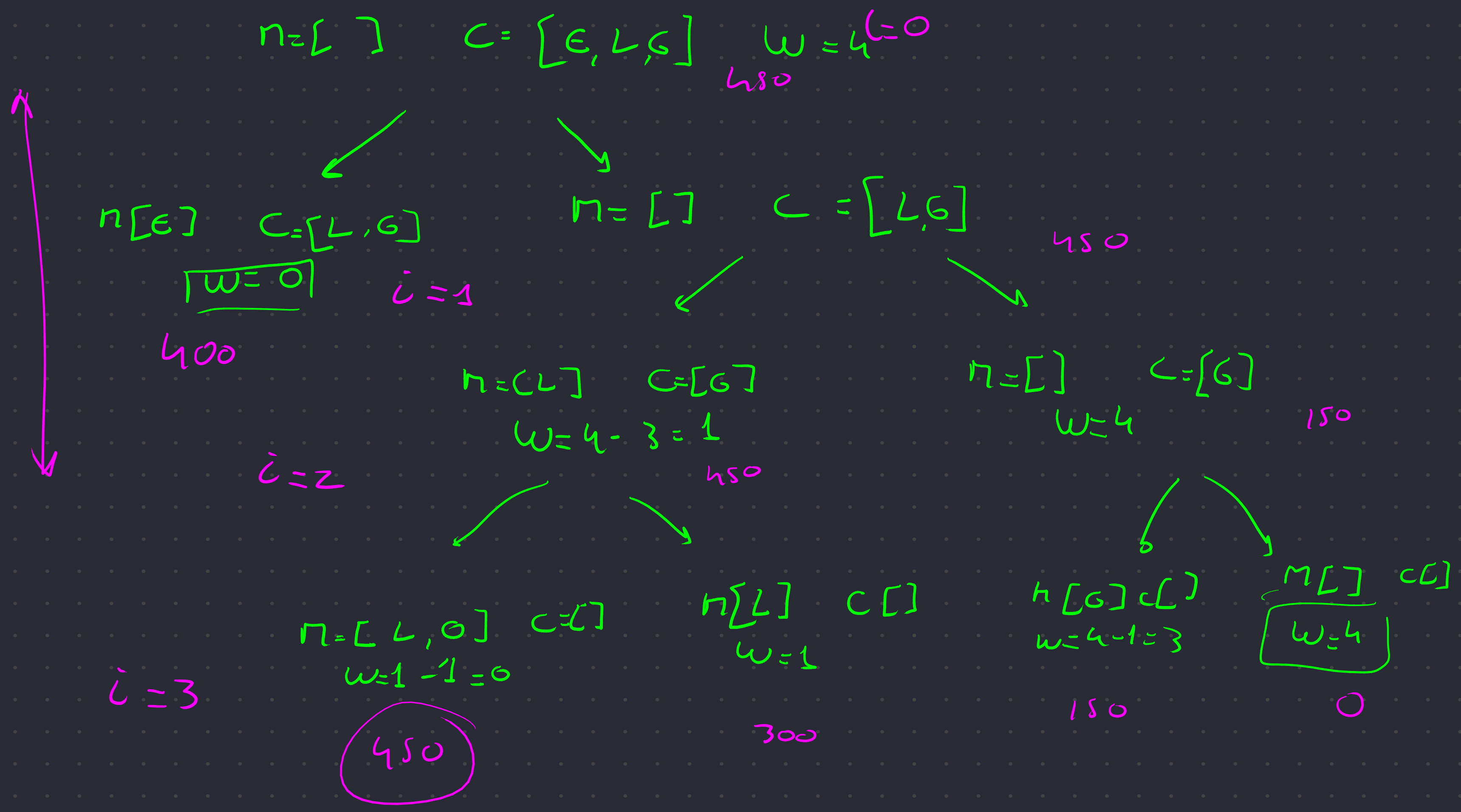
$2^n$

| E (400, 4) | L (300, 3) | G (150, 1) | ✓ |
|---|---|---|---|
| 0 | 0 | 0 | \$0 |
| 0 | 0 | 1 | \$150 |
| 0 | 1 | 0 | \$300 |
| 0 | 1 | 1 | \$450 ◁ |
| 1 | 0 | 0 | \$400 |
| 1 | 0 | 1 | $\varnothing$ |
| 1 | 1 | 0 | $\varnothing$ |
| 1 | 1 | 1 | $\varnothing$ |

| | E | L | G |
|---|---|---|---|
| V | 400 | 300 | 150 |
| w | 4 | 3 | 1 |

$W = 4$

cap?



E L G

0 1 2

↑ ↑ ↑

$W = 4$

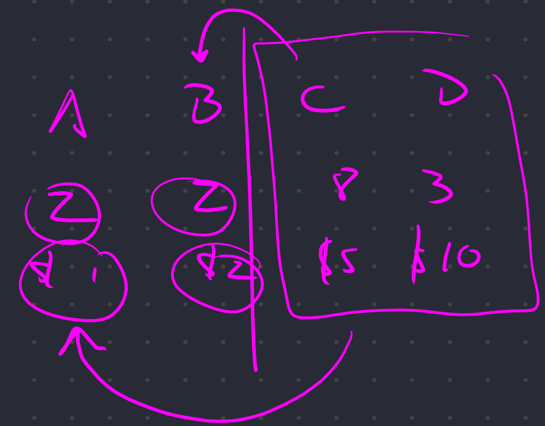
$$
\text{robar}(cap, i) \begin{cases} \emptyset & i = |V| \quad \leftarrow \text{corte} \\ \max \begin{cases} \text{robar}(cap, i+1) \\ v_i + \text{robar}(cap - w_i, i+1) \end{cases} \end{cases}
$$

Top-Down

c/ recursión → Memoria!

$k = (w, i)$

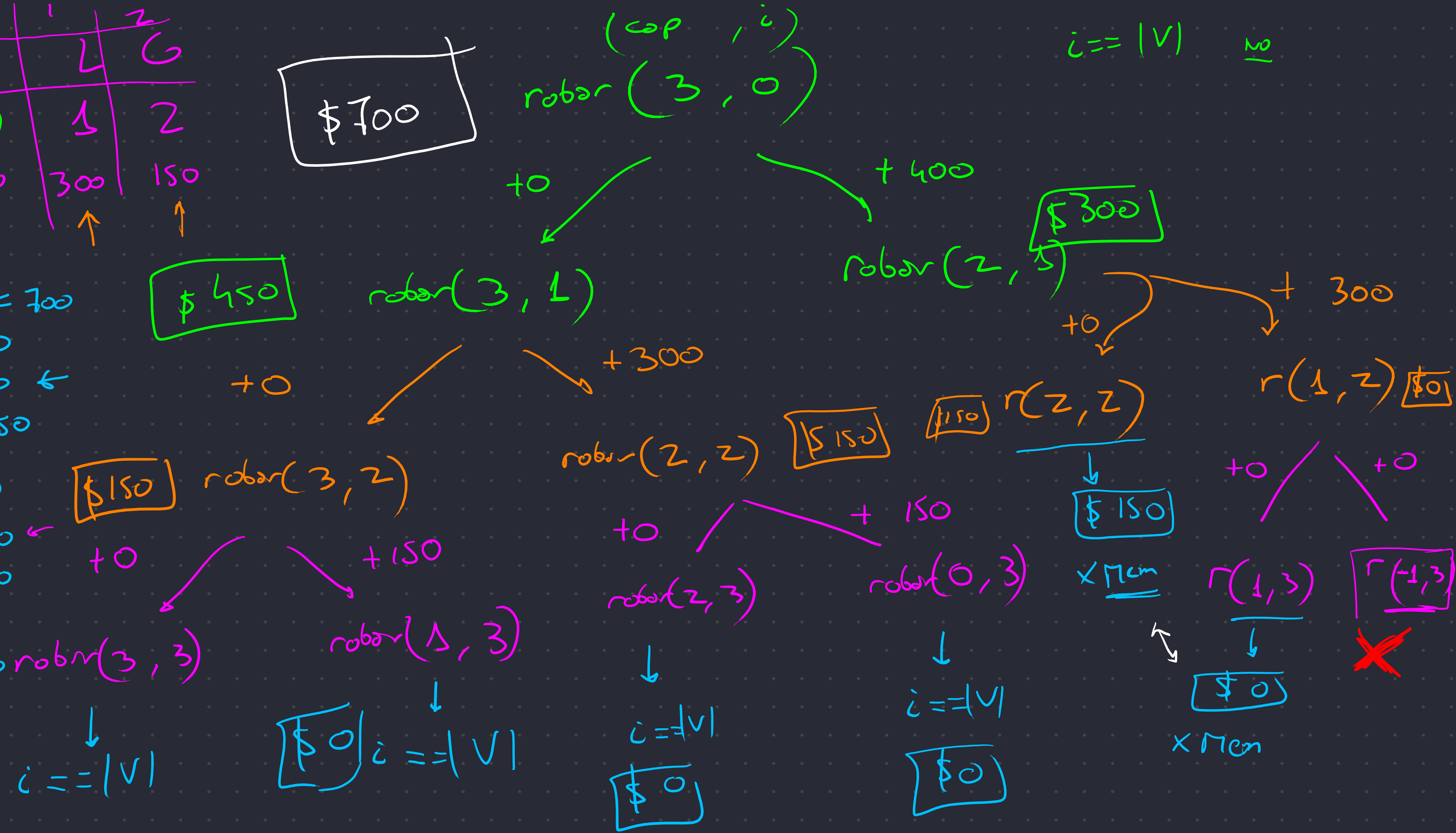
$(W-2, 2)$ → A, → B



| | 0 | 1 | 2 |
|---|---|---|---|
| | E | L | G |
| w | 1 | 1 | 2 |
| v | 400 | 300 | 150 |

W=3

M
3,0 = 700
3,3 = 0
1,3 = 0
3,2 = 150
2,3 = 0
0,3 = 0
2,2 = 150
3,1 = 450
1,2 = 0
2,1 = 300



$V = [400 \quad 300 \quad 150]$

$w = [4 \quad 3 \quad 1]$

$W = 4$

?

$$t[i,j] = \max \begin{cases} t[i-1,j] \\ v_i + t[i-1, j-w_i] \end{cases}$$

| $i$ \ $W$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $400 |
| 1 | +150 0 | +150 0 | +150 0 | +150 $300 | $400 |
| 2 | 0 | 150 | 150 | $300 | $\boxed{\$450}$ |

$V: [20 \quad 30 \quad 15 \quad 25 \quad 10]$

$w: [6 \quad 13 \quad 7 \quad 10 \quad 3]$

$W = 20$

$V_{MAX} = 55$

$$t[i,j] = \max \begin{cases} t[i-1, j] \\ v_i + t[i-1, j - w_i] \end{cases}$$

| i \ W | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $\boxed{0}$ | 0 | 0 | 0 | 0 | 0 | $\boxed{20}$ | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | $\boxed{20}$ | 20 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 30 | 30 | 30 | 30 | 30 | 30 | 50 | 50 |
| 2 | +25 $\boxed{0}$ | 0 | 0 | 0 | 0 | 0 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 35 | 35 | 35 | 35 | 35 | 35 | 50 | 50 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 20 | 20 | 20 | 25 | 25 | 25 | 35 | 35 | 35 | +10 $\boxed{45}$ | 45 | 45 | $\boxed{50}$ | 50 |
| 4 | 0 | 0 | 0 | 10 | 10 | 10 | 20 | 20 | 20 | 30 | 30 | 30 | 30 | 35 | 35 | 35 | 45 | 45 | 45 | 55 | $\boxed{55}$ |