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Q.1

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 \\ 4 & 0 & 0 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ -2 \\ 0 \\ 4 \end{bmatrix}$$

(a) Resolver usando a Eliminação de Gauss com Pivoteamento Parcial.

$Ax = b$

$$\bar{A}^{(0)} = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 2 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad \bar{b}^{(0)} = \begin{bmatrix} 4 \\ -2 \\ 0 \\ 10 \end{bmatrix}$$ permutando a 1ª e 4ª linhas

$$m_{21} = \frac{2}{4} = \frac{1}{2} \qquad m_{31} = \frac{3}{4} \qquad m_{41} = \frac{1}{4}$$

$$A^{(1)} = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 2 & 3 & 3 \end{bmatrix} \qquad b^{(1)} = \begin{bmatrix} 4 \\ -4 \\ -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 2 & 3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -4 \\ -3 \\ 9 \end{bmatrix}$$

• não se faz nada.

$m_{32} = 0$ // $m_{42} = 1$ //

$$A^{(2)} = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 3 & 5 \end{bmatrix}, \quad b^{(2)} = \begin{bmatrix} 4 \\ -4 \\ -3 \\ 13 \end{bmatrix}$$

$\bar{A}^{(2)} = A^{(2)}$ e $\bar{b}^{(2)} = b^{(2)}$ não se faz nada

• $m_{43} = 1$

$$A^{(3)} = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 8 \end{bmatrix}, \quad b^{(3)} = \begin{bmatrix} 4 \\ -4 \\ -3 \\ 16 \end{bmatrix}$$

$$x^* = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$ ← Por substituição reversa!

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$ na matriz identidade 4x4

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 3/4 & 0 & 1 & 0 \\ 1/4 & 1 & 1 & 1 \end{bmatrix} \rightarrow$$ Tomamos os m_{ij} calculados

$$U = \begin{bmatrix} 4 & 0 & 0 & 4 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 8 \end{bmatrix} \rightarrow$$ Matriz resultante da eliminação de Gauss

$$A = \begin{bmatrix} 5 & -1 & w \\ 2 & -4 & 1 \\ -1 & 1 & w \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ 9 \\ 3 \end{bmatrix}$$

a) Usaremos o Critério das linhas

$$\alpha_i = \frac{1}{|a_{ii}|}\left(\sum_{j \neq i} |a_{ij}|\right) < 1 \quad , \quad i \in [1, n]$$

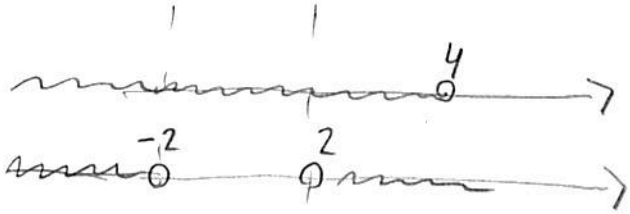
(*) $\alpha_1 = \dfrac{1}{5}(w+1)$

$\alpha_2 = \dfrac{1}{4}(2+1) = 0.75 < 1$ ✓

(**) $\alpha_3 = \dfrac{1}{|w|}(1+1)$

de (*), $\dfrac{w+1}{5} < 1 \rightarrow \underline{w < 4}$

de (**) $\dfrac{2}{|w|} < 1 \rightarrow |w| > 2 \rightarrow w < -2$ ou $w > 2$



Logo, $w < -2$ ou $2 < w < 4$ //

$a = -\infty$, $b = -2$, $c = 2$, $d = 4$ //

$$A = \begin{bmatrix} 2 & -4 & 1 \\ -1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\bar{X}^{(0)} = [0 \quad 0 \quad 0]^T$$

$$\begin{cases} x_1^{(1)} = \left(b_1 - \left(a_{12} x_2^{(0)} + a_{13} x_3^{(0)}\right)\right) / a_{11} \\ x_2^{(1)} = \left(b_2 - \left(a_{21} x_1^{(1)} + a_{23} x_3^{(0)}\right)\right) / a_{22} \\ x_3^{(1)} = \left(b_3 - \left(a_{31} x_1^{(1)} + a_{32} x_2^{(1)}\right)\right) / a_{33} \end{cases}$$

$$\begin{cases} x_1^{(1)} = \left(8 - \left((-1).0 + (3).0\right)\right) / 5 = 8/5 \;// \\ x_2^{(1)} = \left(9 - \left((2)\left(\frac{8}{5}\right) + (1)(0)\right)\right) / -4 = -37/20 \;// \\ x_3^{(1)} = \left(3 - \left((-1)\left(\frac{8}{5}\right) + (1)\left(\frac{-37}{20}\right)\right)\right) / 3 = 129/60 \;// \end{cases}$$

a) $f(x) = 2x + e^x$

$x^{(0)} = -0.35$

$$\varphi(x) = x - \frac{f(x)}{f'(x)}$$

$f'(x) = 2 + e^x$

$$x^{(1)} = \varphi(x^{(0)}) = -0.35 - \left[\frac{2.(-0.35) + e^{-0.35}}{2 + e^{-0.35}}\right] = -0.35173$$

$$|x^{(1)} - x^{(0)}| = 0.00173 < 0.002$$

Parada

$$\xi = x^{(1)} = -0.35173$$

b) Para $f(x) = x^2 - 2x - 3$,

temos $f'(x) = 2x - 2$

Assim, $\varphi(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - 2x - 3}{2x - 2} = \frac{2x^2 - \cancel{2x} - x^2 + \cancel{2x} + 3}{2x - 2}$

$$= \frac{x^2 + 3}{2x - 2}$$

Sabemos também que $x^{(k)} = \varphi(x^{(k-1)})$, $k = 1, 2 \ldots$

Logo, note que $x^{(k)} = \varphi^{(k)}(x^{(0)}) = \varphi^{(k)}(0.9)$ → k-ésima composta!

$$\begin{cases} \\ x_2 = e^{x_1} \end{cases}$$

a) $F(x) = \begin{bmatrix} x_1^2 + x_2 - 4 \\ -e^{x_1} + x_2 \end{bmatrix}$; $J(x) = \begin{bmatrix} 2x_1 & 1 \\ -e^{x_1} & 1 \end{bmatrix}$

$x^{(0)} = \begin{bmatrix} 1 \\ 2.9 \end{bmatrix}$

$$J(x^{(0)})\, s^{(0)} = -F(x^{(0)}) \to \begin{bmatrix} 2 & 1 \\ -e & 1 \end{bmatrix} \begin{bmatrix} s_1^{(0)} \\ s_2^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.1 \\ 0.182 \end{bmatrix}$$

$$s^{(0)} = \begin{bmatrix} 0.06 \\ -0.02 \end{bmatrix}$$ Logo, $x^{(1)} = x^{(0)} + s^{(0)} =$

$$= \begin{bmatrix} 1 \\ 2.9 \end{bmatrix} + \begin{bmatrix} 0.06 \\ -0.02 \end{bmatrix} = \begin{bmatrix} 1.06 \\ 2.88 \end{bmatrix}$$

Note que $\| x^{(1)} - x^{(0)} \|_\infty = \| s^{(0)} \|_\infty = 0.06 < 0.07$

Para

$\therefore \hat{x} = [1.06 \,;\, 2.88]^T$

Note que tanto $F(x) = \begin{bmatrix} x_1^2 + x_2 - 4 \\ -e^{x_1} + x_2 \end{bmatrix}$ e

$$J(x) = \begin{bmatrix} 2x_1 & 1 \\ -e^{x_1} & 1 \end{bmatrix}$$

são contínuas e definidas em todo o espaço $\mathbb{R}$.

Logo, $\forall (x_1, x_2) \in \mathbb{R}^2$, $\exists\, F([x_1, x_2]) \in \mathbb{R}^{2\times 1}$

$\exists\, J([x_1, x_2]) \in \mathbb{R}^{2\times 2}$