1 Osnovne enačbe

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_z \\ \varphi \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v_x \\ v_z \\ \Omega \end{bmatrix} \qquad \mathbf{e}_0 = \begin{bmatrix} -1 \\ 0 \\ -\kappa_0 \end{bmatrix} \qquad \mathbf{d} = \begin{bmatrix} u_x' + \cos\varphi_0 \\ u_z' + \sin\varphi_0 \\ \varphi' \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_x \\ p_z \\ m_y \end{bmatrix}$$

$$\mathbf{R}(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{R}^{(n)}(\varphi) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^n \mathbf{R}(\varphi)$$

$$\mathbf{e} = \mathbf{R}(\varphi)\mathbf{d} + \mathbf{e}_0 \qquad \mathcal{R} = \mathbf{R}^T(\varphi)\mathbf{C}\mathbf{e} \qquad \mathcal{N} = \mathbf{C}\mathbf{e}$$

$$\mathcal{F} = \mathcal{R}' + (\mathbf{p} - \rho_A \dot{\mathbf{v}} + \begin{bmatrix} 0 \\ 0 \\ \mathcal{N}_1\mathbf{e}_2 - (1+\mathbf{e}_1)\mathcal{N}_2 \end{bmatrix}) = 0$$

2 Numerični račun

Poznamo količine v času t_n. Za t_{n+1} predpostavimo hitrosti $\mathbf{v}_{n+1}$.

$$\bar{\mathbf{v}} = (\mathbf{v}_{n+1} + \mathbf{v}_n)/2 \qquad \mathbf{u}_{n+1/2} = \mathbf{u}_n + \frac{h}{2}\bar{\mathbf{v}} \qquad \mathbf{d}_{n+\frac{1}{2}} = \mathbf{d}_n + \frac{h}{2}\bar{\mathbf{v}}'$$

$$\mathbf{u}_{n+1} = \mathbf{u}_n + h\bar{\mathbf{v}} \qquad \mathbf{d}_{n+1} = \mathbf{d}_n + h\bar{\mathbf{v}}'$$

$$\mathbf{e}_{n+\frac{1}{2}} = \mathbf{R}(\tfrac{h}{2}\bar{\Omega})(\mathbf{e}_n - \mathbf{e}_0) + \mathbf{e}_0 + \frac{h}{2}\mathbf{R}(\varphi_n + \tfrac{h}{2}\bar{\Omega})\bar{\mathbf{v}}'$$

$$\mathcal{N}_{n+\frac{1}{2}} = \mathbf{C}\frac{\mathbf{e}_{n+1} + \mathbf{e}_n}{2}$$

$$\bar{\mathcal{R}} = \mathbf{R}^T(\varphi_n + \tfrac{h}{2}\bar{\Omega})\mathcal{N}_{n+\frac{1}{2}}$$

$$\mathcal{F}_{n+\frac{1}{2}} = \int_0^l -\bar{\mathcal{R}}\mathcal{P}_i' + (\mathbf{p}_{n+\frac{1}{2}} - \rho_A(\mathbf{v}_{n+1} - \mathbf{v}_n) + \begin{bmatrix} 0 \\ 0 \\ \mathcal{N}_{n+\frac{1}{2}}[1]\mathbf{e}_{n+\frac{1}{2}}[2] - (1 - \mathbf{e}_{n+\frac{1}{2}}[1])\mathcal{N}_{n+\frac{1}{2}}[2] \end{bmatrix})\mathcal{P}_i \, dx + \bar{\mathcal{R}}\mathcal{P}_i|_0^l = 0$$

3 Linearizacija

$$\delta\mathbf{u}_{n+1/2} = \frac{h}{2}\delta\bar{\mathbf{v}} \qquad \delta\mathbf{d}_{n+1/2} = \frac{h}{2}\delta\bar{\mathbf{v}}'$$

$$\delta\mathbf{e}_{n+1/2} = \frac{h}{2}\delta\bar{\Omega}\mathbf{R}^{(1)}(\tfrac{h}{2}\bar{\Omega})(\mathbf{e}_n - \mathbf{e}_0) + \frac{h^2}{4}\delta\bar{\Omega}\mathbf{R}^{(1)}(\varphi_n + \tfrac{h}{2}\bar{\Omega})\bar{\mathbf{v}}' + \frac{h}{2}\mathbf{R}(\varphi_n + \tfrac{h}{2}\bar{\Omega})\delta\bar{\mathbf{v}}'$$

$$\delta\mathbf{e}_{n+1} = \delta\bar{\Omega}h\mathbf{R}^{(1)}(\varphi_n + h\bar{\Omega})\mathbf{d}_{n+1} + \mathbf{R}(\varphi_n + h\bar{\Omega})h\delta\bar{\mathbf{v}}'$$

$$\delta\mathcal{N}_{n+\frac{1}{2}} = \mathbf{C}\frac{\delta\mathbf{e}_{n+1}}{2}$$

$$\delta\bar{\mathcal{R}} = \frac{h}{2}\delta\bar{\Omega}(\mathbf{R}^{(1)}(\varphi_n + \tfrac{h}{2}\bar{\Omega}))^T\mathcal{N}_{n+\frac{1}{2}} + \mathbf{R}^T(\varphi_n + \tfrac{h}{2}\bar{\Omega})\delta\mathcal{N}_{n+\frac{1}{2}}$$

$$\delta\mathcal{F}_{n+\frac{1}{2}} = \int_0^l -\delta\mathcal{R}\mathcal{P}_i' + \delta\begin{bmatrix} 0 \\ 0 \\ \mathcal{N}_{n+\frac{1}{2}}[1]\mathbf{e}_{n+\frac{1}{2}}[2] - (1 - \mathbf{e}_{n+\frac{1}{2}}[1])\mathcal{N}_{n+\frac{1}{2}}[2] \end{bmatrix}\mathcal{P}_i \, dx$$