

Answer model Intermediate test Calculus B, 3a

1. Solve the following inequality: $\frac{1}{2x-5} > \frac{2}{(x-1)(x-2)}$.

Answer: We ‘move’ everything to the left, turn it into a single fraction and factor the numerator:

$$\frac{(x-3)(x-4)}{(2x-5)(x-1)(x-2)} > 0.$$

We see that the left hand side changes sign for $x \in \{1, 2, 5/2, 3, 4\}$, and is positive on the set $(1, 2) \cup (5/2, 3) \cup (4, \infty)$.

2. Determine the centre of the ellipse with equation $x^2 + 6x + 4y^2 - 4y = 0$.

Answer: We complete the square and find $x^2 + 6x + 9 + 4y^2 - 4y + 1 = 9 + 1 = 10$, so $(x+3)^2 + 4(y-1/2)^2 = 10$, giving the centre $(-3, 1/2)$.

3. Determine $\lim_{x \rightarrow 0} \frac{2x + \sin x}{\tan(4x)}$.

Answer: We divide numerator and denominator by x and use the standard limits for $\sin(x)/x$ and $\tan(4x)/(4x)$, so we get $(2+1)/4 = 3/4$.
Alternative: l'Hôpital $(2 + \cos(x)) \cos^2(4x)/4 \rightarrow 3/4$.

4. Determine an equation of the form $ax + by + cz = d$ for the plane with parametric representation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

Answer: We may compute the triple (a, b, c) using the cross product $(0, 1, 1)^\top \times (2, 1, 0)^\top = (-1, 2, -2)^\top$ and then d from $-1 \cdot 1 + 2 \cdot 1 + -2 \cdot 1 = -1$, so we find $-x + 2y - 2z = -1$, or if you prefer $x - 2y + 2z = 1$.

Answer model Intermediate test Calculus B, 3b

1. Solve the following inequality: $\frac{1}{2x+5} > \frac{-2}{(x+1)(x+2)}$.

Answer: We ‘move’ everything to the left, turn it into a single fraction and factor the numerator:

$$\frac{(x+3)(x+4)}{(2x+5)(x+1)(x+2)} > 0.$$

We see that the left hand side changes sign for $x \in \{-4, -3, -5/2, -2, -1\}$, and is positive on the set $(-4, -3) \cup (-5/2, -2) \cup (-1, \infty)$.

2. Determine the centre of the ellipse with equation $4x^2 - 4x + y^2 + 6y = 0$.

Answer: We complete the square and find $4x^2 - 4x + 1 + y^2 + 6y + 9 = 1 + 9 = 10$, so $4(x - 1/2)^2 + (y + 3)^2 = 10$, giving the centre $(1/2, -3)$.

3. Determine $\lim_{x \rightarrow 0} \frac{x + 3 \sin x}{\tan(2x)}$.

Answer: We divide numerator and denominator by x and use the standard limits for $\sin(x)/x$ and $\tan(2x)/(2x)$, so we get $(1 + 3)/2 = 2$.

Alternative: l'Hôpital $(1 + 3 \cos(x)) \cos^2(2x)/2 \rightarrow 2$.

4. Determine an equation of the form $ax + by + cz = d$ for the plane with parametric representation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$.

Answer: We may compute the triple (a, b, c) using the cross product $(1, 0, 1)^\top \times (0, 2, 1)^\top = (-2, -1, 2)^\top$ and then d from $-2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = -1$, so we find $-2x - y + 2z = -1$, or if you prefer $2x + y - 2z = 1$.