Supplementary Material for "Constraints Always Satisfied Parameters (CASPs) for Fuzzy Sets Optimisation"

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S-I. Supplementary Simulations and Results

A. Supplementary Results using Gradient-Based Optimisation

This section presents supplementary results on the Laser dataset obtained with the gradient-based optimiser (Adam) described in Section IV. Figure S-1 illustrates the initial and optimised membership functions.

B. Results Using Non-Gradient-Based Optimisation

To further assess the generality of the CASPs framework, additional experiments were performed using non-gradient-based optimisers, including the Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO). Both methods were implemented using their respective R packages [1, 2], with all parameters kept in their default settings except for population size and maximum number of iterations, which were set to 30 and 100, respectively, to complete the experiments in a reasonable time. In addition, only one run was conducted for each optimisation method on each dataset. Note that the aim of these experiments is not to achieve the best possible performance but to illustrate the behaviour of the three CASPs variants, that constraints can always be satisfied. Figures S-2, S-3, S-4, and S-5 illustrate the initial and optimised MFs corresponding to the three CASPs variants, obtained on the Electricity and Laser datasets using the GA and PSO optimisers.

C. More Comprehensive Comparisons

This section provides a comprehensive comparison of the proposed CASPs framework against existing fuzzy and non-fuzzy methods from the literature. For each dataset, the results obtained using the three variants of CASPs under three optimisation methods are summarised. Representative baseline results from the literature are also included for reference, covering both conventional and fuzzy-based models, including the Multi-Layer Perceptron (MLP), Linear Regression (LR), k-Nearest Neighbours (KNN), Hierarchical Prototype-based Fuzzy Neural Network (HPFNN), and the recently proposed Self-Constructing Fuzzy System based on Interval Fuzzy Rules (SCFS-IFRs) [3, 4].

In addition, two more datasets from the UCI Machine Learning Repository [5] were used to further examine the empirical behaviour of the three variants of CASPs.

1) Airfoil Dataset: It contains 1,503 samples generated from aerodynamic simulations, where the goal is to predict the scaled sound pressure level based on five input variables related to frequency, angle of attack, chord length, free-stream velocity, and suction side displacement thickness [6]. The data set consists of five inputs and one output. The number of trapezoidal MFs used in the simulations was configured as follows:

- Inputs: 7, 5, 5, 3, and 7 MFs, respectively.
- Output: 3 MFs.

The rule base consisted of 334 rules, generated using the same self-implemented rule generation approach as described in Section IV.

2) Concrete Compressive Strength Dataset: It consists of 1,030 samples and eight input variables describing the composition of concrete (cement, water, coarse and fine aggregate, fly ash, etc.), with one output representing the compressive strength [7]. As discussed in [8], fuzzy systems have difficulty handling high-dimensional input spaces, the input dimensionality was reduced from eight to five using principal component analysis (PCA). The number of trapezoidal MFs was set as:

- Inputs: 5, 5, 3, 5, and 7 MFs, respectively.
- Output: 3 MFs.

The rule base consisted of 349 rules.

The results of the simulations on four datasets are summarised in Tables S-I–S-IV. It is worth noting that the main advantage of the proposed CASPs framework lies in the fact that all predefined constraints can always be satisfied during the optimisation process. Hence, the focus of this work is not on achieving the best possible performance but on demonstrating that the optimisation can be conducted without violating any constraints, regardless of the optimisation method used. Nevertheless, the results show that even with simply defined membership functions and rule bases, the proposed approach can still achieve reasonably competitive performance compared to existing methods in the literature. In particular, on the

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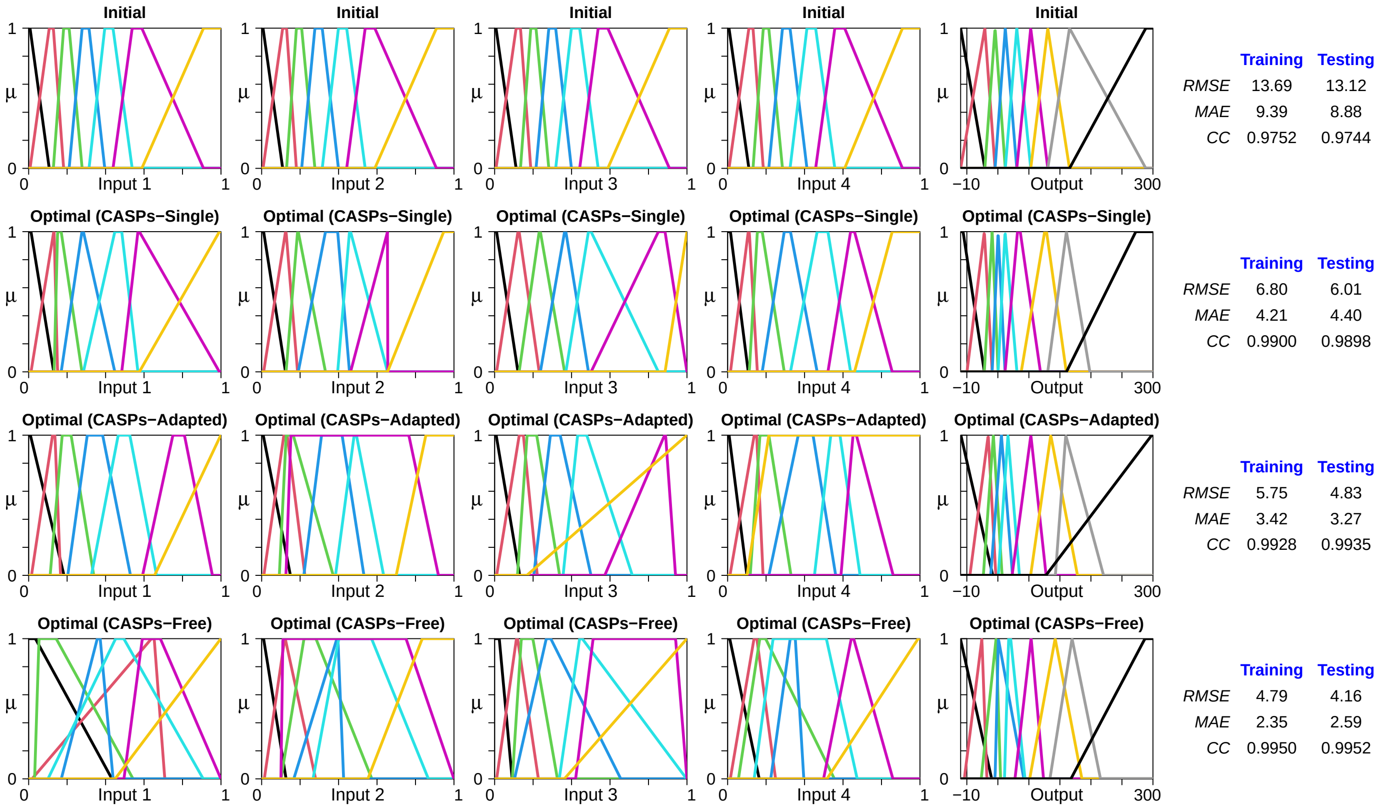

Fig. S-1. The initial and optimised MF plots for the best runs, obtained on the **Laser** dataset using the **Adam** optimiser.

Laser dataset, the CASPs-Free variant optimised using Adam achieved the best performance among all compared methods, including those reported in the literature. These findings further support the consistent pattern previously observed—that a higher degree of freedom in the parameterisation potentially yields better performance—and confirm that this trend holds across different datasets.

References

[1] L. Scrucca, "GA: A Package for Genetic Algorithms in R," *Journal of Statistical Software*, vol. 53, no. 4, pp. 1–37, 2013.

[2] C. Bendtsen, "pso: Particle Swarm Optimization," *CRAN: Contributed Packages*, 2022.

[3] C. Zhang, S.-K. Oh, and Z. Fu, "Hierarchical polynomial-based fuzzy neural networks driven with the aid of hybrid network architecture and ranking-based neuron selection strategies," *Applied Soft Computing*, vol. 113, p. 107865, 2021.

[4] W. Zhou, H. Li, and M. Bao, "Stochastic Configuration Based Fuzzy Inference System with Interpretable Fuzzy Rules and Intelligence Search Process," *Mathematics*, vol. 11, no. 3, 2023.

[5] M. Kelly, R. Longjohn, and K. Nottingham, "The UCI machine learning repository," 2023.

[6] P. D. Brooks Thomas and M. Marcolini, "Airfoil Self-Noise," UCI Machine Learning Repository, 1989.

[7] I.-C. Yeh, "Concrete Compressive Strength," UCI Machine Learning Repository, 1998.

[8] D. Wu, Y. Yuan, J. Huang, and Y. Tan, "Optimize TSK Fuzzy Systems for Regression Problems: Minibatch Gradient Descent With Regularization, DropRule, and AdaBound (MBGD-RDA)," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 5, pp. 1003–1015, 2020.

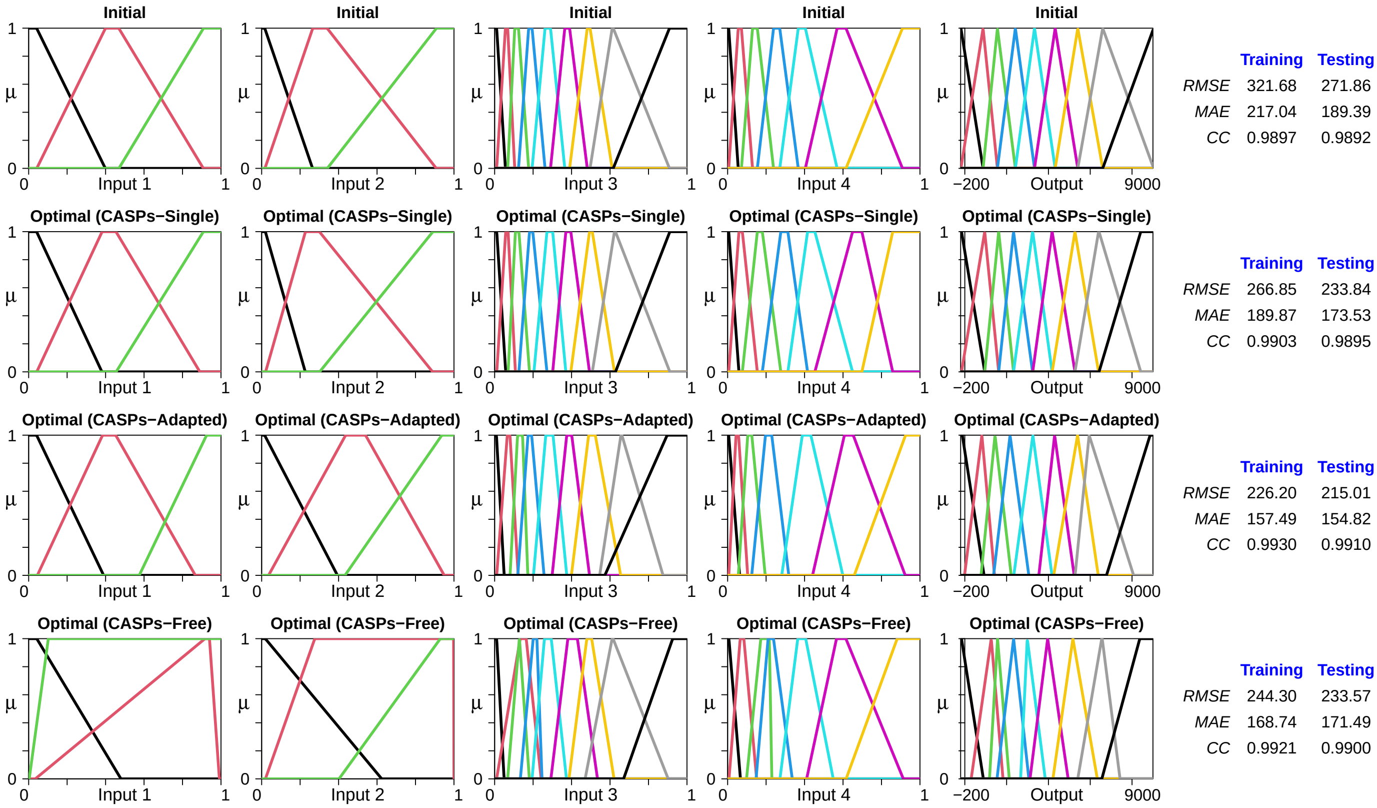

Fig. S-2. The initial and optimised MF plots, obtained on the **Electricity** dataset using the **GA** optimiser.

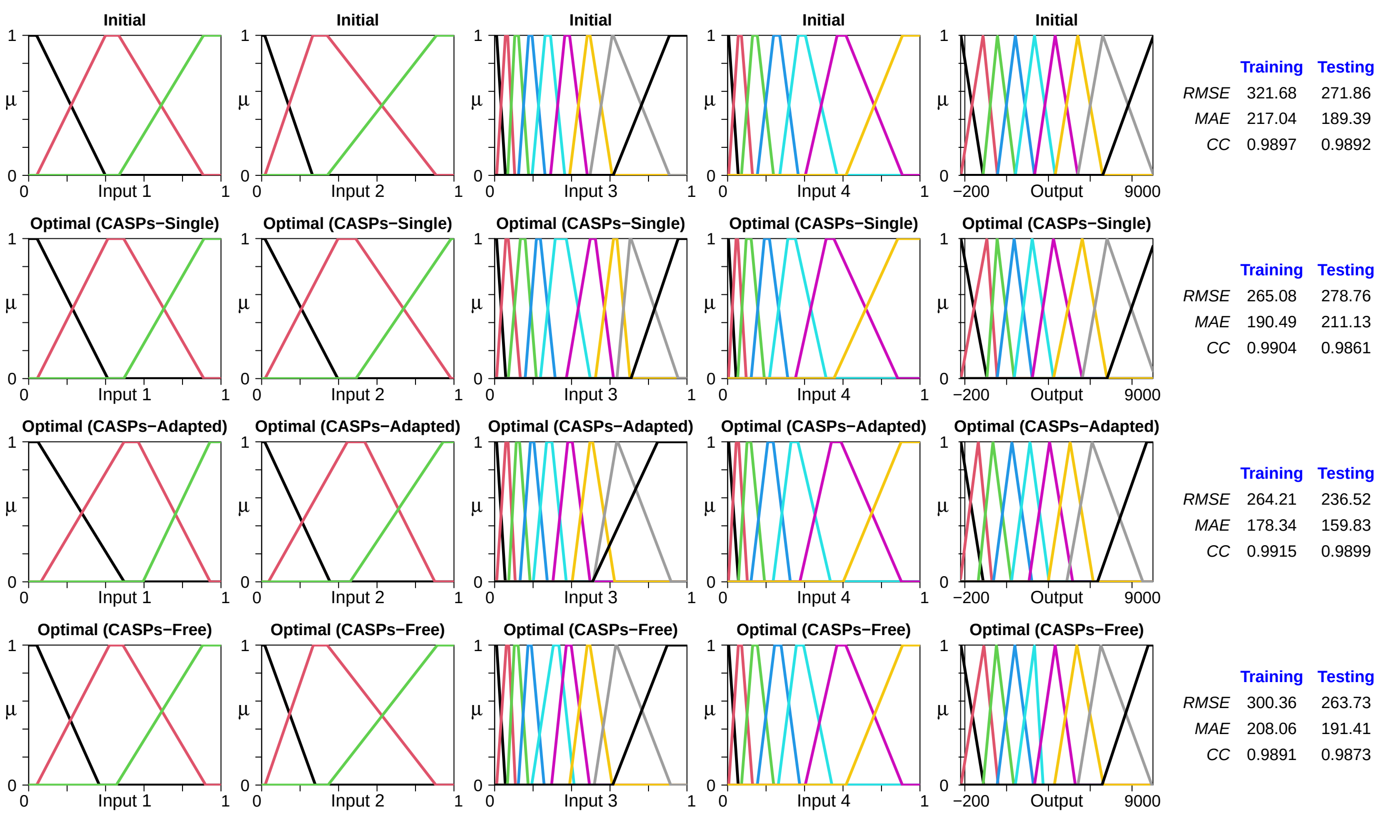

Fig. S-3. The initial and optimised MF plots, obtained on the **Electricity** dataset using the **PSO** optimiser.

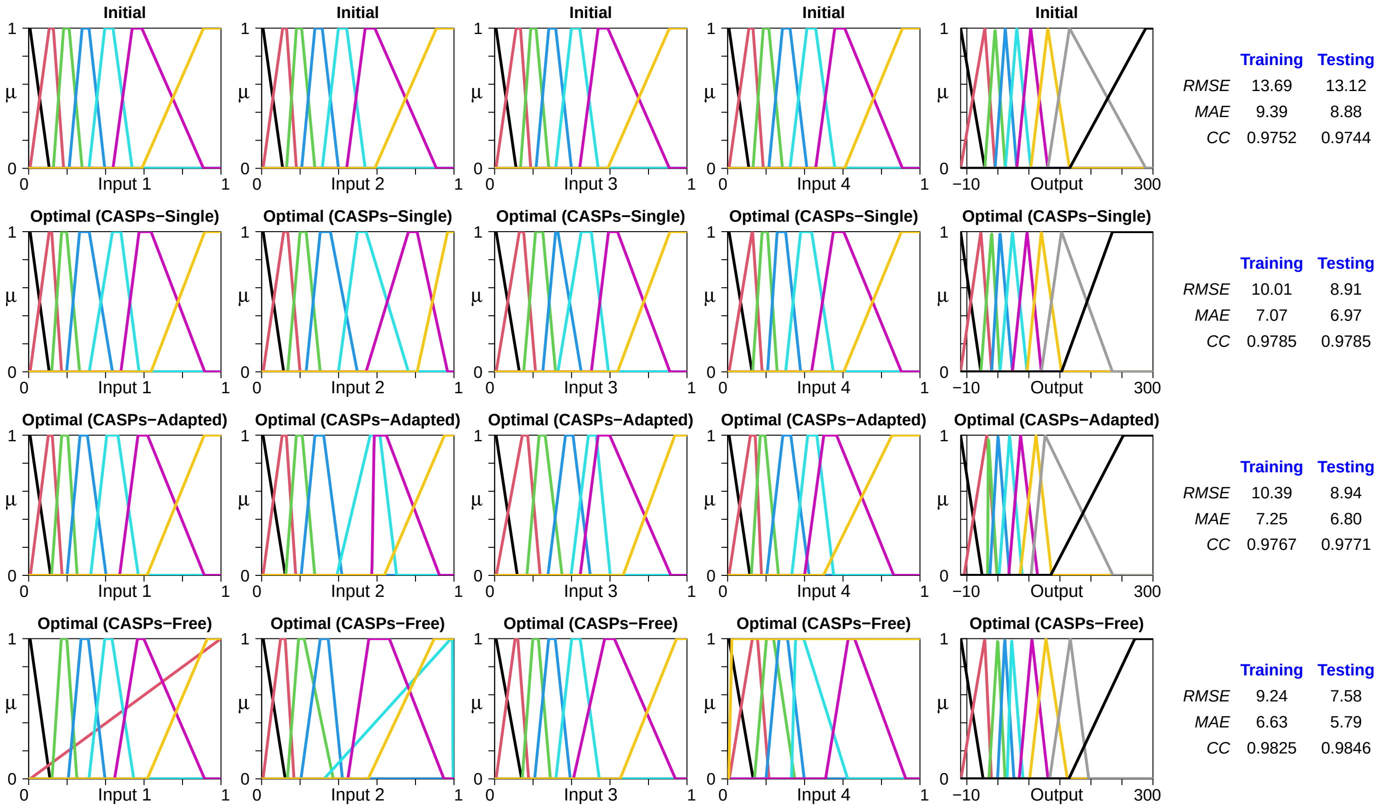

Fig. S-4. The initial and optimised MF plots, obtained on the **Laser** dataset using the **GA** optimiser.

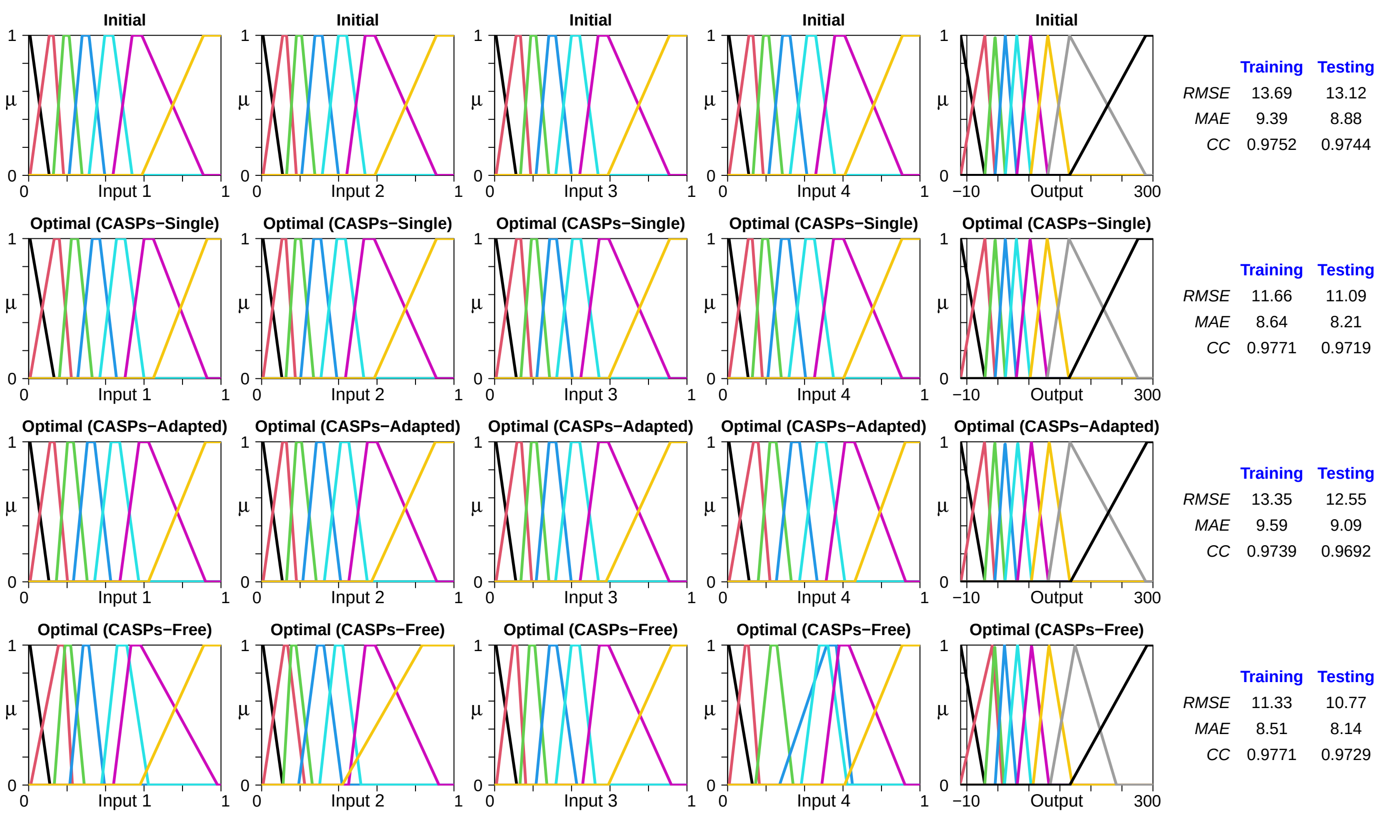

Fig. S-5. The initial and optimised MF plots, obtained on the **Laser** dataset using the **PSO** optimiser.

TABLE S-I

COMPREHENSIVE COMPARISON OF PERFORMANCE METRICS ON THE **ELECTRICITY** DATASET, INCLUDING THE THREE CASPS VARIANTS OPTIMISED USING ADAM, GA, AND PSO, TOGETHER WITH REPRESENTATIVE METHODS FROM THE LITERATURE.

	CASPs−Single			CASPs−Adapted			CASPs−Free			Others				
	PSO	GA	Adam	PSO	GA	Adam	PSO	GA	Adam	MLP	LR	KNN	HPFNN	SCFS−IFRs
RMSE	278.76	233.84	175.93	236.52	215.01	134.31	263.73	233.57	140.36	153.14	164.50	114.95	115.26	80.31
MAE	211.13	173.53	129.01	159.83	154.82	97.16	191.41	171.49	94.58	−	−	−	−	−
CC	0.9861	0.9895	0.9940	0.9899	0.9910	0.9965	0.9873	0.9900	0.9962	−	−	−	−	−

TABLE S-II

COMPREHENSIVE COMPARISON OF PERFORMANCE METRICS ON THE **LASER** DATASET, INCLUDING THE THREE CASPS VARIANTS OPTIMISED USING ADAM, GA, AND PSO, TOGETHER WITH REPRESENTATIVE METHODS FROM THE LITERATURE.

	CASPs−Single			CASPs−Adapted			CASPs−Free			Others				
	PSO	GA	Adam	PSO	GA	Adam	PSO	GA	Adam	MLP	LR	KNN	HPFNN	SCFS−IFRs
RMSE	11.09	8.91	6.01	12.55	8.94	4.83	10.77	7.58	4.16	7.22	23.07	11.75	7.02	5.82
MAE	8.21	6.97	4.40	9.09	6.80	3.27	8.14	5.79	2.59	−	−	−	−	−
CC	0.9719	0.9785	0.9898	0.9692	0.9771	0.9935	0.9729	0.9846	0.9952	−	−	−	−	−

TABLE S-III

COMPREHENSIVE COMPARISON OF PERFORMANCE METRICS ON THE **AIRFOIL** DATASET, INCLUDING THE THREE CASPS VARIANTS OPTIMISED USING ADAM, GA, AND PSO, TOGETHER WITH REPRESENTATIVE METHODS FROM THE LITERATURE.

	CASPs−Single			CASPs−Adapted			CASPs−Free			Others				
	PSO	GA	Adam	PSO	GA	Adam	PSO	GA	Adam	MLP	LR	KNN	HPFNN	SCFS−IFRs
RMSE	8.99	6.44	8.10	7.09	5.42	5.37	7.08	6.31	6.82	4.36	4.82	2.75	3.65	2.75
MAE	7.17	4.60	6.60	5.93	4.13	3.88	5.90	4.23	5.68	−	−	−	−	−
CC	0.3432	0.7325	0.3824	0.3696	0.7948	0.7719	0.4401	0.7760	0.7051	−	−	−	−	−

TABLE S-IV

COMPREHENSIVE COMPARISON OF PERFORMANCE METRICS ON THE **CONCRETE** DATASET, INCLUDING THE THREE CASPS VARIANTS OPTIMISED USING ADAM, GA, AND PSO, TOGETHER WITH REPRESENTATIVE METHODS FROM THE LITERATURE.

	CASPs−Single			CASPs−Adapted			CASPs−Free			Others				
	PSO	GA	Adam	PSO	GA	Adam	PSO	GA	Adam	MLP	LR	KNN	HPFNN	SCFS−IFRs
RMSE	12.00	10.88	11.26	12.46	11.33	10.40	11.91	11.29	10.60	7.74	10.49	9.46	6.79	5.56
MAE	9.41	8.36	8.47	9.78	8.70	8.03	9.37	8.69	8.10	−	−	−	−	−
CC	0.6761	0.7569	0.7185	0.7096	0.7129	0.7744	0.7125	0.7242	0.7616	−	−	−	−	−