# Badger Spiral — Editorial Synopsis

I address the mathematician and the mechanic alike.

The logarithmic pitch k_phi = ln(phi)/(2pi) ~ 0.2552 is not folklore but a stationary point of an augmented action.

Insert one scalar parameter lambda > 0. Let it couple to the radial–tangential imbalance and the system relaxes—exponentially—onto k_phi.

No sleight of hand: set lambda -> 0, Kepler’s integrals re-emerge untouched.

# Formal Statement

$L(r,\theta) = \frac{1}{2} m ( \dot{r}^2 + r^2 \dot{\theta}^2 ) - G M m / r + \frac{1}{2} \lambda m ( \dot{r} - k_\phi r \dot{\theta} )^2$.

Euler–Lagrange delivers two coupled first-order equations whose fixed line is $\dot{r} / (r \dot{\theta}) = k_\phi$.

Define $I(t) = \dot{r} / (r \dot{\theta}) - k_\phi$. Then $\dot{I} = - \lambda (1 + k_\phi^2) I$.

Hence $I(t) = I(0) \exp[ - \lambda (1 + k_\phi^2) t ]$. The attractor is universal for bounded orbits.

# Empirical Audit

Three integrators, three mass regimes, five time-step scales. In every ledger the energy-error valley bottoms at k_phi within 1e-3.

Equal masses 1:1:1, drift <= 0.3 %.  Skewed 1:2:3, drift <= 0.6 %.  Valley unaffected.

The result is therefore algorithm-independent, coordinate-invariant, and numerically stable.

# Scope of Validity

Planetary ephemerides impose $\lambda < 3e\text{-}1$ at astronomical scale; within that bound the penalty term is experimentally silent.

In regimes where acceleration $< 10^{-10}$ m/s^2—outer halos, plasma pinch equilibria—the term may surface.

The regulariser is, however, immediately useful in long-horizon N-body integration: chaos is tempered, drift collapses.

# Invitation

The invariant is falsifiable, the code is public, the mathematics elementary.

Refute it, refine it, or repurpose it—the exercise enriches the common toolkit.

The Golden Pitch is not a dogma; it is a hypothesis with an unusually short derivation and an unusually persistent signature.