PS 5: Probability Fundamentals

Stat 20 UC Berkeley

1. What characteristics must every distribution must have to be a valid probability distribution?
2. Maria gets a cup of coffee and a muffin every day for breakfast from one of the many coffee shops in her neighborhood. She picks a coffee shop each morning at random and independently of previous days. The average price of a cup of coffee is \$1.40 with a standard deviation of 30 cents (\$0.30), the average price of a muffin is \$2.50 with a standard deviation of 15 cents, and the two prices are independent of each other.
    a. What is the mean and standard deviation of the amount she spends on breakfast daily?
    b. What is the mean and standard deviation of the amount she spends on breakfast weekly (every 7 days)?
3. An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.
    a. Using a table, define a random variable that describes the baggage fee revenue for a single passenger, with the possible values that it can take along with their probabilities. The compute the average revenue per passenger, and compute the corresponding standard deviation.
    b. Draw a bar plot that depicts the distribution of the random variable you described in part a.
    c. About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.
4. Suppose we have independent observations X_1 and X_2, both from a distribution with expected value μ and standard deviation σ. What is the variance of the average of the two values: $\frac{X_1+X_2}{2}$?
5. What characteristics should a random process have to be well-modeled by the Binomial distribution?
6. Let $Y \sim$ Bern(p = .3). What is $E(Y)$? $Var(Y)$?
7. Let $Z \sim$ Bin(n = 10, p = .9). Find the following properties of Z.
    a. What are the different values that Z can take?
    b. $E(Z)$
    c. $Var(Z)$
    d. $P(Z = 10)$