1 Osnovne enačbe

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_z \\ \varphi \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v_x \\ v_z \\ \Omega \end{bmatrix} \qquad \mathbf{e}_0 = \begin{bmatrix} -1 \\ 0 \\ -\kappa_0 \end{bmatrix} \qquad \mathbf{d} = \begin{bmatrix} u'_x + \cos\varphi_0 \\ u'_z + \sin\varphi_0 \\ \varphi' \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_x \\ p_z \\ m_y \end{bmatrix}$$

$$\mathbf{R}(\varphi) = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{R}^{(n)}(\varphi) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^n \mathbf{R}(\varphi)$$

$$\mathbf{e} = \mathbf{R}(\varphi)\mathbf{d} + \mathbf{e}_0 \qquad \mathcal{R} = \mathbf{R}^T(\varphi)\mathbf{C}\mathbf{e} \qquad \mathcal{N} = \mathbf{C}\mathbf{e}$$

$$\delta\mathbf{e} = \delta\varphi\ \mathbf{R}^{(1)}(\varphi)\mathbf{d} + \mathbf{R}(\varphi)\ \delta\mathbf{d}$$

$$\delta\mathcal{R} = \delta\varphi\ ((\mathbf{R}^{(1)})^T(\varphi)\mathbf{C}\mathbf{e} + \mathbf{R}^T(\varphi)\mathbf{C}\mathbf{R}^{(1)}(\varphi)\mathbf{d}) + \mathbf{R}^T(\varphi)\mathbf{C}\mathbf{R}(\varphi)\ \delta\mathbf{d}$$

$$\delta\mathcal{N} = \delta\varphi\ \mathbf{C}\mathbf{R}^{(1)}(\varphi)\mathbf{d} + \mathbf{C}\mathbf{R}(\varphi)\ \delta\mathbf{d}$$

$$\mathcal{F} = \int_0^l -\mathcal{R}\mathcal{P}'_i + (\mathbf{p} - \rho_A\dot{\mathbf{v}} + \begin{bmatrix} 0 \\ 0 \\ \mathcal{N}_1\mathbf{e}_2 - (1+\mathbf{e}_1)\mathcal{N}_2 \end{bmatrix})\mathcal{P}_i\ dx + \mathcal{R}\mathcal{P}_i\Big|_0^l$$

$$\delta\mathcal{F} = \int_0^l -\delta\mathcal{R}\mathcal{P}'_i + \begin{bmatrix} 0 \\ 0 \\ \mathbf{e}_2\delta\mathcal{N}_1 + \delta\mathbf{e}_2\mathcal{N}_1 - (1+\mathbf{e}_1)\delta\mathcal{N}_2 - \delta\mathbf{e}_1\mathcal{N}_2 \end{bmatrix}\mathcal{P}_i\ dx$$

Če vzamem $\mathbf{e} = [C, 0, 0]$, kejr je $C > 0$ dobim po formuli $\mathbf{d} = [0, C+1, 0]$.
Recimo, da je element vertikalen, in spodaj podprt. Potem je u_z linearna funkcija, ki pada z koordinato x na elementu. Tako je $\mathbf{d} = [0, A+1, 0]$, kjer je $A < 0$ kar je v protislovju.
Rezultata se uskaldita za $\mathbf{e} = \mathbf{R}^T\mathbf{d} + \mathbf{e}_0$.

2 Numerični račun

Poznamo količine v času t_n. Za t_{n+1} predpostavimo hitrosti $\mathbf{v}_{n+1}$.

$$\bar{\mathbf{v}} = (\mathbf{v}_{n+1} + \mathbf{v}_n)/2 \qquad \bar{\mathbf{u}} = \mathbf{u}_n + \frac{h}{2}\bar{\mathbf{v}} \qquad \bar{\mathbf{d}} = \mathbf{d}_n + \frac{h}{2}\bar{\mathbf{v}}'$$

$$\mathbf{u}_{n+1} = \mathbf{u}_n + h\bar{\mathbf{v}} \qquad \mathbf{d}_{n+1} = \mathbf{d}_n + h\bar{\mathbf{v}}'$$

$$\delta\bar{\mathbf{u}} = \frac{h}{2}\delta\bar{\mathbf{v}} \qquad \delta\bar{\mathbf{d}} = \frac{h}{2}\delta\bar{\mathbf{v}}'$$

$$\bar{\mathcal{R}} = \mathbf{R}^T{}_{(\bar{\varphi})}\mathbf{C}\bar{\mathbf{e}} \qquad \bar{\mathbf{e}} = \mathbf{R}_{(\bar{\varphi})}\bar{\mathbf{d}} - \mathbf{e}_0 \qquad \bar{\mathcal{N}} = \mathbf{C}\bar{\mathbf{e}}$$

$$\delta\bar{\mathcal{R}} = \frac{h}{2}(\delta\bar{\Omega}((\mathbf{R}^{(1)})^T{}_{(\bar{\varphi})}\mathbf{C}\bar{\mathbf{e}} + \mathbf{R}^T{}_{(\bar{\varphi})}\mathbf{C}\mathbf{R}^{(1)}{}_{(\bar{\varphi})}\bar{\mathbf{d}}) + \mathbf{R}^T{}_{(\bar{\varphi})}\mathbf{C}\mathbf{R}_{(\bar{\varphi})}\delta\bar{\mathbf{v}}')$$

$$\delta\bar{\mathbf{e}} = \frac{h}{2}(\delta\bar{\Omega}\mathbf{R}^{(1)}{}_{(\bar{\varphi})}\bar{\mathbf{d}} + \mathbf{R}_{(\bar{\varphi})}\delta\bar{\mathbf{v}}')$$

$$\delta\bar{\mathcal{N}} = \frac{h}{2}(\delta\bar{\Omega}\mathbf{C}\mathbf{R}^{(1)}{}_{(\bar{\varphi})}\bar{\mathbf{d}} + \mathbf{C}\mathbf{R}_{(\bar{\varphi})}\delta\bar{\mathbf{v}}')$$

V programu je napaka ker ne računam količin v t_{n+1} z pravimi u_{n+1}.
Najprej bi moral poračunati hitrosti za $n+1$ in n in od tod hitrosti za $n+\frac{1}{2}$. Z temi izračunam $\mathbf{u}_{n+1}$ in $\mathbf{d}_{n+1}$ in od tod $\mathcal{R}_{n+1}$. Nadaljni postopek bi moral biti pravilen.
Kaj bi moral narediti z energijama?
Če prav razumem sta hitrosti ob časih $n+1$ in n neki funkciji. Zato ker jih ne znamo izvrednostit moramo uporabiti ravnotežne enačbe $\bar{\mathcal{F}}$ in iz njih izraziti diferenco hitrosti (pospešek), kar se pojavi tudi v enačbi energije. Po drugi strani pa je funkcija vmesne hitrosti linearna kombinacija robnih.
Kakšen bi bil pravilen postopek za račun $\delta\bar{\mathcal{R}}$, če za $\bar{\mathcal{R}}$ vzamem povprečnega?
Ker je $\delta\mathcal{R}_n$ neodvisna od $\bar{\mathbf{v}}$ je variacija enaka 0. Razlika med povprečnim in analitično formulo je ta, da so v vseh členih nevariirane količine tiste ob drugem oziroma vmesnem času. Najprej bi raje poiskusil doseči smiselne rezultate z izvorno formulo za $\mathcal{R}$.

3 Implementacija v program

$$\delta\bar{\mathbf{e}} = \frac{h}{2}\mathcal{P}_j \begin{bmatrix} 0 & 0 & | \\ 0 & 0 & \mathbf{R}^{(1)}(\bar{\varphi})\bar{\mathbf{d}} \\ 0 & 0 & | \end{bmatrix} + \frac{h}{2}\mathcal{P}_j'\mathbf{R}(\bar{\varphi})$$

$$\delta\bar{\mathcal{N}} = \frac{h}{2}\mathcal{P}_j \begin{bmatrix} 0 & 0 & | \\ 0 & 0 & \mathbf{C}\mathbf{R}^{(1)}(\bar{\varphi})\bar{\mathbf{d}} \\ 0 & 0 & | \end{bmatrix} + \frac{h}{2}\mathcal{P}_j'\mathbf{C}\mathbf{R}(\bar{\varphi})$$

$$\delta\bar{\mathcal{R}} = \frac{h}{2}\mathcal{P}_j \begin{bmatrix} 0 & 0 & | \\ 0 & 0 & (\mathbf{R}^{(1)})^T(\bar{\varphi})\mathbf{C}\bar{\mathbf{e}} + \mathbf{R}^T(\bar{\varphi})\mathbf{C}\mathbf{R}^{(1)}(\bar{\varphi})\bar{\mathbf{d}} \\ 0 & 0 & | \end{bmatrix} + \frac{h}{2}\mathcal{P}_j'\mathbf{R}^T(\bar{\varphi})\mathbf{C}\mathbf{R}(\bar{\varphi})$$

Za enostaven primer konzole se komponente zadnjega stoplca in vrstice hitro povečujejo. Preveri eačbe ki prispevajo k temu delu.