Answers Intermediate test Calculus, 2WBB3, Monday 3 October 2016, version 1

1. In view of the domain of $\sqrt{\cdot}$, we should have $x \geq -\frac{7}{2}$. Squaring $\sqrt{2x+7} = x-4$ gives $2x+7 = x^2-8x+16$, so $x^2-10x+9 = (x-9)(x-1) = 0$, so $x=1$ or $x=9$. Checking of these values shows that only $x=9$ is a solution. Checking some values for x, such as $x=10$ and $x=0$ gives the answer $x \in [9,\infty)$.

2. Completing the square: $x^2-4x+4-4+2y^2 \leq 0$ so $(x-2)^2+2y^2 \leq 4$. This is a filled ellipse with the x-axis and y-axis as the main axes, with "center" $(2,0)$, passing through the points $(0,0)$, $(4,0)$, and $(2,\pm\sqrt{2})$.

3. Since this is a limit of the type ∞/∞, we divide by the largest quantity of the denominator:

$$\lim_{x\to\infty} \frac{\frac{8}{x} - 8 + \frac{\sin(8x)}{x}}{8 - \frac{\cos(8x)}{x}} = \frac{0-8+0}{8-0} = -1.$$

   Hereby we have used the following: since $-\frac{1}{x} \leq \frac{\sin(8x)}{x} \leq \frac{1}{x}$, we have by the squeeze theorem $\lim_{x\to\infty} \frac{\sin(8x)}{x} = 0$, and the same for $\lim_{x\to\infty} \frac{\cos(8x)}{x} = 0$.

4. The normal to the plane has the direction $(1,-1,1)$. The line through $(1,2,3)$ in this direction has parametrization $(x,y,z) = (1+t, 2-t, 3+t)$. Substitution in the plane gives $1+t-(2-t)+3+t = 5$, so $t=1$, with corresponding point $(2,1,4)$. The distance is the length of $(1,-1,1)$, so $\sqrt{3}$.