

# Solution Sets: Takeaways

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## Concepts

- An inconsistent system has two or more equations that no solution exists when the augmented matrix is in reduced echelon form.

- Example of a inconsistent system:  $\left[ \begin{array}{cc|c} 8 & 4 & 5 \\ 4 & 2 & 5 \end{array} \right]$

- When the determinant is equal to zero, we say the matrix is singular or it contains no inverse.

- Example of a singular matrix:  $\begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}$

- The formula for the determinant of a  $2 \times 2$  square matrix is:

$$\det(A) = ad - bc$$

- If we substitute in the values, we get a determinant of zero:

$$\det(A) = ad - bc = 8 \cdot 2 - 4 \cdot 4 = 16 - 16 = 0$$

- A nonhomogenous system is a system where the constants vector ( $\vec{b}$ ) doesn't contain all zeros.

- Example of a nonhomogenous system:  $\left[ \begin{array}{cc|c} 8 & 4 & 5 \\ 4 & 2 & 5 \end{array} \right]$

- A homogenous system is a system where the constants vector ( $\vec{b}$ ) is equal to the zero vector.

- Example of a homogenous system:  $\left[ \begin{array}{cc|c} 8 & 4 & 0 \\ 4 & 2 & 0 \end{array} \right]$

- A homogenous system always contains the trivial solution: the zero vector.

- For a nonhomogenous system that contains the same number of rows and columns, there are 3 possible solutions:

- No solution.
- A single solution.
- Infinitely many solutions.

- For rectangular (nonsquare, nonhomogenous) systems, there are two possible solutions:

- No solution.
- Infinitely many solutions.

- If  $Ax = b$  is a linear system, then every vector  $\vec{x}$  which satisfies the system is said to be a solution vector of the linear system. The set of solution vectors of the system is called the solution space of the linear system.

- When the solution is a solution space (and not just a unique set of values), it's common to rewrite it into parametric vector form.

- Example a vector in parametric vector form:  $\vec{x} = x_3 \cdot \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$

## Resources

- [Consistent and Inconsistent equations](#)
- [Solution Spaces of Homogenous Linear Systems](#)

