Answer model Intermediate test Calculus B, version 1a

1. Determine an equation of the line passing through the point $(1, 2)$ that is perpendicular (orthogonal) to the line with equation $x + 2y = 5$.

   Answer: the vector $(1, 2)^\top$ is perpendicular to the line, so it is a direction vector for any line perpendicular to it, so these lines have slope 2, so an equation is $y - 2 = 2(x - 1)$.

   Or: Since the vector $(1, 2)^\top$ is perpendicular to the line, the line connecting the origin with the point $(1, 2)$ is perpendicular, with equation $y = 2x$.

2. A triangle ABC is given, with angles α, β and γ and opposite sides of lengths a, b and c. Furthermore $\alpha = \pi/4 = 45°$, $\beta = \pi/6 = 30°$ and $b = 1$. Determine a.

   Answer: The sine law says $\sin(\alpha)/a = \sin(\beta)/b$, so $\frac{1}{2}\sqrt{2}/a = \frac{1}{2}$, and $a = \sqrt{2}$.

3. It is given that $\lim\limits_{x\to 3}\left(\dfrac{1}{x-3} + \dfrac{a}{x^2-9}\right)$ exists (and is finite). Determine a, and the value of the limit.

   Answer: Since $\dfrac{1}{x-3} + \dfrac{a}{x^2-9} = \dfrac{x+3+a}{x^2-9}$, the limit can only exist if the numerator of this fraction is also zero for $x = 3$, that is $a = -6$ and in this case the expression simplifies to $1/(x + 3)$ if $x \neq 3$, so the limit is $1/6$.

4. Determine the real zeros (roots) of the polynomial $x^6 + 7x^3 - 8$ and give a factorization of this polynomial in factors of degree 1 and 2.

   Answer: Write $w = x^3$, then the polynomial becomes $w^2 + 7w - 8$ with factorization $(w - 1)(w + 8)$. Now put back $w = x^3$ and use the fact that $x^3 = a$ has only one real solution for all real (nonzero) a so we get the factorization

   $$(x^3 - 1)(x^3 + 8) = (x - 1)(x^2 + x + 1)(x + 2)(x^2 - 2x + 4).$$

Answer model Intermediate test Calculus B, version 1b

1. Determine an equation of the line passing through the point $(1, 3)$ that is perpendicular (orthogonal) to the line with equation $x + 3y = 10$.

   Answer: the vector $(1, 3)^\top$ is perpendicular to the line, so it is a direction vector for any line perpendicular to it, so these lines have slope 3, so an equation is $y - 3 = 3(x - 1)$.

   Or: Since the vector $(1, 3)^\top$ is perpendicular to the line, the line connecting the origin with the point $(1, 3)$ is perpendicular, with equation $y = 3x$.

2. A triangle ABC is given, with angles α, β and γ and opposite sides of lengths a, b and c. Furthermore $\alpha = \pi/3 = 60°$, $\beta = \pi/4 = 45°$ and $a = 1$. Determine b.

   Answer: The sine law says $\sin(\alpha)/a = \sin(\beta)/b$, so $\frac{1}{2}\sqrt{3} = \frac{1}{2}\sqrt{2}/b$, and $b = \sqrt{2/3}$.

3. It is given that $\lim\limits_{x \to -3} \left(\dfrac{1}{x+3} + \dfrac{a}{x^2 - 9} \right)$ exists (and is finite). Determine a, and the value of the limit.

   Answer: Since $\dfrac{1}{x+3} + \dfrac{a}{x^2-9} = \dfrac{x - 3 + a}{x^2 - 9}$, the limit can only exist if the numerator of this fraction is also zero for $x = -3$, that is $a = 6$ and in this case the expression simplifies to $1/(x - 3)$ if $x \neq -3$, so the limit is $-1/6$.

4. Determine the real zeros (roots) of the polynomial $x^4 - 5x^2 + 4$ and give a factorization of this polynomial in factors of degree 1.

   Answer: Write $w = x^2$, then the polynomial becomes $w^2 - 5w + 4$ with factorization $(w - 1)(w - 4)$. Now put back $w = x^2$ then we get the factorization

   $$(x^2 - 1)(x^2 - 4) = (x - 1)(x + 1)(x - 2)(x + 2).$$