REPORT

Zajęcia: Digital signal processing

Teacher: prof. dr hab. Vasyl Martsenyuk

Lab 3-4

Date

Topic: "3. Dyskretna transformacja Fouriera (DFT): obliczanie DFT sygnałów oraz analiza wyników. 4. Implementacja algorytmów FFT: porównanie wydajności różnych implementacji FFT."

Variant 7

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1. **Problem statement:**

Generate three sine signals of given f1, f2, and f3 and amplitude |x[k]|max for the sampling frequency fs in the range of $0 \leq k < N$. Plot: 1 1. the "normalized" level of the DFT spectra. 2. the window DTFT spectra normalized to their mainlobe maximum. The intervals for f, Ω, and amplitudes should be chosen by yourself for the best interpretation purposes.

2. Input data:

No	f_1	f_2	f_3	$\lvert x[k]\rvert_{\max}$	f_s	N
7	400	400.25	399.75	3	600	3000

3. Commands used (or GUI):

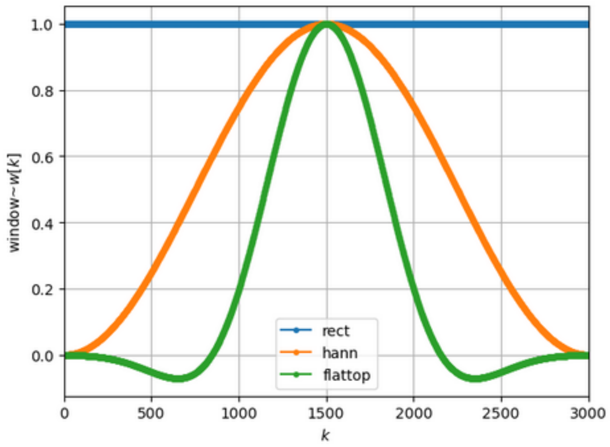
a) source code

b) screenshots

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy.fft import fft , ifft , fftshift
#from s c i p y . f f t import f f t , i f f t , f f t s h i f t
from scipy.signal.windows import hann , flattop
```

In [2]:
```
f1 = 400 # Hz
f2 = 400.25 # Hz
f3 = 399.75 # Hz
fs = 600 # Hz
N = 3000
k = np.arange(N)
x1 = np.sin (2*np.pi * f1 / fs *k)
x2 = np.sin (2*np.pi * f2 / fs *k)
x3 = np.sin (2*np.pi * f3 / fs *k)
```

In [3]:
```
wrect = np.ones (N)
whann = hann(N, sym=False )
wflattop = flattop (N, sym=False )
plt.plot (wrect,'C0o-', ms=3,label='rect')
plt.plot(whann,'C1o-',ms=3,label='hann')
plt.plot (wflattop,'C2o-',ms=3,label='flattop')
plt.xlabel(r'$k$')
plt.ylabel ( r'window~$w[k]$')
plt.xlim (0,N)
plt.legend()
plt.grid(True)
```



In [4]:
```
X1wrect = fft(x1)
X2wrect = fft(x2)
X3wrect = fft(x3)
X1whann = fft(x1*whann)
X2whann = fft(x2*whann)
X3whann = fft(x3*whann)
X1wflattop = fft(x1*wflattop)
X2wflattop = fft(x2*wflattop)
X3wflattop = fft(x3*wflattop)
```

In [5]:
```
def fft2db(X) :
    N=X.size
    Xtmp = 2/N*X # i n d e p e n d e n t o f N, norm f o r s i n e a m p l i t u d e s
    Xtmp[0]*= 1/2 # b i n f o r f =0 Hz i s e x i s t i n g o n l y once ,
    #so c a n c e l *2 from above
    if N % 2 == 0 : # f s /2 i s i n c l u d e d as a b i n
        # f s /2 b i n i s e x i s t i n g o n l y once , so c a n c e l *2 from above
        #6
        Xtmp [N//2] = Xtmp [N//2] / 2
    return 20*np.log10(np.abs(Xtmp))
```

In [6]:
```
df=fs/N
f=np.arange(N)*df

plt.figure(figsize=(16/1.5,10/1.5))
plt.subplot(3,1,1)
plt.plot(f,fft2db(X1wrect),"C0o-",ms=3,label="400Hz")
plt.plot(f,fft2db(X2wrect),"C3o-",ms=3,label="400.25Hz")
plt.plot(f,fft2db(X3wrect),"C1o-",ms=3,label="399.75Hz")
plt.xlim(150,250)
plt.ylim(-60,0)
plt.xticks(np.arange(150,250,25))
plt.yticks(np.arange(-60,10,10))
plt.legend()
#plt.xlabel("f/Hz")
plt.ylabel("A_/_dB")
plt.grid(True)
plt.subplot(3,1,2)
plt.plot(f,fft2db(X1whann),"C0o-",ms=3,label="400Hz")
plt.plot(f,fft2db(X2whann),"C3-",ms=3,label="400.25Hz")
plt.plot(f,fft2db(X3whann),"C1-",ms=3,label="399.75Hz")
plt.xlim(175,225)
plt.ylim(-60,0)
plt.xticks(np.arange(175,230,5))
plt.yticks(np.arange(-60,10,10))
plt.legend()
#plt.xlabel("f/Hz")
plt.ylabel("A_/_dB")
plt.grid(True)
plt.subplot(3,1,3)
plt.plot(f,fft2db(X1wflattop),"C0o-",ms=3,label="400Hz")
plt.plot(f,fft2db(X2wflattop),"C3-",ms=3,label="400.25Hz")
plt.plot(f,fft2db(X3wflattop),"C1-",ms=3,label="399.75Hz")
plt.xlim(175,225)
plt.ylim(-60,0)
plt.xticks(np.arange(175,230,5))
plt.yticks(np.arange(-60,10,10))
plt.legend()
#plt.xlabel("f/Hz")
plt.ylabel("A_/_dB")
plt.grid(True)
```

In [7]:
```
def winDTFTdB(w):
    N=w.size#getwindowlength
    Nz=100*N#zeropaddinglength
    W=np.zeros(Nz)#allocateRAM
    W[0:N]=w#insertwindow
    W=np.abs(fftshift(fft(W)))#fft,fftshiftandmagnitude
    W/=np.max(W)#normalizetomaximum,i.e.themainlobe
    #maximumhere
    W=20*np.log10(W)#getlevelindB
    #getappropriatedigitalfrequencies
    Omega=2*np.pi/Nz*np.arange(Nz)-np.pi#alsoshifted
    return Omega, W
```

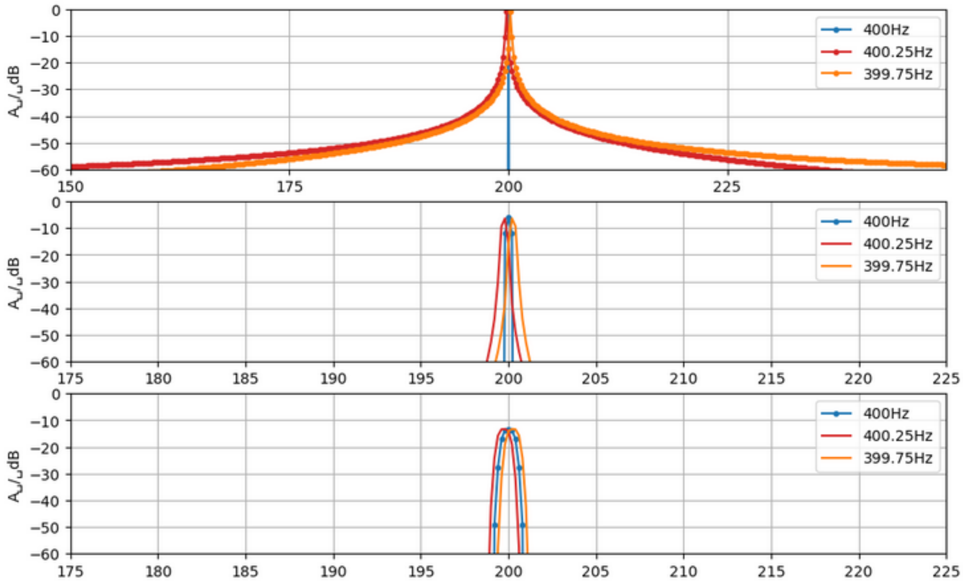
In [ ]:

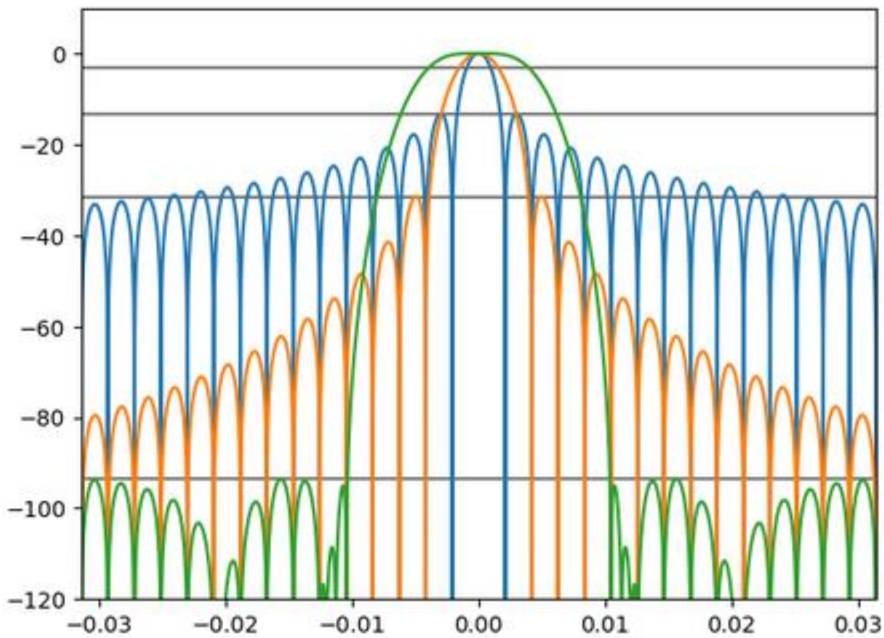
In [8]:
```
plt.plot([-np.pi,+np.pi],[-3.01,-3.01],"gray")#mainlobebandwidth
plt.plot([-np.pi,+np.pi],[-13.3,-13.3],"gray")#rectmaxsidelobe
plt.plot([-np.pi,+np.pi],[-31.5,-31.5],"gray")#hannmaxsidelobe
plt.plot([-np.pi,+np.pi],[-93.6,-93.6],"gray")#flattopmax
#sidelobe
Omega,W=winDTFTdB(wrect)
plt.plot(Omega,W,label="rect")
Omega,W=winDTFTdB(whann)
plt.plot(Omega,W,label="hann")
Omega,W=winDTFTdB(wflattop)
plt.plot(Omega,W,label="flattop")
plt.xlim(-np.pi,np.pi)
plt.ylim(-120,10)
plt.xlim(-np.pi/100,np.pi/100)
```

Link to remote repozytorium (e.g. GitHub)
https://github.com/TomekPietrzyk/DSP_2024_NS

4. Outcomes:

Results from console, screenshots etc.





5. Conclusions: For the reasons given, we conclude that

Using different window functions (e.g., rectangular, Hann, Flattop) for signal analysis impacts the spectral results and reveals important trade-offs:

Frequency Resolution vs. Leakage: The rectangular window provides high frequency resolution but suffers from spectral leakage, especially when the signal frequency (e.g., 400.25 Hz or 399.75 Hz) doesn't align with FFT grid points, creating side lobes or "smearing." This leads to distortions in neighboring frequencies.

Amplitude Accuracy: Flattop windows offer more accurate amplitude measurements, though they reduce frequency resolution due to wider main lobes.

Mitigating Spectral Leakage: Windows like Hann reduce leakage but at the cost of frequency resolution. Increasing the number of FFT points (e.g., by zero-padding) also helps align signal frequencies with FFT bins, reducing "smearing."

Choosing the right window is essential for balancing resolution, leakage, and amplitude accuracy in spectral analysis.