Answers Intermediate test Calculus, 2WBB3, Tuesday 4 October 2016, version 3

1. In view of the domain of $\sqrt{\cdot}$, we should have $x \geq 0$ and $x > 1$, so $x > 1$. Then solve the equality: $\frac{1+\sqrt{x}}{\sqrt{x-1}} = 3$
$1 + \sqrt{x} = 3\sqrt{x-1}$
$1 + 2\sqrt{x} + x = 9(x-1)$
$2\sqrt{x} = 8x - 10$
$\sqrt{x} = 4x - 5$
$x = 16x^2 - 40x + 25$
$16x^2 - 41x + 25 = 0$
$x = \dfrac{41 \pm \sqrt{1681 - 1600}}{32}$, so $x = 1$ and $x = \frac{25}{16}$
because of domain only $x = \frac{25}{16}$, answer $x > \frac{25}{16}$, or $x \in (\frac{25}{16}, \infty)$

2. Repeatedly finding a zero and division: $(x+1)^2(x-1)(x-2)$. For example, find zero $x = 1$ and divide ("staartdeling") $\frac{x^4-x^3-3x^2+x+2}{x-1} = x^3 - 3x - 2$. Then find $x = -1$ and divide $\frac{x^3-3x-2}{x+1} = x^2 - x - 2 = (x-2)(x+1)$.

3. Since this is a limit of the type $0/0$, we apply the square root trick to be able to divide out a factor h:

$$\lim_{h\to 0} \frac{\sqrt{1+5h}-1}{h} \cdot \frac{\sqrt{1+5h}+1}{\sqrt{1+5h}+1} = \lim_{h\to 0} \frac{1+5h-1}{h\left(\sqrt{1+5h}+1\right)} = \lim_{h\to 0} \frac{5}{\sqrt{1+5h}+1} = \frac{5}{2}$$

   Of course, l'Hôpital is also allowed.

4. The normal to the plane has the direction $(2, -2, 1)$. The line through $(1, 2, 3)$ in this direction has parametrization $(x, y, z) = (1 + 2t, 2 - 2t, 3 + t)$. Substitution in the plane gives $2(1 + 2t) - 2(2 - 2t) + 3 + t = 10$, so $t = 1$, with corresponding point $(3, 0, 4)$. The distance is the length of $(2, 2, 1)$, so 3.