* Resolução Extraoficial P1 - MS211 - 1º/2018/VALLE

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Q.1 a) $\hat{A} = \begin{bmatrix} +0.63\times10^0 & +0.5\times10^0 \\ +0.2\times10^1 & +0.14\times10^1 \end{bmatrix}$, $\tilde{b} = \begin{bmatrix} +0.17\times10^1 \\ +0.54\times10^1 \end{bmatrix}$

b) $\bar{A}^{(0)} = \begin{bmatrix} 0.2\times10^1 & 0.14\times10^1 \\ 0.63\times10^0 & 0.5\times10^0 \end{bmatrix}$, $\bar{b}^{(0)} = \begin{bmatrix} 0.54\times10^1 \\ 0.17\times10^1 \end{bmatrix}$ Permutar 1ª c/ 2ª linha

$$m_{21} = \frac{0.63}{2} = 0.32\times10^0$$

$$A^{(1)} = \begin{bmatrix} 0.2\times10^1 & 0.14\times10^1 \\ 0 & 0.5\times10^{-1} \end{bmatrix}, \quad b^{(1)} = \begin{bmatrix} 0.54\times10^1 \\ -0{,}28\cdot10^{-1} \end{bmatrix}$$

Por substituição reversa:

$$\tilde{x} = \begin{bmatrix} 3.1 \\ -0.56 \end{bmatrix}$$

c)

$$A\tilde{x}-b=\begin{bmatrix}1-1/e & 1/2\\ 2 & 1+1/e\end{bmatrix}\begin{bmatrix}3.1\\ -0.56\end{bmatrix}-\begin{bmatrix}e-1\\ 2e\end{bmatrix}=$$

$$=\begin{bmatrix}3.1-\frac{3.1}{e}-\frac{0.56}{2}-e+1\\ 6.2-0.56-\frac{0.56}{e}-2e\end{bmatrix}=\begin{bmatrix}-0{,}03871\\ -0{,}00258\end{bmatrix}$$

$$\|A\tilde{x}-b\|_\infty = 0.03871$$

$$\|b\|_\infty = 2e = 5{,}43656$$

d)

$$\frac{R_r}{cond(A)} \leq E_r \leq \underline{cond(A)R_r}$$

$$E_r \leq 65.5 \cdot \left(\frac{\|A\tilde{x}-b\|_\infty}{\|b_\infty\|}\right) = 65.5\left(\frac{0.03871}{5.43656}\right) =$$

$$= 0.46638$$

1Q. 2

$$C = \begin{bmatrix} 2 & 1 + 1/e \\ 1 - 1/e & 1/2 \end{bmatrix}, \quad g = \begin{bmatrix} 2e \\ e-1 \end{bmatrix}$$

a) Critério de Sassenfeld

$$\beta_i = \frac{1}{|a_{ii}|}\left(\sum_{j=1}^{i-1} |a_{ij}|\beta_j + \sum_{j=i+1}^{n} |a_{ij}|\right) < 1$$

$$\beta_1 = \frac{1}{a_{11}}\left(|a_{12}|\right) = \frac{1}{2}\left(1 + \frac{1}{e}\right) = \frac{e+1}{2e} < 1$$

$$\beta_2 = \frac{1}{|a_{22}|}\left(|a_{21}|\beta_1\right) = \cancel{2}\cdot\left(\left|1 - \frac{1}{e}\right|\right)\cdot\frac{(e+1)}{\cancel{2}e} =$$

$$= \frac{(e-1)}{e^2}\cdot(e+1) = \frac{e^2-1}{e^2} = 1 - \frac{1}{e^2} < 1$$

Critério das Linhas

$$\alpha_i = \frac{1}{|a_{ii}|}\left(\sum_{j \neq i} |a_{ij}|\right)$$

$$\alpha_1 = \frac{1}{2}\cdot\left(1 + \frac{1}{e}\right) = \frac{e+1}{2e} < 1$$

$$\alpha_2 = \frac{1}{1/2}\left(1 - \frac{1}{e}\right) = 2\cdot\left(\frac{e-1}{e}\right) = 2 - \frac{2}{e} > 1$$

b) O método de Gauss-Seidel converge p/ qualquer $\bar{X}^{(0)}$

Nada pode-se afirmar sobre o método de Gauss-Jacobi p/ um $\bar{X}^{(0)}$

c) $X_1^{(K)} = \left(b_1 - \left(a_{12} x_2^{(K-1)}\right)\right) / a_{11}$

$X_2^{(K)} = \left(b_2 - \left(a_{21} X^{(K)}\right)\right) / a_{22}$ P/ $K = 1, 2 \ldots$

d)

$\rightarrow X_1^{(1)} = \left[b_1 - \left(a_{12} X_2^{(0)}\right)\right] / a_{11}$

$\hookrightarrow X_1^{(1)} = \left[2e - \left(\left(1 + 1/e\right)(0)\right)\right] / 2 = e$ //

$\rightarrow X_2^{(1)} = \left[b_2 - \left(a_{21} X_1^{(1)}\right)\right] / a_{22}$

$\hookrightarrow X_2^{(1)} = \left[(e-1) - \left(\left(1 - 1/e\right)(e)\right)\right] / 1/2 = 0$ //

$\bar{X}^{(1)} = \begin{bmatrix} e \\ 0 \end{bmatrix} \longrightarrow \|\bar{X}^{(1)} - \bar{X}^{(0)}\|_\infty = e > 0.1$

$\rightarrow X_1^{(2)} = \left[2e - \left(\left(1 + 1/e\right)(0)\right)\right] / 2 = e$ Sol: $X^{(2)} = \begin{bmatrix} e \\ 0 \end{bmatrix}$ //

$\rightarrow X_2^{(2)} = \left[(e-1) - \left(\left(1 - 1/e\right)(e)\right)\right] / 1/2 = 0$ //

$X_2^{(2)} = \begin{bmatrix} e \\ 0 \end{bmatrix} \longrightarrow \|\bar{X}^{(2)} - \bar{X}^{(1)}\| = 0 \leq 0.1$ Para!

Q.3

$$f(x) = x^3 - 9x + 3$$

a) Método de Newton: $\varphi(x) = x - \frac{f(x)}{f'(x)}$

$$\begin{cases} f(x) = x^3 - 9x + 3 \\ f'(x) = 3x^2 - 9 \end{cases}$$

logo, $\varphi(x) = x - \frac{x^3 - 9x + 3}{3x^2 - 9} =$

$$= \frac{(3x^3 - 9x) - (x^3 - 9x + 3)}{3x^2 - 9} =$$

$$= \frac{2x^3 - 3}{3x^2 - 9} \quad //$$

Do gráfico, temos uma raiz entre (2, 4)

Chute: $x^{(0)} = 3$

$$x^{(1)} = \varphi(x^{(0)}) = \frac{2 \cdot 3^3 - 3}{3 \cdot 3^2 - 9} = \frac{51}{18} = 2.8333$$

$$x^{(2)} = \varphi(x^{(1)}) = 2.8171$$

$$x^{(3)} = \varphi(x^{(2)}) = 2.8169 \quad \text{PARA}$$

Note que $|x^{(3)} - x^{(2)}| \leq 0.01$

$$\xi = x^{(3)} = 2.8169 \;!$$

b)

$$|\varphi'(x)| \leq M < 1 \rightarrow$$ Critério de convergência do ponto fixo

se $\varphi(x) = \dfrac{x^3+3}{9}$

$\varphi'(x) = \dfrac{x^2}{3}$

Assim, se $x \in (\overset{a}{\overset{\shortparallel}{-\sqrt{3}}}, \overset{b}{\overset{\shortparallel}{\sqrt{3}}})$,

$$|\varphi'(x)| \leq M < 1$$

Q.4

$$\begin{cases} x^2 - y^2 = 0 \\ 2xy = 1 \end{cases}$$

a) $F(x) = \begin{bmatrix} x^2 - y^2 \\ 2xy - 1 \end{bmatrix}$

$J(x) = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$

P/ $x^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- $J(x^{(0)})\, s^{(0)} = -F(x^{(0)}) \to \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} s_1^{(0)} \\ s_2^{(0)} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$s^{(0)} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \to x^{(1)} = x^{(0)} + s^{(0)} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

- $J(x^{(1)})\, s^{(1)} = -F(x^{(1)}) \to \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$

$s^{(1)} = \begin{bmatrix} 1/4 \\ 1/4 \end{bmatrix} \longrightarrow x^{(2)} = x^{(1)} + s^{(1)} = \begin{bmatrix} 3/4 \\ 3/4 \end{bmatrix}$

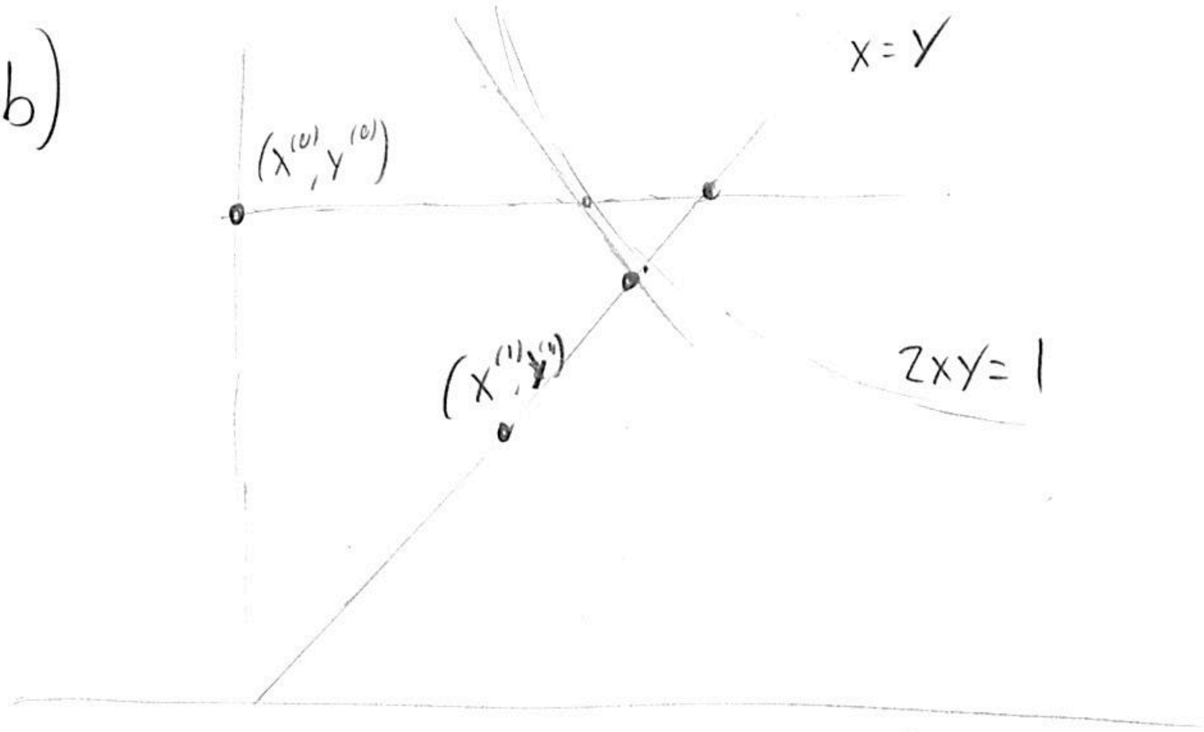
$$J(x^{(2)})\, s^{(2)} = -F(x^{(2)})$$

$$\begin{bmatrix} 3/2 & -3/2 \\ 3/2 & 3/2 \end{bmatrix} \begin{bmatrix} s_1^{(2)} \\ s_2^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ -1/8 \end{bmatrix}$$

$$s^{(2)} = \begin{bmatrix} -1/24 \\ -1/24 \end{bmatrix} \longrightarrow x^{(3)} = x^{(2)} + s^{(2)} = \begin{bmatrix} 17/24 \\ 17/24 \end{bmatrix}$$

PARA!

b)



Calculam-se as tangentes de cada curva e analisamos sua interseção.