Criteria

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10 exercises · 50.0 points

1 20.0 POINTS · 8 QUESTIONS

Exercise 1 (Multiple Choice Questions)

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a MULTIPLE CHOICE · 2.5 POINTS

The function/ De functie $f(x) = \dfrac{\sin(x)}{1+\cos(x)}$ is

even	0.0
odd / oneven	2.5
even and odd / even en oneven	0.0
neither even nor odd/ noch even noch oneven	0.0

b MULTIPLE CHOICE · 2.5 POINTS

$\cos\left(\arctan\left(\frac{1}{2}\right)\right) =$

$\frac{1}{\sqrt{5}}$	0.0
$\sqrt{\frac{3}{5}}$	0.0
$\frac{4}{5}$	0.0
$\sqrt{\frac{4}{5}}$	2.5

c MULTIPLE CHOICE · 2.5 POINTS

The slope of the tanget line through the point $(3,1)$ at the curve $yx^2 + e^y = x + 6 + e$ equals
De richtingsco\"efficient van raaklijn door $(3,1)$ aan de kromme $yx^2 + e^y = x + 6 + e$ is

$-\frac{5}{9+e}$	2.5
$-\frac{5+e}{9}$	0.0
$\frac{1}{9+e}$	0.0
$\frac{5+e}{9}$	0.0

d MULTIPLE CHOICE · 2.5 POINTS

Determine the distance from $(1,2,2)$ to the line with parameter representation
Bepaal de afstand van het punt $(1,2,2)$ tot de lijn met parametervoorstelling

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

0	0.0
$\sqrt{5}$	2.5
$\sqrt{7}$	0.0
3	0.0

e MULTIPLE CHOICE · 2.5 POINTS

The limit / de limiet

$$\lim_{x\to\infty}\left(\frac{x^2}{x+1} - \sqrt{x^2+1}\right)$$

doesn't exist (neither as a real number, nor as $\pm\infty$) / bestaat niet (noch als re\"eel getal, noch als $\pm\infty$)	0.0
exists and has value -2 / bestaat en heeft waarde -2	0.0
exists and has value -1 / bestaat en heeft waarde -1	2.5
exists and has value 2 / bestaat en heeft waarde 2	0.0

f MULTIPLE CHOICE · 2.5 POINTS

The function / De functie

$\big(2 \qquad x = 0$

is

continuous and differentiable / continu en differentieerbaar	0.0
continuous but not differentiable / continu maar niet differentieerbaar	0.0
differentiable but not continuous / differentieerbaar maar niet continu	0.0
neither continuous nor differentiable / noch continu noch differentieerbaar	2.5

(g) MULTIPLE CHOICE · 2.5 POINTS

Consider the function $f(x) = \int_{x^3}^{2} \frac{\sin(t)}{3t}\, dt$. The derivative of f is

Gegeven is de functie $f(x) = \int_{x^3}^{2} \frac{\sin(t)}{3t}\, dt$. De afgeleide van de functie f is

$-\frac{\sin(x^3)}{x}$	2.5
$\frac{\sin(x^3)}{x}$	0.0
$-\frac{\sin(x^3)}{3x^3}$	0.0
$\frac{\sin(x^3)}{3x^3}$	0.0

(h) MULTIPLE CHOICE · 2.5 POINTS

The integrals / de integralen

$$\mathrm{I}: \quad \int_1^\infty \sqrt{\frac{x^7}{e^{3x}}}\, dx, \qquad \mathrm{II}: \quad \int_0^1 \frac{1}{x^{3/2}}\, dx$$

are / zijn

I: convergent, II: convergent	0.0
I: divergent, II: convergent	0.0
I: convergent, II: divergent	2.5
I: divergent, II: divergent	0.0

(2) 6.0 POINTS · 3 QUESTIONS

Exercise 2

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(a) POINTS · 2.0 POINTS · 2 CRITERIA

Specify for every $a \in \mathbb{R}$ how many zeros the function f_a has.

Geef voor iedere $a \in \mathbb{R}$ het aantal nulpunten van f_a.

$f_a(x) = x(x^2 + ax + 1) = 0 \quad \Longleftrightarrow \quad x = 0 \vee x^2 + ax + 1 = 0.$ — 1.0

The quadratic equation $x^2 + ax + 1 = 0$ has discriminant $a^2 - 4$, with solutions

$$x_\pm = \frac{-a \pm \sqrt{a^2 - 4}}{2}.$$

Therefore we find — 1.0

1 solution if $|a| < 2$

2 solutions if $|a| = 2$

3 solutions if $|a| > 2$

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(b) POINTS · 2.0 POINTS · 1 CRITERION

Show that f_0 is injective.

Toon aan dat f_0 injectief is.

$f_0'(x) = 3x^2 + 1 \geq 1 > 0$, so that f_0 is injective. — 2.0

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(c) POINTS · 2.0 POINTS · 2 CRITERIA

Determine/Bepaal $(f_0^{-1})'(-10)$.

$f_0(-2) = -10$, and therefore — 1.0

$f_0^{-1})'(-10) = \frac{1}{f_0'(-2)} = \frac{1}{13}.$ — 1.0

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3 4.0 POINTS · 1 QUESTION
Exercise 3
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a POINTS · 4.0 POINTS · 4 CRITERIA
Determine the Taylor polynomial of degree 2 around $x = \frac{\pi}{3}$ of the function
Bepaal het Taylorpolynoom van graad 2 rond $x = \frac{\pi}{3}$ van de functie

$$f(x) = e^{\cos(x)} .$$

Hint: Use the definition.

Take the definition: 1.0
$f(\pi/3) = e^{1/2}$.

$f'(x) = -\sin x e^{\cos x}$, so $f'(\pi/3) = -\frac{1}{2}\sqrt{3}e^{1/2}$ 1.0

$f''(x) = (-\cos x + \sin^2 x)e^{\cos x}$, so $f''(\pi/3) = (-\frac{1}{2} + \frac{3}{4})e^{1/2} = \frac{1}{4}e^{1/2}$. 1.0

Then 1.0

$$P_2(x) = e^{1/2}\Big\{1 - \frac{1}{2}\sqrt{3}\Big(x - \frac{\pi}{3}\Big) + \frac{1}{8}\Big(x - \frac{\pi}{3}\Big)^2\Big\}.$$

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4 4.0 POINTS · 1 QUESTION
Exercise 4
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a POINTS · 4.0 POINTS · 4 CRITERIA
Consider the function $f(x) = |2\sin x - 1|$ on the interval $[0, 2\pi]$.
Determine the local and absolute (global) extreme values as well as the range of f.
Beschouw de functie $f(x) = |2\sin(x) - 1|$ op het interval $[0, 2\pi]$.
Bepaal zowel de lokale en absolute (globale) extreme waarden als ook het bereik van f.

For $x \in [0, 2\pi]$ we have $2\sin(x) - 1 = 0 \Leftrightarrow \sin(x) = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$. 1.0
On the interval $[\frac{\pi}{6}, \frac{5\pi}{6}]$ the function $\sin(x)$ takes all values between $\frac{1}{2}$ and 1.
Outside the interval $\sin(x)$ takes all values between $\frac{1}{2}$ and -1.

For $x \in [\frac{\pi}{6}, \frac{5\pi}{6}]$ we have $f(x) = 2\sin(x) - 1$ which is minimal with value 0 at the endpoints and maximal at $x = \frac{\pi}{2}$ with value $2 - 1 = 1$. Of course 0 is an absolute minimum and 1 a local maximum. 1.0

For $x \in [0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$ we have $f(x) = 1 - 2\sin(x)$ which is maximal at $x = \frac{3\pi}{2}$ with value $1 - 2 \cdot (-1) = 3$ (an absolute maximum) and minimal at $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$ with value 0. 1.0

The range of f is therefore $[0, 3]$. 1.0

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5 8.0 POINTS · 2 QUESTIONS · STARTS AT NEW PAGE
Exercise 5
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a POINTS · 4.0 POINTS · 3 CRITERIA

$$\int_3^5 \frac{13 - 5x}{x^3 - 2x^2 + 2x + 5}\, dx$$

$x^3 - 2x^2 + 2x + 5 = (x + 1)(x^2 - 3x + 5)$, so we search for A, B, C such that 1.0

$$\frac{13 - 5x}{x^3 - 2x^2 + 2x + 5} = \frac{A}{x + 1} + Bx + Cx^2 - 3x + 5 = \frac{A(x^2 - 3x + 5) + (Bx + C)(x + 1)}{(x + 1)(x^2 - 3x + 5)},$$

so we find that $A + B = 0$, $-3A + B + C = -5$, and $5A + C = 13$. This yields $A = 2$, $B = -2$, and $C = 3$. 1.0

Then 2.0

$$\begin{aligned}\textstyle\int_3^5 \frac{13 - 5x}{x^3 - 2x^2 + 2x + 5}\, dx &= \Big[2\ln|x + 1| - \ln(x^2 - 3x + 5)\Big]_3^5 \\ &= 2\ln(6) - \ln(15) - 2\ln(4) + \ln(5) \\ &= \ln(3) - 2\ln(2).\end{aligned}$$

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b POINTS · 4.0 POINTS · 2 CRITERIA

$$\int \sin^4(x)\cos^5(x)\, dx.$$

$\sin^4 x \cos^5 x = \sin^4 x(1 - \cos^2 x)^2 \cos x = (\sin^4 x - 2\sin^6 x + \sin^8 x)\cos x$, and therefore 2.0
with $u = \sin x$ we have
$\int \sin^4 x \cos^5 x dx = \int u^4 - 2u^6 + u^8 du$

2.0

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6 4.0 POINTS · 1 QUESTION
Exercise 6

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a POINTS · 4.0 POINTS · 3 CRITERIA
Determine the solution y of the differential equation
Bepaal de oplossing y van de differentiaalvergelijking

$$\frac{dy}{dx} = \frac{y^{2/3}}{(x-2)^2}, \qquad y(1) = 0.$$

With sepration of the variables: 2.0
$y^{-2/3}y' = \frac{1}{(x-2)^2} \quad \Longleftrightarrow \quad 3y^{1/3}(x) = \frac{-1}{x-2} + C.$

Since $y(1) = 0$ we find $C = -1$, and therefore 2.0

$$y(x) = -\frac{1}{27}\left(\frac{1}{x-2}+1\right)^3 = -\frac{1}{27}\left(\frac{x-1}{x-2}\right)^3.$$

Alternative 4.0
There is a second solution. The function $y(x) = 0$.
If a students finds this solution, then (s)he gets the full score.

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7 4.0 POINTS · 1 QUESTION
Exercise 7

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a POINTS · 4.0 POINTS · 4 CRITERIA
Show with the Mean Value Theorem
Bewijs met de Middelwaardestelling

$1 + 2\ln(a) < a^2 \qquad \text{for all / voor alle } a > 1.$

Hint: interval $[1, a]$.

Apply the Mean Value Theorem to $f(x) = 2\ln x - x^2$ on the interval $[1, a]$: 1.0
there exists $c \in (1, a)$ such that

$$\frac{f(a) - f(1)}{a-1} = f'(c),$$

So 1.0

$$\frac{2\ln a - a^2 + 1}{a-1} = \frac{2}{c} - 2c.$$

Because $c > 1$, we have $2/c - 2c < 0$, 1.0

and therefore 1.0

$$2\ln a - a^2 + 1 < 0.$$

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8 · 1 QUESTION
Extra Space

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a POINTS · 0.0 POINTS · 0 CRITERIA
Please indicate clearly on which exercise you are working.

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