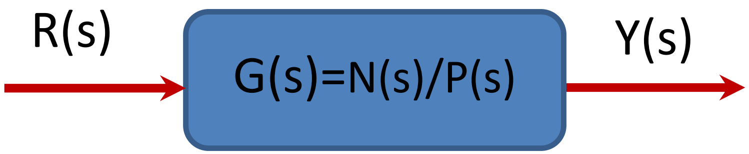
Control y Servomecanismos A

Control Automático I

Tema: Estabilidad – Diagramas de Bode

Cursada Virtual 2020
F. Valenciaga

Conceptos de Estabilidad

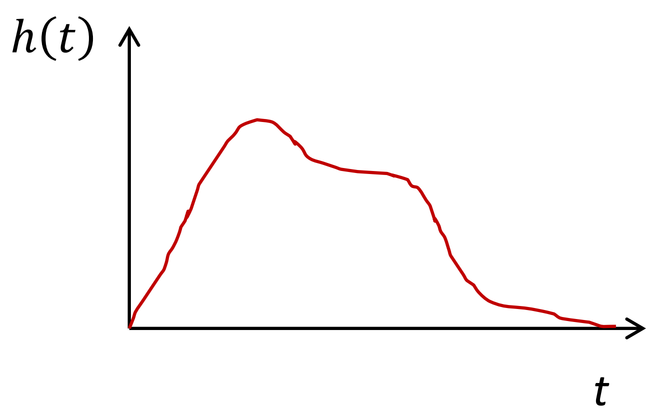


Entrada Acotada – Salida Acotada

Criterios de ESTABILIDAD → BIBO

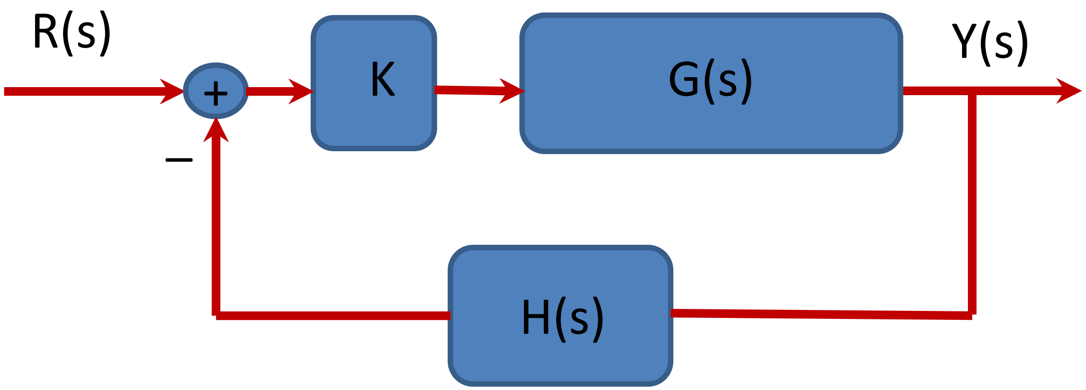
(Bounded Input – Bounded Output)

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \quad donde \quad h(t) = \mathcal{L}^{-1}\{\mathrm{G(s)}\} \longrightarrow \lim_{t\to\infty} h(t) = 0$$



Conceptos de Estabilidad

Transferencia Lazo Cerrado



$$T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}$$

Ec. Característica

$$1 + KG(s)H(s) = 0$$

Análisis de ESTABILIDAD

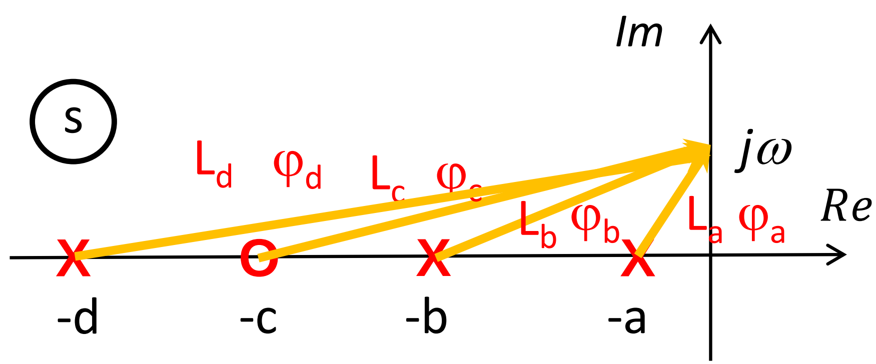
- Criterio de Routh
- Lugar de las Raíces
- Criterio de Bode
- Criterio de Nyquist

Criterio de BODE

- Permite establecer márgenes de estabilidad relativa de un sistema a lazo cerrado a partir de la gráfica de respuesta en frecuencia de la transferencia GH(s)

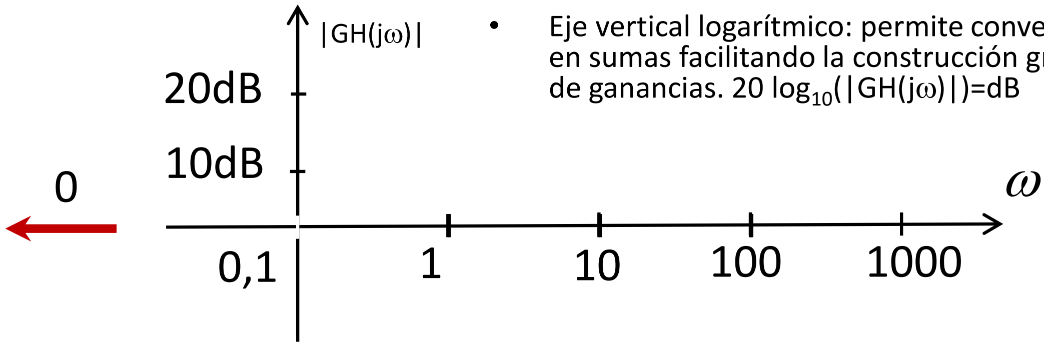
Gráfica de Bode:

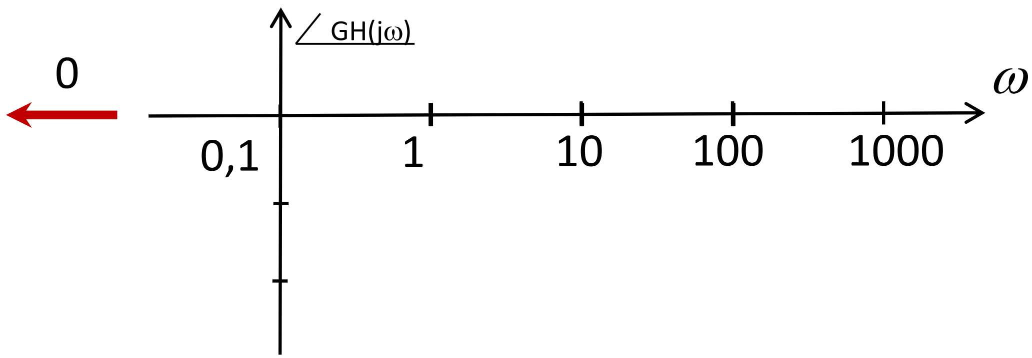
- Gráfica de Módulo $\dfrac{KL_c}{L_a L_b L_d}$
- Gráfica de Fase $(-\varphi_a - \varphi_b - \varphi_d + \varphi_c)$



GRÁFICA DE BODE

- Eje horizontal logarítmico (log $_{10}$ (ω)): permite graficar un gran espectro de frecuencias en un gráfico reducido
- Eje vertical logarítmico: permite convertir las operaciones de productos de factores en sumas facilitando la construcción gráfica. Además permite comprimir la escala de ganancias. 20 $\log_{10}(|GH(j\omega)|)$=dB





$$KG(j\omega)H(j\omega) = \frac{KL_c}{L_aL_bL_d} e^{-j(\varphi_a+\varphi_b+\varphi_d-\varphi_c)}$$

$Módulo$ → 20 $log_{10}(|GH(j\omega)|)$=20 log K+20 $log(L_c)$-20 $log(L_a)$-20 $log(L_b)$-20 $log(L_d)$

$Fase$ → $-\varphi_a - \varphi_b - \varphi_d + \varphi_c$

GRÁFICA DE BODE

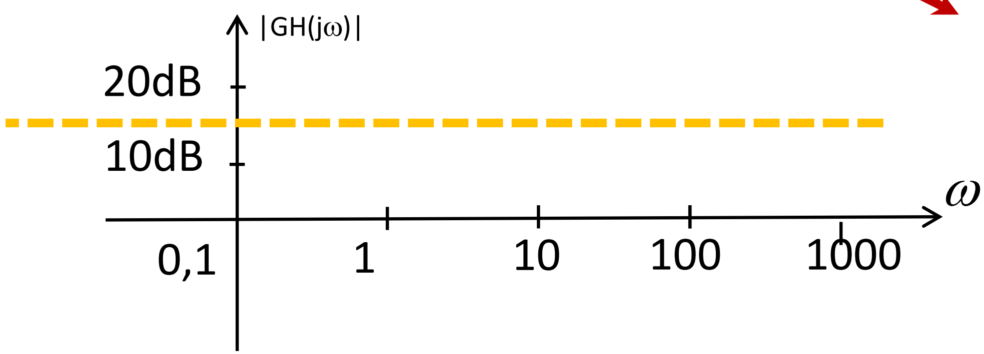
$$KG(s)H(s) = \frac{\pm K s^{n_1}\left(1 \pm \frac{s}{z_1}\right)^{n_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{n_3}}{s^{m_1}\left(1 \pm \frac{s}{p_1}\right)^{m_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{m_3}}$$

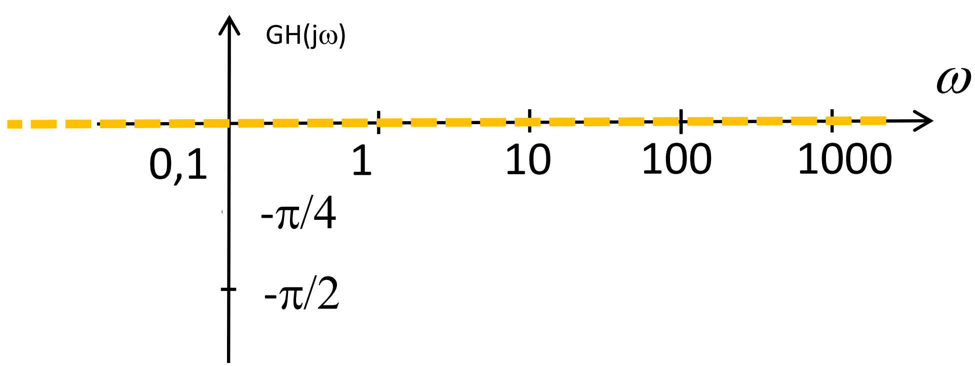
→ Forma de Bode

- Factor Ganancia → $20 \log_{10}(|K|)$= xx dB

K>1 positivo en dB

K<1 negativo en dB





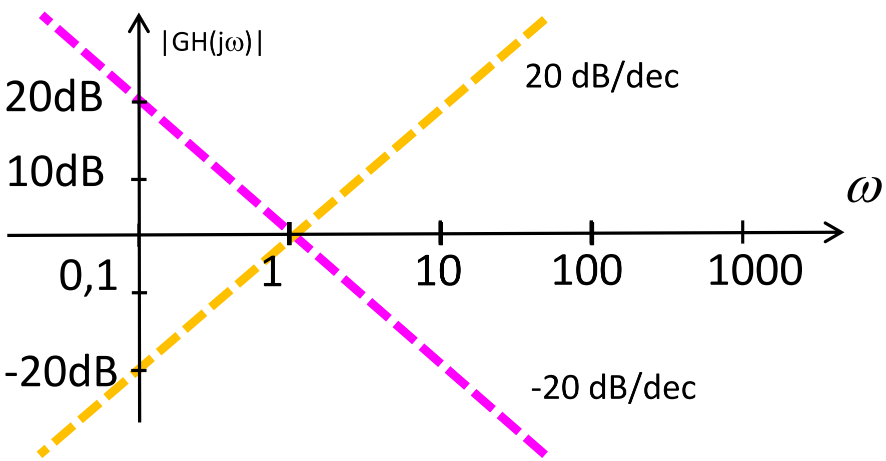
GRÁFICA DE BODE

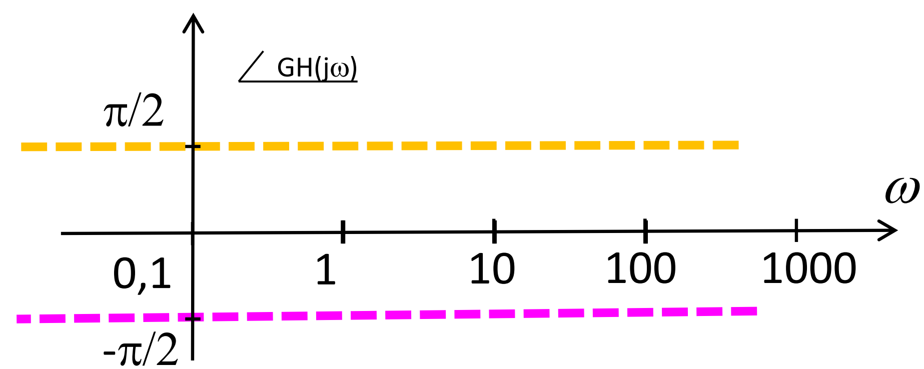
$$KG(s)H(s) = \frac{\pm K s^{n_1}\left(1+\frac{s}{z_1}\right)^{n_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n}+1\right)^{n_3}}{s^{m_1}\left(1+\frac{s}{p_1}\right)^{m_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n}+1\right)^{m_3}} \rightarrow \text{Forma de Bode}$$

- Factores Integral/ Derivativo $\rightarrow s^{\pm 1}$

20 log (|jω|)=20 log (ω) ó 20 log (|jω|$^{-1}$) = - 20 log (ω)

tg^{-1} (ω/0)= π/2 ó tg^{-1} (-ω/0) = -π/2





GRÁFICA DE BODE

$$KG(s)H(s) = \frac{\pm K s^{n_1}\left(1 \pm \frac{s}{z_1}\right)^{n_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{n_3}}{s^{m_1}\left(1 \pm \frac{s}{p_1}\right)^{m_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{m_3}} \rightarrow \text{Forma de Bode}$$

- Factores 1er. Orden → $(1 + s/a)^{\pm 1}$

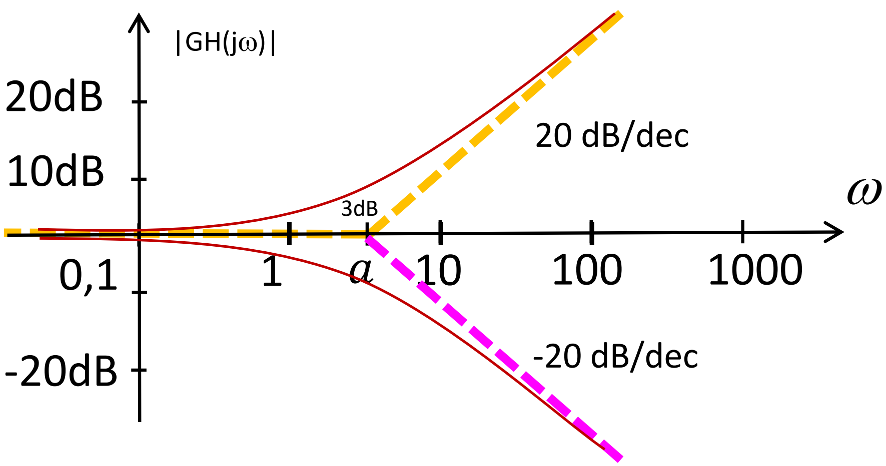
  Fase Mínima

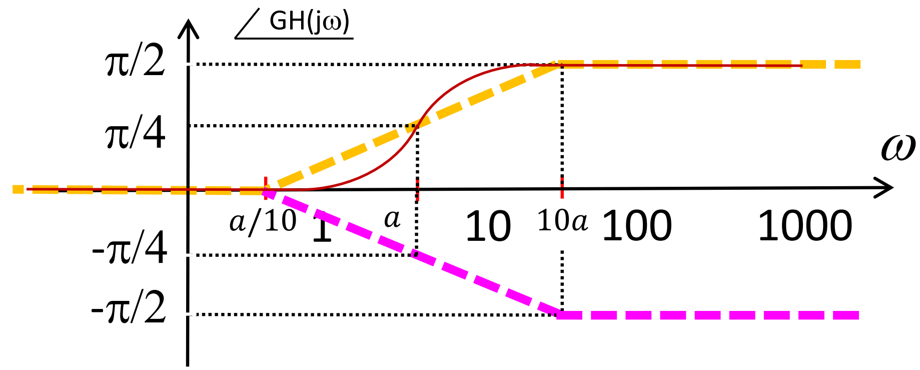
$\omega \gg a$ 20 log (|jω/a|)=20 log (ω/a)

$\omega \ll a$ 20 log (|1|) = 0dB

$tg^{-1}(\omega/a)$ →

$\omega = 0$	0
$\omega = a$	$\pi/4$
$\omega = \infty$	$\pi/2$





GRÁFICA DE BODE

$$KG(s)H(s) = \frac{\pm K s^{n_1}\left(1 \pm \frac{s}{z_1}\right)^{n_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{n_3}}{s^{m_1}\left(1 \pm \frac{s}{p_1}\right)^{m_2}\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^{m_3}}$$ → Forma de Bode

- Factores 1er. Orden → $(1 - s/a)^{\pm 1}$

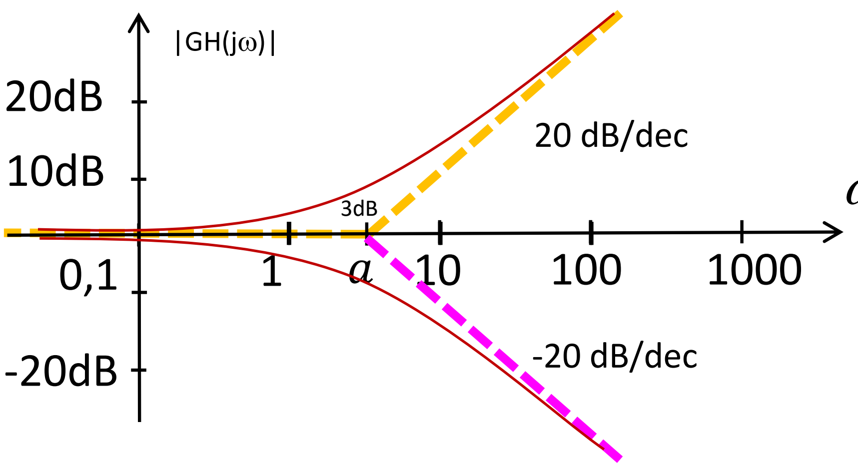
  Fase No Mínima

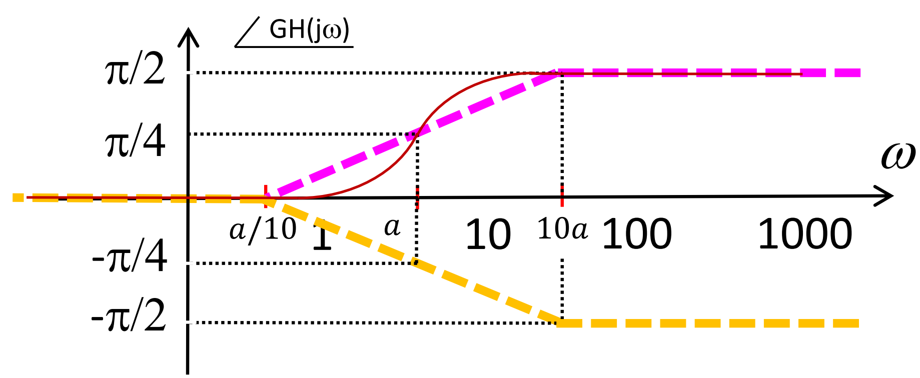
$\omega \;\; >> a$ 20 log ($|-j\omega/a|$)=20 log (ω/a)

$\omega \;\; <<$ ó 20 log ($|1|$) = 0dB

$\pm$ tg^{-1} ($-\omega/a$) →

$\omega = 0$	0	0
$\omega = a$	$-\pi/4$	$\pi/4$
$\omega = \infty$	$-\pi/2$	$\pi/2$
	cero	polo





GRÁFICA DE BODE

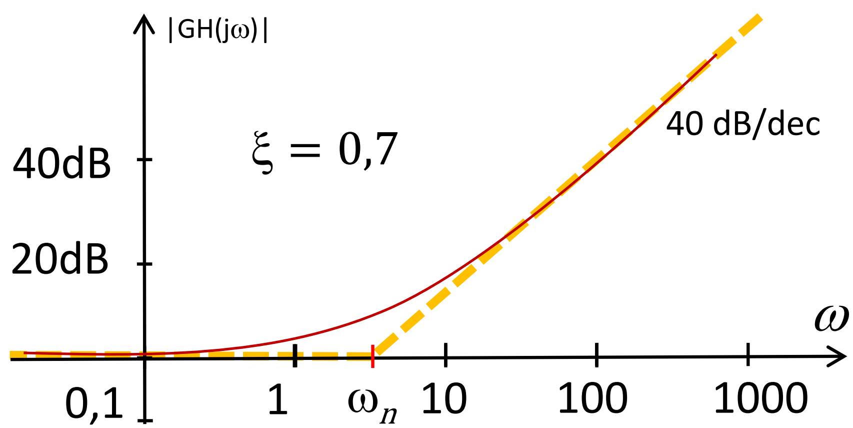
- Factores 2do.Orden $\rightarrow$ $\left(\frac{s^2}{{\omega_n}^2} \pm \frac{2\xi s}{\omega_n} + 1\right)^1$ $\rightarrow$ P. Real $\left(1 - \frac{\omega^2}{{\omega_n}^2}\right)$
  P Imag. $\left(\pm \frac{2j\xi\omega}{\omega_n}\right)$

❖ Ceros mínima o no mínima fase

Módulo $\rightarrow$ $20\log \sqrt[2]{\left(1 - \frac{\omega^2}{{\omega_n}^2}\right)^2 + \left(\pm \frac{2\xi\omega}{\omega_n}\right)^2}$

$\omega \gg \omega_n$ $\quad 20 \log \frac{\omega^2}{{\omega_n}^2} = 40 \log \frac{\omega}{\omega_n}$

$\omega \ll \omega_n$ $\quad 20 \log 1 = 0\text{dB}$



GRÁFICA DE BODE

- Factores 2do.Orden $\rightarrow$ $\left(\dfrac{s^2}{{\omega_n}^2} \pm \dfrac{2\xi s}{\omega_n} + 1\right)^{-1}$ $\rightarrow$

P. Real $\left(1 - \dfrac{\omega^2}{{\omega_n}^2}\right)$

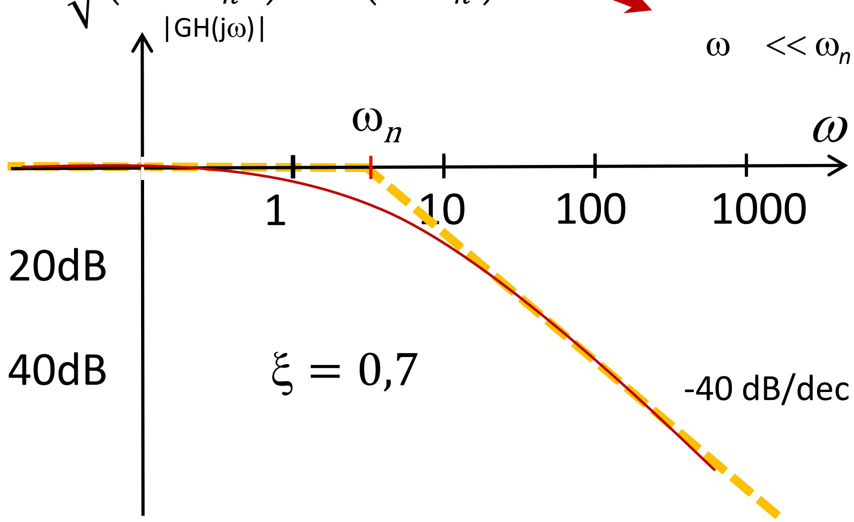
P Imag. $\left(\pm \dfrac{2j\xi\omega}{\omega_n}\right)$

❖ Polos mínima o no mínima fase

Módulo $\rightarrow$ $-20\log \sqrt[2]{\left(1 - \dfrac{\omega^2}{{\omega_n}^2}\right)^2 + \left(\pm \dfrac{2\xi\omega}{\omega_n}\right)^2}$

$\omega \gg \omega_n$ $-20 \log \dfrac{\omega^2}{{\omega_n}^2} = -40 \log \dfrac{\omega}{\omega_n}$

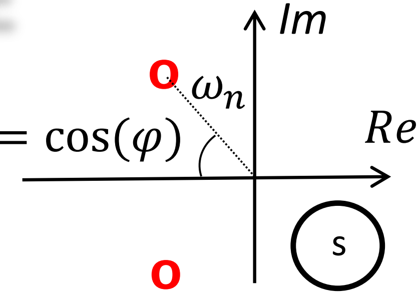
$\omega \ll \omega_n$ $-20 \log 1 = 0\text{dB}$



GRÁFICA DE BODE

- Factores 2do.Orden → $\left(\frac{s^2}{{\omega_n}^2}+\frac{2\xi s}{\omega_n}+1\right)^1$ → P. Real $\left(1-\frac{\omega^2}{{\omega_n}^2}\right)$ P Imag. $\left(\frac{2j\xi\omega}{\omega_n}\right)$

$\xi = \cos(\varphi)$



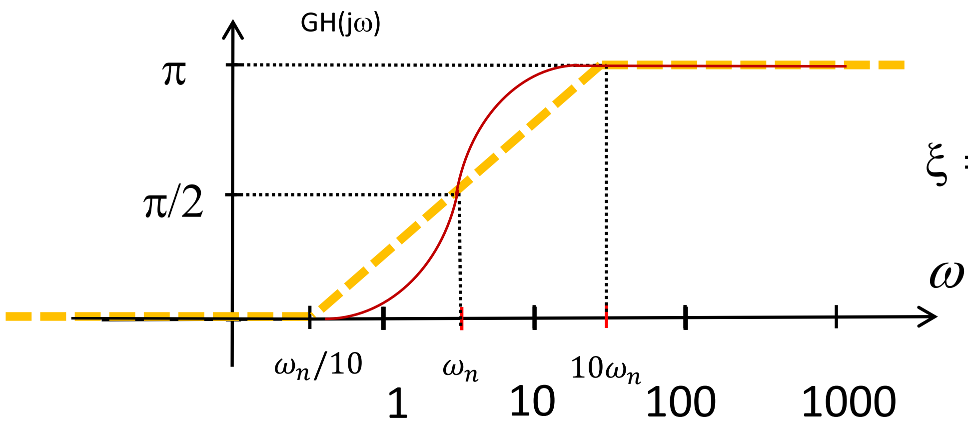
❖ Ceros Fase Mínima

Fase → $tg^{-1}\frac{\left(\frac{2\xi\omega}{\omega_n}\right)}{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)}$

$\omega = 0$ $\quad 0$

$\omega = \omega_n$ $\quad tg^{-1}\frac{2\xi}{0}$ → $\pi/2$

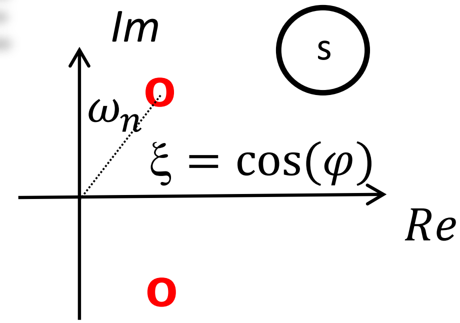
$\omega = \infty$ $\quad tg^{-1}\frac{\left(\frac{2\xi\omega}{\omega_n}\right)}{\left(-\frac{\omega^2}{{\omega_n}^2}\right)}$ → π



$\xi = 0{,}7$

GRÁFICA DE BODE

- Factores 2do.Orden $\longrightarrow \left(\frac{s^2}{{\omega_n}^2} - \frac{2\xi s}{\omega_n} + 1\right)^1 \longrightarrow$ P. Real $\left(1 - \frac{\omega^2}{{\omega_n}^2}\right)$ P Imag. $\left(-\frac{2j\xi\omega}{\omega_n}\right)$



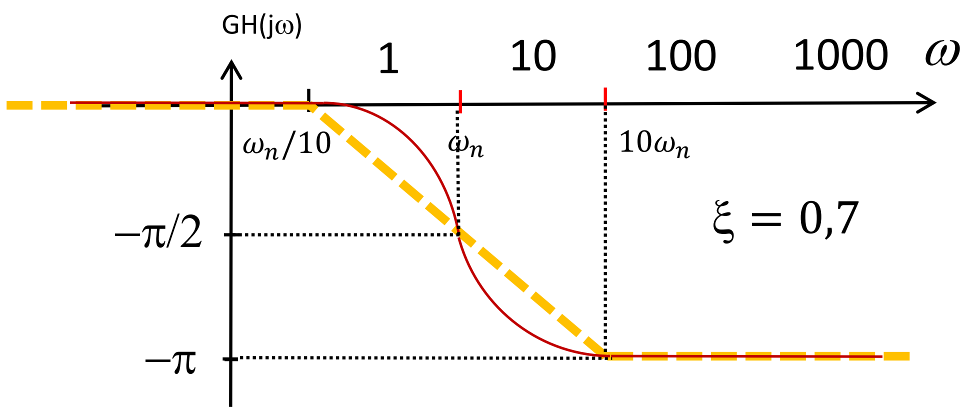
❖ Ceros Fase No Mínima

Fase $\longrightarrow tg^{-1}\frac{\left(-\frac{2\xi\omega}{\omega_n}\right)}{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)}$

$\omega = 0 \qquad 0$

$\omega = \omega_n \qquad tg^{-1}\left(\frac{-2\xi}{0}\right) \longrightarrow -\pi/2$

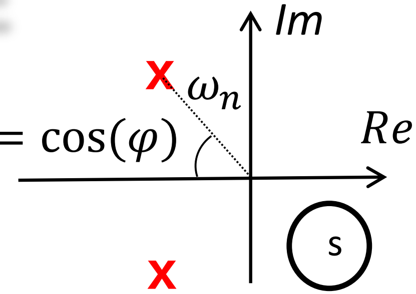
$\omega = \infty \qquad tg^{-1}\frac{\left(-\frac{2\xi\omega}{\omega_n}\right)}{\left(-\frac{\omega^2}{{\omega_n}^2}\right)} \longrightarrow -\pi$



GRÁFICA DE BODE

- Factores 2do.Orden $\rightarrow \left(\frac{s^2}{{\omega_n}^2}+\frac{2\xi s}{\omega_n}+1\right)^{-1} \rightarrow$ P. Real $\left(1-\frac{\omega^2}{{\omega_n}^2}\right)$ P Imag. $\left(\frac{2j\xi\omega}{\omega_n}\right)$

$\xi = \cos(\varphi)$



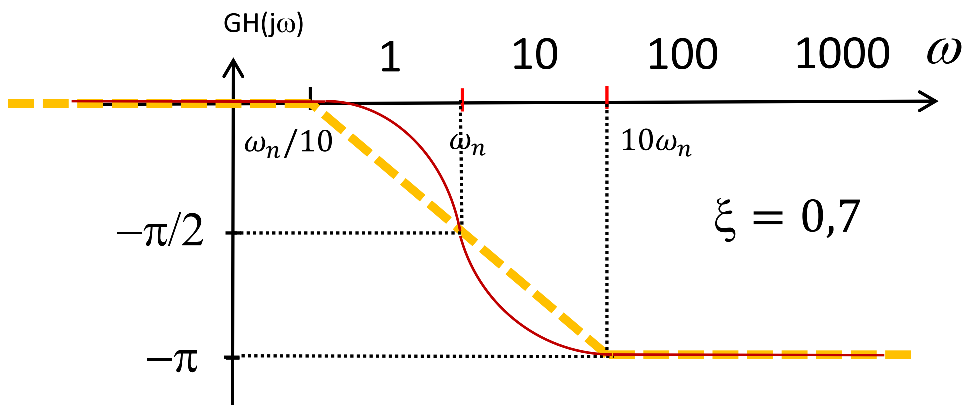
❖ Polos Fase Mínima

Fase $\rightarrow -tg^{-1}\frac{\left(\frac{2\xi\omega}{\omega_n}\right)}{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)}$

$\omega = 0 \qquad 0$

$\omega = \omega_n \qquad -tg^{-1}\frac{2\xi}{0} \rightarrow -\pi/2$

$\omega = \infty \qquad -tg^{-1}\frac{\left(\frac{2\xi\omega}{\omega_n}\right)}{\left(-\frac{\omega^2}{{\omega_n}^2}\right)} \rightarrow -\pi$



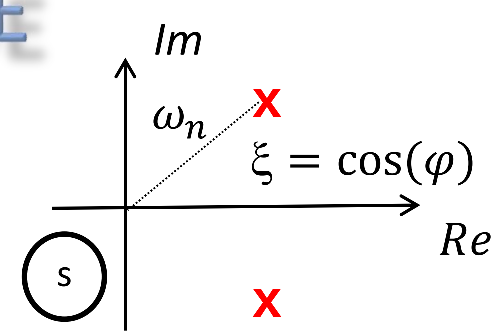
GRÁFICA DE BODE

- Factores 2do.Orden

$$\left(\frac{s^2}{{\omega_n}^2}+\frac{2\xi s}{\omega_n}+1\right)^{-1}$$

P. Real $\left(1-\frac{\omega^2}{{\omega_n}^2}\right)$

P Imag. $\left(\frac{2j\xi\omega}{\omega_n}\right)$



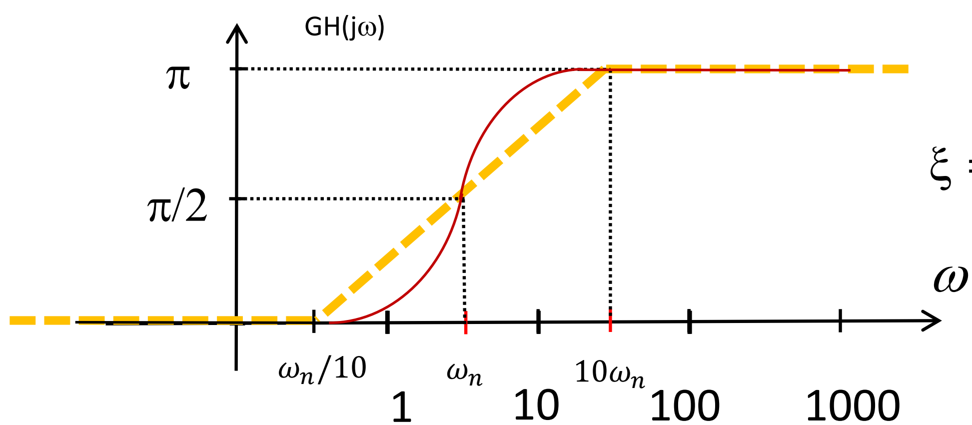
❖ Polos Fase No Mínima

Fase $\quad -tg^{-1}\dfrac{\left(-\frac{2\xi\omega}{\omega_n}\right)}{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)}$

$\omega = 0 \qquad 0$

$\omega = \omega_n \qquad tg^{-1}\frac{2\xi}{0} \rightarrow \pi/2$

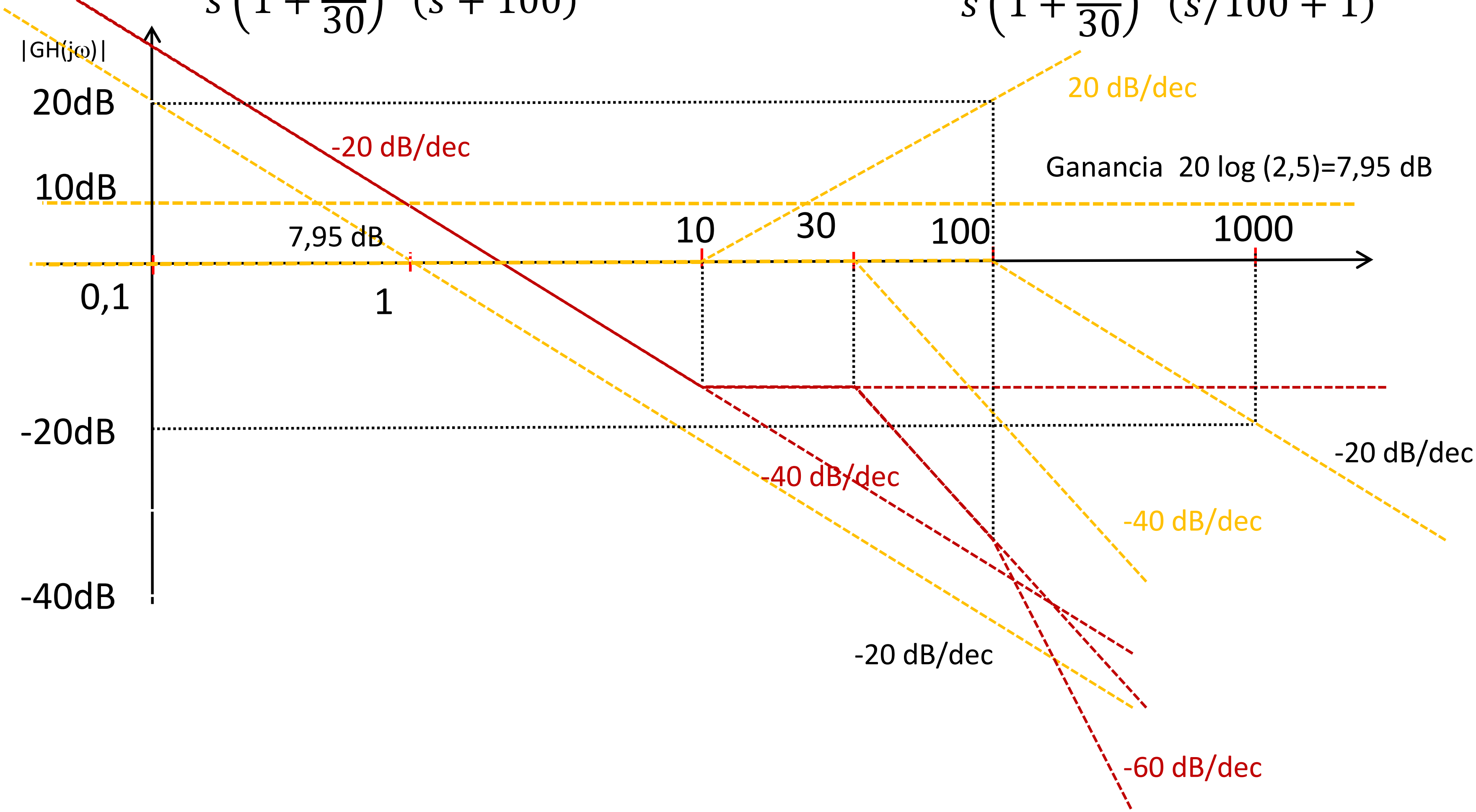
$\omega = \infty \qquad tg^{-1}\dfrac{\left(\frac{2\xi\omega}{\omega_n}\right)}{\left(-\frac{\omega^2}{{\omega_n}^2}\right)} \rightarrow \pi$



$\xi = 0{,}7$

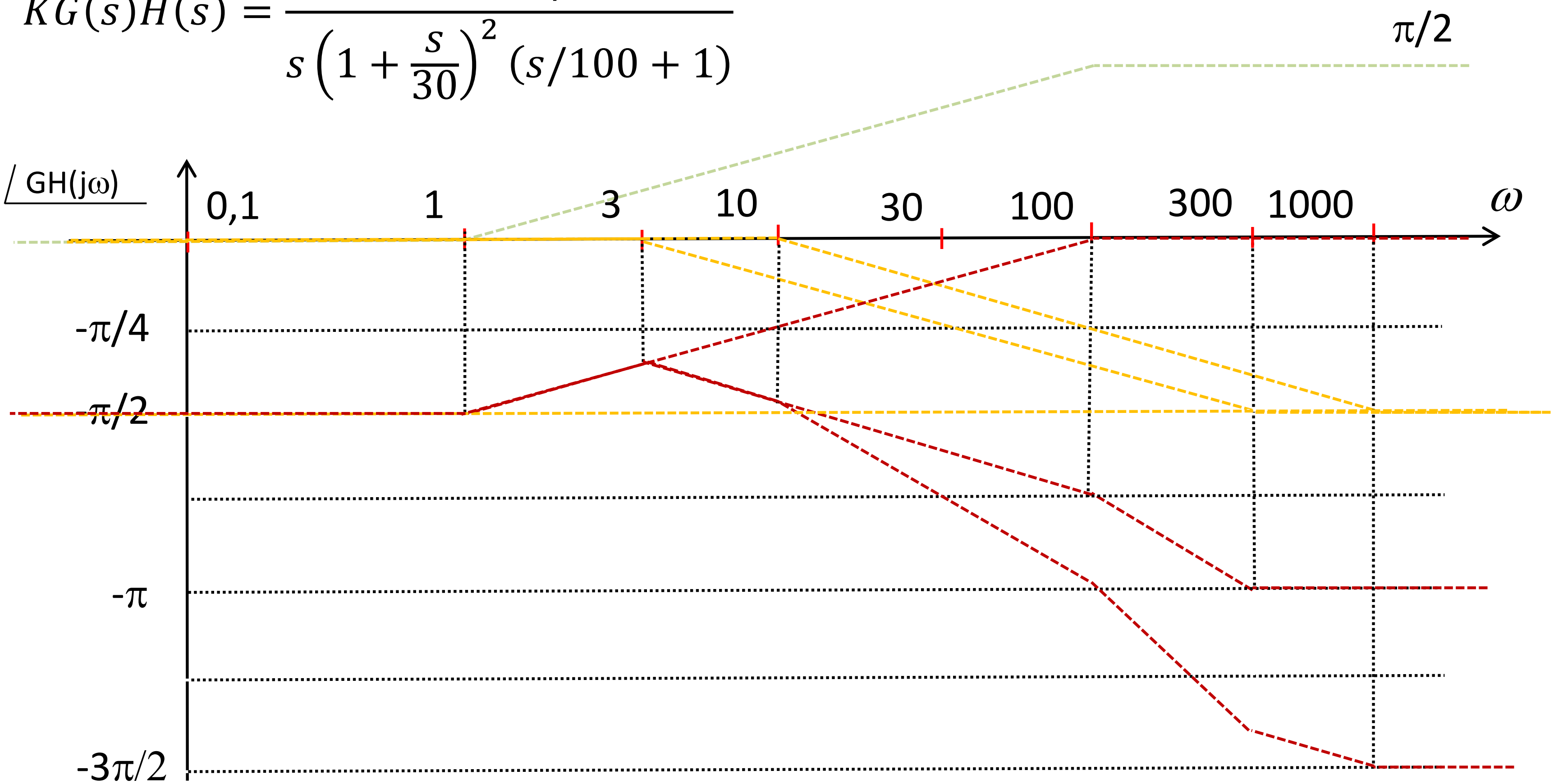
GRÁFICA DE BODE

$$KG(s)H(s) = \frac{25(10+s)}{s\left(1+\frac{s}{30}\right)^2(s+100)} \rightarrow KG(s)H(s) = \frac{2{,}5(1+s/10)}{s\left(1+\frac{s}{30}\right)^2(s/100+1)}$$



GRÁFICA DE BODE

$$KG(s)H(s) = \frac{2,5(1 + s/10)}{s\left(1 + \frac{s}{30}\right)^2 (s/100 + 1)}$$



ESTABILIDAD POR BODE

$$T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}$$

Ec. Característica

$$1 + KG(s)H(s) = 0$$

$$KG(s)H(s) = -1$$

$$|KG(s)H(s)| = 1$$ Frecuencia de transición de ganancia

$$KG(s)H(s) = -180º$$ Frecuencia de transición de fase

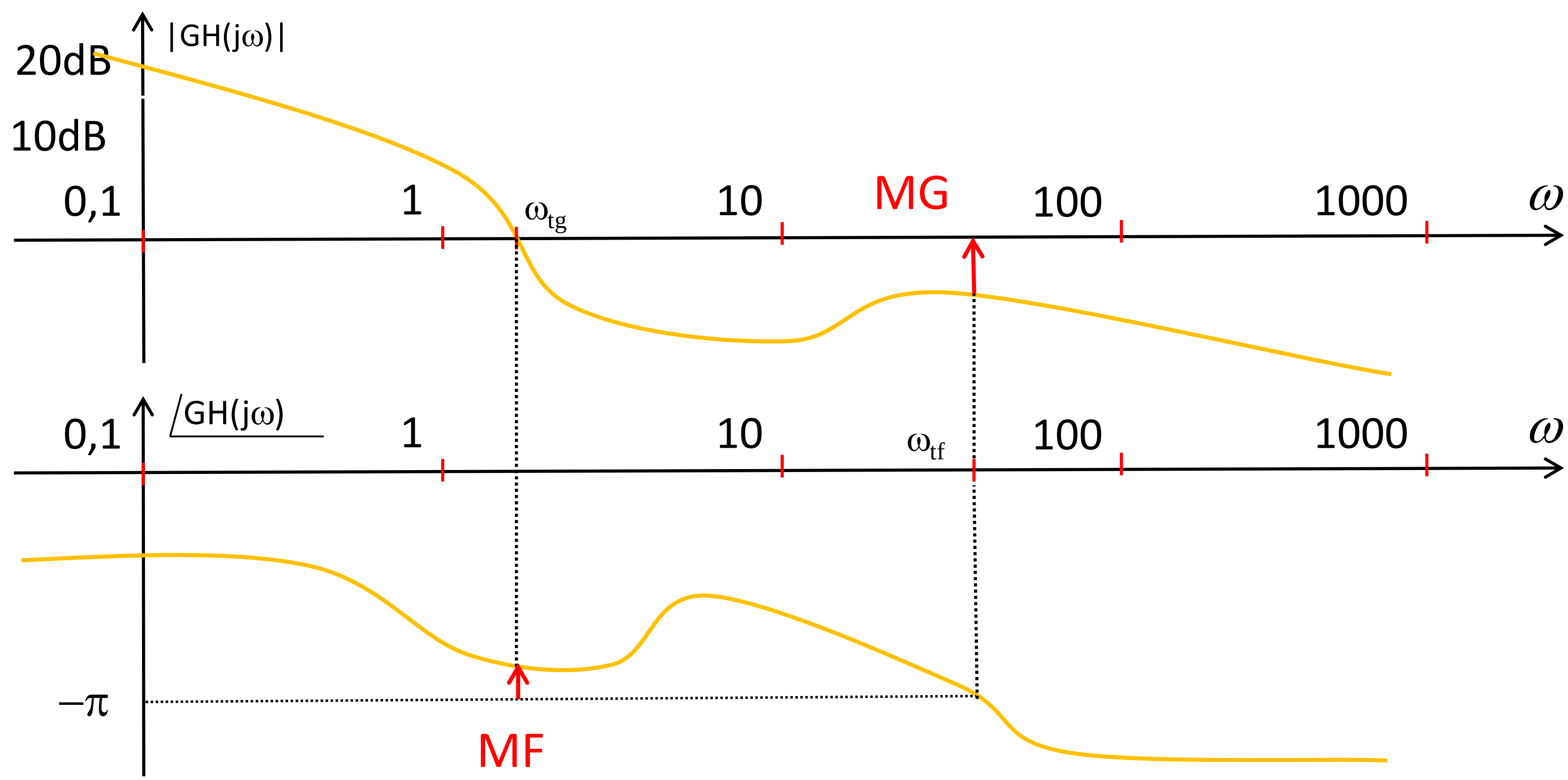
Margen de Ganancia: ganancia que debe aplicarse al sistema para que |G(s)H(s)|=1 en la frecuencia de transición de fase.

Margen de Fase: fase que debe adicionarse al sistema para que en la frecuencia de transición de ganancia posea -180º de fase.

ESTABILIDAD POR BODE

Margen de Ganancia: ganancia que debe aplicarse al sistema para que |G(s)H(s)|=1 en la frecuencia de transición de fase.

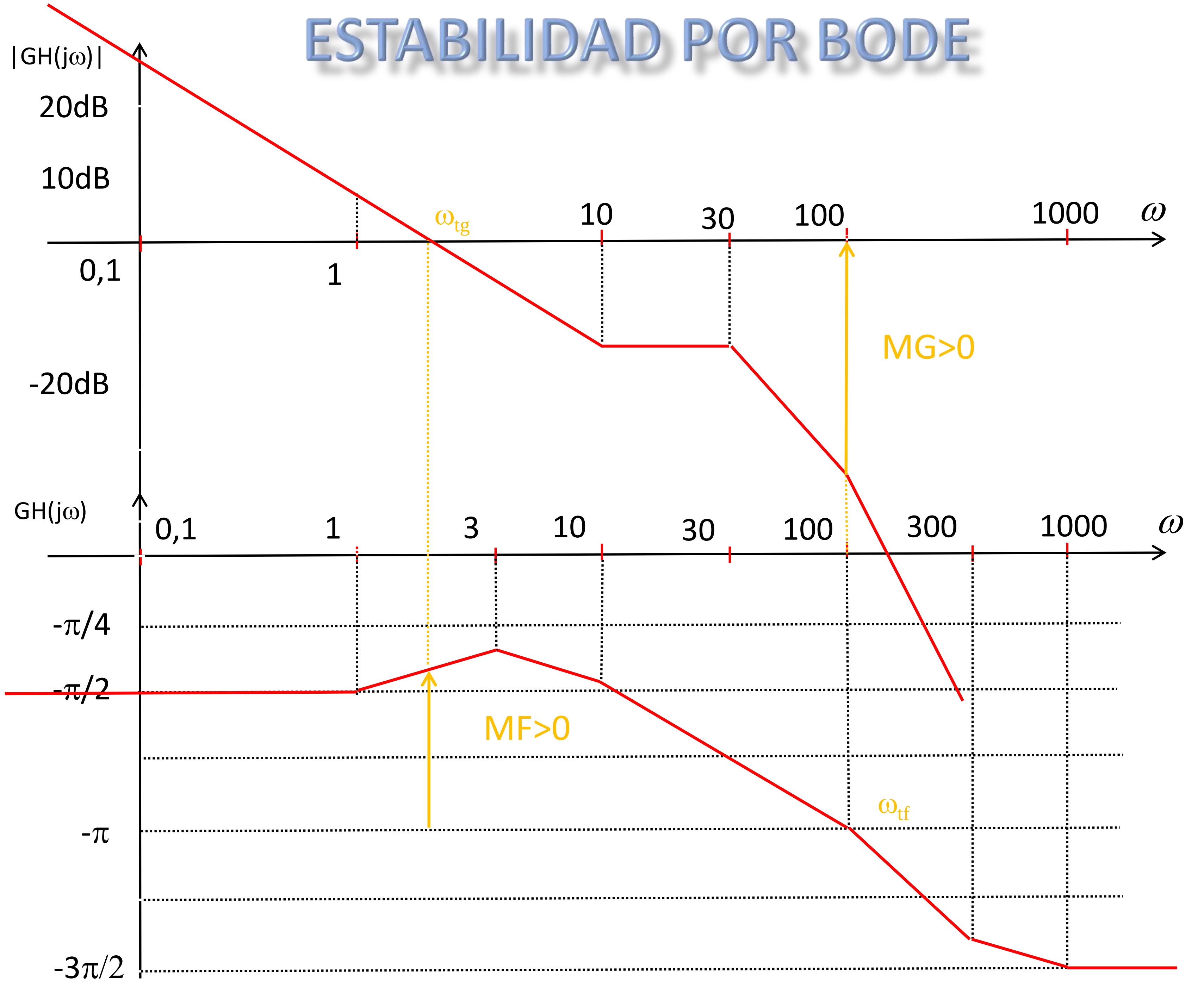
Margen de Fase: fase que debe adicionarse al sistema para que en la frecuencia de transición de ganancia posea -180º de fase.



ESTABILIDAD POR BODE

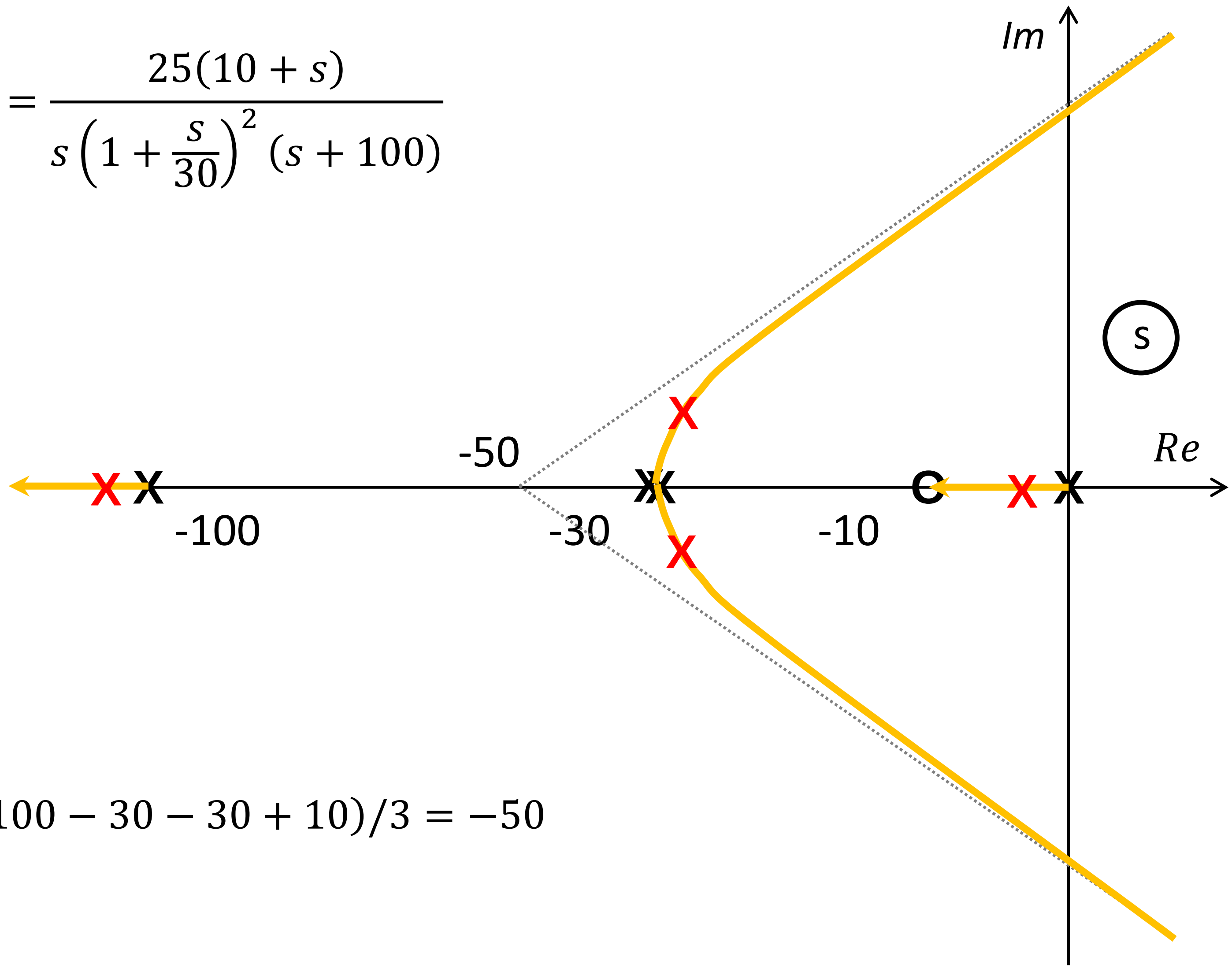
	MG>0	**MG<0**
MF>0	Estable	?
MF<0	?	Inestable

ESTABILIDAD POR BODE



RELACION CON LR

$$KG(s)H(s) = \frac{25(10+s)}{s\left(1+\frac{s}{30}\right)^2 (s+100)}$$



$$\sigma_a = (-100 - 30 - 30 + 10)/3 = -50$$