

## Topics

- Elementary Boolean operators
- Duality
- Basic theorems
- Algebraic minimization

## Problems

- 1 Recall the truth table of the elementary Boolean operators. Solve the following system of equations for the variables A, B, C and D

$$\begin{cases} A' + A.B &= 0 \\ A.C &= A.B \\ A.B + A.C' + C.D &= C'.D \end{cases}$$

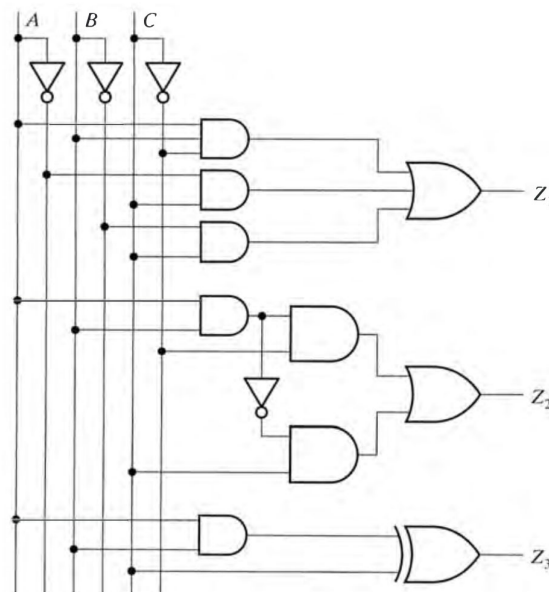
- 2 Verify, by perfect induction, the following simplification theorems. Write the dual counterpart of each theorem.

a)  $x + x.y = x$

b)  $x + x'.y = x + y$

c)  $x.y + x.y' = x$

- 3 Show that  $x'.y'.z' + x'.y'.z + x'.y.z' + x'.y.z + x.y.z' + x.y.z = x' + y$
- 4 Show that  $x.y + x'.z + y.z = x.y + x'.z$ . Write the dual version of the previous expression.
- 5 Write the truth table of the XOR operation  $x \oplus y$ . Express this operator as an elementary sum of logic products.
- 6 Consider the logic circuits in the figure and show, by algebraic methods, that  $Z_1 = Z_2 = Z_3$ .



- 7 Use DeMorgan's laws to obtain the complement of:
- a)  $(x \cdot y' + x' \cdot y)$                       b)  $(x \cdot y + z \cdot (x + y') + z \cdot y)$
- 8 Show that  $(a' \cdot b + a \cdot c) \cdot (a + b') \cdot (a' + c') = 0$
- 9 Show that the dual of an XOR is an XNOR, that is  $(x \oplus y)^D = (x \oplus y)'$ .
- 10 Implement the XOR operation with NAND gates. Assume that both uncomplemented and complemented inputs are available.
- 11 Consider the following Boolean functions:
- $$S = x \oplus y \oplus c_i$$
- $$C_o = x \cdot y + c_i \cdot (x + y)$$
- a) Draw the logic circuit.
- b) Redraw the circuit using only NAND gates.
- 12 The Majority function  $M(x, y, z)$ , is 1 whenever there are at least two inputs equal to 1.
- a) Write the truth table for  $M(x, y, z)$ .
- b) From the truth table propose a Boolean expression for  $M(x, y, z)$ .
- c) Draw the corresponding logic circuit.
- d) Show that using the set  $S = \{M(x, y, z), NOT, "0"\}$  we can express any logic function. Suggestion: show how to implement the fundamental Boolean operators  $\{+, \cdot, '\}$  using the elements of  $S$ .