

# Thoughts on Mandelbrots

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## Abstract

We live in a world of number, both real and imaginary. This simple and seemingly benign truth can lead to peculiar and interesting properties, and nothing demonstrates this much more clearly than a Mandelbrot set. While the numbers that fall into the range are hardly “unpredictable” in any kind of literal sense, that can certainly be described as *unexpected* from the (admittedly flawed) human perception. These are my thoughts on how Mandelbrots work, and perhaps some ideas on how there might be legitimate use for them.

## 1 Iterative Function

The mathematics for what works in a Mandelbrot series is actually boils down to one equation:

$$f_{n+1}(x) = x^2 + c \quad (1)$$

One may accurately infer that this is just a standard quadratic formula that was learned in seventh grade, and that wouldn’t be too far off, but the key difference comes down to the  $f_{n+1}$  in the front.

This notation is known as “iterated function” syntax, and works as a sort of “infinite self-composition”. An example of an iterated function would be something like this.

$$f(x) = x^2$$

$$f(2) = 4$$

$$f(4) = 16$$

$$f(16) = 256$$

...

Squaring a larger number generally makes it larger, but perhaps more interesting are the numbers that don’t go to infinity. For example, in the first supplied equation, if we substitute the value -2, and start the iteration at 0, an interesting thing happens:

$$f(0) = 0^2 - 2 = -2$$

$$f(-2) = (-2)^2 - 2 = 2$$

$$f(2) = (2)^2 - 2 = 2$$

$$f(2) = (2)^2 - 2 = 2$$

...

-2 is a bizarre number. As you can clearly see, no matter how many times we iterate, it always returns back 2.

This is the crux of what the Mandelbrot says: There are numbers that don’t go to infinity after iteration in the iterated function of  $f(x) = x^2 + c$

## 2 What is $c$ ?

It's impossible to not wonder what  $c$  actually is in this iteration, and that is simple: it's the complex part of iterated function.

Most people have taken any kind of advanced math class knows this definition:

$$i^2 = -1$$

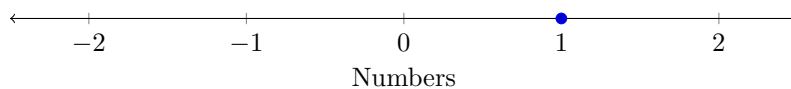
$i$  is simply name for the imaginary number, and it's an important thing to realize: there exists a separate bit of numbers that survive on a layer “on top of” the number line, and those are the imaginary numbers.

So then, what is a complex number? It simply is a number that has both a real and imaginary part:

$$1 + 3i$$

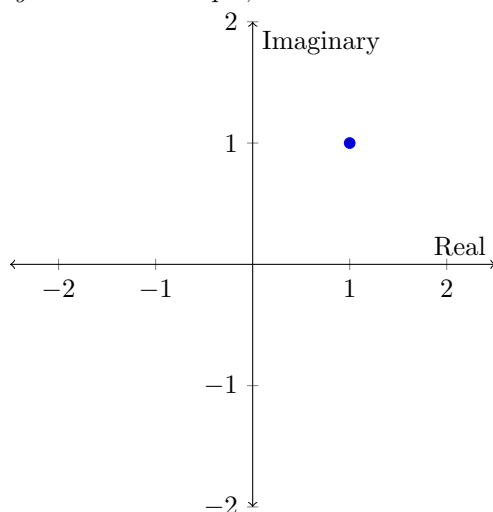
## 3 Projecting Our numbers

Projecting a number on the number line is easy:



This is easy enough to conceptualize, but where do complex numbers fit in?

In sort of a by-product of not having an accurate way to display an imaginary number on a one-dimensional grid, we traditionally plot imaginary numbers on the  $y$  axis. For example, here is a chart for the complex number  $1 + i$ :



## 4 Making the Mandelbrot

The Mandelbrot is simply a plot of all the numbers that “survive” the iteration cycle and returns a value below 2.