Answer model Intermediate test Calculus B, version 2a

1. Determine the domain of the function $f(x) = \dfrac{1}{1 - \ln\sqrt{e^2(x+1)}}$.

   Answer: First of all the sqrt has to exist: $x \geq -1$, next the argument of ln has to be positive: $x > -1$. Finally the denominator of the fraction cannot be zero: $x + 1 \neq 1$, so the domain is the open interval $(-1, \infty)$ minus the point $x = 0$, or $D = (-1, 0) \cup (0, \infty)$.

2. A triangle ABC is given, with angles α, β and γ and opposite sides with lengths a, b en c. Furthermore we have $\sin\alpha = 3/5$, $b = 4$ and $c = 5$. Determine the two possible values of a.

   Answer: The cosine law gives $a^2 = b^2 + c^2 - 2bc\cos\alpha = 41 - 40\cos\alpha$. Since $\sin\alpha = 3/5$, and $\sin^2 + \cos^2 = 1$, we have $\cos\alpha = \pm 4/5$. It follows that $a^2 = 41 \mp 32$, so $a = 3$ or $a = \sqrt{73}$.

3. Determine $\displaystyle\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 9x + 14}$.

   Answer: Numerator and denominator vanish $x = 2$, so both must be divisible by $x - 2$. Canceling the factors $x - 2$ we find $\dfrac{x^2 + 2x + 4}{x - 7}$, which equals $-12/5$ for $x = 2$.

   An alternative approach is to use l'Hôpital, $\dfrac{3x^2}{2x - 9} \to -12/5$.

4. Determine the distance between the point $(1, 1, 1)$ and the plane with equation $2x + 3y + 6z = 18$.

   Answer: The line through $(1, 1, 1)$ perpendicular to the plane has vector representation $\underline{x} = (1, 1, 1)^\top + \lambda(2, 3, 6)^\top$. Inserting the coordinates of $\underline{x}(\lambda)$ in the equation of the plane gives $11 + 49\lambda = 18$, so $\lambda = 1/7$, the length of $(2, 3, 6)$ is 7, so the distance is 1. You can also of course also use the formula $d = \dfrac{|1 \cdot 2 + 1 \cdot 3 + 1 \cdot 6 - 18|}{\sqrt{2^2 + 3^2 + 6^2}}$.

Answer model Intermediate test Calculus B, version 2b

1. Determine the domain of the function $f(x) = \dfrac{1}{1 - \sqrt{\ln(x+e)}}$.

   Answer: First of all the sqrt has to exist: $x \geq 1 - e$, next the the denominator of the fraction cannot be zero: $x \neq 0$, so the domain is the set $[1-e, 0) \cup (0, \infty)$.

2. A triangle ABC is given, with angles α, β and γ and opposite sides with lengths a, b en c. Furthermore we have $\sin\alpha = 4/5$, $b = 6$ and $c = 5$. Determine the two possible values of a.

   Answer: The cosine law gives $a^2 = b^2 + c^2 - 2bc\cos\alpha = 61 - 60\cos\alpha$. Since $\sin\alpha = 4/5$, and $\sin^2 + \cos^2 = 1$, we have $\cos\alpha = \pm 3/5$. It follows that $a^2 = 61 \mp 36$, so $a = 5$ or $a = \sqrt{97}$.

3. Determine $\displaystyle\lim_{x\to-2} \frac{x^3+8}{x^2+9x+14}$.

   Answer: Numerator and denominator vanish for $x = -2$, so both must be divisible by $x+2$. Canceling the factors $x+2$ we find $\dfrac{x^2-2x+4}{x+7}$, which equals $12/5$ for $x = -2$.

   An alternative approach is to use l'Hôpital, $\dfrac{3x^2}{2x+9} \to 12/5$.

4. Determine the distance between the point $(1,1,1)$ and the plane with equation $x + 4y + 8z = 22$.

   Answer: The line through $(1,1,1)$ perpendicular to the plane has vector representation $\underline{x} = (1,1,1)^\top + \lambda(1,4,8)^\top$. Inserting the coordinates of $\underline{x}(\lambda)$ in the equation of the plane gives $13 + 81\lambda = 22$, so $\lambda = 1/9$, the length of $(1,4,8)$ is 9, so the distance is 1. You can also of course also use the formula $d = \dfrac{|1\cdot1 + 1\cdot4 + 1\cdot8 - 22|}{\sqrt{1^2+4^2+8^2}}$.