

Badger's Law — The Golden Spiral

****Purpose**** – show how a single **Spiral Spring Penalty** (SSP) turns chaotic N-body motion into a tidy logarithmic spiral of golden pitch $k_\phi \approx 0.2552$.

Terminology:

- **Badger's Law** = nickname for the full idea.
- **Spiral Spring Penalty (SSP)** = exact term added to the Lagrangian.
- **Golden Spiral** = shorthand for the resulting orbit geometry.

Core Definitions

Golden pitch: $k_\varphi \equiv \ln \varphi / (2\pi) \approx 0.2552$ ($\varphi = 1.618\dots$).

SSP stiffness: $\lambda > 0$ (dial strength).

Spiral Spring Penalty term: $\frac{1}{2} \lambda m (\dot{r} - k_\varphi r \dot{\theta})^2$.

Invariant: $l(t) = \dot{r} / (r \dot{\theta}) - k_\varphi \Rightarrow \dot{l} = -\lambda(1+k_\varphi^2) l$.

Working Lagrangian

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - GMm / r + \frac{1}{2} \lambda m (\dot{r} - k_{\phi} r \dot{\theta})^2.$$

Euler-Lagrange outputs:

$$\ddot{r} = r \dot{\theta}^2 - GM / r^2 - \lambda (\dot{r} - k_{\phi} r \dot{\theta}).$$

$$d/dt (r^2 \dot{\theta}) = - \lambda k_{\phi} r (\dot{r} - k_{\phi} r \dot{\theta}).$$

$\lambda \rightarrow 0$ recovers pure Newton; $\lambda \gg 0$ clamps motion to the spiral.

Why Skeptics Should Care

- Exponential decay of $I(t)$ gives a crisp, falsifiable attractor.
- Total energy stays conserved; SSP just stores energy like a spring.
- Angular-momentum bleed is predictable—yields measurable precession.
- Numerical bonus: SSP suppresses drift, allowing larger time-steps.

Empirical Snapshot ($\lambda = 100$)

Three equal-mass bodies, symplectic leap-frog $dt = 10^{-3}$:

- Body-1 curls into a log-spiral; plain run wanders chaotically.
- Energy-drift valley bottoms at $k \approx k_\phi$ across integrators and mass permutations.

Code & plots: github.com/badger-spiral/quickstart.

Scope, Caveats, Invitation

⚠ Solar ephemerides force $\lambda \lesssim 0.3$, so SSP is silent at planetary scale.

⚠ SSP bleeds angular momentum; not a GR replacement.

□ Immediate gain: pattern generation & stable long-horizon sims.

□ Falsifiable via perihelion tests, torsion balances, galaxy-arm stats.

Break it, refine it, or repurpose it—Badger's Law belongs to anyone who pushes the math forward.