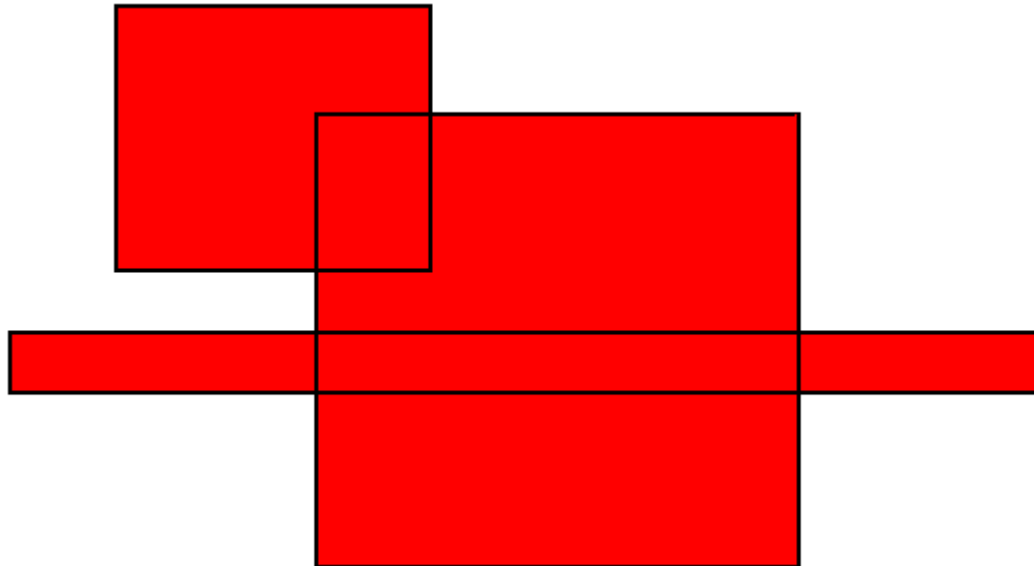


# Computational Geometry



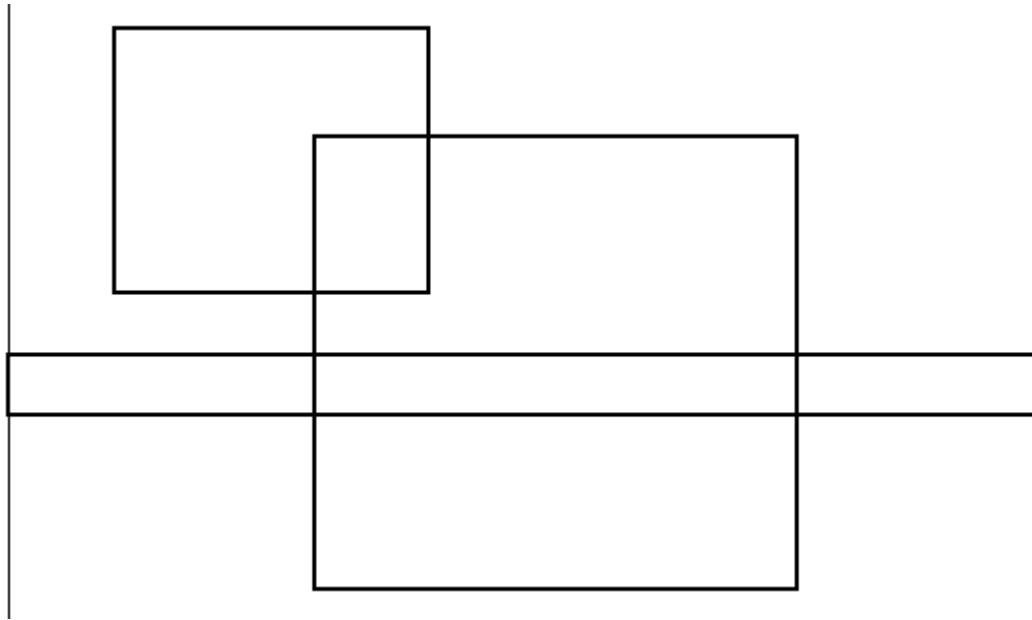
- Area of the union of rectangles problem  
( $n$  rectangles or  $4n$  lines or  $4n$  points)



# Computational Geometry



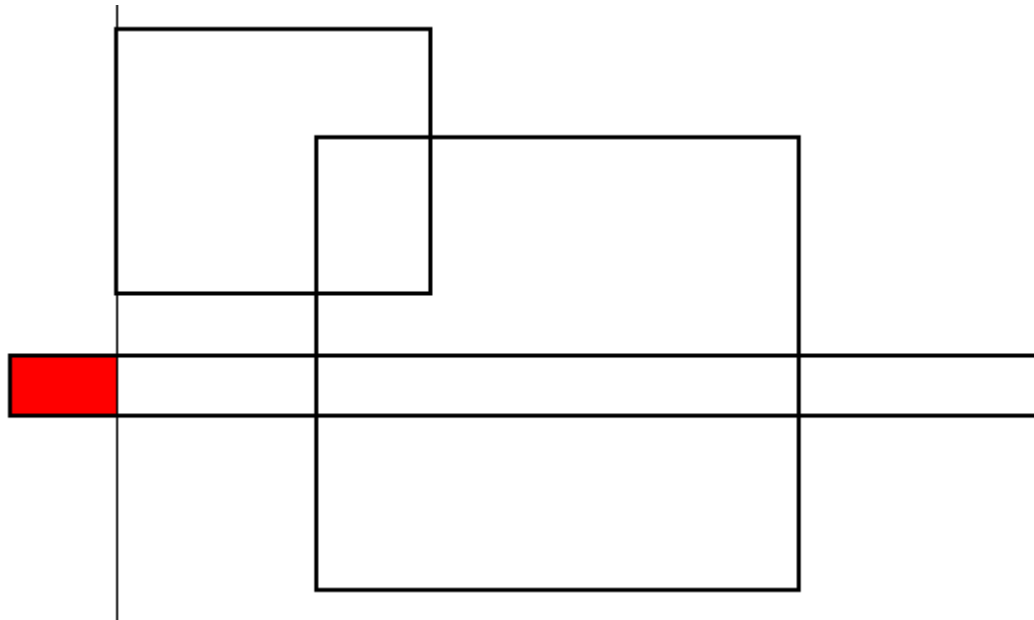
- Area of the union of rectangles problem



# Computational Geometry



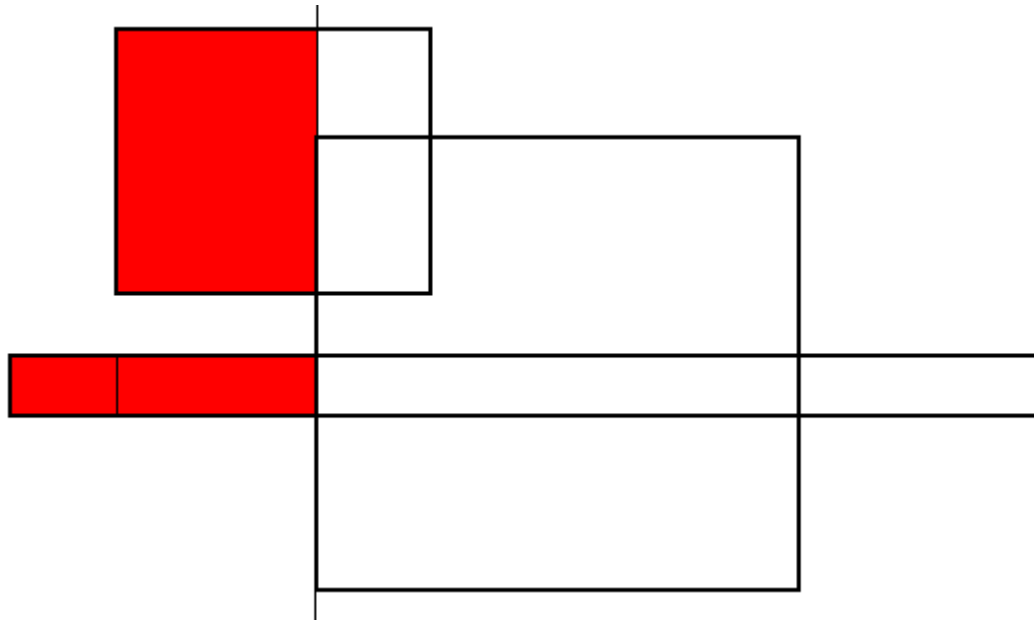
- Area of the union of rectangles problem



# Computational Geometry



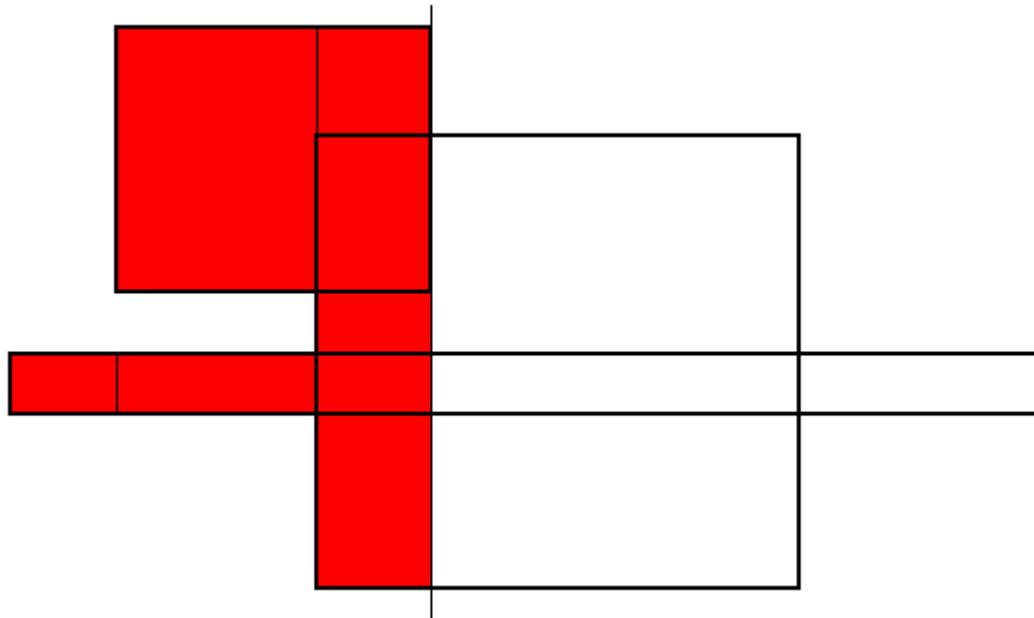
- Area of the union of rectangles problem



# Computational Geometry



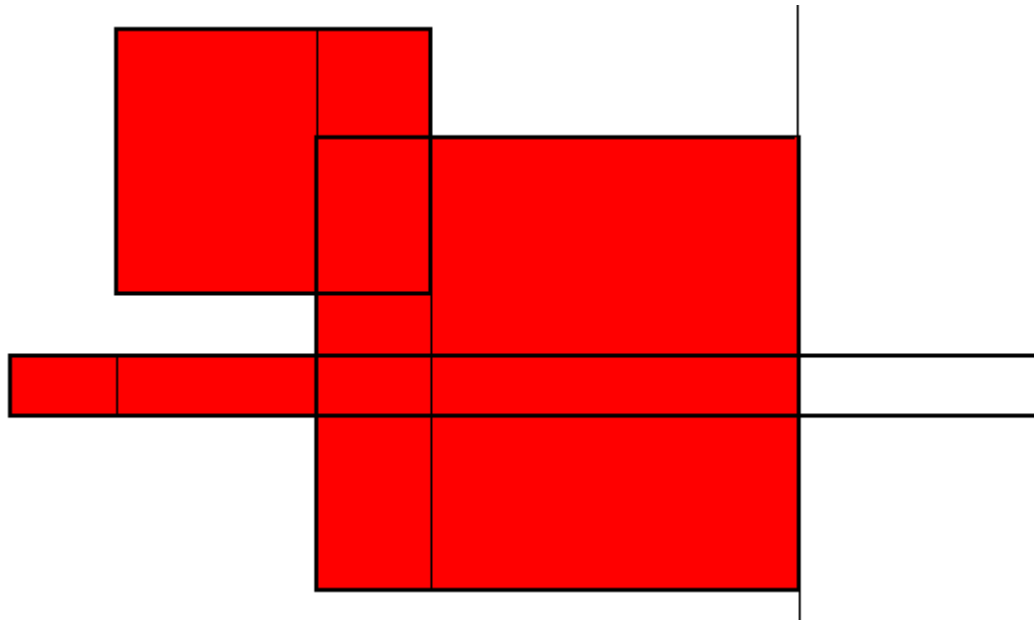
- Area of the union of rectangles problem



# Computational Geometry



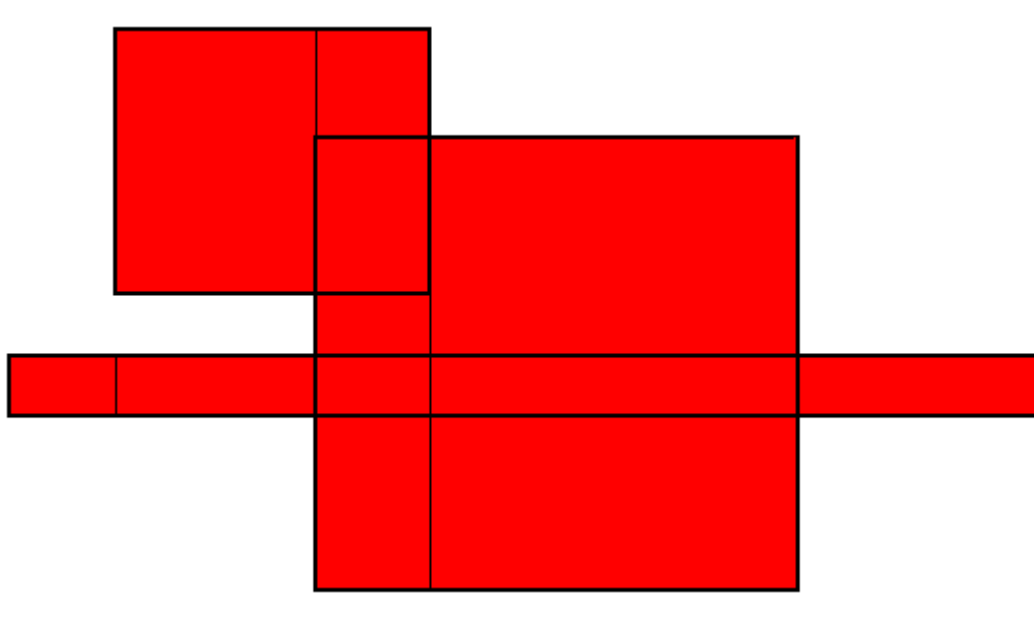
- Area of the union of rectangles problem



# Computational Geometry



- Area of the union of rectangles problem



# Computational Geometry



- Area of the union of rectangles problem

Step 1: Sort points in x-coordinate

Step 2: For each vertical segment  $l$ :

Step 2.1: Update the area from the last visited segment to  $l$



# Computational Geometry



- $P_x = \{p_x^1, \dots, p_x^{2n}\}$  is the set of vertices sorted wrt  $x$ -axis
- $P_y = \{p_y^1, \dots, p_y^{2n}\}$  is the set of vertices sorted wrt  $y$ -axis
- $A$ : area of the union of rectangles
- $Q$ : list of active rectangles between two successive points in  $P_x$
- $v$ : height of the union of rectangles in  $Q$

**Invariant:** At a point  $p_x^i$  the area until that point is

$$A = A + v (p_x^i - p_x^{i-1})$$

# Computational Geometry



$A = 0$

$Q = \{r\}$

*// r is the leftmost rectangle*

*for  $i=2$  to  $|P_x|$*

*...*

*Vertical sweep at point  $p_x^{i-1}$*

*...*

$A = A + v(p_x^i - p_x^{i-1})$

*if  $p_x^i$  is at the left side of rectangle  $r$*

$Q = Q \cup \{r\}$

*else if  $p_x^i$  is at the right side of rectangle  $r$*

$Q = Q \setminus \{r\}$

*Return  $A$*

# Computational Geometry



Vertical sweep at point  $p_x^{i-1}$

$v = 0, l = 0, S = 0$

for  $j=1$  to  $|P_y|$

if  $p_y^j$  is at the top of rectangle  $r$  in  $Q$

if  $S > 0$

$v = v + (l - p_y^j)$

$S = S + 1$

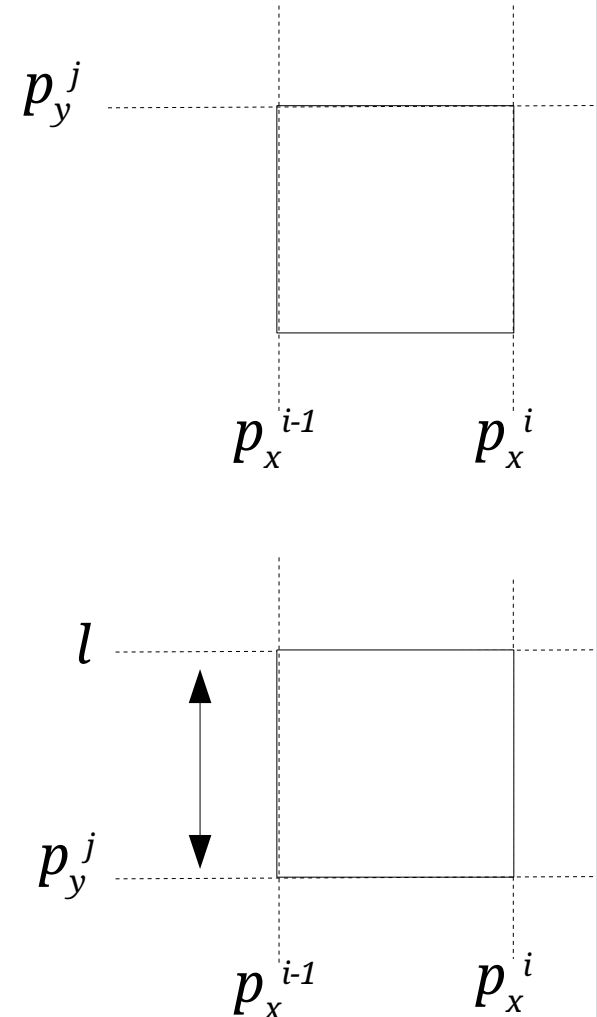
$l = p_y^j$

else if  $p_y^j$  is at the bottom of rectangle  $r$  in  $Q$

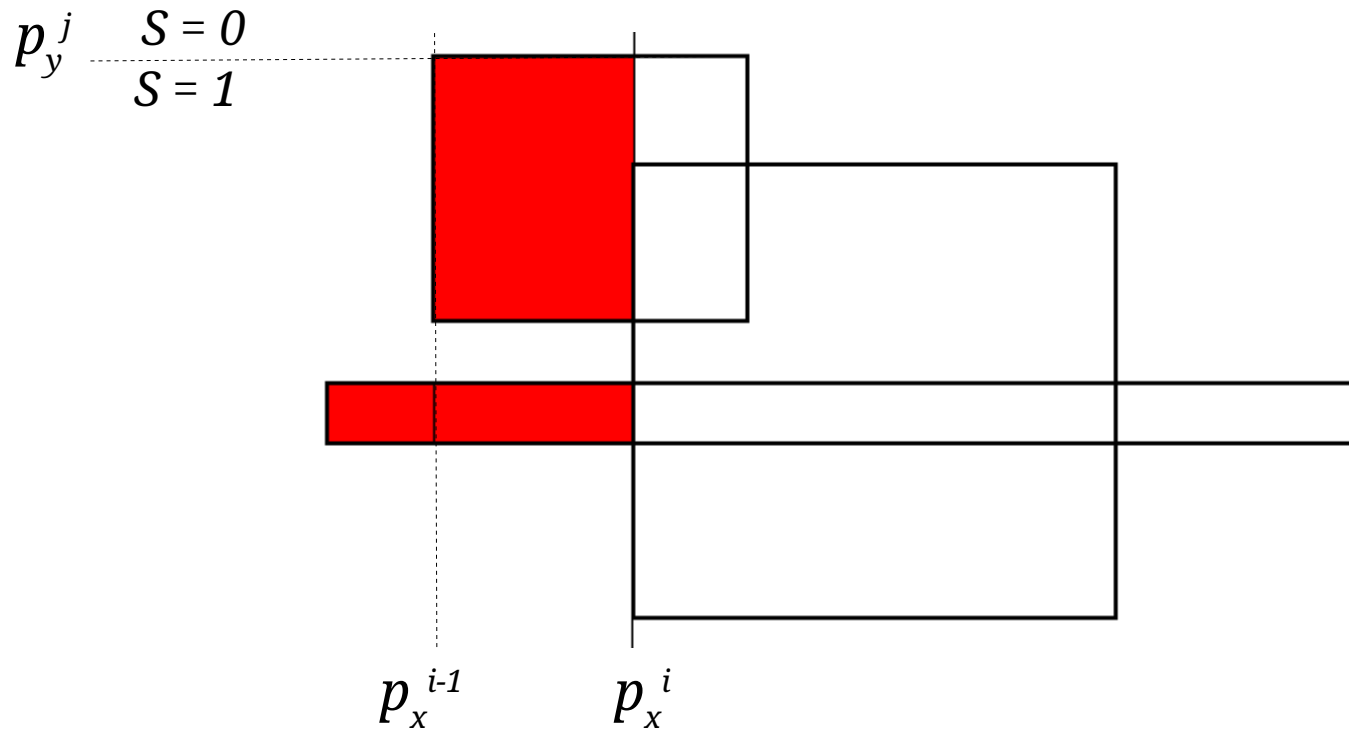
$v = v + (l - p_y^j)$

$S = S - 1$

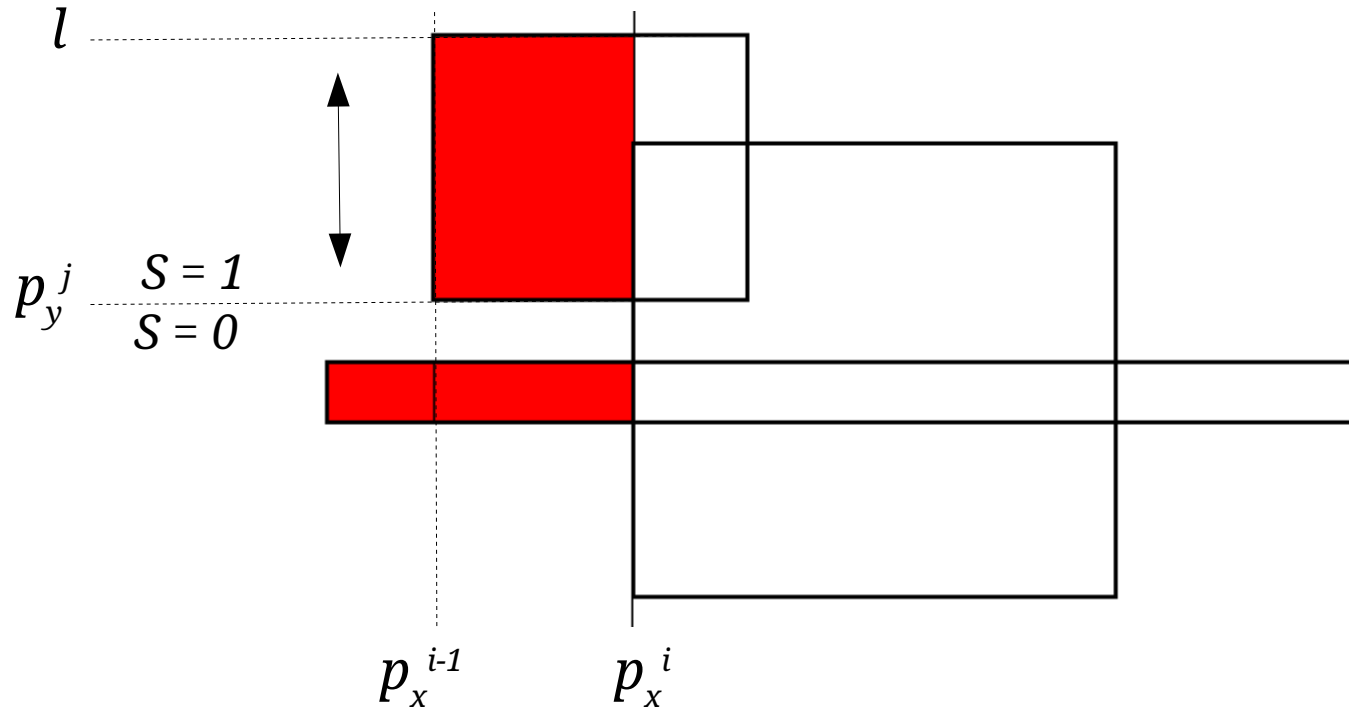
$l = p_y^j$



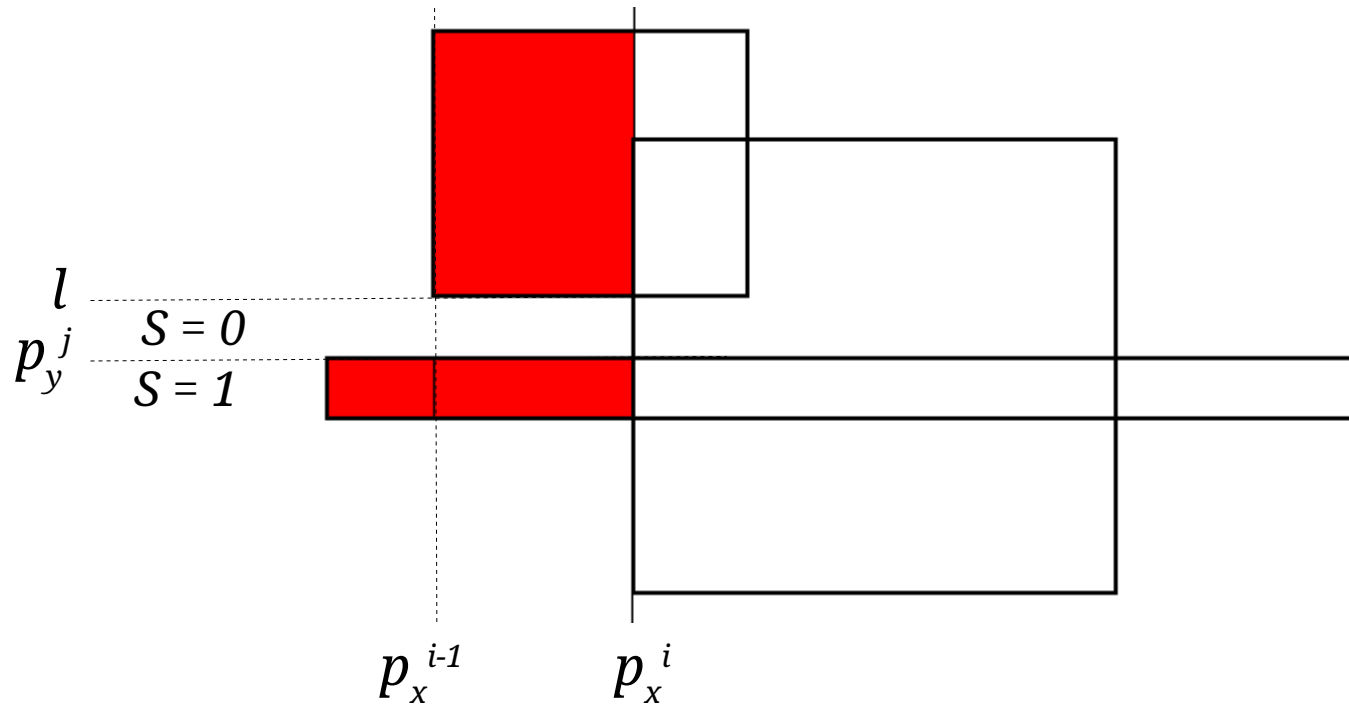
# Computational Geometry



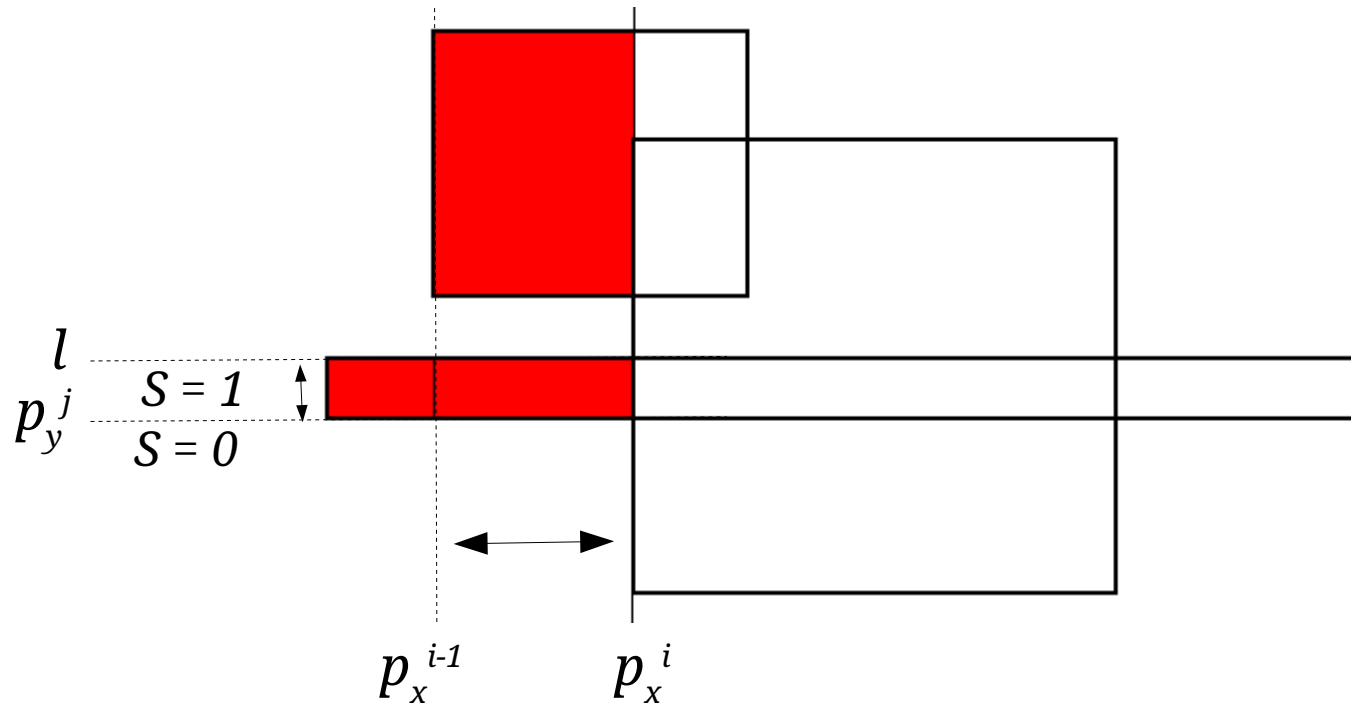
# Computational Geometry



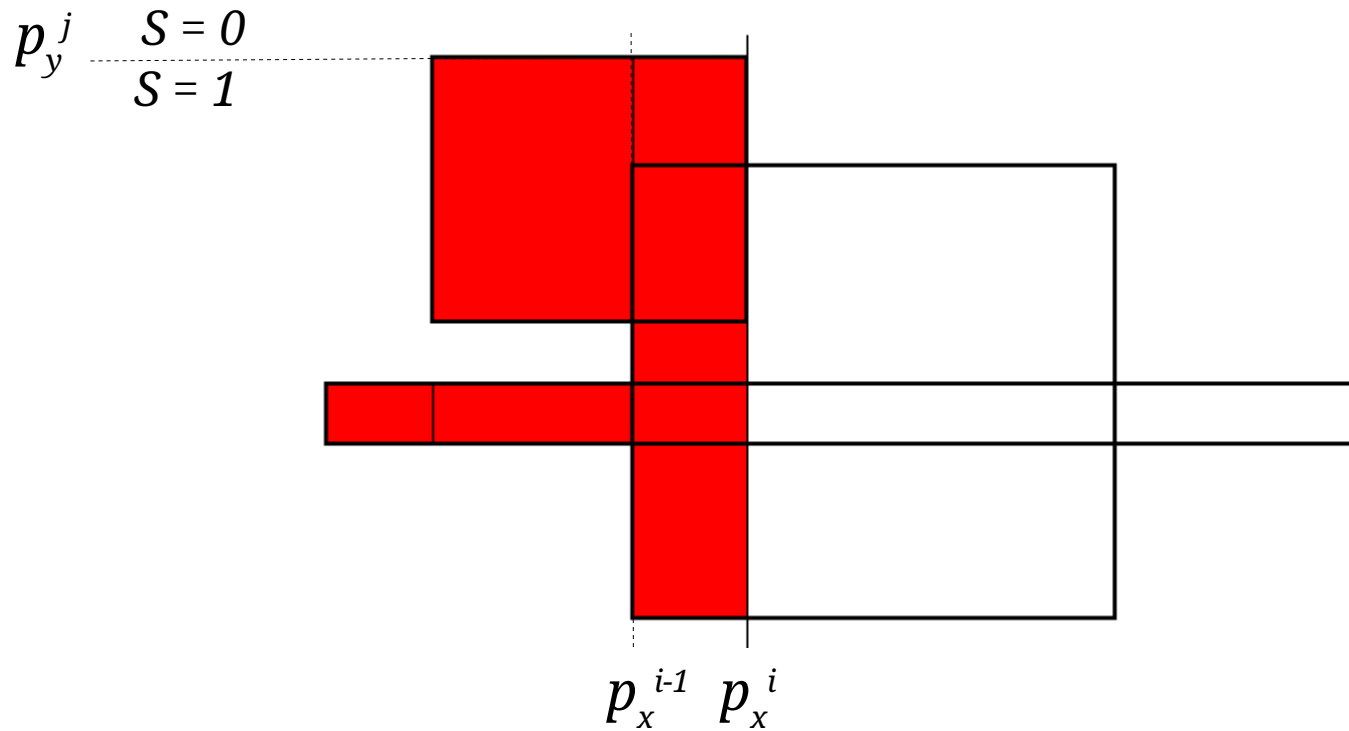
# Computational Geometry



# Computational Geometry

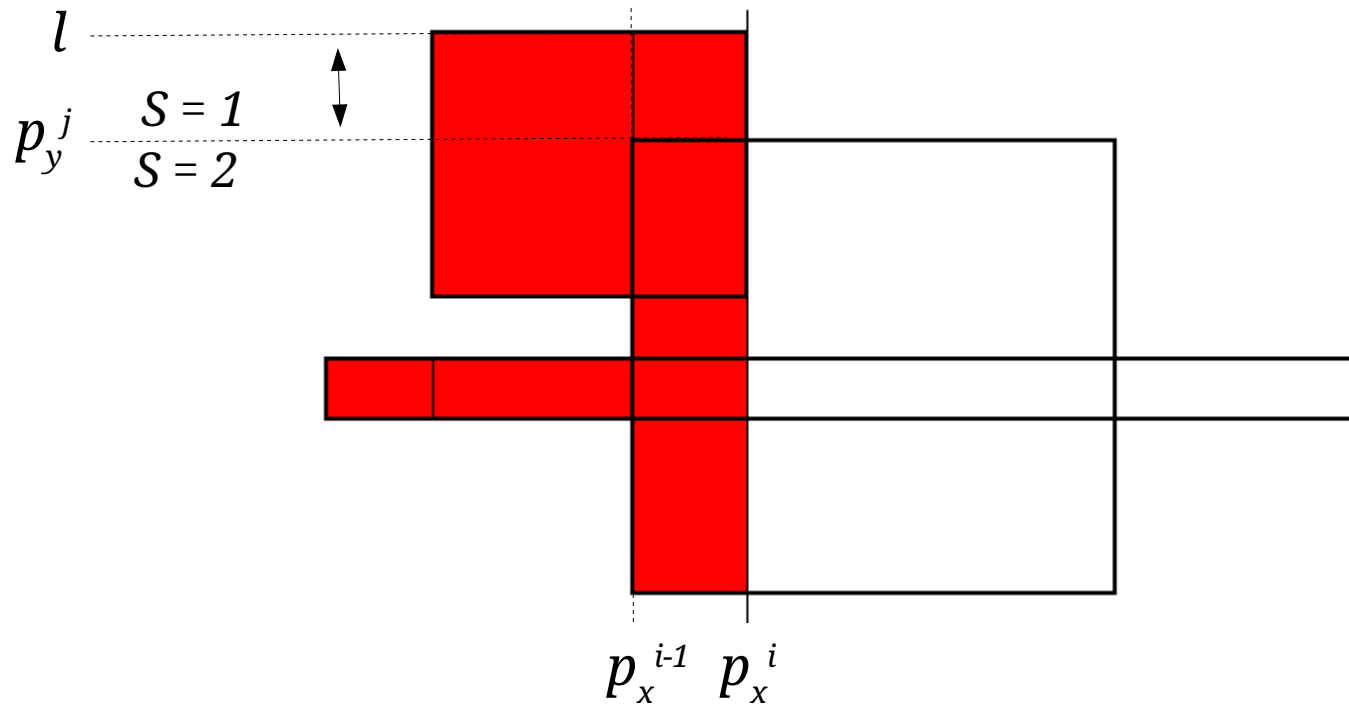


# Computational Geometry

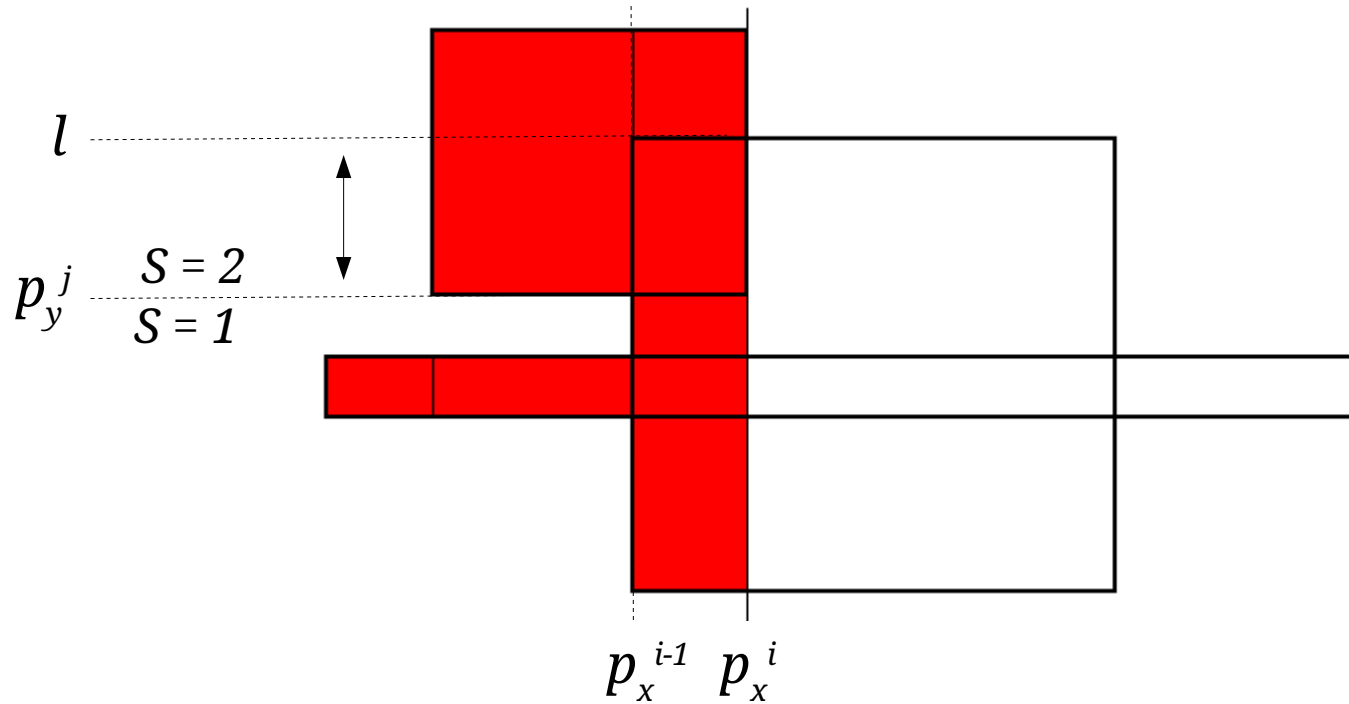




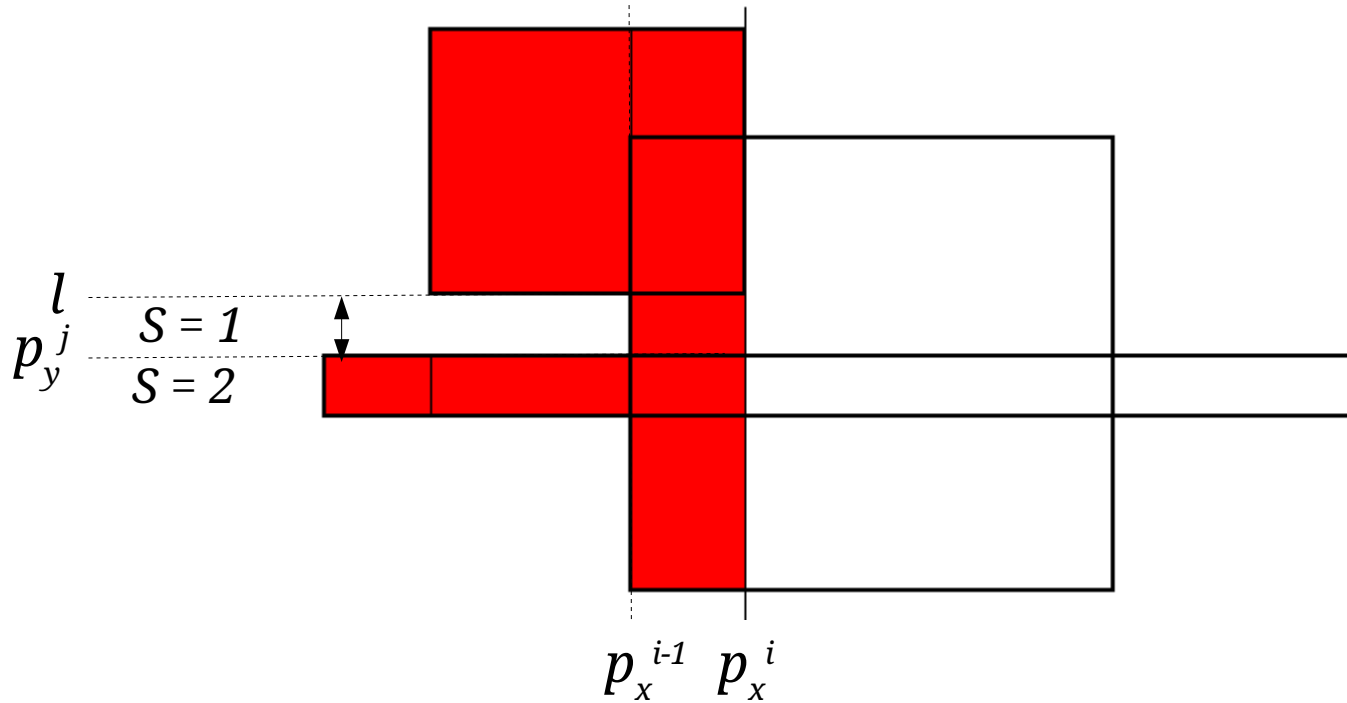
# Computational Geometry



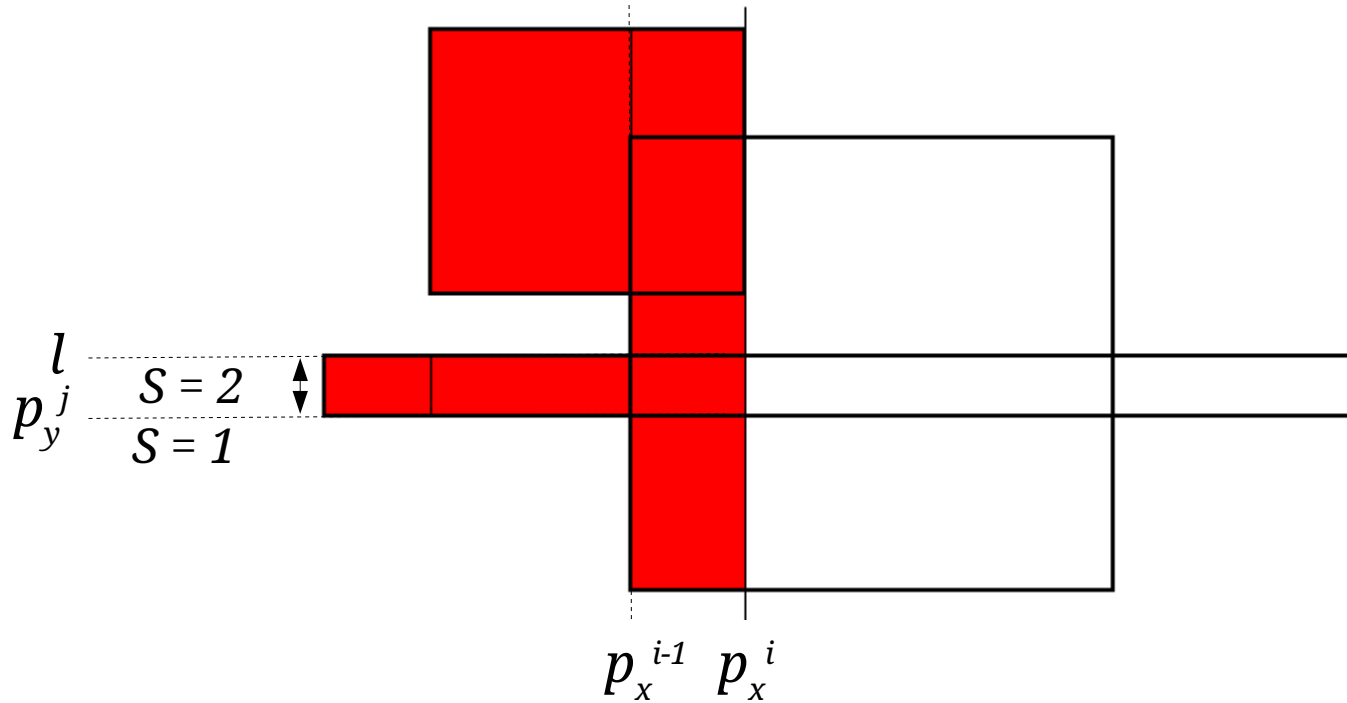
# Computational Geometry



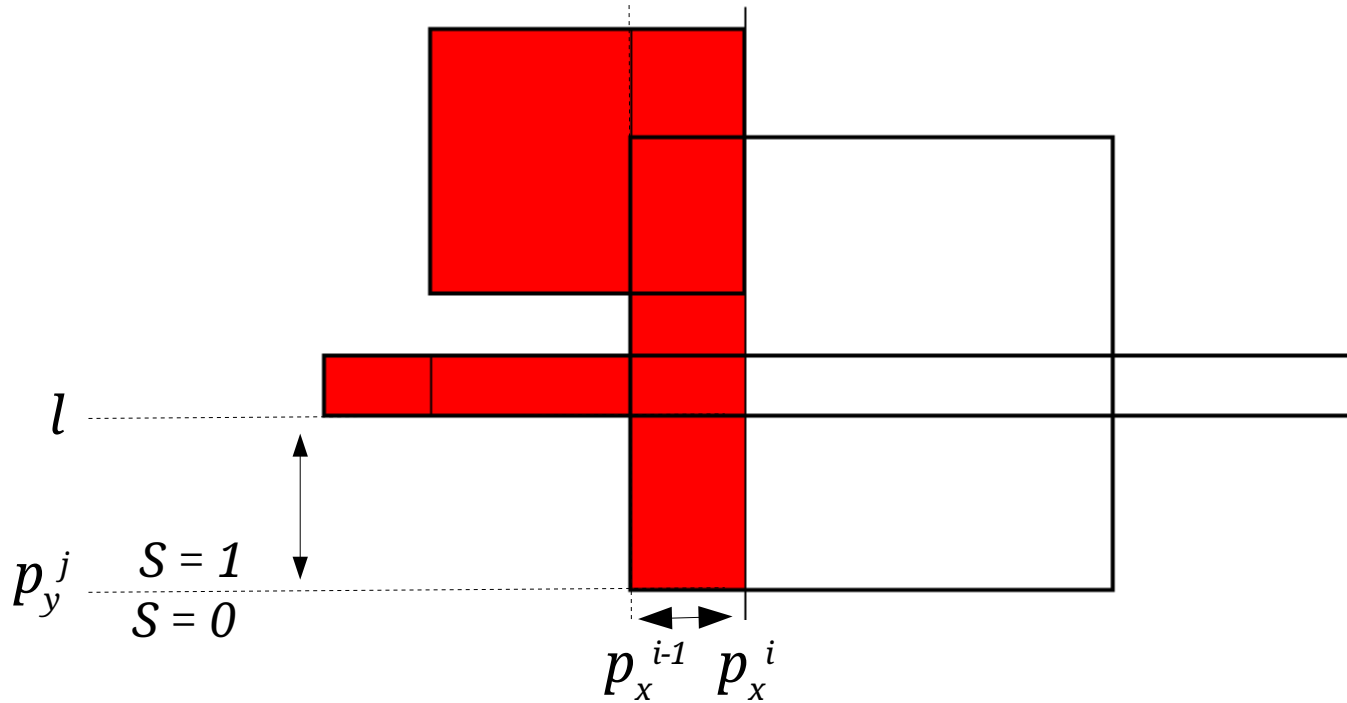
# Computational Geometry



# Computational Geometry



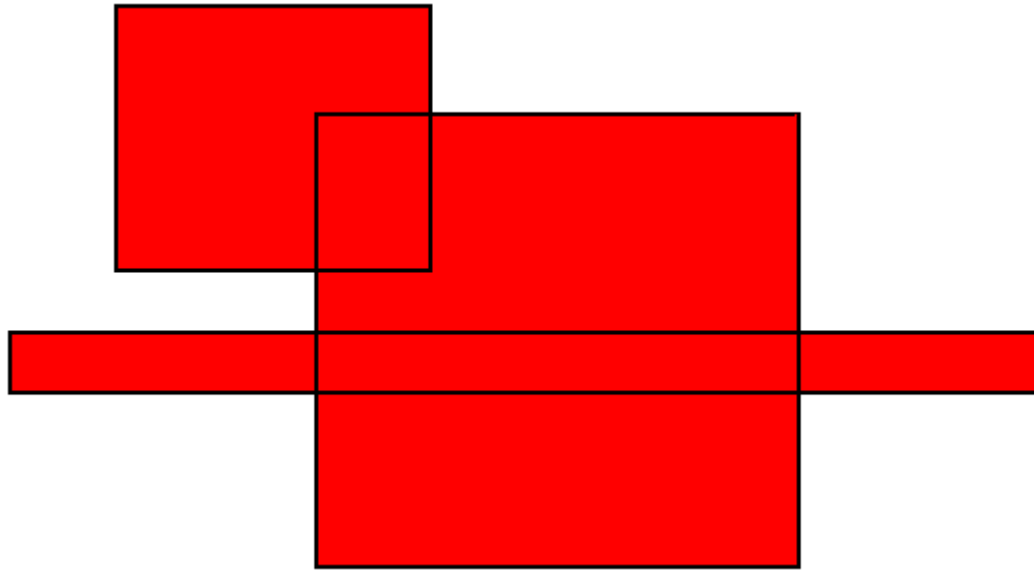
# Computational Geometry



# Computational Geometry



- Perimeter of the union of rectangles problem



1. Compute the length of the horizontal edges

# Computational Geometry



$P = \emptyset$

$Q = \{r\}$

*// r is the leftmost rectangle*

for  $i=2$  to  $|P_x|$

...

*Vertical sweep at point  $p_x^{i-1}$*

...

*if  $p_x^i$  is at the left side of rectangle  $r$*

$Q = Q \cup \{r\}$

*else if  $p_x^i$  is at the right side of rectangle  $r$*

$Q = Q \setminus \{r\}$

*Return  $P$*

# Computational Geometry



Vertical sweep at point  $p_x^i$

$S = 0$

for  $j=1$  to  $|P_y|$

if  $p_y^j$  is at the top of rectangle  $r$  in  $Q$

if  $S = 0$

$P = P + (p_x^i - p_x^{i-1})$

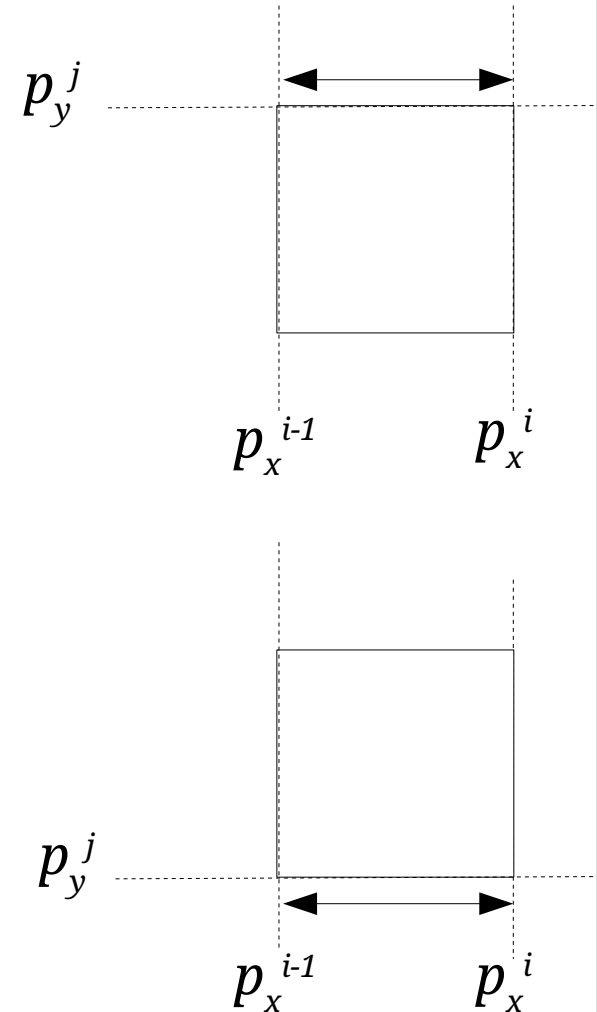
$S = S + 1$

else if  $p_y^j$  is at the bottom of rectangle  $r$  in  $Q$

$S = S - 1$

if  $S = 0$

$P = P + (p_x^i - p_x^{i-1})$

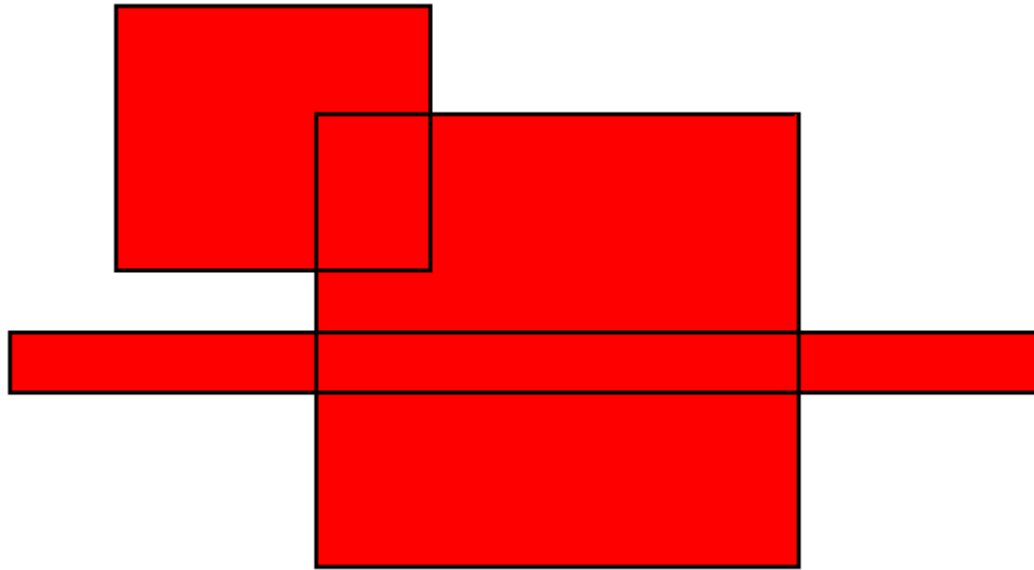




# Computational Geometry

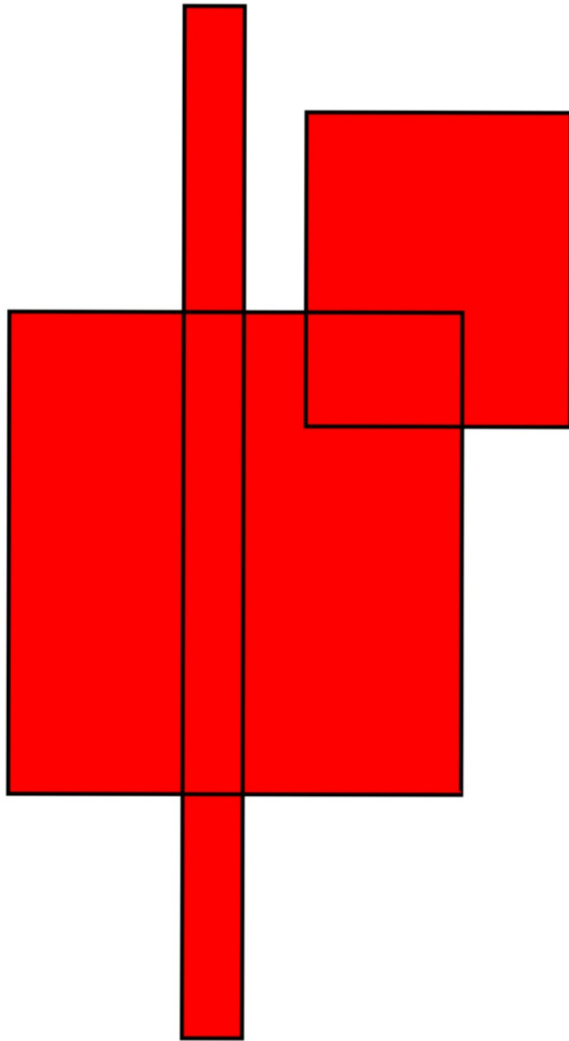


- Perimeter of the union of rectangles problem



2. Compute the length of the vertical edges (how)?

# Computational Geometry



Rotate the Figure!