REPORT

Zajęcia: Analog and digital electronic circuits
Teacher: prof. dr hab. Vasyl Martsenyuk

Lab 1
29.09.2023
Topic: Spectral Analysis of Deterministic Signals
Variant 1

Imię Nazwisko
Informatyka II stopień,
stacjonarne,
1 semestr,
Gr.1B

1. Problem statement:

The objective is to use discrete Fourier transform and its implementation with the help of matrix multiplication. Synthesize a discrete-time signal by using the IDFT in matrix notation for different values of N. Show the matrices W and K. Plot the signal synthesized.

2. Input data:

$$\mathbf{x}_\mu = [6, 2, 4, 3, 4, 5, 0, 0, 0, 0]^{\mathrm{T}}$$

3. Commands used (or GUI):

Remote repository: https://github.com/TomaszSteblik/Aadec_1

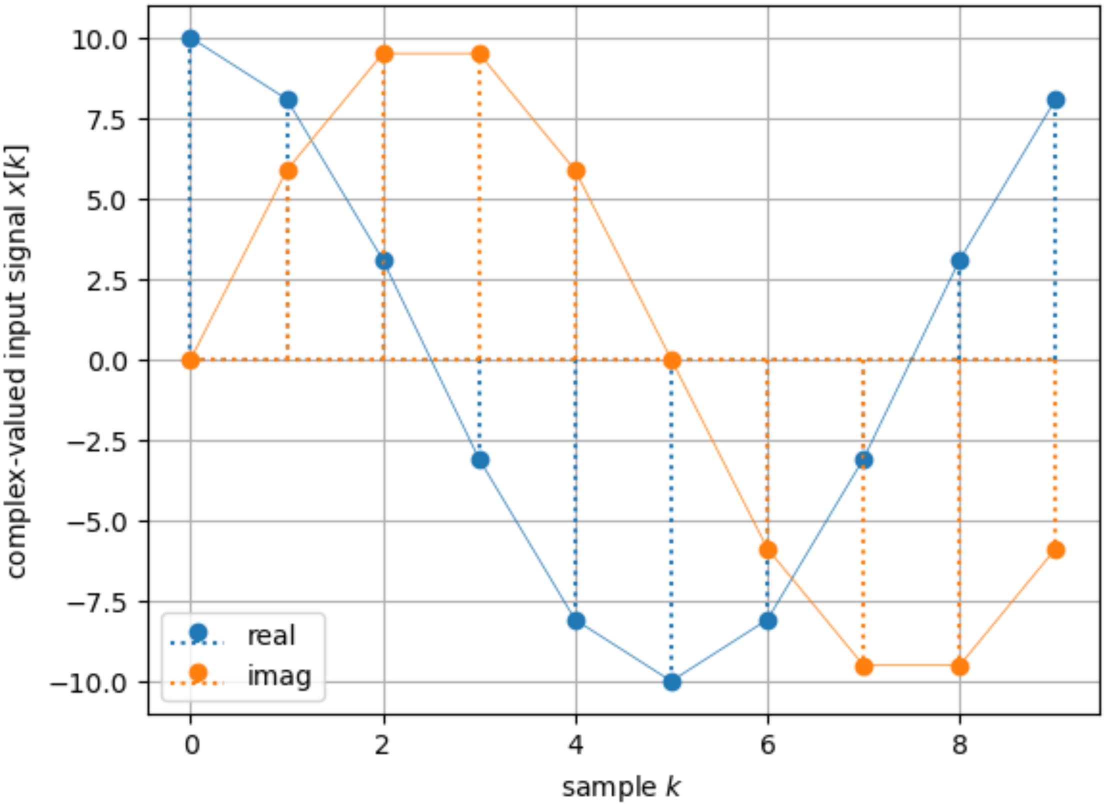
a) source code

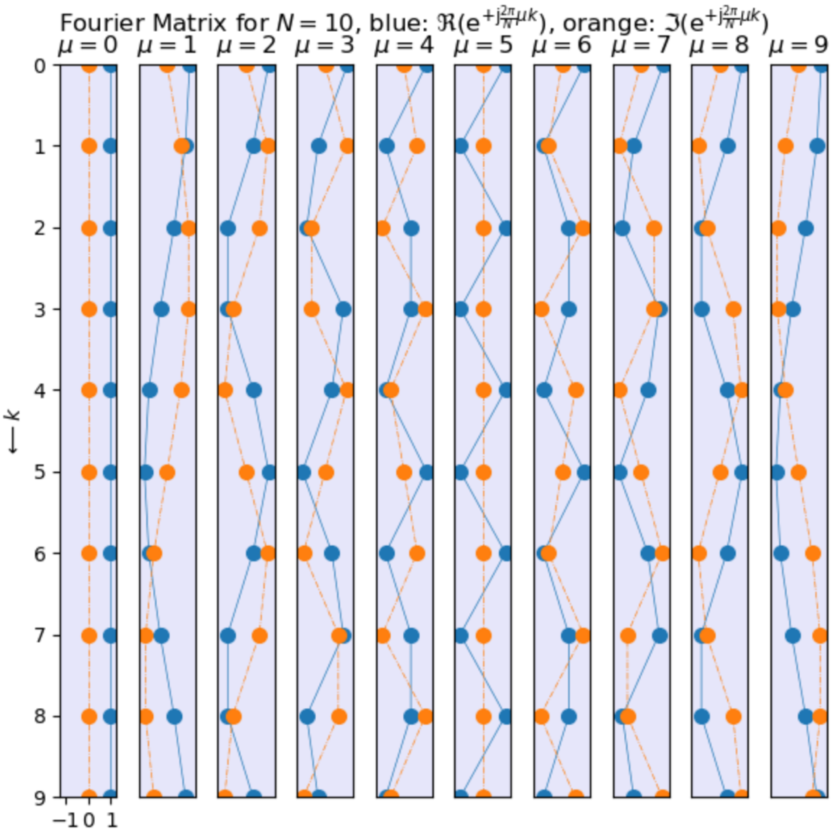
Untitled-2.py — Edytowany

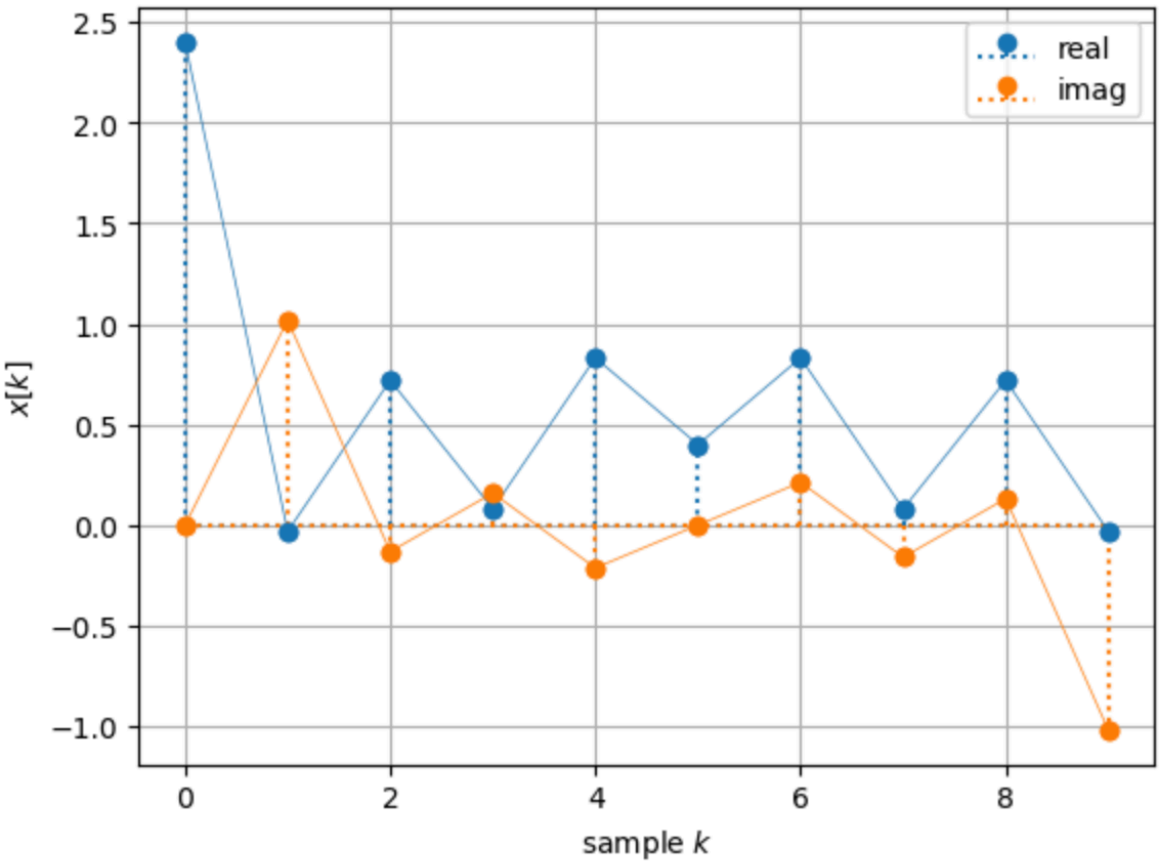
```python
import numpy as np
import matplotlib.pyplot as plt
from numpy.linalg import inv
from numpy.fft import fft, ifft

N = 10  # signal block length
k = np.arange(N)  # all required sample/time indices
A = 10  # signal amplitude
tmpmu = 2-1/2  # DFT eigenfrequency worst case
tmpmu = 1  # DFT eigenfrequency best case
x = A * np.exp(tmpmu * +1j*2*np.pi/N * k)
plt.stem(k, np.real(x), markerfmt='C0o',
         basefmt='C0:', linefmt='C0:', label='real')
plt.stem(k, np.imag(x), markerfmt='C1o',
         basefmt='C1:', linefmt='C1:', label='imag')
plt.plot(k, np.real(x), 'C0-', lw=0.5)
plt.plot(k, np.imag(x), 'C1-', lw=0.5)
plt.xlabel(r'sample $k$')
plt.ylabel(r'complex-valued input signal $x[k]$')
plt.legend()
plt.grid(True)
X_ = np.zeros((N, 1), dtype=complex)  # alloc RAM, init with zeros
for mu_ in range(N):  # do for all DFT frequency indices
    for k_ in range(N):  # do for all sample indices
        X_[mu_] += x[k_] * np.exp(-1j*2*np.pi/N*k_*mu_)
x_ = np.zeros((N, 1), dtype=complex)  # alloc RAM, init with zeros
for k_ in range(N):
    for mu_ in range(N):
        x_[k_] += X_[mu_] * np.exp(+1j*2*np.pi/N*k_*mu_)
x_ *= 1/N  # normalization in the IDFT stage
mu = np.arange(N)
K = np.outer(k, mu)  # get all possible entries k*mu in meaningful arrangement
W = np.exp(+1j * 2*np.pi/N * K)  # analysis matrix for DFT
fig, ax = plt.subplots(1, N)
fig.set_size_inches(6, 6)
fig.suptitle(
    r'Fourier Matrix for $N=$%d, blue: $\Re(\mathrm{e}^{+\mathrm{j} \frac{2\pi}{N} \mu k})$,
orange: $\Im(\mathrm{e}^{+\mathrm{j} \frac{2\pi}{N} \mu k})$' % N)
for tmp in range(N):
    ax[tmp].set_facecolor('lavender')
    ax[tmp].plot(W[:, tmp].real, k, 'C0o-', ms=7, lw=0.5)
    ax[tmp].plot(W[:, tmp].imag, k, 'C1o-.', ms=7, lw=0.5)
    ax[tmp].set_ylim(N-1, 0)
    ax[tmp].set_xlim(-5/4, +5/4)
    if tmp == 0:
        ax[tmp].set_yticks(np.arange(0, N))
        ax[tmp].set_xticks(np.arange(-1, 1+1, 1))
        ax[tmp].set_ylabel(r'$\longleftarrow k$')
    else:
        ax[tmp].set_yticks([], minor=False)
        ax[tmp].set_xticks([], minor=False)
    ax[tmp].set_title(r'$\mu=$%d' % tmp)
fig.tight_layout()
fig.subplots_adjust(top=0.91)
fig.savefig('fourier_matrix.png', dpi=300)
X_test = np.array([6, 2, 4, 3, 4, 5, 0, 0, 0, 0])
# x_test = 1/N*W@X_test  # >= Python3.5
x_test = 1/N * np.matmul(W, X_test)
plt.stem(k, np.real(x_test), label='real',
         markerfmt='C0o', basefmt='C0:', linefmt='C0:')
plt.stem(k, np.imag(x_test), label='imag',
         markerfmt='C1o', basefmt='C1:', linefmt='C1:')
plt.plot(k, np.real(x_test), 'C0o-', lw=0.5)
plt.plot(k, np.imag(x_test), 'C1o-', lw=0.5)
plt.xlabel(r'sample $k$')
plt.ylabel(r'$x[k]$')
plt.legend()
plt.grid(True)
print(np.allclose(ifft(X_test), x_test))
print('DC is 1 as expected: ', np.mean(x_test))
x_test2 = X_test[0] * W[:, 0] + X_test[1] * W[:, 1] + X_test[2] * W[:, 2]
x_test2 *= 1/N
print(np.allclose(x_test, x_test2))  # check with result before
W
K
```

b) screenshots







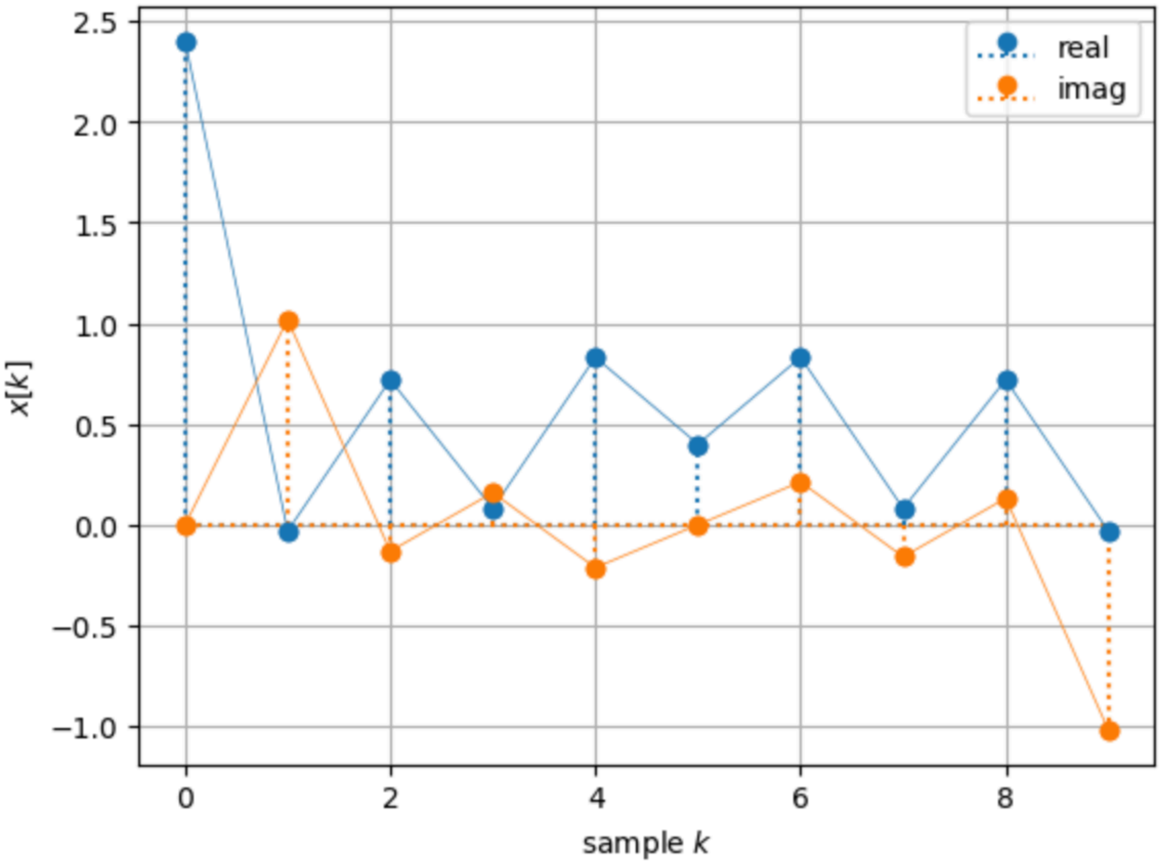
4. Outcomes:

```
In [63]: K

Out[63]: array([[ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
                [ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
                [ 0,  2,  4,  6,  8, 10, 12, 14, 16, 18],
                [ 0,  3,  6,  9, 12, 15, 18, 21, 24, 27],
                [ 0,  4,  8, 12, 16, 20, 24, 28, 32, 36],
                [ 0,  5, 10, 15, 20, 25, 30, 35, 40, 45],
                [ 0,  6, 12, 18, 24, 30, 36, 42, 48, 54],
                [ 0,  7, 14, 21, 28, 35, 42, 49, 56, 63],
                [ 0,  8, 16, 24, 32, 40, 48, 56, 64, 72],
                [ 0,  9, 18, 27, 36, 45, 54, 63, 72, 81]])
```

In [62]:
```
W
```

Out[62]:
```
array([[ 1.        +0.00000000e+00j,  1.        +0.00000000e+00j,
         1.        +0.00000000e+00j,  1.        +0.00000000e+00j,
         1.        +0.00000000e+00j,  1.        +0.00000000e+00j,
         1.        +0.00000000e+00j,  1.        +0.00000000e+00j,
         1.        +0.00000000e+00j,  1.        +0.00000000e+00j],
       [ 1.        +0.00000000e+00j,  0.80901699+5.87785252e-01j,
         0.30901699+9.51056516e-01j, -0.30901699+9.51056516e-01j,
        -0.80901699+5.87785252e-01j, -1.        +1.22464680e-16j,
        -0.80901699-5.87785252e-01j, -0.30901699-9.51056516e-01j,
         0.30901699-9.51056516e-01j,  0.80901699-5.87785252e-01j],
       [ 1.        +0.00000000e+00j,  0.30901699+9.51056516e-01j,
        -0.80901699+5.87785252e-01j, -0.80901699-5.87785252e-01j,
         0.30901699-9.51056516e-01j,  1.        -2.44929360e-16j,
         0.30901699+9.51056516e-01j, -0.80901699+5.87785252e-01j,
        -0.80901699-5.87785252e-01j,  0.30901699-9.51056516e-01j],
       [ 1.        +0.00000000e+00j, -0.30901699+9.51056516e-01j,
        -0.80901699-5.87785252e-01j,  0.80901699-5.87785252e-01j,
         0.30901699+9.51056516e-01j, -1.        +3.67394040e-16j,
         0.30901699-9.51056516e-01j,  0.80901699+5.87785252e-01j,
        -0.80901699+5.87785252e-01j, -0.30901699-9.51056516e-01j],
       [ 1.        +0.00000000e+00j, -0.80901699+5.87785252e-01j,
         0.30901699-9.51056516e-01j,  0.30901699+9.51056516e-01j,
        -0.80901699-5.87785252e-01j,  1.        -4.89858720e-16j,
        -0.80901699+5.87785252e-01j,  0.30901699-9.51056516e-01j,
         0.30901699+9.51056516e-01j, -0.80901699-5.87785252e-01j],
       [ 1.        +0.00000000e+00j, -1.        +1.22464680e-16j,
         1.        -2.44929360e-16j, -1.        +3.67394040e-16j,
         1.        -4.89858720e-16j, -1.        +6.12323400e-16j,
         1.        -7.34788079e-16j, -1.        +8.57252759e-16j,
         1.        -9.79717439e-16j, -1.        +1.10218212e-15j],
       [ 1.        +0.00000000e+00j, -0.80901699-5.87785252e-01j,
         0.30901699+9.51056516e-01j,  0.30901699-9.51056516e-01j,
        -0.80901699+5.87785252e-01j,  1.        -7.34788079e-16j,
        -0.80901699-5.87785252e-01j,  0.30901699+9.51056516e-01j,
         0.30901699-9.51056516e-01j, -0.80901699+5.87785252e-01j],
       [ 1.        +0.00000000e+00j, -0.30901699-9.51056516e-01j,
        -0.80901699+5.87785252e-01j,  0.80901699+5.87785252e-01j,
         0.30901699-9.51056516e-01j, -1.        +8.57252759e-16j,
         0.30901699+9.51056516e-01j,  0.80901699-5.87785252e-01j,
        -0.80901699-5.87785252e-01j, -0.30901699+9.51056516e-01j],
       [ 1.        +0.00000000e+00j,  0.30901699-9.51056516e-01j,
        -0.80901699-5.87785252e-01j, -0.80901699+5.87785252e-01j,
         0.30901699+9.51056516e-01j,  1.        -9.79717439e-16j,
         0.30901699-9.51056516e-01j, -0.80901699-5.87785252e-01j,
        -0.80901699+5.87785252e-01j,  0.30901699+9.51056516e-01j],
       [ 1.        +0.00000000e+00j,  0.80901699-5.87785252e-01j,
         0.30901699-9.51056516e-01j, -0.30901699-9.51056516e-01j,
        -0.80901699-5.87785252e-01j, -1.        +1.10218212e-15j,
        -0.80901699+5.87785252e-01j, -0.30901699+9.51056516e-01j,
         0.30901699+9.51056516e-01j,  0.80901699+5.87785252e-01j]])
```



5. Conclusions:

In summary, at less technical effort the reproducibility of DSP is higher compared to analog signal processing. DSP system are typically cheaper compared to their analog counterparts. This is due to the tremendous efforts that have been spend in the last decades in the development and production of digital circuits.