

PS 5: Probability Fundamentals

Stat 20 UC Berkeley

1. What characteristics must every distribution must have to be a valid probability distribution?
2. Maria gets a cup of coffee and a muffin every day for breakfast from one of the many coffee shops in her neighborhood. She picks a coffee shop each morning at random and independently of previous days. The average price of a cup of coffee is \$1.40 with a standard deviation of 30 cents (\$0.30), the average price of a muffin is \$2.50 with a standard deviation of 15 cents, and the two prices are independent of each other.
 - a. What is the mean and standard deviation of the amount she spends on breakfast daily?
 - b. What is the mean and standard deviation of the amount she spends on breakfast weekly (every 7 days)?
3. An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.
 - a. Using a table, define a random variable that describes the baggage fee revenue for a single passenger, with the possible values that it can take along with their probabilities. The compute the average revenue per passenger, and compute the corresponding standard deviation.
 - b. Draw a bar plot that depicts the distribution of the random variable you described in part a.
 - c. About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.
4. Suppose we have independent observations X_1 and X_2 , both from a distribution with expected value μ and standard deviation σ . What is the variance of the average of the two values: $\frac{X_1+X_2}{2}$?
5. What characteristics should a random process have to be well-modeled by the Binomial distribution?
6. Let $Y \sim \text{Bern}(p = 3)$. What is $E(Y)$? $Var(Y)$?
7. Let $Z \sim \text{Bin}(n = 10, p = .9)$. Find the following properties of Z .
 - a. What are the different values that Z can take?
 - b. $E(Z)$
 - c. $Var(Z)$
 - d. $P(Z = 10)$