PRESENTATION OF PROJECT

math for classmates

About project
Preview of coursebook
Preview of workbook

You could see us in media

Česká televize

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LIDOVÉ NOVINY

ČT 24

DRTINOVÁ VESELOVSKÝ TV

math
for classmates

PRESENTATION OF PROJECT

comprehensibility

expertise

preparations for graduation exam

modern graphics

Now you are holding preview of coursebooks in your hands, which are created in association with teachers and students from high schools.

We would like to introduce you the project math for classmates in several steps

We will start short and clear

Within the frame of collaboration of **high school teachers and students**, we have written our own coursebooks of mathematics. With their help we want to improve students‘ relationship to mathematics, to give another way of their perception and approach to it. Thanks to this we want to get rid of fear, sometimes even aversion from mathematics as such.

Now in details

Our coursebooks will prepare students for the graduation exam.

The project was made during our mutual study at **Gymnasium of J. K. Tyl** in Hradec Králové. We wrote the coursebooks in student language, thanks to collaborations with teachers expertise was maintained and we created graphics to fit actual trends.

Coursebooks went through long-term testing on several high schools and hundreds of students had them in hands. Thanks to their feedback and mainly suggestions from the side of the teachers, the coursebooks have earned their final form.

In addition, they are enriched by supplementary workbooks, which will help students in practising ongoing topic in use. We believe, that we finally created coursebooks with which it will be pleasure to teach.

Chart of comparison of our coursebooks and recent coursebooks on the market.

	math for classmates	current coursebook
Teachers' guarantee	✓	✓
Word problems from our practice	✓	✗
Collaboration of high school teachers and students	✓	✗
Prime examples from graduation exam	✓	✗
Summary of primary school curriculum	✓	✗
Well arranged graphics	✓	✗

We will answer you directly

Questions

We have prepared some questions and answers for you, which we consider very important and maybe they already popped up in your head. Everything in style of the coursebook.

Will the coursebooks be used in education?

Our coursebooks and workbooks cover all phases of the education.

In coursebook there is introduction, theory and solved problems. In the workbook the student will practice the topic, do homework and will revise the coursebook, the summary and final practice during self-study.

Will you prepare the students for the graduation exam?

The coursebooks are designed in a manner to prepare every high school student for the graduation exam from mathematics.

In addition, workbooks contain selected problems from past exams.

Do we need the coursebooks and also the workbooks?

The coursebooks and the workbooks bond together into a complex supplementary study material.

The workbooks are based on the coursebooks and contain a lot of references to them.

Syllabus of which schools do the coursebooks fulfil?

We are covering whole high school curriculum of mathematics according to general education program. Topics and problems within the subheads are sorted ascending according to difficulty.

Therefore everyone has an option to choose what fits his needs.

How did you bring the coursebooks closer to the students?

The coursebooks were and are written by students and corrected by teachers.

Thanks to this, the approach is student-friendly, we are approaching them like classmates.

We use student language, we write in second person.

What kind of tactics are you using to explain the topic?

The coursebooks contain a large number of prime examples which are solved progressively and are commented step by step.

Everything is complemented for a pleasing graphic elements.

Guarantee and opinions of teachers

We have prepared the opinions of high school teachers for you, which we asked to evaluate our materials.

Mgr. Miroslav Novák

Gymnasium of J. K. Tyl, Hradec Králové

“The project is absolutely unique for offering the students detailed and comprehensively described process of solution of each problem, namely in two, colour distinguished levels. First level contains mathematical notation of progress, on the second level it is possible to find very detailed description of every modification from level one, in addition, it is commented in student language. Both levels are overlapping, so it’s up to every reader how deep he wants to go.”

Mgr. Jaroslav Kerner

deputy director, SPŠ, SOŠ and SOU, Hradec Králové

“The coursebook math for classmates is complementing offer of coursebooks for students of high schools with interest in mathematics in an appropriate way. It is made in high-quality, modern and interesting way and could be very good aid in preparation of pupils for graduation exam from mathematics.“

RNDr. Jaroslav Šolc

director of Lepař Gymnasium in Jičín

“The coursebook is well arranged, segmented in an appropriate way, explanations of problems are comprehensible. Adding more problems in form of the workbook is ideal.“

PhDr. Ivo Králíček

Bishop gymnasium of Bohuslav Balbín, Hradec Králové

“New coursebooks of mathematics are offering high school knowledge of this department in a unique way. I see that great advantage of this coursebooks is, that it is written in collaboration with actual students for their classmates.“

Mgr. Jaroslava Mohelníková

High school of services, business and gastronomy, Hradec Králové

“We are not dealing with ordinary coursebook with briefly described process of problem solving, which pupils often don’t understand. The solution is always complemented by detailed description in language, which is easily comprehensible for pupils. I also positively evaluate the graphic appeal of coursebook, which greatly contributes to clarity of this whole book.“

Mgr. Leoš Bílek

High school of services, business and gastronomy, Hradec Králové

“These coursebooks are absolutely different from all the other coursebooks on our market. They are not sapiency books souped-up with lots of definitions, but they are trying to explain mathematical phenomenons with own student language. Another one of their goals is to clarify to the reader, what mathematics is for in real life.”

Our sets of educational materials

Detailed tables of contents of coursebooks can be found at
www.ucebnicematiky.cz/obsahy

1

coursebook and workbook

Review from elementary school / Basic knowledge

coursebook and workbook

Equations and inequations

2

September 2016

coursebook and workbook

Planimetry

coursebook and workbook

Functions

coursebook and workbook

Goniometry

3

April 2017

coursebook and workbook

Stereometry

coursebook and workbook

Analytic geometry

coursebook and workbook

Sequences and series

4

September 2017

coursebook and workbook

Differential and integral calculus

coursebook and workbook

Combinatorics
Probability
Statistics

coursebook and workbook

Complex numbers
Revision from high school

Intervals

Basic knowledge | Theory of sets

What will I want from you today?

In this subhead I will teach you to work with intervals, note them down properly and graph them on real number line. You will know, what types of intervals do exist and what is the difference between them. In the end, I will show you how to make an intersection and a union of two intervals, because this skill is really very important, so don't underestimate this topic.

What will you once need it for?

You can find the answer to this question in the end of this subhead, where I will tell you, in which particular cases in life you will meet with intervals, everything comes to him who waits.

How it will be related in mathematics?

Intervals are used really almost everywhere and so their knowledge is necessary. You will meet with them for example in determining conditions for variable in Equations and inequations or in solving a system of linear inequations.

What is an interval?

Definition says, that interval is a subset of set of all real numbers, which is **limited** from both sides by two endpoints (endpoint can be even the infinity).

Interval is therefore a set of real numbers, which are **higher** (or equal) to given number (or minus infinity) and in the same time **lower** (or equal) to **another** number (or plus infinity), for instance higher than 5 and lower or equal to 17, in math language as (5; 17]. First of all don't forget that interval exists only in real numbers.

There are two types of intervals:

a) **finite interval** – the type of interval, which is **limited from both sides** by given numbers (not by symbol of infinity), e.g. (−3; 2].

These intervals are further distinguished into interval closed, right-closed, left-closed and open. I will tell you more detailed information in a moment.

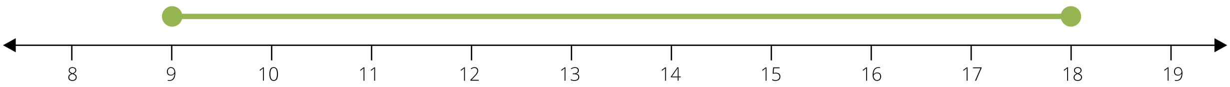
b) **Infinite interval**

Infinite interval is that kind of interval, which is limited by exact value **at most** (maximally) from **one side**, e.g. (−∞; 1) or (−∞; ∞).

Even this type of interval is further divided, that is interval left-bounded and left closed, interval left-bounded and left-open, interval right-bounded and right-closed, interval interval right-bounded and right-open and interval unbounded (this is used rarely). Although there are lots of them, there is nothing difficult about it. The chart, which you will find on the page 92, where are these types of intervals shown, will tell you more.

What is interval good for?

If you need to delineate **some section of real numbers**, then you'll use interval. For example, you want to say that opening hours of store "Coolshop" are from nine (included) until eighteen (included) o'clock. With interval you would note down such fact as 9;18. For example, number 19 o'clock **is not in** this interval, therefore you cannot buy anything in this shop, because it's **closed**. While number 12,5 (i.e. half past twelve) is included in interval, so you can shop.



A common problem are brackets, so whether if there will be square, i.e. „[" or „]" or round, i.e. „(" or „)". But I will tell you more about this on page 95, where I will show it to you on examples from real life.

How to note down an interval?

You can note down an interval in three ways. Most often you will meet with the first one, it's a typical notation of interval, where two numbers are are in brackets divided by semicolon (sometimes only comma is used, which may be misleading and may be confused with decimal number, so we won't use it in this book).

The first type of notation looks like it has got two endpoints in brackets. Point which is on the left side must be lower than the one on the right side. For example interval (1; 5) is written correctly, but interval (5; 1) is **wrong**.

Of course the brackets may vary, so you can note down four types of intervals, and those are $(a; b)$, $[a; b)$, $(a; b]$, or $[a; b]$. It depends, if you want the endpoint to be part of the interval. If the point is part of the interval, you will use square brackets, i.e. "[" or "]". If the point is not part of the interval, you will use round bracket, thus "(" or ")". I will tell you more about brackets later at finite intervals.

$$(a;\ b)$$

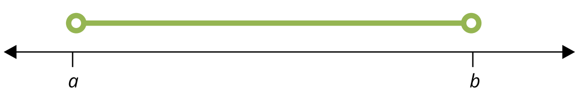
Endpoint (so called lower limit)   Endpoint (so called upper limit)

The second type of notation, which is called a set-builder notation, is used primarily for notation of sets. It works in the way that initially you will write the lower endpoint, and then sign of inequality (either "≤" → endpoint belongs to the interval, or "<" → does not belong to the interval). Later you will note down the variable (it's up to you how you name it, usually it's x) and again you will write the sign of inequality (symbol "≤" or "<", it depends if endpoint belongs or not to the interval). Finally you will write the value of the higher endpoint. Notation $1 < x < 5$ can be read as "variable x is higher than one and at the same time lower than five."

$$a < x < b;\ x \in \mathbb{R}$$

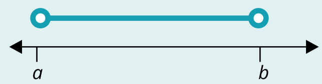
Endpoint (so called lower limit)   variable (name is arbitrary)   Endpoint (so called upper limit)

Third type is graphing on real number line. This way is suitable especially in the moments, when you need to imagine more intervals at once, so you can for instance see, which part they have in common. I will tell you more about graphing under subhead Finite intervals in more detail.



Preview of finite intervals

In the next chart (where a, b are real numbers applies, that $a < b$) there is a preview of finite intervals. If any of these types is not clear or you don't know how to graph the interval on the real number line right or write it with the set-builder notation, take a look at the next lines under the chart, where I will explain it thoroughly. In case it is clear, you can leaf through to the page 92, where is the preview of finite intervals.

Type	Interval notation	Set-builder notation	Graphing on line
Closed interval	$[a; b]$	$a \le x \le b;\ x \in \mathbb{R}$	
Right-closed interval	$(a; b]$	$a < x \le b;\ x \in \mathbb{R}$	
Left-closed interval	$[a; b)$	$a \le x < b;\ x \in \mathbb{R}$	
Open interval	$(a; b)$	$a < x < b;\ x \in \mathbb{R}$	

Finite intervals in more detail

Finite intervals are further divided according to **where they are closed**.

a) **closed interval**

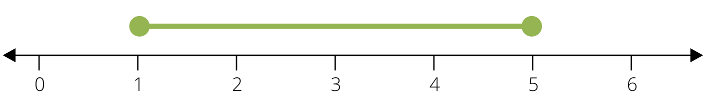
Closed interval is that kind of interval, which **is closed on both sides** with given values, which belong to the interval. It means that there are **square** brackets on both sides of the interval, e.g. [1; 5].

$$[\underbrace{1}_{\text{Endpoint (so called lower limit)}};\ \underbrace{5}_{\text{Endpoint (so called upper limit)}}]$$

$$\underbrace{1}_{\text{Endpoint (so called lower limit)}} \le \underbrace{x}_{\text{variable (name is arbitrary)}} \le \underbrace{5}_{\text{Endpoint (so called upper limit)}};\ x \in \mathbb{R}$$

You will note down the interval [1; 5] using the set-builder notation as $1 \le x \le 5$ (read: "number x is higher or equal to one, and at the same time lower or equal to five"). In other types of intervals you should focus on changing sign of inequality (i.e. "<" and "≤") while using the set-builder notation.

You can graph this type of interval on real number line. You will graph both endpoints on line with **filled-in circle**, because they belong to interval, what is indicated by square bracket → [1; 5]. If the points wouldn't belong there, which means there would be round bracket at the endpoints, then it would be graphed by open circles on real number line, but we will get to it later.

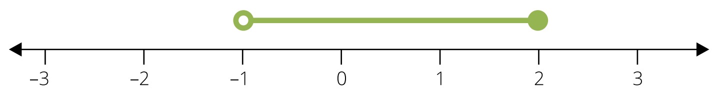


b) **right-closed interval**

Right-closed interval (and left-open interval, depends on how you look at it) is that kind of interval, which is **closed** on the right side (square bracket indicates, that the endpoint **belongs** to the interval) and **open** on the left side (round bracket indicates, that the endpoint **doesn't belong** to the interval), e.g. interval (-1; 2].

You will note down the interval (-1; 2] using set-builder notation as: $-1 < x \leq 2$ (read: "number x is higher than minus one, and at the same time lower or equal to two"). As you can see, if the bracket at the endpoint is round, you will use sign "<". If there is square bracket in the assignment, you will use sign „≤".

This interval is noted in the way, that over the value of endpoint with square bracket you will draw a filled-in circle on the real number line. Over the endpoint with round bracket in assignment of interval will be an open circle. Real number line below will tell you more.



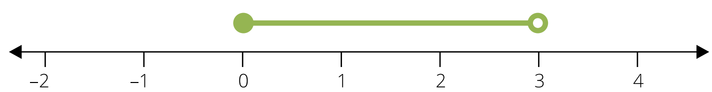
Square bracket in assignment of the interval and filled-in circle on the real number line indicates, that this point belongs to the interval. Whereas round bracket in the assignment of the interval and open circle on the real number line says, that this point doesn't belong to the interval, the real number line above will tell you more.

c) **left-closed interval**

Left-closed interval (also right-open interval) is that kind of interval, which is closed on the left side (square bracket) and open on the right side (round bracket), e.g. interval [0; 3).

You will note down the interval [0; 3) using set-builder notation as: $0 \leq x < 3$. (read: "number x is higher or equal to zero, and at the same time lower than three"). There, where is round bracket, you will put "<" and where is square bracket, you will write "≤".

You can of course graph this type of interval on the real number line. Endpoint limiting interval from the left belongs to the interval (therefore it's marked on the line with filled-in circle), while endpoint, which is limiting interval from the right, doesn't belong to the interval (therefore it's marked on the line with open circle).

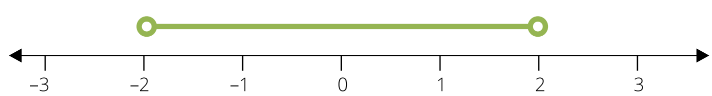


d) **open interval**

Open interval is that kind of interval, which is **opened** on the both sides (has got only round brackets), e.g. (-2; 2).

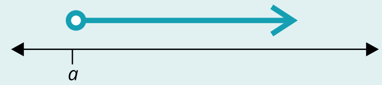
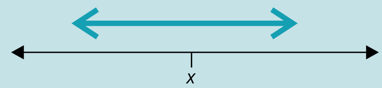
You will note down the interval (-2; 2) using set-builder notation as $-2 < x < 2$ (read: "number x is higher than minus two, and at the same time lower than two").

You will graph this type of interval on the real number line in the way, that at the both endpoints will be an **open circle**, because they don't belong to the interval, there are round brackets in the interval assignment.



Summary of the infinite intervals

As I said in the beginning of the chapter, even **infinite** intervals can be further divided. In the chart below (where a is a real number) you can find summary of these infinite intervals. Rules for graphing, for instance, that round bracket is noted by an open circle or that square bracket is filled-in circle, apply here too.

Type	Interval notation	Set-builder notation	Graphing on line
Interval left-bounded and left closed	$[a; \infty)$	$x \geq a; x \in \mathbb{R}$	
Interval left-bounded and left-open	$(a; \infty)$	$x > a; x \in \mathbb{R}$	
Interval right-bounded and right-closed	$(-\infty; a]$	$x \leq a;\ x \in \mathbb{R}$	
Interval right-bounded and right-open	$(-\infty; a)$	$x < a;\ x \in \mathbb{R}$	
Interval unbounded	$(-\infty; \infty)$	$-\infty < x < \infty;\ x \in \mathbb{R}$	

Exercise 1

Interpret the notation $K = \{x \in \mathbb{R}; 1 \leq x < 6\}$ by roster and after that by interval notation.

Process

You have the set K, which is using the set-builder notation for its notation, write by roster and then by interval. So let's do it!

The set K cannot be noted by roster.

You cannot write this kind of notation by roster, because the result is all real numbers (that is determined by sign $\mathbb{R}$) between numbers one and six (says notation $1 \leq x < 6$), and that is **endlessly many numbers** (it can be for example number 1.6; 1.789; 2.86; 3.885469 etc.). And that is why we need some easier notation, therefore the way of notation using interval.

$K = [1; 6)$

In interval notation it is very important to know which brackets to use, either square or round. It depends, if there are signs $<$, $>$, $\leq$ or $\geq$ in the assignment of inequation. At the first two signs you will put a round bracket, which indicates, that given number does not belong to the interval anymore. Two remaining signs have a square bracket indicating, that given number belongs to the interval.

You will write the interval notation for this set as [1; 6). In assignment at the number one, is the sign "≤" , so you will use square bracket, and at the number six will be round bracket, because there is the sign "<". This bounded interval is technically called left-closed.

Wow, intersection of intervals!

If you need to make intersection of two or more intervals, it is best to graph all the intervals on one real number line. From subhead Sets you know, that result of intersection is set (interval) **containing all** of the elements, which both of the sets **share**. This means, that if you want the intersection of two intervals, the result will be the interval with numbers, that belong to the both intervals at the same time.

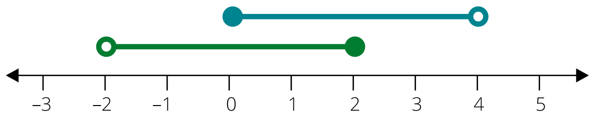
Exercise 2

Determine the intersection of interval $A = (-2; 2]$ and $B = [0; 4)$.

Process

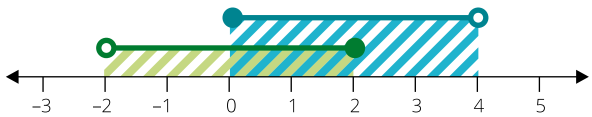
Therefore you will make an intersection of two intervals A and B and you will get a new interval, which will consist of numbers that are in interval A, and at the same time in interval B.

$A = (-2; 2]$ a $B = [0; 4)$



Initially you will graph both of the intervals on one real number line. You will start with the endpoints. If there is round bracket at the endpoint, then on the real number line will be an open circle. If there is square bracket at the endpoint, you will draw a filled-in circle. You will connect the endpoints from one interval with a line.

$A \cap B$



After that you will hatch the intersection of these two intervals, namely, what they have **in common** (= all numbers, which are contained in both intervals at the same time). On the line you see graphed intersection as hatched area (crossed lines). It is that area, where both of the intervals are graphed "below each other". In this case all the numbers from zero (included) to two (included) belong to the intersection. "Included" because points belong to the assigned intervals (there is a square bracket next to them in the interval assignment).

$K = A \cap B = (-2; 2] \cap [0; 4) = [0; 2]$

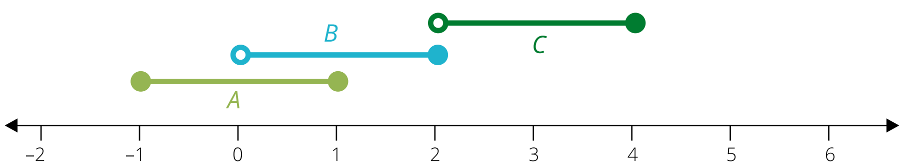
The sign used for intersection is in math language "∩". The result is therefore an interval from zero (included) to two (included).

Exercise 3

Determine the intersection of intervals $A \cap B \cap C$, if $A = [-1; 1]$, $B = (0; 2]$ and $C = (2; 4]$.

Process

You will solve this exercise in a similar way like exercise 2. The result of the intersection is a new interval, which will consist of elements, which are in all three intervals at the same time.



Initially you will graph the endpoints of intervals on the real number line. Filled-in circle will be given to that point, which has square bracket. Open circle represents point, which is at the round bracket. Finally you will connect the points from each interval. Therefore the intersection is that, what all of the points have in common (on the real number line it will be the area, where all of the three lines are below each other). As it is clear from the line above, they have nothing in common (there are at most two "lines" below each other).

$A \cap B \cap C = \emptyset$

For notation of intersection sign "∩" is used and if there is no intersection, sign "∅" is used, which indicates empty set (if you don't know what does it mean, look at the page 69).

Union of intervals

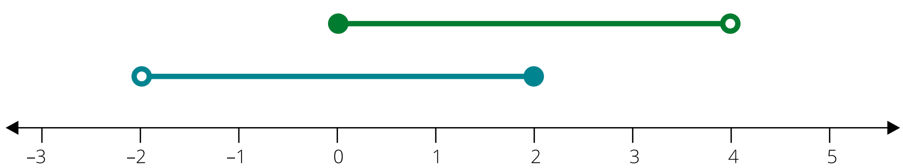
From the subhead Sets you definitely know, that result of intersection is a new **set** (interval) containing all numbers, which are **at least** (minimally) in one set. In two intervals the union will be a new interval, which will contain all numbers, which are at least in one of two intervals.

Exercise 4

Determine the union of intervals $A = (-2; 2]$ and $B = [0; 4)$.

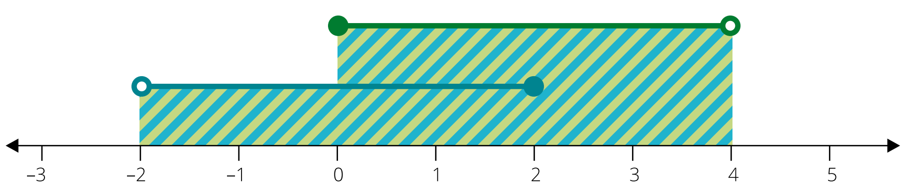
Process

Therefore you will make a union of two intervals *A* and *B*, and you will get a new interval, which will contain numbers, which are at least in interval *A* or *B*.



$A = (-2; 2]$ a $B = [0; 4)$

Initially you will graph both of the intervals on the real number line, or rather their endpoints in the form of filled-in (square bracket) or open (round bracket) circles and you will connect the corresponding circles.



$A \cup B$

After that, you will mark (e.g. hatch) the area, which belongs to at least (minimally) one of the intervals. In this case it is all numbers from minus two (without) to four (without). "Without" because in the assignment of intervals, are round brackets at the numbers -2 and 4, which means that given point doesn't belong to the interval.

$K = A \cup B = (-2; 2] \cup [0; 4) = (-2; 4)$

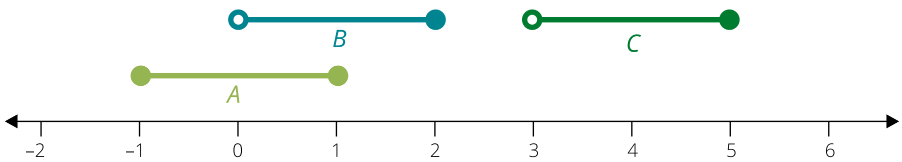
In math language the sign "∪" is used for notation of union. The result of this exercise is therefore interval from minus two (without) to four (without).

Exercise 5

Determine the union of intervals $A = [-1; 1]$, $B = (0; 2]$ a $C = (3; 5]$.

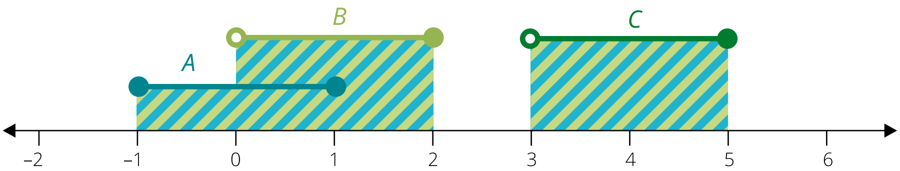
Process

The process will be very similar to the exercise 4. The result of the union will be a new interval, which will contain numbers, which will be in at least one of three intervals.



You will draw the intervals on the real number line, or rather their endpoints. Point at the round bracket has got an open circle and point at the square bracket is graphed as a filled-in circle. In the end you connect drawn points from each interval.

$A \cup B \cup C$



Union is all the numbers, which are at least in one of the intervals, therefore everywhere, where “line” leads on the real number line. The union on the picture above is indicated by hatched area.

$A \cup B \cup C = [-1; 1] \cup (0; 2] \cup (3; 5]$

You can write the result either as all the intervals, which you have to unite and put a sign of union between them, thus „∪“. But mathematically more correct is to discard the matching values, which are in each of the intervals. For example number 1 is contained either in the first and in the second interval, and that is not absolutely correct.

$A \cup B \cup C = [-1; 1] \cup (0; 2] \cup (3; 5] = [-1; 2] \cup (3; 5]$

So the right result is this. You will find it out easily when you look at the real number line and copy directly what you see. Initially you will write point –1, because the first interval starts with it. Then you will continue to the right on the line. You will come across point 0, which has got an open circle. But point 0 is contained in the interval *A* (“line” is crossing it). Thus point zero is a part of it and you won’t note it down. The next point is number 1, which actually belongs to both intervals (to *A*, as well as to *B*), therefore you won’t focus on this point either. In the end, there is point 2, which is situated at the end of the interval, so it is necessary to note it down. It ends the interval. There will be a square bracket next to it, because it has an open circle. Therefore you have first interval, which is [-1;2]. The last interval is just two points, where is nothing to modify, so you will just copy it and you have the final result.

What are intervals good for in real life?

Although you don’t realise it, you are **using intervals every day**. Even your daily routine is one interval. At six in the morning you get up and at twenty two o’clock you are going to sleep. This could be noted as interval (6; 22), while round brackets can be square, because you don’t know if you will get up exactly at six, or fall asleep exactly at twenty two. This leads us to a problem...

In the real life it doesn’t matter, if you get up exactly at six (if you get up at 6:01, nothing will happen, except you’d miss the bus), but in mathematics it has to be determined exactly when the person gets up. It means that round brackets are very important. Look at the next examples, which will almost every mortal encounter with.

“Opening hours are from 9:00 to 18:00.”

It means that you can come in this time interval. Mathematically noted: [9; 18], (9; 18), [9; 18) or (9; 18], depends, if they close and open the shop exactly.

“Children up to age of 15 have entry for free.”

Again it depends if hosts thought, that even children, which are exactly 15 years old have entry for free, thus mathematically noted: (0; 15] “or” (0; 15). Another possibility is, that if someone is 15 years and 5 months old, he is legally still 15, therefore he should have discount too. Then it would be mathematically noted like this: (0; 16).

“Weight limit of platform is 1 000 kg.”

Will the platform carry even 1000 kg, or at most 999,99... kg. Mathematically it has to be determined exactly, so either [0; 1 000] or [0; 1 000). In real life, for example weight limit of a lift is much higher, than it’s stated, because it is needed to reckon with various physical factors, thus in real life variant [0; 1 000] applies, so don’t worry that lift wouldn’t be able to carry you.

“No thoroughfare for vehicles with height over 3,5m”

In this exercise it **doesn’t matter** on the agreement of people, here it isn’t reckoned with large excess (the sign says 3,5 m, so real height of the bridge is just a little bit higher, at most in the order of centimeters), therefore here should be used interval with square brackets [0; 3,5].

It always matters how people make an agreement, but at least in mathematics, it always must be clearly stated, and that’s why brackets at intervals are so **important**, so please focus on it.

Don’t worry, I will leave you alone soon!

- **Interval** is a set of real numbers, which are higher (or equal) to given number (or minus infinity) and at the same time lower (or equal) to another number (or infinity).

- Types of intervals:

  Finite interval is limited from both sides by given values, e.g. (-3; 2].

  Interval infinite is limited at most from one side, e.g. (2; ∞) or (−∞; ∞).

- **Graphing on real number line:**

  Filled-in circle on real number line or square bracket in the assignment of interval indicates, that the endpoint **is part** of stated interval.

  Open circle on real number line or round bracket in the assignment of interval indicates, that the endpoint **doesn’t belong** to stated interval..

- **By union of intervals** a new interval will be created, which contains numbers, which are at least in one of them.

- **Intersection of intervals** is a new interval, which contains numbers that all intervals have in common.

Practice

48 In the most simple way note down sets:

a) $(2; 6) \cap [4; \infty)$

b) $(2; 6) \cup [4; \infty)$

49 Graph on the real number line and determine an intersection and a union of intervals:

a) $(2; 7)$ a $(5; 9)$ b) $\left(-\frac{8}{7}; 7\right)$ a $\left\langle -4; \frac{11}{2} \right\rangle$

c) $(-\infty; -5)$ a $(-7; 0]$ d) $(-5; -2)$ a $[-1; 3)$

50 On the real number line note down these sets as interval:

a) $A = \{x \in \mathbb{R};\ -7 < x \leq -2\}$ b) $B = \{x \in \mathbb{R};\ 5 \leq x < 12{,}5\}$

c) $C = \{x \in \mathbb{R};\ x > 0\}$ d) $D = \{x \in \mathbb{R};\ x \leq 1\}$

51 You are given intervals $A = [-3; 2]$, $B = (-\infty; 2)$, $C = (0; 10]$. Note down the the following sets using interval:

a) $A \cup B \cup C$ b) $A \cap B \cap C$

c) $(B \cap C) \cup A$ d) $(A \cap B) \cup C$

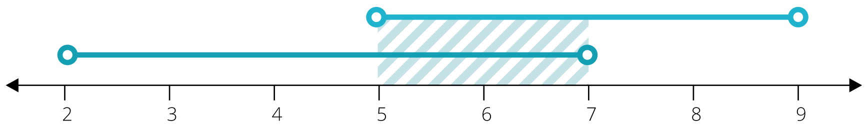
52 On the way from Hradec to Pardubice you are going over three bridges. First has weight limit 15 t, second has weight limit 20 t and third 30 t. Note down with interval (in tons) how heavy the cars can be to use this route.

You can find the process of solution at www.pocitame.si

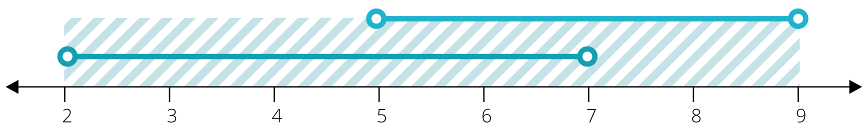
Results

48 a) $[4; 6)$ b) $(2; \infty)$

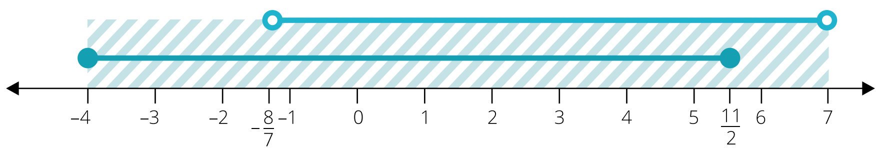
49 a) $(2; 7) \cap (5; 9) =$



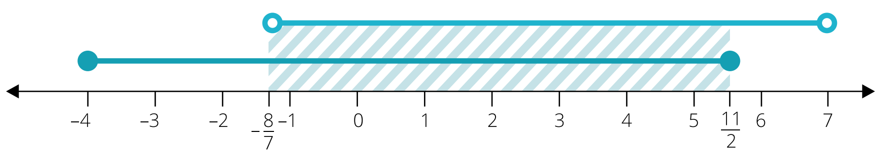
$(2; 7) \cup (5; 9) =$



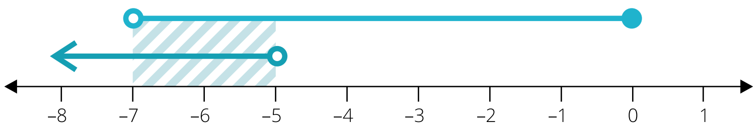
b) $\left(-\frac{8}{7}; 7\right) \cap \left\langle -4; \frac{11}{2} \right\rangle =$



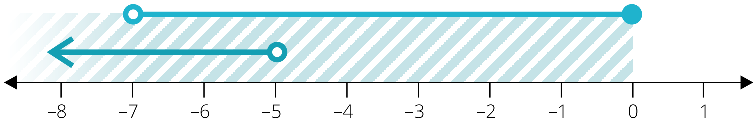
$\left(-\frac{8}{7}; 7\right) \cup \left\langle -4; \frac{11}{2} \right\rangle =$



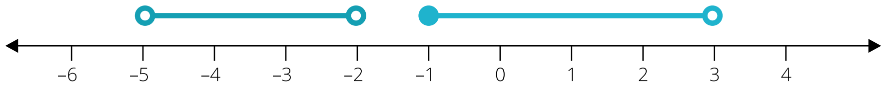
c) (−∞; −5) ∩ (−7; 0] =



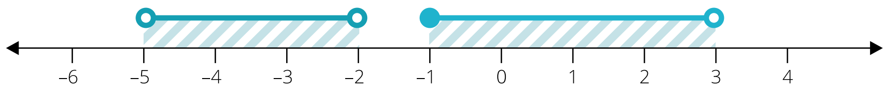
(−∞; −5) ∪ (−7; 0] =



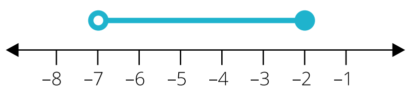
d) (−5; −2) ∩ [−1; 3) =



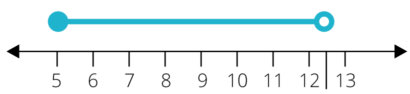
(−5; −2) ∪ [−1; 3) =



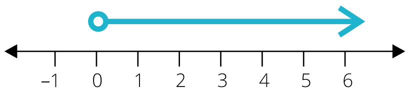
50 a) A = (−7; 2]



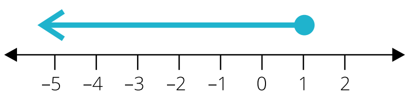
b) B = [5; 8,5)



c) C = (0; ∞)



d) D = (−∞; 1]



51 a) (−∞; 10] b) (0; 2) c) ⟨−3; 2⟩ d) ⟨−3; 10⟩

52 (0; 15]

chapter 2

section 3

Intervals

Basic knowledge > Theory of sets

Excerpts

- **Interval** is a subset of set of all real numbers, which is limited from both sides with two endpoints (endpoint can even be an infinity).
- **The result of intersection** of two intervals is a new set (interval) containing all of the elements, which are common for both intervals.
- **The result of the union** of intervals is a new set (interval) containing all numbers, which are at least (minimally) in one interval.
- **Graphing on real number line:**

  Filled-in circle on the real number line or square bracket in the assignment of the interval indicates, that endpoint belongs to the stated interval.

  Open circle on the real number line or round bracket in the assignment of the interval indicates, that endpoint doesn't belong to the stated interval.

Exercise 1

You can find the process of how to do it in the coursebook on page 93

In the simplest way note down following sets:

a) $A = (-4; 2) \cup (0; 5)$ ★☆☆

b) $B = (-7; -4) \cup (-10; 0] \cap (-2; 2)$ ★★☆

c) $C = (-2; 5) \cap [0; 5) \cap [-1; 3]$ ★★☆

d) $D = [5; 7) \cap (0; 4]$ ★★☆

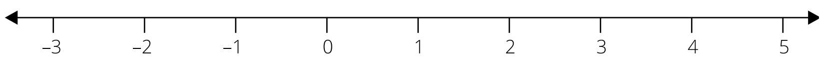
Notes

Exercise 2

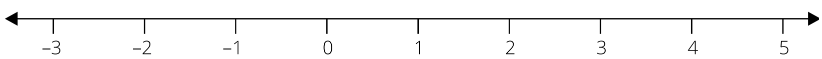
You don't know? You can find the process in the coursebook on page 93

Graph on the real number line and determine the intersection or union of intervals:

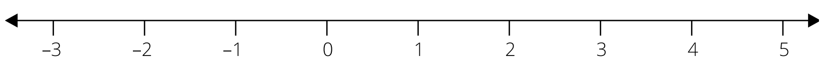
a) $A = \left[\frac{1}{2}; 5\right) \cup (-2; 3]$



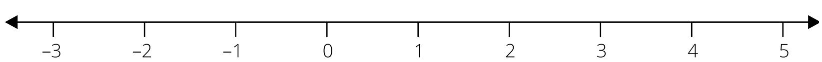
b) $B = \left[-\frac{3}{2}; 2\right) \cap (-1;1)$



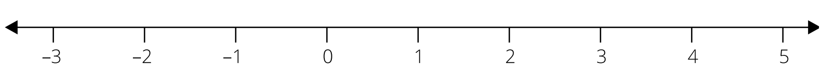
c) $C = (-1{,}7; 0] \cup (0; 3) \cup [3; \infty)$



d) $D = (-\infty; 1] \cap (-5; 2] \cap [-3; 3)$



e) $E = \left(-\infty; -\frac{1}{5}\right] \cap \left[-\frac{1}{4}; \infty\right) \cup \{0\} \cap \left(-5; -\frac{3}{2}\right]$



Notes

Exercise 3

Do you have a problem? Have a peek at the coursebook on page 96

You are given intervals $A = (-5; 1)$, $B = (-\infty; 4]$ and $C = [-3; 3)$. Note down the following sets using interval:

a) $A \cup B \cap C$ ★☆☆

b) $A \cap B \cup C$ ★☆☆

c) $(B \cup C) \cap A$ ★☆☆

d) $A \cap C \cap (B \cup A)$ ★★☆

e) $(A \cap B) \cup (B \cap C)$ ★★☆

f) $(A \cup B \cap C) \cup (C \cap B)$ ★★★

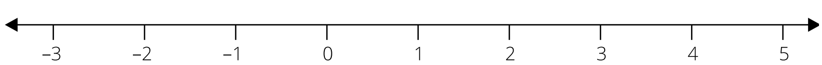
g) $(B \cup C \cap A) \cap (C \cap B \cup A)$ ★★★

Exercise 4

What's not in the head, that is in the coursebook on page 92

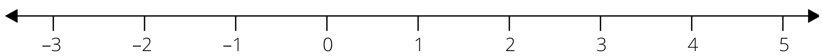
On the real number line graph and note down the following sets as interval:

a) $BŽ = \{x \in \mathbb{R}; -5 < x \leq -3\}$ ★★☆

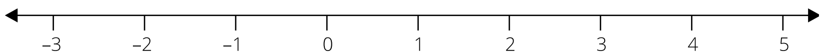


Notes

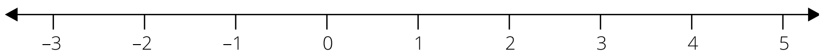
b) $FŇ = \{x \in \mathbb{R};\ x > 0\}$



c) $PŠT = \{x \in \mathbb{R};\ 5 \leq x \leq 14{,}5\}$



d) $HUŠ = \left\{x \in \mathbb{R};\ 0 < x \leq \frac{3}{2}\right\}$



Exercise 5

You can find how to do it in the coursebook on page 92

The company GEM has 6 employees. It is necessary to make an appointment of all employees. Find out the times when it’s possible to make an appointment, which all of the employees need to attend.

Time availability of each employee is:

Employee	Time availability
Mark	9:30 – 19:30
Tom	11:00 – 18:15
Anette	9:00 – 14:30
Jacob	10:30 – 17:30
Roman	13:30 – 19:30
Kate	8:30 – 15:00

Notes

Notes

Exercise 6

15 min.

Note the following sets by roster:

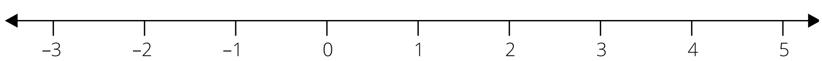
a) $FUJ = \{x \in \mathbb{Z}_0^+; x < 5\}$

b) $HMM = \left\{x \in \mathbb{N}; \dfrac{10}{n} \in \mathbb{N}\right\}$

Exercise 7

3 min.

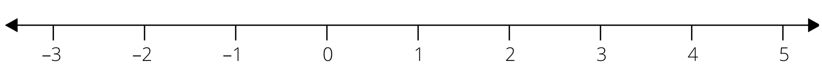
On the real number line graph interval $[x + 1; x + 3)$ where $x = -2$.



Exercise 8

4 min.

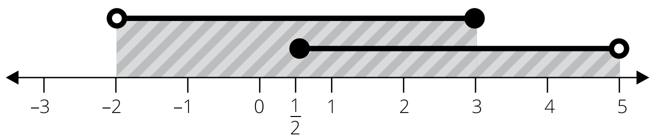
Find the smallest natural number x, for which interval $[x - 3; 1 - x)$ exists , and graph this interval on the real number line.



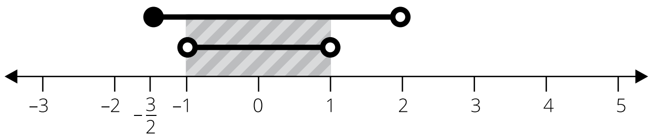
Results

1 a) $A = (-4; 5)$ b) $B = [-2; 0]$ c) $C = [0; 3]$ d) $D = \emptyset$

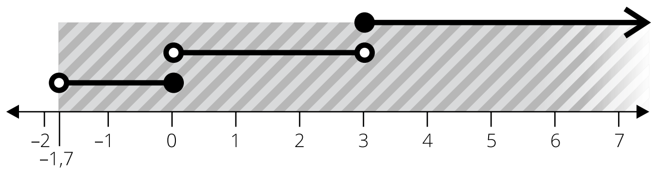
2 a)



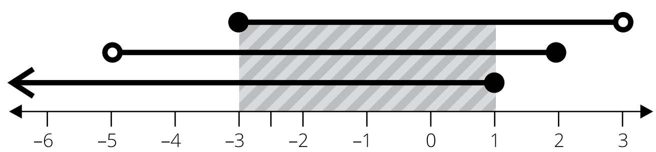
b)



c)



d)



e) $E = \emptyset$

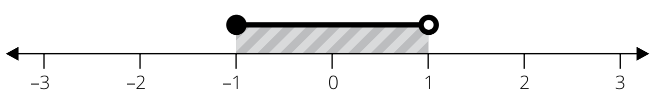
3 a) $[-3; 3]$ b) $(-5; 3)$ c) $(-5; 1)$ d) $(-5; 1)$ e) $[-3; 3)$ f) $(-5; 3)$ g) $(-5; 1)$

4 a) $BŽ = (-5; -3]$ b) $FŇ = (0; \infty)$ c) $PŠT = [5; 14{,}5]$ d) $HUŠ = \left(0; \dfrac{3}{2}\right]$

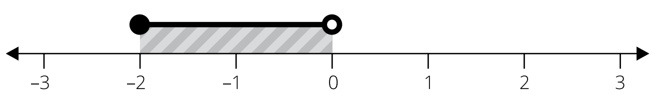
5 An appointment can be made at the time from 13:30 to 14:30.

6 a) $FUJ = \{0; 1; 2; 3; 4\}$ b) $HMM = \{1; 2; 5; 10\}$

7



8 x = 1



Notes

Summary & contacts

- collaboration of students and teachers
- preparation for graduation exam
- supplementary workbook for free with every coursebook
- large amount of solved problems
- covering of whole high school curriculum
- an option of long-term collaboration
- actual form thanks to feedback from teachers and students

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