TÉCNICO LISBOA

Instituto Superior Técnico

Bachelor's Degree in Aerospace Engineering

Distributed Predictive Control and Estimation

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Laboratory Work - 2nd Report

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The group of students identified above guarantees that the text of this report and all the software and results delivered were entirely carried out by the elements of the group, with a significant participation of all of them, and that no part of the work or the software and results presented was obtained from other people or sources.

1 Control of the nonlinear inverted pendulum with MPC

As a way to illustrate the uses of MPC in Engineering, the aim is to control and stabilise an inverted pendulum, which constitutes a reasonable approximation for a real-life problem: the landing of a re-usable rocket or rocket booster.

In order to simulate the control of the nonlinear inverted pendulum with MPC, the *Simulink* model used for the previous exercise was updated with the plant model of the inverted pendulum provided in the lab handout.

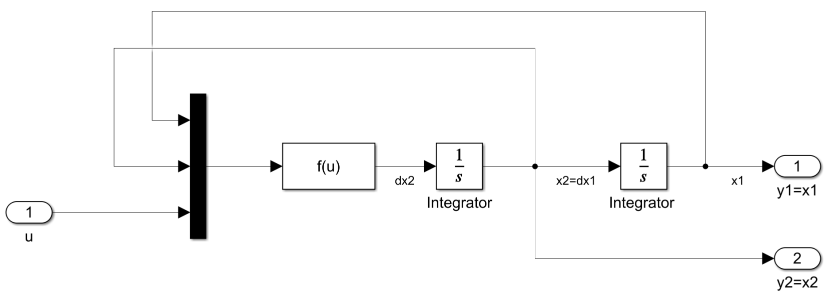

Figure 1: Block diagram which simulates an inverted pendulum.

The model was then redefined in discrete time using MATLAB function `c2d()`. This was necessary since the MPC is based on a linearised, discrete time model, and the simulation tests are made with a nonlinear, continuous time model.

The aim of this controller is, at first, to drive the measured angle θ to 0. After some generalisation, it should allow to act on the torque in such a way that the angle tracks a reference of small amplitude. Note that the angle is measured from the vertical position, i.e., the initial aim is to keep the pendulum in a vertical position.

To test the controller, the pendulum was set with an initial angular position of $\theta = 30°$, and a reference of $\theta_{ref} = 0°$ was given.

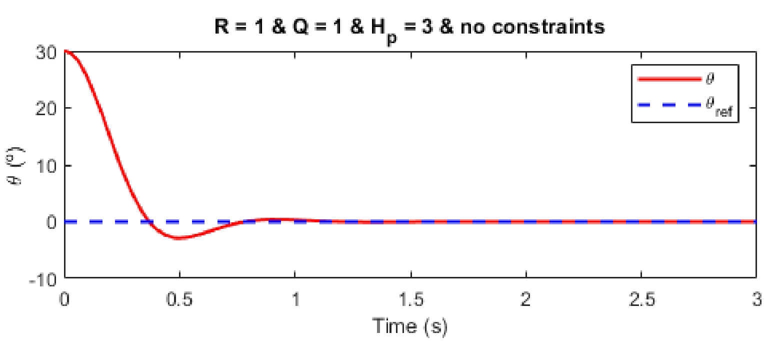

Figure 2: Controller response without constraints.

As seen in figure 2 the controller is able to drive the pendulum to the reference position in approximately 1 second, with a small overshoot.

2 Configuration of the MPC parameters

In this section the different possible configurations of the MPC controller are analysed. Firstly, Q is taken as the identity matrix, while the saturations are too large to ever have an effect. As such, the initial study regards only the effect of the weight matrix R and the horizon H_p on the response.

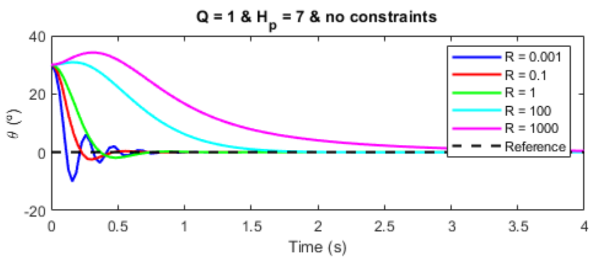

Figure 3: Effect of R on the response.

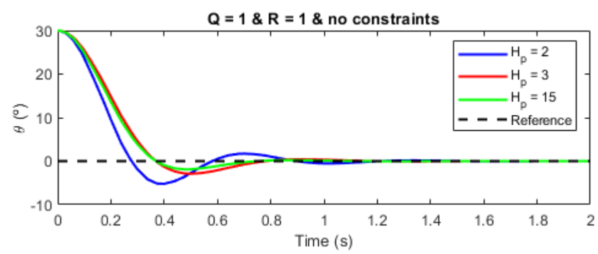

Figure 4: Effect of H_p on the response.

From figures 3 and 4, one can see that R has significant influence in the overshoot and settling time. The settling time increases with R, which is the opposite of what happens with the overshoot, although for large R there is no overshoot and instead the angle increases before going to 0. There is no noticeable effect of H_p on the settling time and it has a smaller effect on the overshoot than R.

Next, multiple different configurations of R and H_p were tested for overshoot and settling time, T_s, through the MATLAB function `stepinfo()`, as shown below, with the goal of obtaining the combination of parameters to be used in the complete controller. Note that the overshoot plots do not actually plot the overshoot, instead they plot the *settling max*, θ_s, which is equal to the overshoot multiplied by the initial position θ_0.

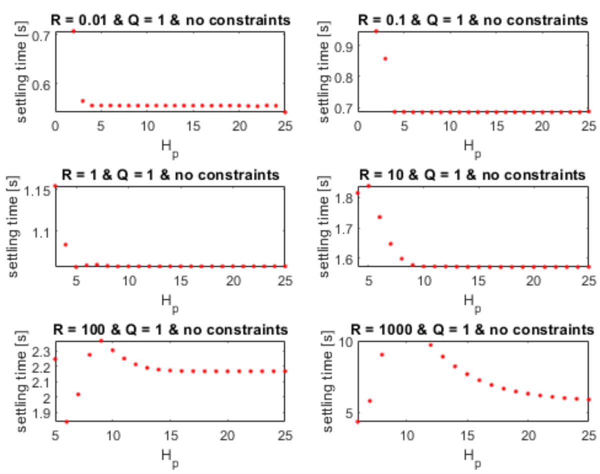

Figure 5: Effect of R and H_p on the settling time.

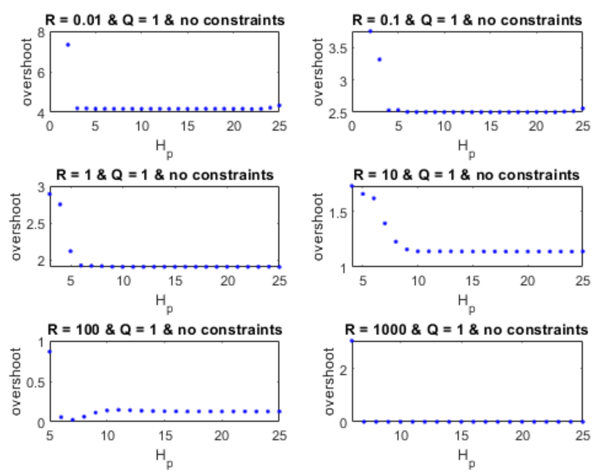

Figure 6: Effect of R and H_p on the overshoot.

These plots show the need of a trade-off between a fast settling time and a small overshoot. As valuable as these plots are, in order to get a complete picture of this trade-off, a *figure of merit* (FoM) was defined according to the formula $\text{FoM} = 10/(2T_s + \theta_s)$.

This FoM assigns a value to a certain configuration that takes into account both the slightly more important settling time and the overshoot. A better controller will have a larger FoM, while a worse controller will have a smaller value of this *figure of merit*.

Though it could somewhat be seen in figures 5 and 6 that the "sweet spot" was located somewhere in the ballpark of $R = 0,1$ and $H_p = 10$, the plots shown in figure 7 show a much clearer picture. Thus, these two values were chosen.

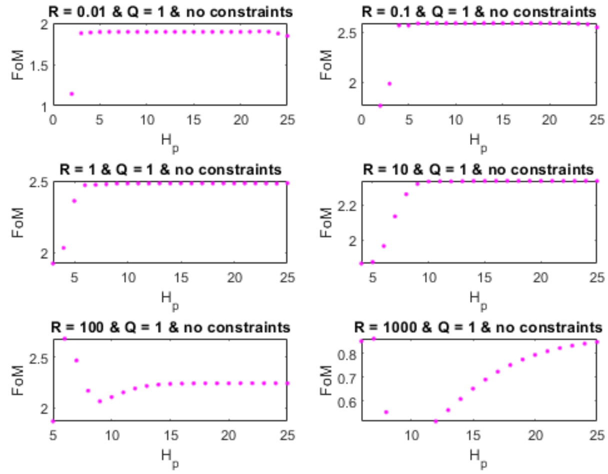

Figure 7: Figure of merit plots.

The final step in this section was to show the response with the selected combination of R and H_p with two different objectives: one to drive the pendulum to zero, the other starting from zero and guiding it to a specific angle.

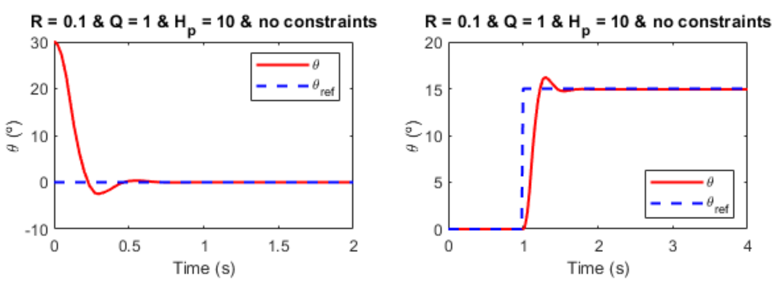

Figure 8: Response with the chosen values and without constraints

3 Limitations imposed by hard-limiting the control action

In this section, the difference between the limitations imposed by hard-limiting the control action when MPC or LQ control is used are analysed and discussed.

Through the study of the MPC parameters in the previous section, the best combination of values was chosen. Constraints were now added in order to understand the impact that they have on the control action of each controller.

In figure 9, the increments of the control input variable are varied and, as expected, the bigger the increments allowed, the faster the system's response. The overshoot will also decrease since the system is able to react more quickly. An initial increase in the value of θ is seen for smaller increments as the controllers struggle to reach the necessary actuation to drive the angle to 0. In figure 10, the control input variable is varied. Here, only the settling time is affected, which increases when harder constraints are applied on the maximum torque allowed.

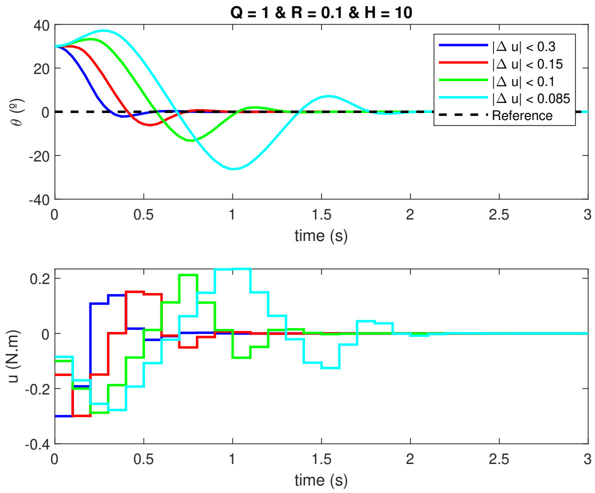

Figure 9: Effect of constraints on the control input variable increments.

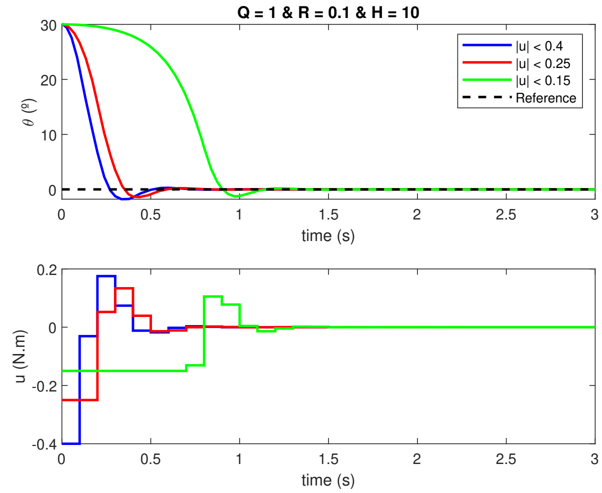

Figure 10: Effect of constraints on the control input variable.

As seen in figures 11 and 12, the MPC controller is substantially faster at reaching the reference than the LQ controller, which also leads to it having a small overshoot that is not present in the LQ controller's response. Additionally, both controllers react the same way to variations in the control input variable constraints.

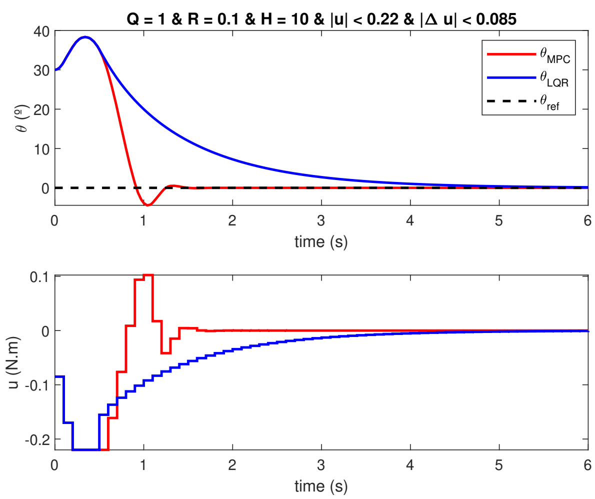

Figure 11: Comparison between MPC and LQ controllers with hard-limited constraints.

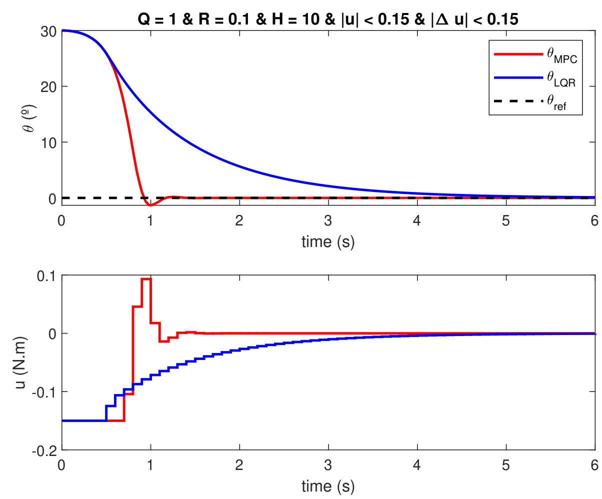

Figure 12: Comparison between MPC and LQ controllers with hard-limited constraints.

These plots use a different value of R than computed in the previous section for paedagogical purposes. Comparing figures 11 and 13 one notices that for higher values of R the response of the LQR controller becomes much faster, which is the opposite of what happens with the MPC controller's response, leading to an almost identical response from both controllers. In addition, the MPC controller's response no longer has an overshoot, which is a consequence of a slower response due to actuation being discouraged with the increase of R.

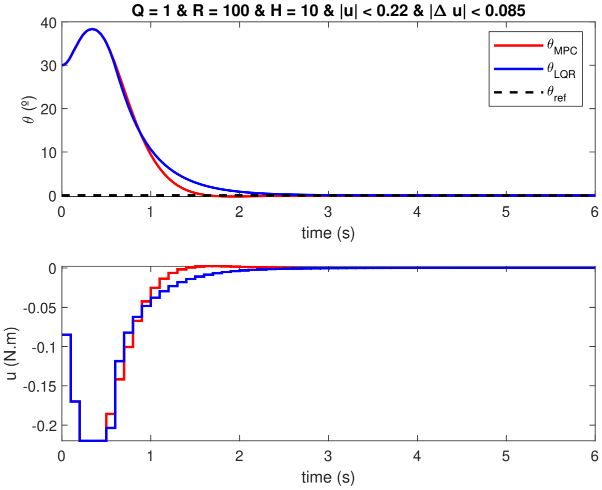

Figure 13: Comparison between MPC and LQ controllers with hard-limited constraints.

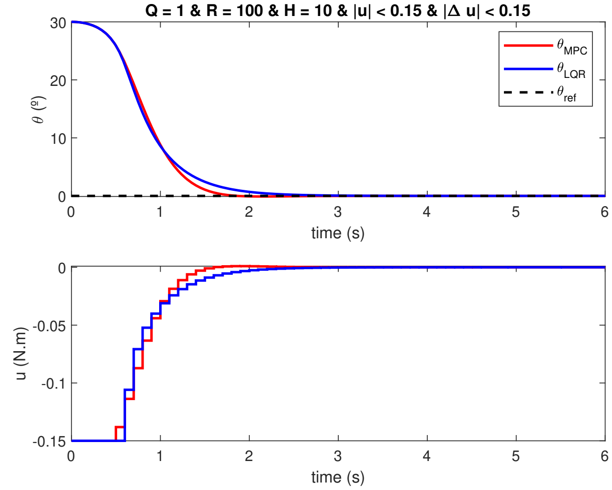

Figure 14: Comparison between MPC and LQ controllers with hard-limited constraints.

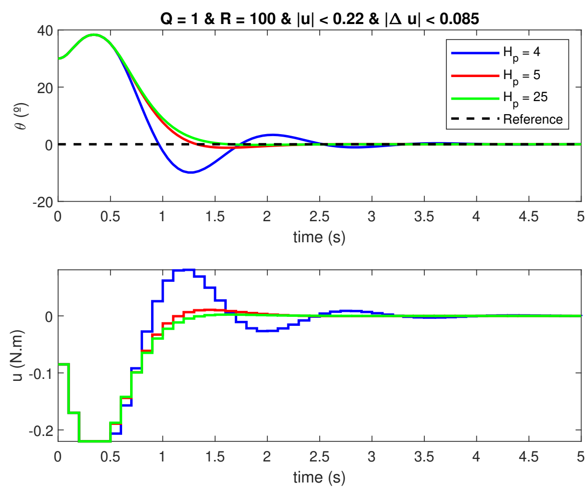

Figure 15: Effect of changing the horizon on the MPC controller's response

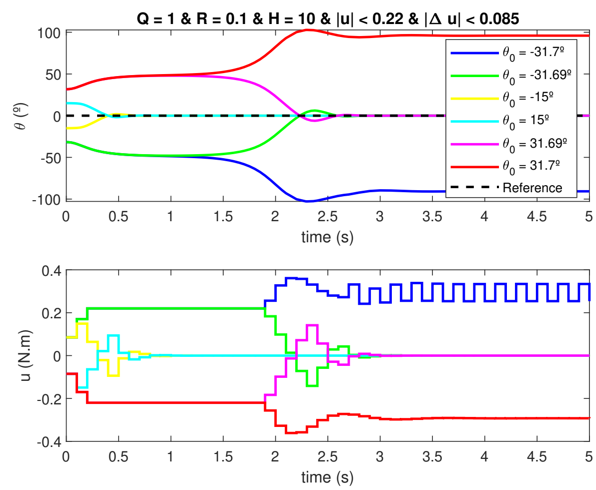

Figure 16: Effect of changing the initial condition on the MPC controller's response

In figure 15, the effect of the horizon on the hard-limited MPC controller's response is represented. A clear change is noticeable by increasing the horizon from 4 to 5 time-steps, as the response becomes much faster, having almost no overshoot. However, increasing the horizon more than that has little to no effect on the response, leading only to a higher need for computational power while introducing no additional benefits.

Next, a quick study of the attraction basin of the reference $\theta_{ref} = 0°$ with the constraints and parameters shown in table 1 is depicted in figure 16. Through an iterative process, the boundary of the attraction basin of θ_{ref} was found to be $\theta_0 = \pm 31,69°$. As expected, the bigger the amplitude of θ_0, the longer it takes for the controller to drive the pendulum to 0.

An interesting result comes from setting $\theta_0 = \pm 31,7°$, as not only is the response asymmetric but the MPC controller no longer follows the constraints imposed on the control input variable. This is presumably a limitation of this controller, that comes from using initial conditions that lay in the boundary of the attraction basin of the unstable equilibrium point $\theta = 0°$.

Table 1: Hard-limiting MPC parameters.

Q	R	H	$\lvert u\rvert_{max}$	$\lvert\Delta u\rvert_{max}$
1	0,1	10	0,22	0,085

4 Effect of disturbances

The final section of this work attempts to simulate the effect of disturbances in the system, most notably that of a wind gust, which would best be translated as a sudden increase in torque in a small interval of time.

For the modelling of disturbances, the MATLAB function `gauspuls()` provides a Gaussian-modulated sinusoidal RF pulse that can be tailored in terms of width and amplitude. Two of these pulses were applied at different times, one while the controller was still driving the inverted pendulum to its final position, and the other after the pendulum was already at its intended position.

Figure 17 shows the response in both situations considered, as well as the modelled disturbance itself. Comparing with the response without disturbance, the disturbance forces the controller to counteract it. However, it is quickly mitigated and the system returns to its unperturbed state with ease.

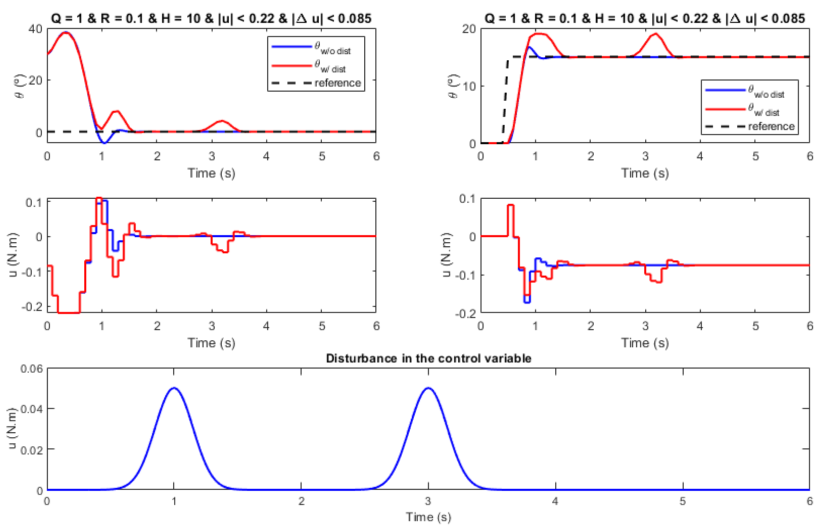

Figure 17: Effect of disturbances.

5 Conclusion

With this work the differences between MPC and LQ control were clearly established leading to a better understanding of the applications and implications of each one and how to manipulate their parameters for optimal results. Furthermore, through the use of the inverted pendulum example, a sight of how they may be used in a real world was given, leaving an appropriate base for similar problems that might come in the future.