Algoritmos e Estruturas de Dados

Sorting Algorithms

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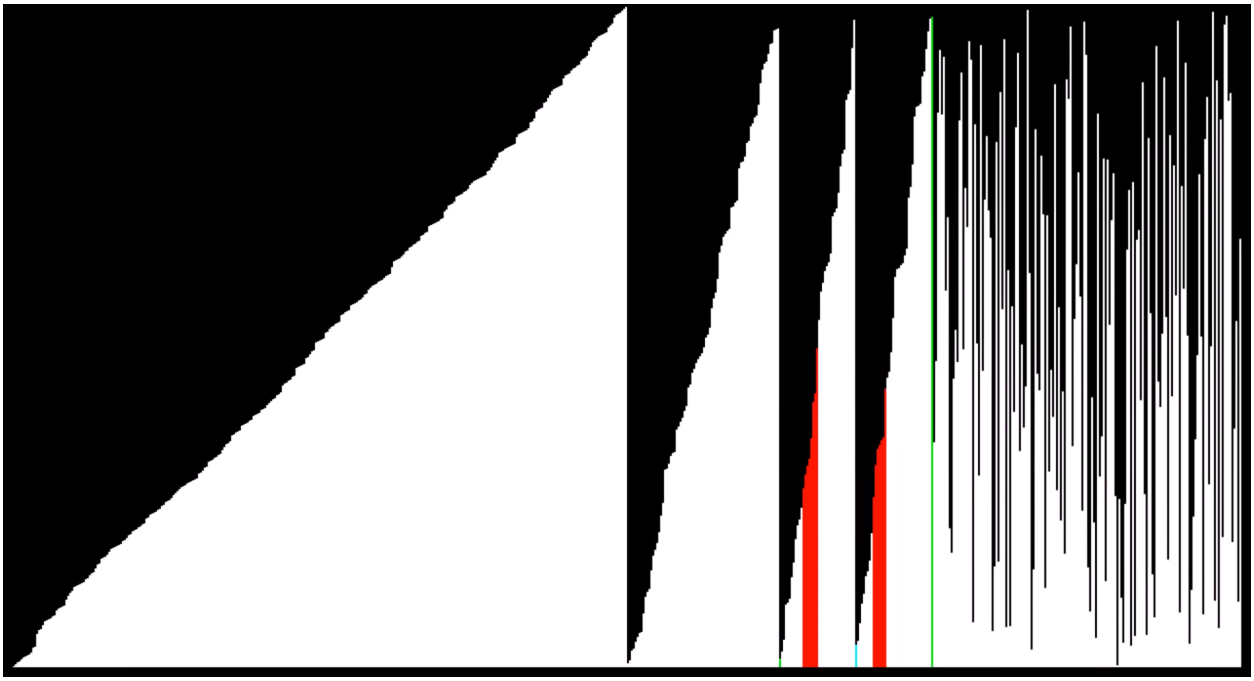
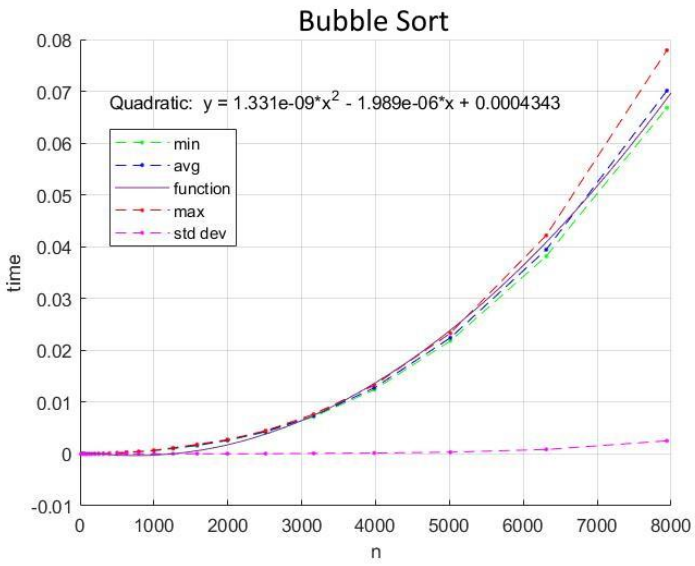
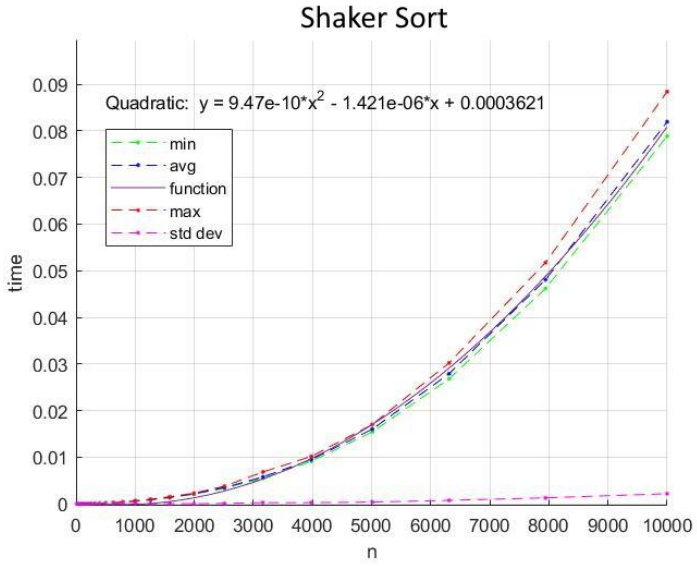
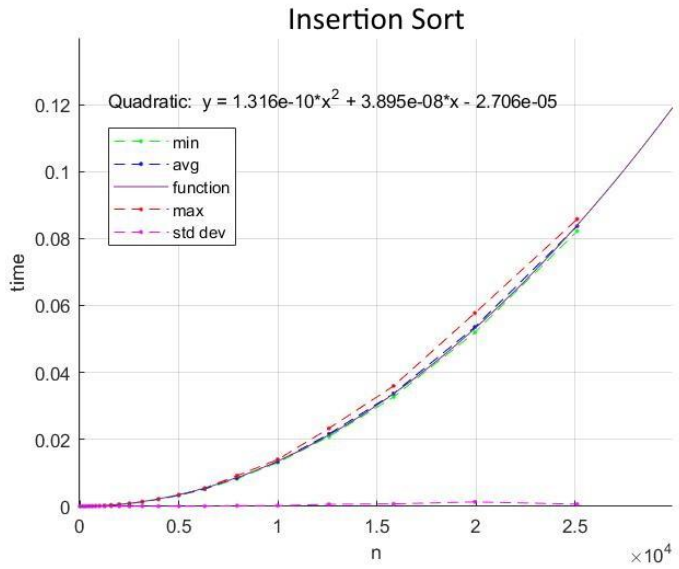
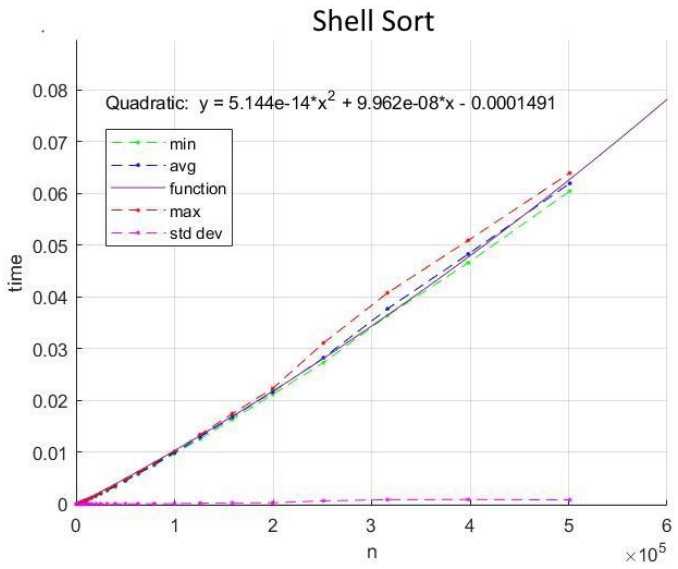


Gráficos de todos os algoritmos, com tempos de execução mínimos, médios e máximo em função de n (tamanho do array), bem como o desvio padrão. *function* é a função quadrática que mais se aproxima aos *data points*.

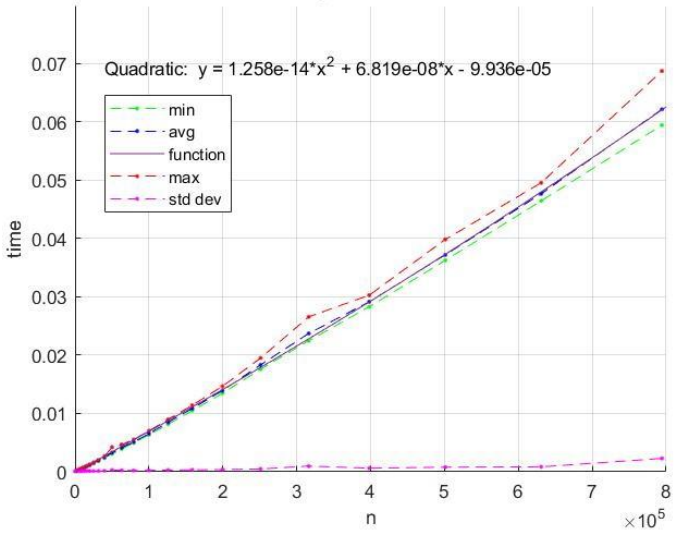




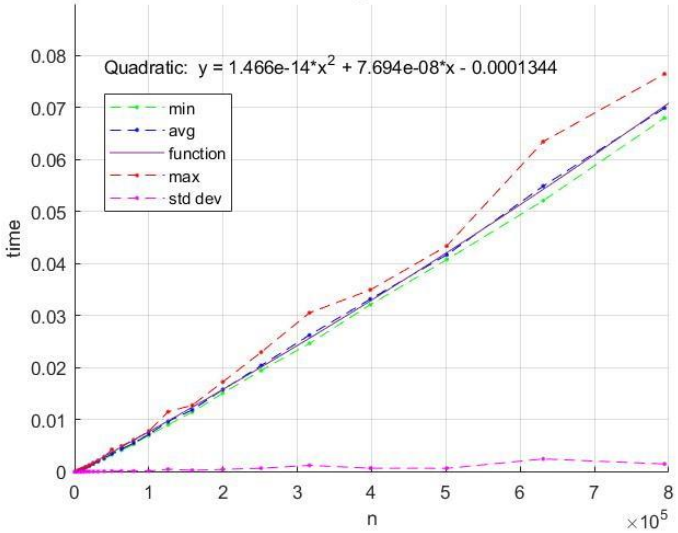




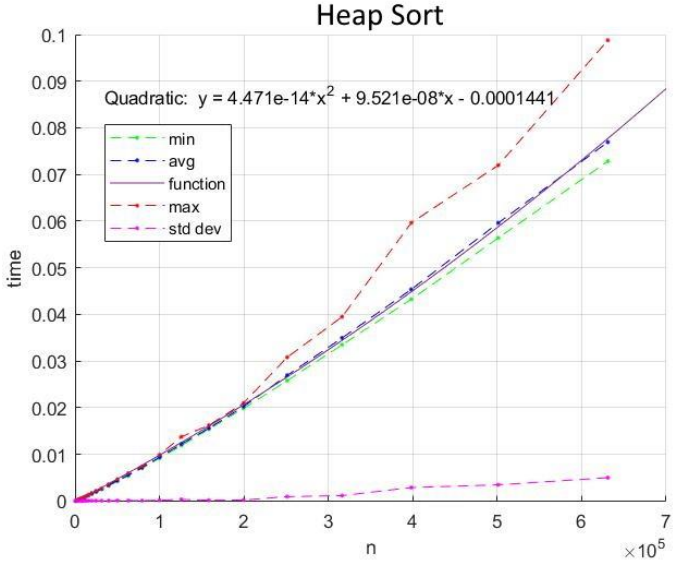
Quick Sort



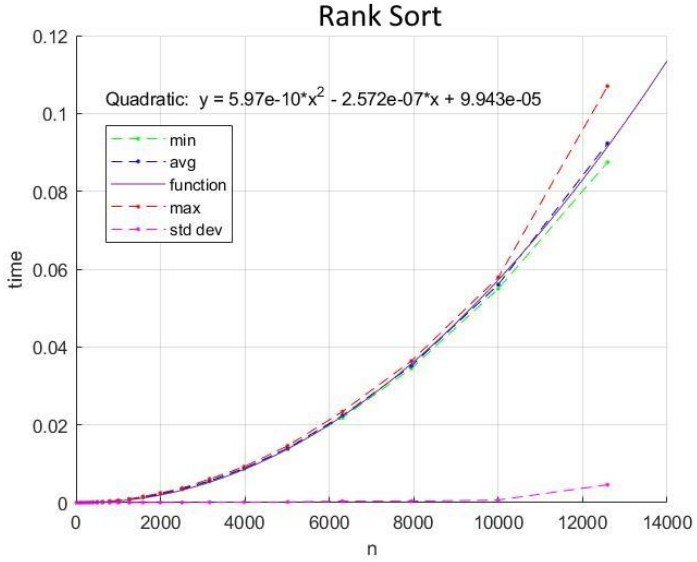
Merge Sort



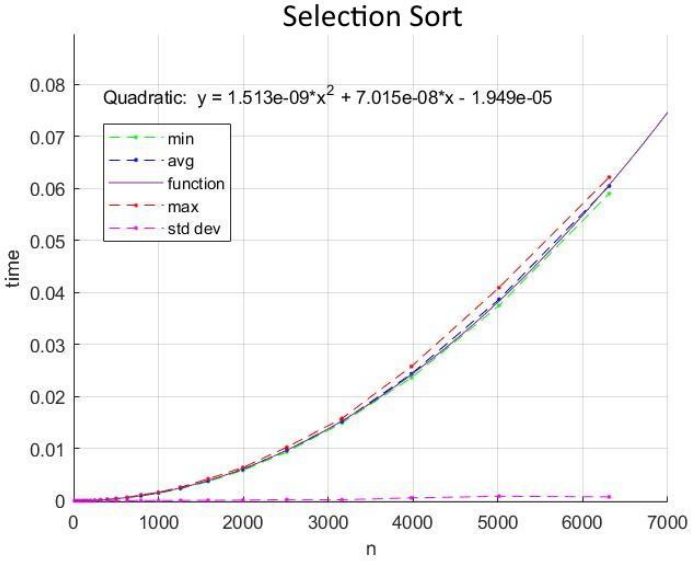
Heap Sort



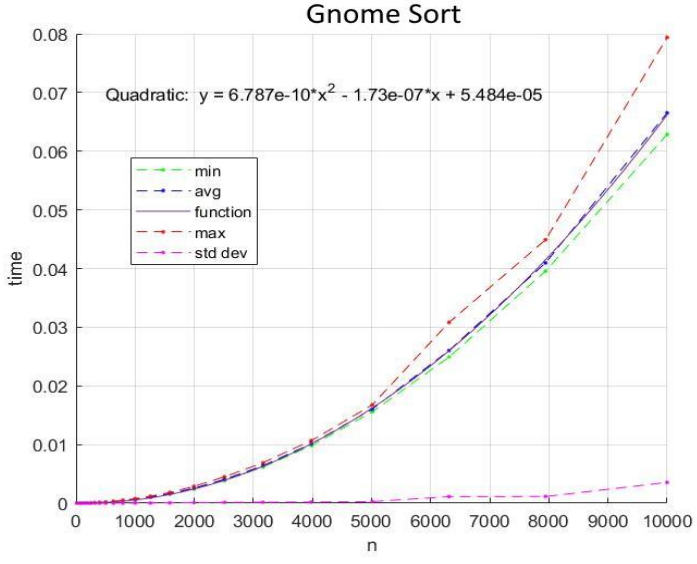
Rank Sort



Selection Sort



Gnome Sort



O programa *sorting_methods.c* foi executado sob Windows Subsystem for Linux num computador ASUS G550JK com um processador i7-4710HQ. O resultado do *Gnome Sort* foi obtido numa execução separada (no mesmo computador).
Os resultados de cada *sort* foram copiados para ficheiros de texto separados, que depois foram carregados no MatLab para criação dos gráficos.

Sort	Melhor caso experimental	Pior caso experimental
Bubble Sort	$O(n^2)$	$O(n^2)$
Shaker Sort	$O(n^2)$	$O(n^2)$
Insertion Sort	$O(n^2)$	$O(n^2)$
Shell Sort	$O(n^?)$	$O(n^?)$
Quick Sort	$O(n * log(n))$	$O(n * log(n))$
Merge Sort	$O(n * log(n))$	$O(n * log(n))$
Heap Sort	$O(n * log(n))$	$O(n * log(n))$
Rank Sort	$O(n^2)$	$O(n^2)$
Selection Sort	$O(n^2)$	$O(n^2)$
Gnome Sort	$O(n^2)$	$O(n^2)$

Desenhámos uma reta nos gráficos de cada algoritmo para comparar com uma complexidade $O(n)$. Concluímos que, nos primeiros 3 e nos últimos 3 algoritmos tanto os melhores casos obtidos como os piores aparentam ter uma complexidade $O(n^2)$. Isto não se verificou nos algoritmos *Quick Sort*, *Merge Sort* e *Heap Sort*, que deduzimos que têm uma complexidade $O(n * log(n))$e também no *Shell Sort*, que aparenta ser diferente de $O(n^2)$ e $O(n * log(n))$, pelo que pensamos que seja da forma $O(n^k)$, com *k* desconhecido mas entre 1 e 2.

Que algoritmo escolheríamos?

Pela análise dos resultados, o *Quick Sort* aparenta ser o mais eficiente. Teoricamente, no pior caso, tem complexidade $O(n^2)$. No entanto, o pior caso é quando é sempre escolhido o pior pivot, o que muito dificilmente acontece na realidade. Mesmo assim, se pretendêssemos ter a certeza absoluta de que o pior caso correria em complexidade $O(n * log(n))$ ou utilizar um *sort* estável, escolheríamos o *Merge Sort*, que, nos resultados, aparenta ser o segundo melhor.

Mesmo para arrays de pequena dimensão, não se verificou vantagem de tempo de execução , o *Quick Sort* demonstrou ser o melhor método quando comparado com os outros sorts.

Quando comparado com outros *sorts* o *Quick Sort* demonstra ser o mais rápido porque os arrays usados para o estudo só variam no número de elementos e não na sua distribuição.
Por exemplo: o *Insertion Sort*, o *Bubble Sort* e o *Shaker Sort* são, teoricamente, mais eficientes nos seus melhores casos (array já ordenado ou quase ordenado); e o *Merge Sort* e o *Heap Sort* são mais eficientes perante os piores casos para o *Quick Sort* (muitos elementos repetidos).

Os algoritmos menos eficientes como o *Bubble Sort*, *Insertion Sort* e *Selection Sort* são, sem dúvida, mais fáceis de implementar do que os melhores. No entanto, muitas linguagens de programação já possuem funções de ordenação eficientes (por exemplo, *qsort*, em C), pelo que não há verdadeiro incentivo para utilizar os algoritmos mais simples.

Tabela de tempos de execução de cada *sort*

	Bubble Sort					Shaker Sort			
n	Min. time	Max. time	Avg. time	Std. dev.		Min. time	Max. time	Avg. time	Std. dev.
10	6.04E-07	6.98E-07	6.47E-07	2.21E-08		6.30E-07	8.50E-07	6.97E-07	3.54E-08
13	6.82E-07	8.02E-07	7.44E-07	2.88E-08		6.85E-07	9.63E-07	7.50E-07	4.83E-08
16	7.72E-07	9.34E-07	8.53E-07	3.80E-08		7.66E-07	9.03E-07	8.31E-07	3.29E-08
20	9.51E-07	1.16E-06	1.05E-06	5.17E-08		9.04E-07	1.09E-06	9.91E-07	4.41E-08
25	1.23E-06	1.50E-06	1.37E-06	6.22E-08		1.11E-06	1.36E-06	1.23E-06	6.00E-08
32	1.70E-06	2.07E-06	1.89E-06	9.09E-08		1.48E-06	1.81E-06	1.65E-06	7.82E-08
40	2.33E-06	2.84E-06	2.60E-06	1.19E-07		2.01E-06	2.49E-06	2.25E-06	1.11E-07
50	3.34E-06	5.04E-06	3.72E-06	2.07E-07		2.81E-06	3.49E-06	3.14E-06	1.51E-07
63	4.94E-06	6.10E-06	5.42E-06	2.42E-07		4.09E-06	4.94E-06	4.51E-06	2.02E-07
79	7.34E-06	8.97E-06	8.01E-06	3.42E-07		5.96E-06	7.12E-06	6.52E-06	2.72E-07
100	1.12E-05	1.63E-05	1.22E-05	5.90E-07		8.94E-06	1.05E-05	9.69E-06	3.63E-07
126	1.67E-05	2.00E-05	1.80E-05	7.00E-07		1.33E-05	1.61E-05	1.43E-05	5.59E-07
158	2.43E-05	3.65E-05	2.65E-05	1.91E-06		1.92E-05	2.99E-05	2.10E-05	1.36E-06
200	3.54E-05	5.71E-05	3.86E-05	3.58E-06		2.92E-05	4.27E-05	3.17E-05	2.48E-06
251	5.25E-05	8.14E-05	5.69E-05	5.33E-06		4.40E-05	6.41E-05	4.76E-05	3.52E-06
316	7.82E-05	1.14E-04	8.37E-05	7.28E-06		6.68E-05	9.15E-05	7.34E-05	5.55E-06
398	1.17E-04	1.62E-04	1.26E-04	1.03E-05		9.95E-05	1.35E-04	1.07E-04	7.36E-06
501	1.77E-04	2.33E-04	1.89E-04	1.31E-05		1.51E-04	1.97E-04	1.65E-04	1.16E-05
631	2.70E-04	3.46E-04	2.88E-04	1.82E-05		2.24E-04	2.86E-04	2.42E-04	1.39E-05
794	4.11E-04	4.94E-04	4.39E-04	2.21E-05		3.42E-04	4.23E-04	3.70E-04	2.01E-05
1000	6.38E-04	7.33E-04	6.76E-04	2.21E-05		5.46E-04	6.80E-04	5.84E-04	2.51E-05
1259	1.02E-03	1.20E-03	1.08E-03	3.71E-05		8.39E-04	1.00E-03	8.90E-04	3.07E-05
1585	1.60E-03	1.84E-03	1.68E-03	4.79E-05		1.32E-03	1.53E-03	1.38E-03	4.22E-05
1995	2.58E-03	2.78E-03	2.65E-03	4.30E-05		2.08E-03	2.29E-03	2.15E-03	4.44E-05
2512	4.23E-03	4.50E-03	4.33E-03	5.64E-05		3.34E-03	3.80E-03	3.50E-03	1.06E-04
3162	7.16E-03	7.69E-03	7.37E-03	1.25E-04		5.47E-03	6.84E-03	5.79E-03	2.52E-04
3981	1.24E-02	1.33E-02	1.28E-02	1.89E-04		9.13E-03	1.02E-02	9.50E-03	2.62E-04
5012	2.18E-02	2.33E-02	2.24E-02	3.51E-04		1.54E-02	1.70E-02	1.60E-02	3.74E-04
6310	3.82E-02	4.22E-02	3.95E-02	8.74E-04		2.68E-02	3.02E-02	2.79E-02	7.54E-04
7943	6.68E-02	7.79E-02	7.01E-02	2.54E-03		4.62E-02	5.17E-02	4.81E-02	1.29E-03
10000						7.89E-02	8.84E-02	8.19E-02	2.18E-03

	Insertion Sort					Shell Sort			
n	Min. time	Max. time	Avg. time	Std. dev.		Min. time	Max. time	Avg. time	Std. dev.
10	5.76E-07	6.21E-07	5.98E-07	1.08E-08		6.09E-07	6.97E-07	6.46E-07	1.80E-08
13	6.01E-07	6.61E-07	6.23E-07	1.35E-08		6.72E-07	9.43E-07	7.22E-07	5.21E-08
16	6.35E-07	6.90E-07	6.59E-07	1.30E-08		7.26E-07	1.05E-06	7.96E-07	6.26E-08
20	6.82E-07	7.42E-07	7.10E-07	1.40E-08		8.32E-07	1.12E-06	8.95E-07	4.57E-08
25	7.50E-07	8.26E-07	7.87E-07	1.76E-08		9.58E-07	1.13E-06	1.03E-06	3.57E-08
32	8.50E-07	9.47E-07	8.93E-07	2.15E-08		1.14E-06	1.57E-06	1.24E-06	7.35E-08
40	9.80E-07	1.09E-06	1.03E-06	2.52E-08		1.39E-06	2.07E-06	1.72E-06	1.90E-07
50	1.17E-06	1.31E-06	1.24E-06	3.18E-08		1.70E-06	2.12E-06	1.82E-06	6.97E-08
63	1.45E-06	1.63E-06	1.54E-06	4.30E-08		2.14E-06	2.43E-06	2.26E-06	6.27E-08
79	1.87E-06	2.11E-06	1.98E-06	5.87E-08		2.67E-06	3.03E-06	2.82E-06	7.26E-08
100	2.57E-06	2.92E-06	2.73E-06	8.36E-08		3.44E-06	3.78E-06	3.60E-06	7.92E-08
126	3.54E-06	4.01E-06	3.77E-06	1.11E-07		4.43E-06	4.81E-06	4.61E-06	8.93E-08
158	4.84E-06	5.52E-06	5.16E-06	1.55E-07		5.74E-06	6.25E-06	5.99E-06	1.26E-07
200	7.22E-06	8.16E-06	7.68E-06	2.15E-07		7.43E-06	8.10E-06	7.70E-06	1.36E-07
251	1.04E-05	1.17E-05	1.10E-05	2.91E-07		9.63E-06	1.06E-05	9.96E-06	1.82E-07
316	1.53E-05	2.46E-05	1.63E-05	6.42E-07		1.26E-05	2.17E-05	1.30E-05	4.46E-07
398	2.30E-05	3.45E-05	2.46E-05	1.92E-06		1.65E-05	2.59E-05	1.70E-05	9.05E-07
501	3.49E-05	5.56E-05	3.73E-05	2.68E-06		2.17E-05	3.22E-05	2.25E-05	1.54E-06
631	5.42E-05	7.82E-05	5.78E-05	3.97E-06		2.84E-05	4.02E-05	2.96E-05	2.03E-06
794	8.39E-05	1.20E-04	8.98E-05	7.45E-06		3.61E-05	5.51E-05	3.80E-05	2.78E-06
1000	1.31E-04	1.78E-04	1.41E-04	1.04E-05		4.71E-05	7.53E-05	4.96E-05	4.25E-06
1259	2.05E-04	2.64E-04	2.20E-04	1.49E-05		6.17E-05	9.22E-05	6.53E-05	5.69E-06
1585	3.24E-04	3.92E-04	3.46E-04	1.80E-05		8.11E-05	1.16E-04	8.61E-05	7.39E-06
1995	5.09E-04	5.94E-04	5.43E-04	2.14E-05		1.06E-04	1.47E-04	1.12E-04	8.60E-06
2512	8.16E-04	9.06E-04	8.56E-04	1.93E-05		1.38E-04	1.88E-04	1.46E-04	1.14E-05
3162	1.30E-03	1.42E-03	1.35E-03	2.92E-05		1.79E-04	2.33E-04	1.90E-04	1.37E-05
3981	2.06E-03	2.21E-03	2.12E-03	3.45E-05		2.42E-04	2.98E-04	2.58E-04	1.31E-05
5012	3.26E-03	3.45E-03	3.35E-03	4.65E-05		3.09E-04	3.81E-04	3.32E-04	1.72E-05
6310	5.11E-03	5.44E-03	5.23E-03	6.57E-05		4.00E-04	4.86E-04	4.26E-04	2.05E-05
7943	8.21E-03	9.16E-03	8.48E-03	1.93E-04		5.21E-04	6.15E-04	5.46E-04	2.12E-05
10000	1.31E-02	1.40E-02	1.33E-02	1.95E-04		6.77E-04	7.84E-04	7.07E-04	2.43E-05
12589	2.08E-02	2.33E-02	2.16E-02	6.07E-04		8.91E-04	1.01E-03	9.35E-04	2.39E-05
15849	3.27E-02	3.60E-02	3.37E-02	7.50E-04		1.18E-03	1.30E-03	1.22E-03	2.91E-05
19953	5.20E-02	5.77E-02	5.35E-02	1.31E-03		1.53E-03	1.67E-03	1.59E-03	3.42E-05
25119	8.22E-02	8.58E-02	8.37E-02	6.70E-04		2.00E-03	2.15E-03	2.07E-03	3.54E-05
31623						2.61E-03	2.79E-03	2.69E-03	4.32E-05
39811						3.36E-03	3.62E-03	3.47E-03	5.76E-05
50119						4.44E-03	4.71E-03	4.57E-03	6.18E-05
63096						5.77E-03	6.06E-03	5.90E-03	6.57E-05
79433						7.49E-03	7.86E-03	7.65E-03	8.33E-05
100000						9.76E-03	1.02E-02	9.94E-03	9.94E-05
125893						1.26E-02	1.34E-02	1.29E-02	1.97E-04
158489						1.64E-02	1.74E-02	1.68E-02	2.34E-04
199526						2.12E-02	2.23E-02	2.17E-02	2.53E-04
251189						2.73E-02	3.11E-02	2.82E-02	6.25E-04
316228						3.64E-02	4.08E-02	3.77E-02	8.56E-04
398107						4.66E-02	5.09E-02	4.83E-02	8.82E-04
501187						6.04E-02	6.39E-02	6.19E-02	8.22E-04

	Quick Sort					Merge Sort			
n	Min. time	Max. time	Avg. time	Std. dev.		Min. time	Max. time	Avg. time	Std. dev.
10	5.59E-07	6.24E-07	5.89E-07	1.69E-08		5.73E-07	6.21E-07	5.97E-07	1.14E-08
13	5.85E-07	6.54E-07	6.15E-07	1.79E-08		6.17E-07	6.69E-07	6.36E-07	1.02E-08
16	6.16E-07	6.95E-07	6.52E-07	2.08E-08		6.55E-07	8.49E-07	6.78E-07	2.31E-08
20	7.20E-07	8.43E-07	7.70E-07	2.73E-08		7.02E-07	9.03E-07	7.27E-07	1.88E-08
25	7.99E-07	9.26E-07	8.52E-07	3.10E-08		7.58E-07	1.10E-06	8.14E-07	7.09E-08
32	9.40E-07	1.10E-06	1.01E-06	3.77E-08		8.73E-07	1.17E-06	9.17E-07	4.24E-08
40	1.11E-06	1.29E-06	1.19E-06	3.98E-08		1.12E-06	1.24E-06	1.17E-06	2.64E-08
50	1.33E-06	1.53E-06	1.42E-06	4.61E-08		1.31E-06	1.43E-06	1.36E-06	2.89E-08
63	1.66E-06	1.91E-06	1.78E-06	5.95E-08		1.54E-06	1.74E-06	1.63E-06	3.98E-08
79	2.10E-06	2.39E-06	2.23E-06	6.73E-08		1.98E-06	2.15E-06	2.06E-06	4.08E-08
100	2.65E-06	2.99E-06	2.81E-06	7.91E-08		2.54E-06	2.77E-06	2.65E-06	5.08E-08
126	3.34E-06	3.72E-06	3.52E-06	9.42E-08		3.20E-06	3.49E-06	3.31E-06	6.22E-08
158	4.25E-06	4.76E-06	4.49E-06	1.12E-07		4.23E-06	4.52E-06	4.36E-06	6.69E-08
200	5.55E-06	6.18E-06	5.82E-06	1.37E-07		5.61E-06	6.06E-06	5.78E-06	8.99E-08
251	7.19E-06	7.93E-06	7.52E-06	1.66E-07		7.12E-06	7.53E-06	7.30E-06	9.07E-08
316	9.38E-06	1.02E-05	9.77E-06	1.98E-07		9.34E-06	1.02E-05	9.65E-06	1.86E-07
398	1.22E-05	1.35E-05	1.27E-05	2.49E-07		1.24E-05	1.40E-05	1.27E-05	2.39E-07
501	1.57E-05	2.59E-05	1.66E-05	1.16E-06		1.58E-05	1.88E-05	1.63E-05	3.26E-07
631	2.09E-05	2.31E-05	2.16E-05	3.83E-07		2.11E-05	3.14E-05	2.18E-05	1.20E-06
794	2.65E-05	3.66E-05	2.77E-05	9.42E-07		2.79E-05	3.85E-05	2.85E-05	1.34E-06
1000	3.44E-05	4.76E-05	3.60E-05	2.08E-06		3.57E-05	4.68E-05	3.66E-05	2.00E-06
1259	4.49E-05	6.15E-05	4.71E-05	2.97E-06		4.73E-05	6.01E-05	4.84E-05	2.48E-06
1585	5.86E-05	8.13E-05	6.12E-05	3.38E-06		6.27E-05	7.67E-05	6.41E-05	2.69E-06
1995	7.63E-05	1.05E-04	7.96E-05	4.51E-06		8.02E-05	1.11E-04	8.36E-05	6.45E-06
2512	9.93E-05	1.34E-04	1.04E-04	5.77E-06		1.05E-04	1.39E-04	1.08E-04	4.93E-06
3162	1.29E-04	1.66E-04	1.35E-04	7.16E-06		1.39E-04	1.74E-04	1.43E-04	7.11E-06
3981	1.68E-04	2.27E-04	1.78E-04	1.20E-05		1.78E-04	2.21E-04	1.85E-04	9.57E-06
5012	2.18E-04	2.71E-04	2.30E-04	1.20E-05		2.32E-04	2.85E-04	2.42E-04	1.52E-05
6310	2.83E-04	3.42E-04	2.96E-04	1.27E-05		3.08E-04	3.74E-04	3.23E-04	1.83E-05
7943	3.65E-04	4.47E-04	3.85E-04	1.67E-05		3.93E-04	4.67E-04	4.13E-04	2.09E-05
10000	4.75E-04	5.67E-04	5.01E-04	2.08E-05		5.08E-04	6.01E-04	5.46E-04	1.71E-05
12589	6.16E-04	7.32E-04	6.49E-04	2.58E-05		6.85E-04	7.60E-04	7.24E-04	1.32E-05
15849	7.98E-04	9.41E-04	8.40E-04	3.26E-05		8.71E-04	1.00E-03	9.14E-04	2.66E-05
19953	1.04E-03	1.18E-03	1.08E-03	3.01E-05		1.13E-03	1.31E-03	1.17E-03	3.70E-05
25119	1.35E-03	1.51E-03	1.41E-03	3.59E-05		1.49E-03	1.73E-03	1.56E-03	4.83E-05
31623	1.76E-03	1.92E-03	1.82E-03	3.69E-05		1.92E-03	2.20E-03	2.00E-03	5.60E-05
39811	2.26E-03	2.60E-03	2.37E-03	7.61E-05		2.47E-03	2.91E-03	2.60E-03	8.80E-05
50119	2.95E-03	4.12E-03	3.19E-03	2.19E-04		3.28E-03	4.31E-03	3.48E-03	1.56E-04
63096	3.82E-03	4.59E-03	4.04E-03	1.70E-04		4.18E-03	4.97E-03	4.40E-03	1.66E-04
79433	4.88E-03	5.46E-03	5.03E-03	1.18E-04		5.32E-03	6.12E-03	5.52E-03	1.66E-04
100000	6.24E-03	6.94E-03	6.46E-03	1.48E-04		7.03E-03	7.82E-03	7.23E-03	1.70E-04
125893	8.07E-03	8.89E-03	8.40E-03	2.04E-04		9.03E-03	1.15E-02	9.65E-03	5.19E-04
158489	1.04E-02	1.13E-02	1.08E-02	2.44E-04		1.15E-02	1.28E-02	1.19E-02	3.06E-04
199526	1.34E-02	1.46E-02	1.38E-02	2.95E-04		1.51E-02	1.73E-02	1.58E-02	4.91E-04
251189	1.75E-02	1.94E-02	1.83E-02	3.96E-04		1.95E-02	2.30E-02	2.04E-02	6.94E-04
316228	2.24E-02	2.65E-02	2.36E-02	8.87E-04		2.47E-02	3.05E-02	2.63E-02	1.24E-03
398107	2.82E-02	3.02E-02	2.91E-02	5.29E-04		3.21E-02	3.49E-02	3.32E-02	7.13E-04
501187	3.62E-02	3.98E-02	3.71E-02	7.00E-04		4.08E-02	4.34E-02	4.17E-02	6.82E-04
630957	4.64E-02	4.95E-02	4.76E-02	7.68E-04		5.21E-02	6.34E-02	5.49E-02	2.48E-03
794328	5.94E-02	6.87E-02	6.21E-02	2.20E-03		6.79E-02	7.64E-02	6.98E-02	1.52E-03

	Heap Sort					Rank Sort			
n	Min. time	Max. time	Avg. time	Std. dev.		Min. time	Max. time	Avg. time	Std. dev.
10	6.20E-07	8.14E-07	6.58E-07	2.89E-08		5.84E-07	6.33E-07	6.00E-07	1.09E-08
13	6.72E-07	8.98E-07	7.20E-07	2.90E-08		6.23E-07	6.58E-07	6.34E-07	9.41E-09
16	7.45E-07	8.52E-07	7.95E-07	2.46E-08		6.65E-07	7.02E-07	6.79E-07	9.84E-09
20	8.48E-07	9.91E-07	9.07E-07	3.06E-08		7.41E-07	7.85E-07	7.58E-07	1.15E-08
25	9.83E-07	1.13E-06	1.06E-06	3.39E-08		8.54E-07	9.61E-07	8.81E-07	1.70E-08
32	1.22E-06	1.39E-06	1.29E-06	3.98E-08		1.07E-06	1.14E-06	1.10E-06	1.54E-08
40	1.51E-06	1.69E-06	1.59E-06	4.22E-08		1.36E-06	1.46E-06	1.41E-06	2.23E-08
50	1.86E-06	2.08E-06	1.97E-06	5.13E-08		1.85E-06	1.99E-06	1.92E-06	3.39E-08
63	2.37E-06	2.62E-06	2.49E-06	5.99E-08		2.63E-06	2.86E-06	2.74E-06	5.21E-08
79	2.99E-06	3.27E-06	3.13E-06	6.68E-08		3.82E-06	4.18E-06	3.98E-06	8.14E-08
100	3.79E-06	4.21E-06	3.99E-06	9.63E-08		5.79E-06	6.38E-06	6.05E-06	1.34E-07
126	4.91E-06	5.47E-06	5.13E-06	1.24E-07		8.85E-06	9.81E-06	9.25E-06	2.05E-07
158	6.47E-06	7.03E-06	6.71E-06	1.22E-07		1.36E-05	1.50E-05	1.42E-05	2.98E-07
200	8.23E-06	8.86E-06	8.50E-06	1.38E-07		2.15E-05	3.24E-05	2.26E-05	1.50E-06
251	1.08E-05	1.17E-05	1.11E-05	1.70E-07		3.30E-05	4.58E-05	3.47E-05	2.06E-06
316	1.40E-05	1.50E-05	1.44E-05	1.84E-07		5.23E-05	8.26E-05	5.55E-05	4.55E-06
398	1.79E-05	2.79E-05	1.86E-05	1.27E-06		8.26E-05	1.17E-04	8.77E-05	7.21E-06
501	2.34E-05	3.42E-05	2.44E-05	1.78E-06		1.31E-04	1.76E-04	1.41E-04	1.03E-05
631	3.05E-05	4.17E-05	3.18E-05	2.03E-06		2.07E-04	2.64E-04	2.21E-04	1.42E-05
794	3.97E-05	5.81E-05	4.14E-05	2.45E-06		3.28E-04	3.98E-04	3.49E-04	1.81E-05
1000	5.17E-05	8.10E-05	5.38E-05	4.26E-06		5.20E-04	6.12E-04	5.54E-04	2.24E-05
1259	6.72E-05	9.71E-05	7.01E-05	6.17E-06		8.41E-04	9.40E-04	8.80E-04	2.04E-05
1585	8.73E-05	1.20E-04	9.17E-05	6.75E-06		1.34E-03	1.58E-03	1.41E-03	4.69E-05
1995	1.14E-04	1.51E-04	1.20E-04	8.88E-06		2.15E-03	2.47E-03	2.25E-03	6.53E-05
2512	1.47E-04	1.97E-04	1.55E-04	1.15E-05		3.41E-03	3.74E-03	3.52E-03	6.90E-05
3162	1.90E-04	2.47E-04	2.02E-04	1.37E-05		5.42E-03	6.07E-03	5.60E-03	1.25E-04
3981	2.46E-04	3.04E-04	2.59E-04	1.59E-05		8.61E-03	9.25E-03	8.81E-03	1.32E-04
5012	3.17E-04	3.85E-04	3.35E-04	1.73E-05		1.37E-02	1.46E-02	1.40E-02	1.74E-04
6310	4.08E-04	4.89E-04	4.31E-04	2.00E-05		2.18E-02	2.34E-02	2.24E-02	3.89E-04
7943	5.29E-04	6.08E-04	5.57E-04	2.08E-05		3.45E-02	3.64E-02	3.52E-02	3.96E-04
10000	6.83E-04	7.83E-04	7.24E-04	2.28E-05		5.49E-02	5.79E-02	5.60E-02	6.94E-04
12589	9.07E-04	1.05E-03	9.39E-04	2.62E-05		8.75E-02	1.07E-01	9.23E-02	4.60E-03
15849	1.15E-03	1.34E-03	1.21E-03	3.75E-05					
19953	1.50E-03	1.72E-03	1.57E-03	4.27E-05					
25119	1.94E-03	2.20E-03	2.02E-03	5.18E-05					
31623	2.51E-03	2.75E-03	2.59E-03	4.80E-05					
39811	3.26E-03	3.58E-03	3.36E-03	6.92E-05					
50119	4.21E-03	4.66E-03	4.36E-03	9.54E-05					
63096	5.41E-03	5.97E-03	5.59E-03	1.10E-04					
79433	7.01E-03	7.41E-03	7.16E-03	8.80E-05					
100000	9.15E-03	9.90E-03	9.39E-03	1.73E-04					
125893	1.19E-02	1.38E-02	1.22E-02	3.47E-04					
158489	1.54E-02	1.62E-02	1.56E-02	1.73E-04					
199526	1.98E-02	2.10E-02	2.03E-02	2.08E-04					
251189	2.58E-02	3.08E-02	2.69E-02	9.43E-04					
316228	3.35E-02	3.95E-02	3.50E-02	1.16E-03					
398107	4.32E-02	5.96E-02	4.54E-02	2.89E-03					
501187	5.64E-02	7.20E-02	5.96E-02	3.47E-03					
630957	7.28E-02	9.88E-02	7.69E-02	5.00E-03					

	Selection Sort					Gnome Sort			
	Min. time	Max. time	Avg. time	Std. dev.		Min. time	Max. time	Avg. time	Std. dev.
n	6.64E-07	1.05E-06	8.00E-07	8.84E-08		6.33E-07	7.39E-07	6.77E-07	2.40E-08
10	7.60E-07	1.18E-06	8.63E-07	1.13E-07		7.13E-07	8.65E-07	7.89E-07	3.89E-08
13	8.95E-07	1.09E-06	9.25E-07	3.63E-08		8.04E-07	9.89E-07	8.96E-07	4.41E-08
16	1.12E-06	1.18E-06	1.14E-06	1.27E-08		9.38E-07	1.18E-06	1.06E-06	6.06E-08
20	1.46E-06	1.50E-06	1.48E-06	1.02E-08		1.16E-06	1.44E-06	1.30E-06	6.53E-08
25	2.05E-06	2.13E-06	2.09E-06	1.31E-08		1.51E-06	2.07E-06	1.76E-06	1.30E-07
32	2.93E-06	3.78E-06	3.11E-06	1.76E-07		1.94E-06	2.56E-06	2.22E-06	1.40E-07
40	4.28E-06	4.99E-06	4.62E-06	1.92E-07		2.65E-06	3.77E-06	3.01E-06	2.13E-07
50	6.39E-06	7.36E-06	6.65E-06	1.84E-07		3.72E-06	6.17E-06	4.42E-06	5.92E-07
63	9.70E-06	1.20E-05	1.01E-05	2.97E-07		5.40E-06	6.96E-06	6.02E-06	3.47E-07
79	1.52E-05	1.71E-05	1.56E-05	2.73E-07		7.99E-06	1.22E-05	9.13E-06	9.44E-07
100	2.34E-05	3.37E-05	2.46E-05	1.77E-06		1.17E-05	2.28E-05	1.33E-05	1.53E-06
126	3.65E-05	4.74E-05	3.81E-05	2.02E-06		1.73E-05	2.81E-05	1.93E-05	1.57E-06
158	5.80E-05	7.84E-05	6.12E-05	3.70E-06		2.67E-05	4.03E-05	2.97E-05	2.46E-06
200	9.01E-05	1.22E-04	9.46E-05	5.92E-06		4.10E-05	5.99E-05	4.53E-05	3.68E-06
251	1.41E-04	1.99E-04	1.51E-04	1.11E-05		6.32E-05	9.67E-05	7.06E-05	6.81E-06
316	2.23E-04	2.86E-04	2.39E-04	1.48E-05		9.83E-05	1.41E-04	1.09E-04	9.37E-06
398	3.54E-04	4.36E-04	3.82E-04	1.84E-05		1.54E-04	2.02E-04	1.69E-04	1.06E-05
501	5.63E-04	6.66E-04	6.02E-04	2.20E-05		2.43E-04	3.18E-04	2.66E-04	1.69E-05
631	9.18E-04	1.14E-03	9.73E-04	4.21E-05		3.82E-04	5.07E-04	4.17E-04	2.45E-05
794	1.45E-03	1.64E-03	1.51E-03	3.69E-05		6.08E-04	7.77E-04	6.56E-04	3.18E-05
1000	2.31E-03	2.60E-03	2.41E-03	5.60E-05		9.67E-04	1.19E-03	1.03E-03	4.43E-05
1259	3.69E-03	4.21E-03	3.84E-03	1.02E-04		1.53E-03	1.84E-03	1.62E-03	6.48E-05
1585	5.84E-03	6.39E-03	6.03E-03	1.22E-04		2.42E-03	2.86E-03	2.57E-03	9.38E-05
1995	9.37E-03	1.03E-02	9.70E-03	2.27E-04		3.85E-03	4.50E-03	4.06E-03	1.38E-04
2512	1.49E-02	1.58E-02	1.52E-02	2.02E-04		6.15E-03	6.90E-03	6.41E-03	1.68E-04
3162	2.36E-02	2.58E-02	2.44E-02	5.50E-04		9.80E-03	1.07E-02	1.01E-02	2.03E-04
3981	3.75E-02	4.09E-02	3.87E-02	8.69E-04		1.56E-02	1.68E-02	1.60E-02	2.71E-04
5012	5.90E-02	6.21E-02	6.04E-02	7.49E-04		2.49E-02	3.08E-02	2.60E-02	1.12E-03
6310						3.96E-02	4.49E-02	4.10E-02	1.14E-03
7943						6.29E-02	7.94E-02	6.65E-02	3.55E-03

Código de implementação do *Gnome Sort*

```
void gnome_sort(T *data, int first, int one_after_last) {
   int i = first;

   while (i < one_after_last) {
       if (i == first) // se estivermos no 1º elemento
           i++;        // passar para o 2º (porque se compara com i - 1)
       if (data[i] >= data[i - 1]) // se estiverem ordenados
           i++;                    // passar para o seguinte
       else {                      // caso não estejam
           T tmp = data[i - 1];    // trocar os elementos
           data[i - 1] = data[i];
           data[i--] = tmp;        //i-- para recuar
       }
   }
}
```

O *Gnome Sort* é um algoritmo semelhante ao *Insertion Sort* (opera sobre um item de cada vez) mas coloca o item no lugar certo através de uma série de trocas, como o *Bubble Sort*.

O algoritmo encontra o primeiro par de elementos que estão fora de ordem e troca-os. Tira proveito do facto de que a troca pode introduzir um par desordenado junto ao par trocado. Apenas precisa de verificar a posição diretamente atrás dos elementos trocados, os elementos anteriores a essa posição já estarão ordenados.

Segue-se um exemplo passo a passo do funcionamento da rotina. A cor azul representa `i == first` ou `i == one_after_last`, a cor verde indica que os elementos em comparação estão ordenados e a cor vermelha indica que os elementos em comparação estão fora de ordem.

0	1	2	3	i	i - 1	Passo
4	2	3	1	0	-1	i == first -> i++
4	2	3	1	1	0	data[i] < data[i-1] -> troca, i--
2	4	3	1	0	-1	i == first -> i++
2	4	3	1	1	0	data[i] >= data[i-1] -> i++
2	4	3	1	2	1	data[i] < data[i-1] -> troca, i--
2	3	4	1	1	0	data[i] >= data[i-1] -> i++
2	3	4	1	2	1	data[i] >= data[i-1] -> i++
2	3	4	1	3	2	data[i] < data[i-1] -> troca, i--
2	3	1	4	2	1	data[i] < data[i-1] -> troca, i--
2	1	3	4	1	0	data[i] < data[i-1] -> troca, i--
1	2	3	4	0	-1	i == first -> i++
1	2	3	4	1	0	data[i] >= data[i-1] -> i++
1	2	3	4	2	1	data[i] >= data[i-1] -> i++
1	2	3	4	3	2	data[i] >= data[i-1] -> i++
1	2	3	4	4	3	i == one_after_last -> terminou

Código MATLAB utilizado para gerar os gráficos

```
bubble_sort = load("bubble_sort.txt");
heap_sort = load("heap_sort.txt");
insertion_sort = load("insertion_sort.txt");
merge_sort = load("merge_sort.txt");
quick_sort = load("quick_sort.txt");
rank_sort = load("rank_sort.txt");
selection_sort = load("selection_sort.txt");
shaker_sort = load("shaker_sort.txt");
shell_sort = load("shell_sort.txt");
gnome_sort = load("gnome_sort.txt");

sorts = {bubble_sort, heap_sort, insertion_sort, merge_sort, quick_sort, rank_sort, selection_sort,
shaker_sort, shell_sort, gnome_sort};

for i = 1 : length(sorts)
      figure(i);
      analyse(sorts{i});
end

function [] = analyse(sort)
      grid on;

      n = sort(:, 1);
      min = sort(:, 2);
      max = sort(:, 3);
      avg = sort(:, 4);
      std_dev = sort(:, 5);

      hold on;

      plot(n, min, 'g.--');
      plot(n, avg, 'b.--');
      plot(n, max, 'r.--');
      plot(n, std_dev, 'm.--');
      xlabel("n");
      ylabel("time");

      hold off;
end
```

Exemplo de um dos ficheiros de texto usados no código MATLAB

Editor - C:\Users\tomec\MATLAB Drive\bubble_sort.txt

bubble_sort.txt

```
10 6.040e-07 6.980e-07 6.471e-07 2.206e-08
13 6.820e-07 8.020e-07 7.436e-07 2.883e-08
16 7.720e-07 9.340e-07 8.533e-07 3.796e-08
20 9.510e-07 1.159e-06 1.054e-06 5.165e-08
25 1.233e-06 1.497e-06 1.369e-06 6.215e-08
32 1.702e-06 2.074e-06 1.889e-06 9.086e-08
40 2.325e-06 2.840e-06 2.595e-06 1.194e-07
50 3.339e-06 5.041e-06 3.724e-06 2.069e-07
63 4.942e-06 6.100e-06 5.424e-06 2.416e-07
79 7.339e-06 8.965e-06 8.008e-06 3.416e-07
100 1.115e-05 1.630e-05 1.220e-05 5.903e-07
126 1.666e-05 1.995e-05 1.804e-05 7.004e-07
158 2.431e-05 3.653e-05 2.647e-05 1.909e-06
200 3.541e-05 5.705e-05 3.860e-05 3.582e-06
251 5.252e-05 8.135e-05 5.686e-05 5.328e-06
316 7.819e-05 1.135e-04 8.368e-05 7.280e-06
398 1.170e-04 1.620e-04 1.259e-04 1.033e-05
501 1.768e-04 2.329e-04 1.891e-04 1.314e-05
631 2.695e-04 3.456e-04 2.875e-04 1.819e-05
794 4.110e-04 4.941e-04 4.391e-04 2.213e-05
1000 6.376e-04 7.327e-04 6.757e-04 2.207e-05
1259 1.024e-03 1.201e-03 1.075e-03 3.711e-05
1585 1.604e-03 1.841e-03 1.680e-03 4.791e-05
1995 2.578e-03 2.782e-03 2.654e-03 4.304e-05
2512 4.233e-03 4.502e-03 4.332e-03 5.640e-05
3162 7.162e-03 7.685e-03 7.366e-03 1.253e-04
3981 1.240e-02 1.325e-02 1.275e-02 1.891e-04
5012 2.182e-02 2.332e-02 2.243e-02 3.508e-04
6310 3.818e-02 4.220e-02 3.945e-02 8.744e-04
7943 6.682e-02 7.794e-02 7.012e-02 2.541e-03
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