About
Exercises
Criteria
Objectives
Review
Results
Analytics

Export

Criteria

10 exercises · 50.0 points

(1) 20.0 POINTS · 8 QUESTIONS
Exercise 1 (Multiple Choice Questions)

New version available

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(a) MULTIPLE CHOICE · 2.5 POINTS
$\tan(\arccos(-\frac{5}{9})) =$

○ $\frac{1}{5}\sqrt{14}$	0.0
○ $\frac{1}{9}\sqrt{56}$	0.0
○ $\frac{56}{81}$	0.0
○ $-\frac{2}{5}\sqrt{14}$	2.5

(b) MULTIPLE CHOICE · 2.5 POINTS
The slope of the tangent line at point $(-3,1)$ of curve $x^2y+xy^2=6$ is:
De richtingsco\"efficient van de raaklijn in het punt $(-3,1)$ aan de kromme $x^2y+xy^2=6$ is:

○ 1	0.0
○ $-\frac{3}{5}$	0.0
○ $\frac{5}{3}$	2.5
○ -2	0.0

Jump to
In $x=1$ the piecewise defined function
In $x=1$ is de stuksgewijs gedefinieerde functie

$$f(x)=\begin{cases} x^2+3x-4 \text{ if } x>1 \\ x^2+2x-3 \text{ if } x\leq 1 \end{cases}$$

○ \begin{minipage}{6.5cm}is not continuous and not differentiable \\ niet continue en niet differentieerbaar\\\end{minipage}	0.0
○ \begin{minipage}{6.5cm}is continuous but not differentiable \\ continue, maar niet differentieerbaar\\\end{minipage}	2.5
○ \begin{minipage}{6.5cm}is not continuous but differentiable\\ niet continu maar wel differentieerbaar\end{minipage}	0.0
○ \begin{minipage}{6.5cm} is continuous and differentiable\\ continu en differentieerbaar\end{minipage}	0.0

(d) MULTIPLE CHOICE · 2.5 POINTS
The derivative of $\arctan(x^2)-\arctan(\frac{1}{x^2})$ with respect to x equals:
De afgeleide van $\arctan(x^2)-\arctan(\frac{1}{x^2})$ naar x is gelijk aan:

○ $\frac{4x}{1+x^4}$	2.5
○ 0	0.0
○ $\frac{2x+2x^3}{1+x^4}$	0.0
○ $\frac{2x-2x^3}{1+x^4}$	0.0

(e) MULTIPLE CHOICE · 2.5 POINTS
Linearisation of $f(x)=\sqrt{x}$ in $x=49$ yields the following approximation of $\sqrt{50}$:
Linearisatie van $f(x)=\sqrt{x}$ in $x=49$ geeft de volgende benadering van $\sqrt{50}$:

○ 7	0.0
○ $\frac{50}{7}$	0.0
○ $\frac{99}{14}$	2.5
○ $\frac{197}{28}$	0.0

(f) MULTIPLE CHOICE · 2.5 POINTS
For $0\leq x<1$ we have
Voor $0\leq x<1$ geldt

Then $f'(x) =$
Dan is $f'(x) =$

- x — 0.0
- $x\cos(x)$ — 2.5
- $\frac{x}{\sqrt{1-x^2}}$ — 0.0
- $x\sin(x)$ — 0.0

(g) MULTIPLE CHOICE · 2.5 POINTS

The plane V is given by the vector representation
Het vlak V is gegeven door parameter voorstelling

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}.$$

The distance of the piont $(4,2,3)$ to V equals
De afstand van het punt $(4,2,3)$ tot V is gelijk aan

- 3 — 2.5
- $\sqrt{14}$ — 0.0
- $\sqrt{5}$ — 0.0
- 2 — 0.0

(h) MULTIPLE CHOICE · 2.5 POINTS

$\int^{e^{\circ}} \quad 1 \quad ,$

Jump to

- $\ln(\frac{1}{4})$ — 0.0
- $\ln(\frac{1}{3})$ — 0.0
- $\ln(3)$ — 2.5
- $\ln(4)$ — 0.0

2 · 3.0 POINTS · 1 QUESTION

Exercise 2

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(a) POINTS · 3.0 POINTS · 4 CRITERIA

Give a vector representation for the intersection line ℓ of the planes V, given by the equation\\ $3x-2y+4z=5$, and W, given by the equation $x+y+3z=5$.
Geef een vectorvoorstelling voor de snijlijn ℓ van de vlakken V, gegeven door de vergelijking\\ $3x-2y+4z=5$, en W, gegeven door de vergelijking $x+y+3z=5$.

A direction vector of ℓ is a vector perpendicular to the normal vectors $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ of V and $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ of W. — 1.0

So, any nonzero scalar multiple of $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ can be used as direction vector. — 1.0

The point $(1,1,1)$ is on both planes and hence on ℓ. — 1.0
Thus a vector representation for ℓ is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \text{ with } \lambda \in \mathbb{R}.$$

ALTERNATIVE — 3.0
Let $z=\lambda$.
Then $3x-2y=5-4\lambda$, and $x+y=5-3\lambda$.
So, $5x=15-10\lambda$ and $x=3-2\lambda$.
But then $y=5-3\lambda-x=2-\lambda$.
So, we have

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \text{ with } \lambda \in \mathbb{R}.$$

+ ADD POINTS | + COPY POINTS

3 · 4.0 POINTS · 2 QUESTIONS · STARTS AT NEW PAGE

Exercise 3

Ⓐ POINTS · 2.0 POINTS · 2 CRITERIA
Show that the function f is one-to-one.
Bewijs dat de functie f injectief is.

$f'(x) = 2x + \cos(x) - x\sin(x).$ 1.0

Now $x(2-\sin(x)) + \cos(x) \geq x + \cos(x) > 0$ for $x \geq 0$.
So, f is strictly increasing and hence one-to-one. 1.0

+ ADD POINTS | + COPY POINTS

Ⓑ POINTS · 2.0 POINTS · 2 CRITERIA
Determine $(f^{-1})'(\frac{\pi^2}{4})$.
Bepaal $(f^{-1})'(\frac{\pi^2}{4})$.

$f(\frac{\pi}{2}) = \frac{\pi^2}{4}$. 1.0

So, $(f^{-1})'(\frac{\pi^2}{4}) = \frac{1}{f'(\frac{\pi}{2})} = \frac{1}{\pi - \pi/2} = \frac{2}{\pi}$. 1.0

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④ 8.0 POINTS · 2 QUESTIONS
Exercise 4

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Ⓐ POINTS · 4.0 POINTS · 2 CRITERIA
$\lim_{x\to 0} \frac{e^{(x^4)} - \cos(x^2)}{x^2(\cos(x)-1)}$.

$$\lim_{x\to 0} \frac{e^{(x^4)} - \cos(x^2)}{x^2(\cos(x)-1)} = \lim_{x\to 0} \frac{1 + x^4 + \mathcal{O}(x^8) - (1 - \frac{x^4}{2!} + \mathcal{O}(x^6))}{x^2(1 - \frac{x^2}{2!} - 1 + \mathcal{O}(x^4))}$$ 3.0

Jump to

(one point for each correct Taylor polynomial)

$$= \lim_{x\to 0} \frac{\frac{3x^4}{2} + \mathcal{O}(x^6)}{-\frac{x^4}{2} + \mathcal{O}(x^6)} = -3$$ 1.0

+ ADD POINTS | + COPY POINTS

Ⓑ POINTS · 4.0 POINTS · 4 CRITERIA
$\lim_{x\to\infty} \frac{\ln(f(x))}{x^2 - x}$,
where f is a function satisfying $\frac{1}{2}e^{3x^2-x} \leq f(x) \leq 2e^{3x^2+5x}$ for all $x > 0$.
waarbij f een functie is met $\frac{1}{2}e^{3x^2-x} \leq f(x) \leq 2e^{3x^2+5x}$ voor alle $x > 0$.

Since 1.0

$$\lim_{x\to\infty} \frac{\ln(\frac{1}{2}e^{3x^2-x})}{x^2 - x} = \lim_{x\to\infty} \frac{3x^2 - x - \ln(2)}{x^2 - x} = \lim_{x\to\infty} \frac{3 - 1/x - \ln(2)/x^2}{1 - 1/x} = 3$$

and 1.0

$$\lim_{x\to\infty} \frac{\ln(2e^{3x^2+5x})}{x^2 + x} = \lim_{x\to\infty} \frac{3x^2 + 5x + \ln(2)}{x^2 - x} = \lim_{x\to\infty} \frac{3 + 5/x + \ln(2)/x^2}{1 - 1/x} = 3,$$

we find, using the Squeeze Theorem, that 1.0

$$\lim_{x\to\infty} \frac{\ln(f(x))}{3x^2 - x} = 3$$ 1.0

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⑤ 8.0 POINTS · 2 QUESTIONS · STARTS AT NEW PAGE
Exercise 5

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Ⓐ POINTS · 4.0 POINTS · 4 CRITERIA
$\int_0^{\pi} x^2 \cos(x)\, dx$.

Integration by parts yields 1.0

$$= x^2\sin(x) - (2x(-\cos(x)) + \int 2\cos(x)dx)$$

1.0

$$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C.$$

1.0

So

$$\int_0^\pi x^2\cos(x)\,dx = [x^2\sin(x) + 2x\cos(x) - 2\sin(x)]_0^\pi = -2\pi.$$

1.0

+ ADD POINTS + COPY POINTS

(b) POINTS · 4.0 POINTS · 3 CRITERIA

$\int_0^\infty \frac{1}{x^2+3x+2}\,dx.$

If $\frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$, then $A(x+2) + B(x+1) = (A+B)x + (2A+B) = 1$.
So, $B = -A$ and $2A + B = A = 1$. We find $A = 1$ and $B = -1$.

1.0

So,

$$\begin{aligned}\int \frac{1}{x^2+3x+2}dx &= \int \tfrac{1}{x+1} - \tfrac{1}{x+2}dx \\ &= \ln(x+1) - \ln(x+2) + C\end{aligned}$$

1.0

So

$$\begin{aligned}\int_0^\infty \frac{1}{x^2+3x+2}dx &= [\ln(\tfrac{x+1}{x+2})]_0^\infty \ (1pt) \\ &= \lim_{x\to\infty}\ln(\frac{x+1}{x+2}) - \ln(\frac{1}{2}) \\ &= \ln(1) - \ln(\tfrac{1}{2}) = \ln(2).\ (1pt)\end{aligned}$$

2.0

+ ADD POINTS + COPY POINTS

(6) 3.0 POINTS · 1 QUESTION

Exercise 6

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(a) POINTS · 3.0 POINTS · 3 CRITERIA

Show that for all a, b in $\mathbb{R}$ we have:
Bewijs dat voor alle a, b in $\mathbb{R}$ geldt dat

$$|\sin^2(b) - \sin^2(a)| \leq |b - a|.$$

Hint: Mean-Value Theorem.

Without loss of generality we can assume $a < b$.
Consider the function $f(x) = \sin^2(x)$ on the interval $[a, b]$.
Then the Mean-Value Theorem states that $\frac{f(b)-f(a)}{b-a} = f'(c)$ for some $c \in (a, b)$.

1.0

But $|f'(c)| = |2\sin(c)\cos(c)| = |\sin(2c)| \leq 1$.

1.0

So, we find

$$|\frac{\sin^2(b) - \sin^2(a)}{b-a}| \leq 1,$$

which clearly implies $|\sin^2(b) - \sin^2(a)| \leq |b - a|$.

1.0

+ ADD POINTS + COPY POINTS

(7) 4.0 POINTS · 1 QUESTION

Exericse 7

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(a) POINTS · 4.0 POINTS · 3 CRITERIA

Find a solution to the following differential equation satisfying the given initial condition:
Vind een oplossing voor de volgende differentiaalvergelijking die voldoet aan de gegeven beginvoorwaarde:

$$\frac{dy}{dx} = x^2 - 1 + \frac{y}{x+1}, \quad y(0) = 2.$$

This is a linear differential equation.
We first consider the homogeneous part:

$$y' = \frac{y}{x+1}.$$

1.0

Now we use variation of the constant.
So, we try $y = C(x)(x+1)$ and find

$$y' = C(x) + C'(x)(x+1) = x^2 - 1 + C(x).$$

We deduce that $C'(x) = \frac{x^2-1}{x+1} = x - 1,$ so $C(x) = \frac{1}{2}x^2 - x + D$ and $y(x) = (\frac{1}{2}x^2 - x + D)(x+1).$ 2.0

Since $y(0) = D = 2$ we find $y(x) = (\frac{1}{2}x^2 - x + 2)(x+1).$ 1.0

+ ADD POINTS
+ COPY POINTS

8 · 1 QUESTION · STARTS AT NEW PAGE
Extra Space

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a POINTS · 0.0 POINTS · 0 CRITERIA
Please indicate clearly on which exercise you are working.

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