

**Answers Intermediate test Calculus, 2WBB3, Monday 3 October 2016, version 2**

1. In view of the domain of  $\sqrt{\cdot}$ , we should have  $x \geq 3$ . Squaring  $\sqrt{x-3} = x-5$  gives  $x-3 = x^2-10x+25$ , so  $x^2-11x+28 = (x-4)(x-7) = 0$ , so  $x = 4$  or  $x = 7$ . Checking of these values shows that only  $x = 7$  is a solution. Checking some values for  $x$ , such as  $x = 4$  and  $x = 8$  gives the answer  $x \in [3, 7)$ .
2. Completing the square:  $x^2-6x+9-9+y^2+8y+16-16 \geq 0$  so  $(x-3)^2+(y+4)^2 \geq 25$ . This is everything outside (and including the border of) the circle with center  $(3, -4)$  and radius 5, which passes through the origin  $(0, 0)$ .
3. Since this is a limit of the type  $\infty/\infty$ , we divide by the largest quantity of the denominator:

$$\lim_{x \rightarrow \infty} \frac{\frac{7}{x} - 7 + \frac{\cos(7x)}{x}}{7 - \frac{\sin(7x)}{x}} = \frac{0 - 7 + 0}{7 - 0} = -1.$$

Hereby we have used the following: since  $-\frac{1}{x} \leq \frac{\sin(7x)}{x} \leq \frac{1}{x}$ , we have by the squeeze theorem  $\lim_{x \rightarrow \infty} \frac{\sin(7x)}{x} = 0$ , and the same for  $\lim_{x \rightarrow \infty} \frac{\cos(7x)}{x} = 0$ .

4. The normal to the plane has the direction  $(1, 2, -2)$ . The line through  $(1, -1, 1)$  in this direction has parametrization  $(x, y, z) = (1+t, -1+2t, 1-2t)$ . Substitution in the plane gives  $1+t+2(-1+2t)-2(1-2t)=6$ , so  $t=1$ , with corresponding point  $(2, 1, -1)$ . The distance is the length of  $(1, 2, -2)$ , so 3.