Connectivity of Graphs

Definition

Graph connectivity refers to the ability to travel or navigate between vertices in a graph

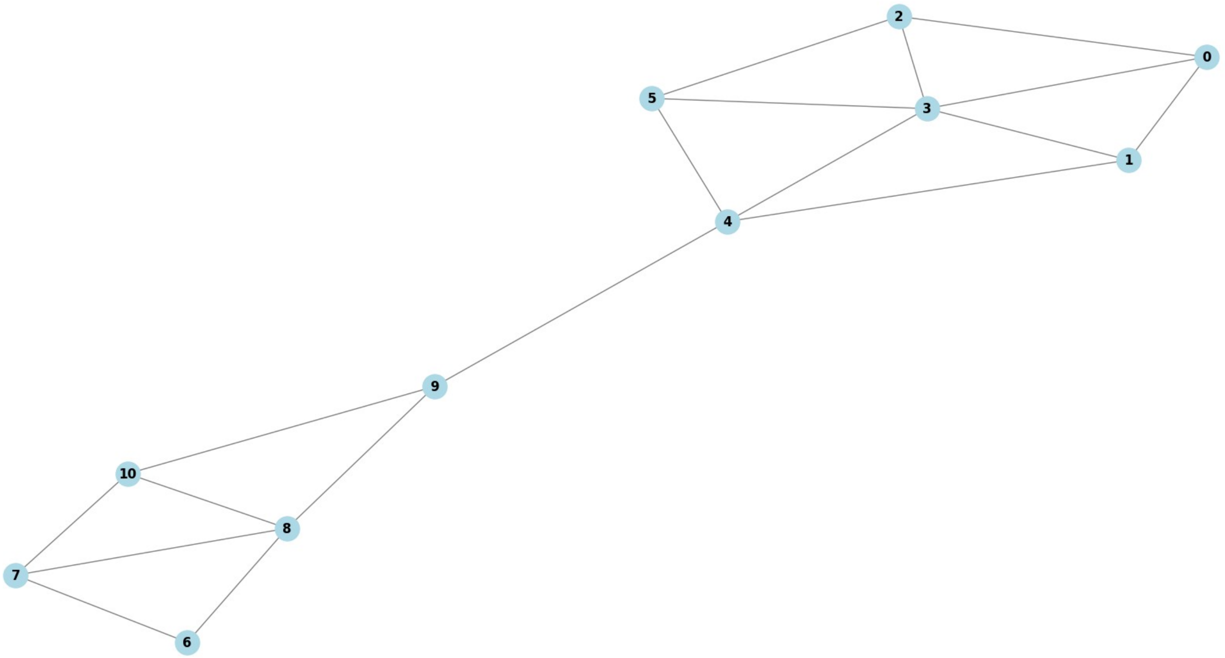
Classification

- ***Strong Connectivity***: A directed graph is strongly connected if there is a directed path between any two vertices in the graph.
- ***Weak Connectivity***: A directed graph is weakly connected if replacing all the directed edges with undirected edges produces a connected undirected graph.
- ***Disconnected***: No path is possible even by directed or undirected edges.

Graph connectivity with eigen values

- The connectivity of a graph can be determined using the Laplacian matrix. By calculating the eigenvalues of the Laplacian matrix and examining the second smallest eigenvalue, you can determine whether the graph is disconnected or has multiple connected components.
- Theorem: The second smallest eigenvalue $\lambda 2$ tells you about the connectivity of the graph. If the graph has two disconnected components, $\lambda 2 = 0$. And if $\lambda 2$ is small, this suggests the graph is nearly disconnected, with two components
- that are not very connected to each other.

Connected graph



Disconnected Graph

Applications of Laplacian Eigen Values

- **Graph Partitioning**: The eigenvalues of the Laplacian matrix can be used in graph partitioning algorithms.
- **Spectral Graph Theory:** The Laplacian matrix and its eigenvalues play a significant role in spectral graph theory. Spectral graph theory studies the properties of graphs using linear algebraic techniques.
- **Network Analysis**:The theorem can be applied in the analysis of various networks, such as social networks, transportation networks, and communication networks.

Overview of Laplacian Eigen Values

In the context of graph theory, the Laplacian eigenvalues are defined for a graph, which is a collection of vertices (nodes) connected by edges. The Laplacian matrix of a graph is a square matrix that encodes the graph’s connectivity information.

Types of Eigen Values for the Laplacian Matrix

- **Zero Eigenvalue:** As mentioned earlier, a Laplacian matrix always has a zero eigenvalue. It corresponds to a constant eigenvector, where all elements of the vector are the same.
- **Positive Eigenvalues:** The positive eigenvalues of the Laplacian matrix provide information about the connectivity and expansion properties of the graph. The magnitude of these eigenvalues indicates how fast information can spread or diffuse across the graph.

Types of Eigen Values for the Laplacian Matrix

- **Negative Eigenvalues:** In some cases, the Laplacian matrix may have negative eigenvalues. The presence of negative eigenvalues indicates that the graph has a non-trivial bipartite structure or a sign of imbalance in the graph’s connectivity.

Spectral Gap

- The difference between consecutive eigenvalues is referred to as the ”spectral gap.” The spectral gap provides insights into the connectivity and conductance properties of the graph. A larger spectral gap indicates a graph with stronger connectivity and better conductance between its components.

Uses

- **Graph partitioning and clustering**
- **Graph Drawing and Visualization**
- **Graph based image Segmentation**
- **Community Detection and Social Network Analysis**
- **Random Walks on Graphs**
- **Machine Learning and Data Analysis**
- **Network Resilience and Robustness**

THANK YOU

for more details, you can go through the pdf