Temas a tratar

- Modulación exponencial
- PM / FM
- Modulación
- Demodulación (algunos esquemas)

VEX-1B de la CONAE en su base de lanzamiento en Punta Indio, Pcia. Bs.As.

Modulación de Onda Continua

$$X(t) = Re\left\{A(t)\, e^{j2\pi f_p t + \phi(t)}\right\}$$

$$X(t) = A(t)\cos(2\pi f_p t + \phi(t))$$

$A(t)$: amplitud

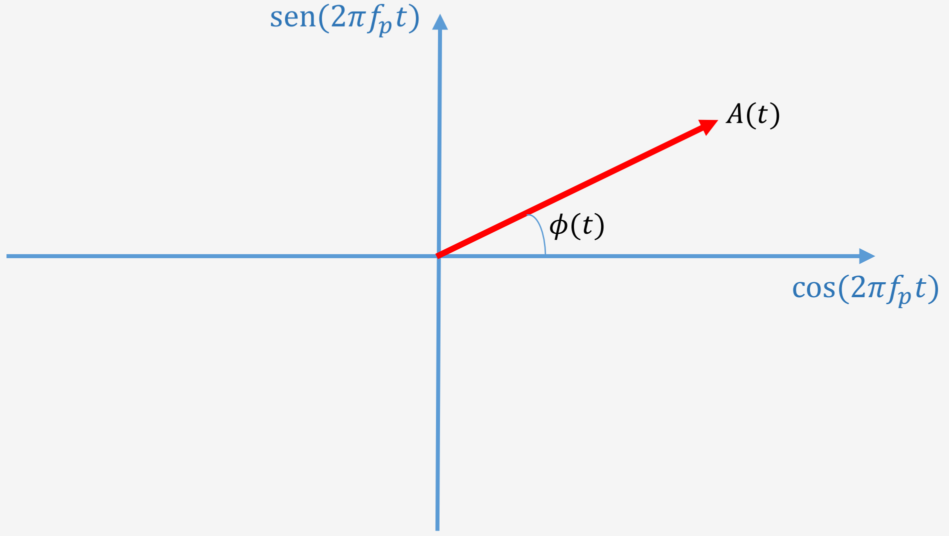
f_p: frecuencia de portadora (carrier)

$\varphi(t) = 2\pi f_p t + \phi(t)$ $\phi(t)$: desviación de fase

$$f(t) = \frac{1}{2\pi}\frac{d\varphi}{dt} = f_p + \frac{1}{2\pi}\frac{d\phi}{dt}$$

$$f_d(t) = \frac{1}{2\pi}\frac{d\phi}{dt}$$

$f_d(t)$: desviación de frecuencia



- Modulación de amplitud
- Modulación de fase/frecuencia
- Modulación IQ (amplitud - fase)

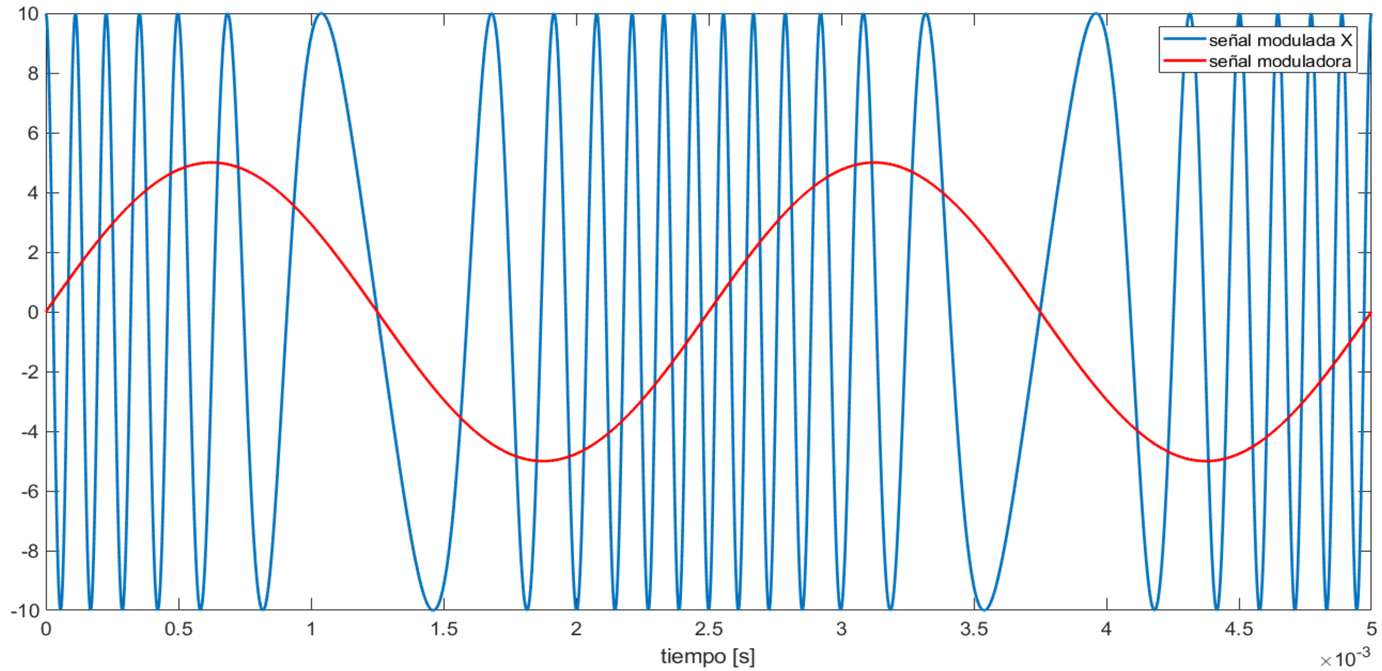
Modulación en Fase (PM)

$M(t)$: mensaje, $BW = W$

$\phi(t) = k_p\, M(t)$ $\xrightarrow{\text{MODULACIÓN DE FASE (PM)}}$ $X(t) = A\cos(2\pi f_p t + k_p\, M(t))$ $\qquad P_X = \dfrac{A^2}{2}$

k_p: constante de desviación de fase [rad./unidad de M]

$$f_d(t) = \frac{k_p}{2\pi}\,\dot{M}(t)$$



Modulación en Frecuencia (FM)

$f_d(t) = k_f\ M(t)$

k_f: constante de desviación de frecuencia [Hz/unidad de M]

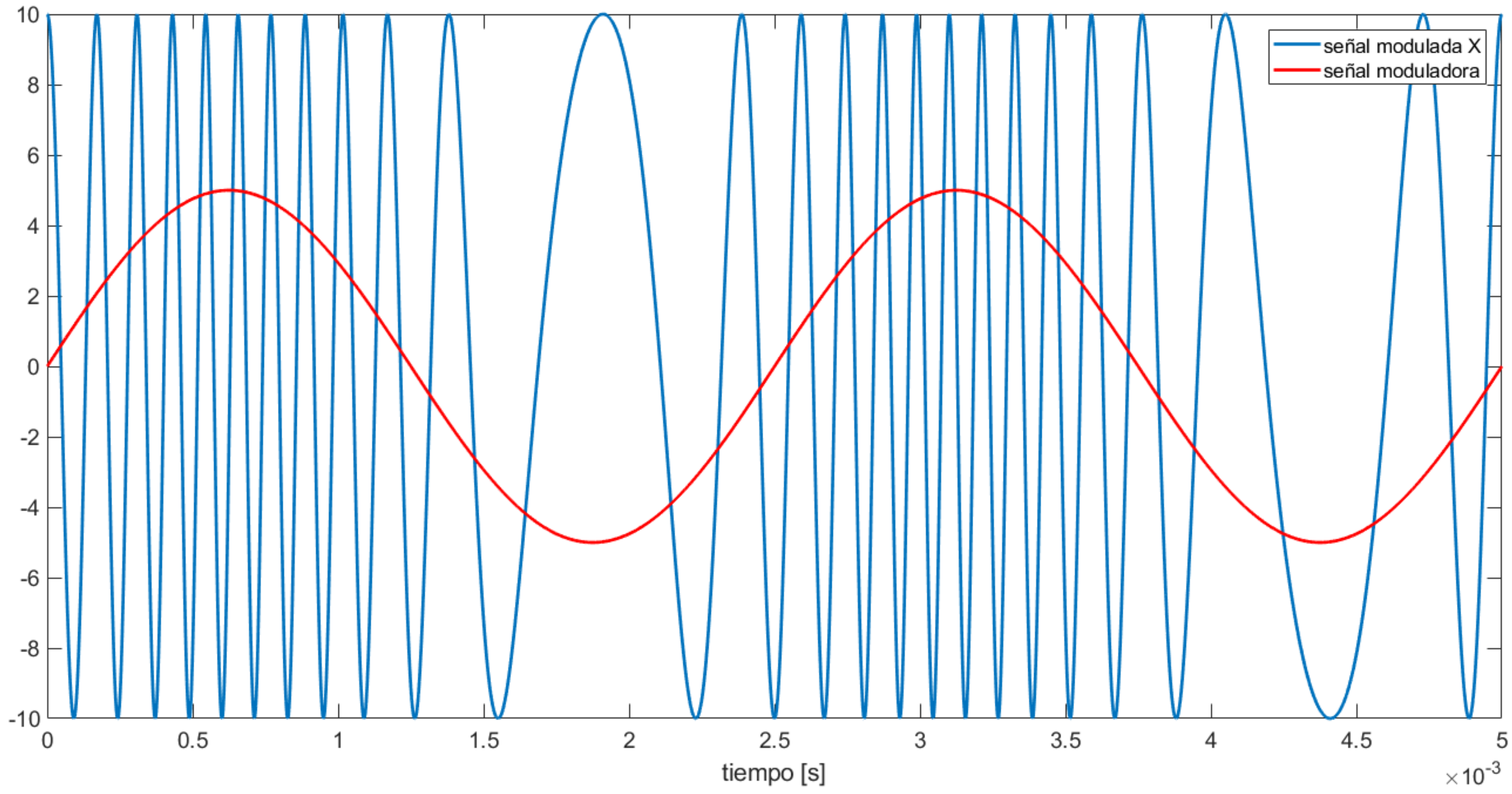
$f_d(t) = k_f\ M(t)$

$$\frac{1}{2\pi}\frac{d\phi}{dt} = k_f M(t) \xrightarrow{\text{MODULACIÓN EN FRECUENCIA (FM)}} X(t) = A\cos(2\pi f_p t\ + 2\pi k_f \int_{-\infty}^{t} M(\lambda)\, d\lambda) \qquad P_X = \frac{A^2}{2}$$

$f_{d_{max}} = k_f \max|M(t)| = \Delta f\ = f_d$

Si llamamos $M_n(t) = M(t)/|M(t)|$ la señal modulada $X(t) = A\cos(2\pi f_p t\ + 2\pi f_d \int_{-\infty}^{t} M_n(\lambda)\, d\lambda)$

Modulación en Frecuencia (FM)



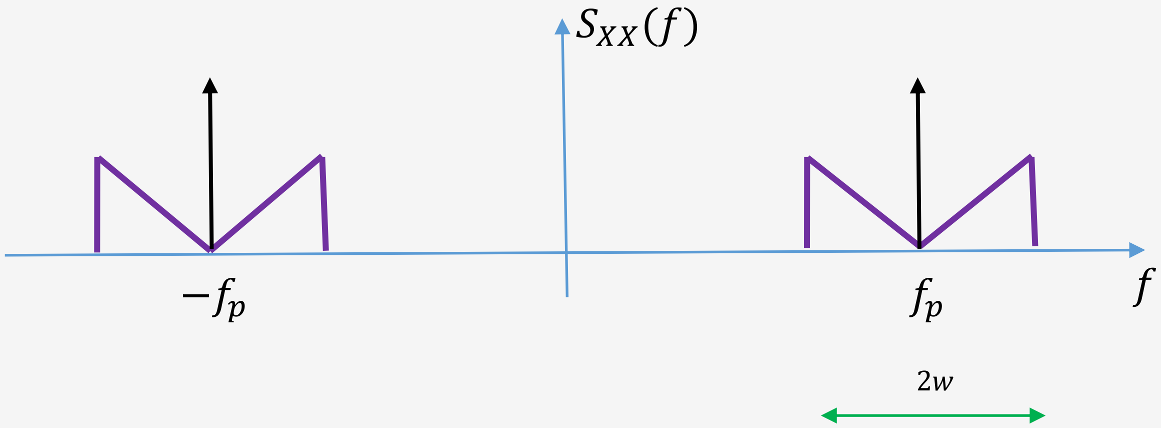
PM/FM de Banda Angosta (N-PM/N-FM)

$$X(t) = A\cos\left(2\pi f_p t \ + \phi(t)\right)$$

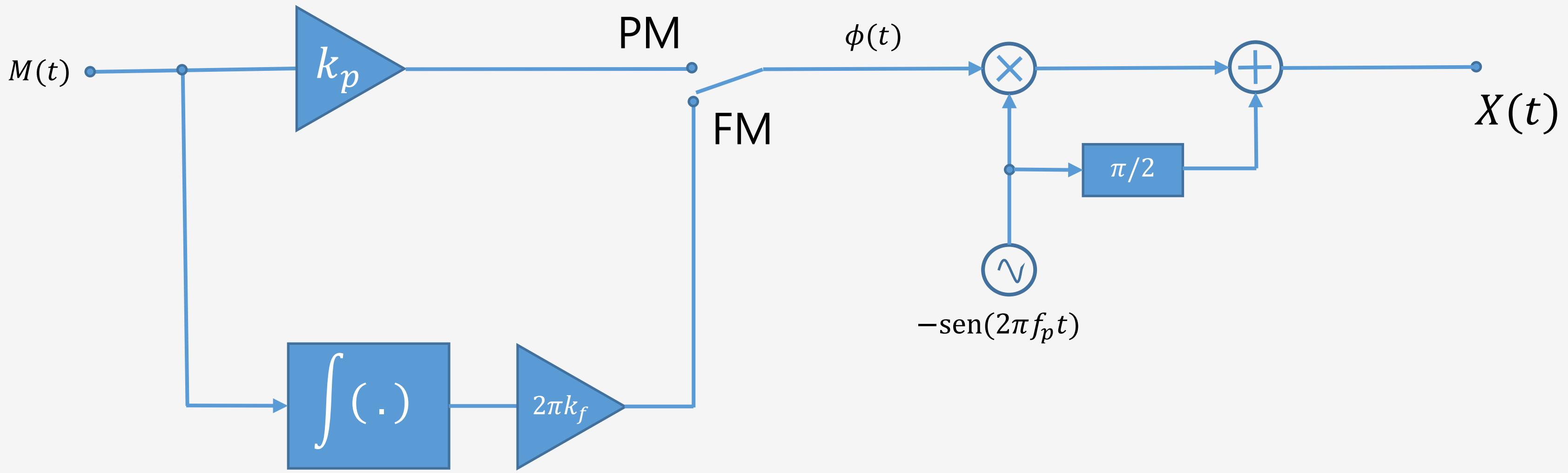
$$X(t) = A\left[\,\cos\left(2\pi f_p t\right)\cos\phi(t) \ - \ \mathrm{sen}\left(2\pi f_p t\right)\mathrm{sen}\,\phi(t)\,\right]$$

$$\beta \triangleq \max|\phi(t)|$$

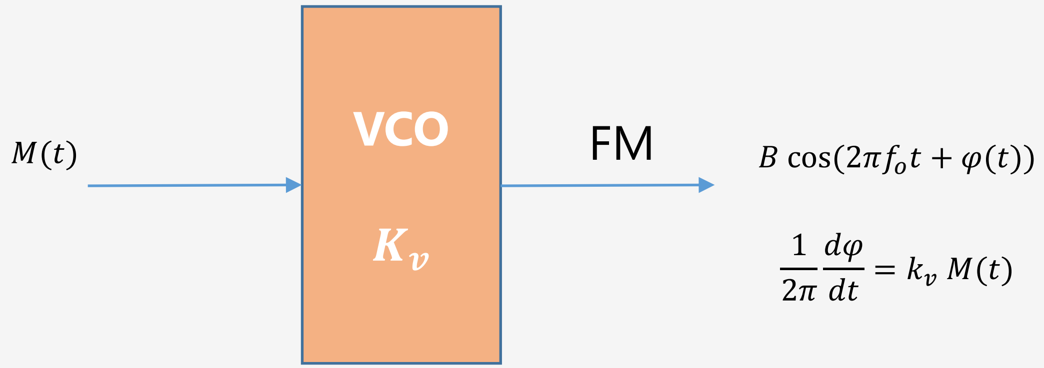
Para valores de $\beta \ll 1$, $X(t) \cong A\left[\,\cos\left(2\pi f_p t\right) \ - \ \mathrm{sen}\left(2\pi f_p t\right)\phi(t)\,\right]$



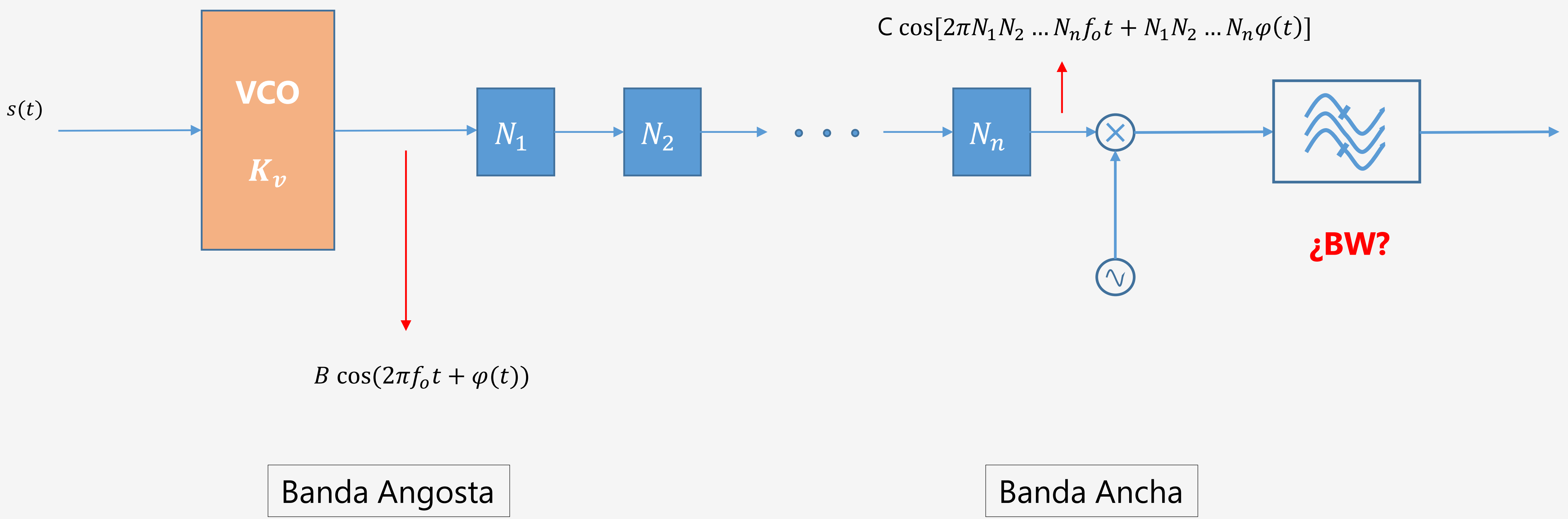
PM/FM de Banda Angosta (N-PM/N-FM)



Oscilador controlado por tensión



PM/FM de banda ancha (W-PM/W-FM)



DEP

Modulación sinusoidal: $M(t) = A_m \cos(2\pi f_m t)$

FM

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} A_m \cos(2\pi f_m \lambda)\, d\lambda = \frac{A_m\, k_f}{f_m} \operatorname{sen}(2\pi f_m t)$$

$$\beta = \frac{A_m\, k_f}{f_m}$$

$$f_d(t) = A_m k_f \cos(2\pi f_m t)$$

$$\phi(t) = \beta \operatorname{sen}(2\pi f_m t)$$

$f_{d_{max}} = \Delta f = A_m\, k_f$ entonces $\beta = \frac{f_{d_{max}}}{f_m}$

PM

$$\phi(t) = k_p\, M(t) = k_p A_m \cos(2\pi f_m t)$$

$$\beta = k_p\, A_m$$

$$f_d(t) = -k_p\, A_m\, f_m \operatorname{sen}(2\pi f_m t)$$

$$\phi(t) = \beta \cos(2\pi f_m t)$$

$f_{d_{max}} = \Delta f = k_p\, A_m\, f_m$ entonces $\beta = \frac{f_{d_{max}}}{f_m}$

¿BW?

DEP

La señal modulada: $X(t) = A\cos\left(2\pi f_p t + \phi(t)\right) = A\,\mathrm{Re}\{e^{j2\pi f_p t}\, e^{j\phi(t)}\}$ $\phi(t)$ es β cos(.) o β sen(.)

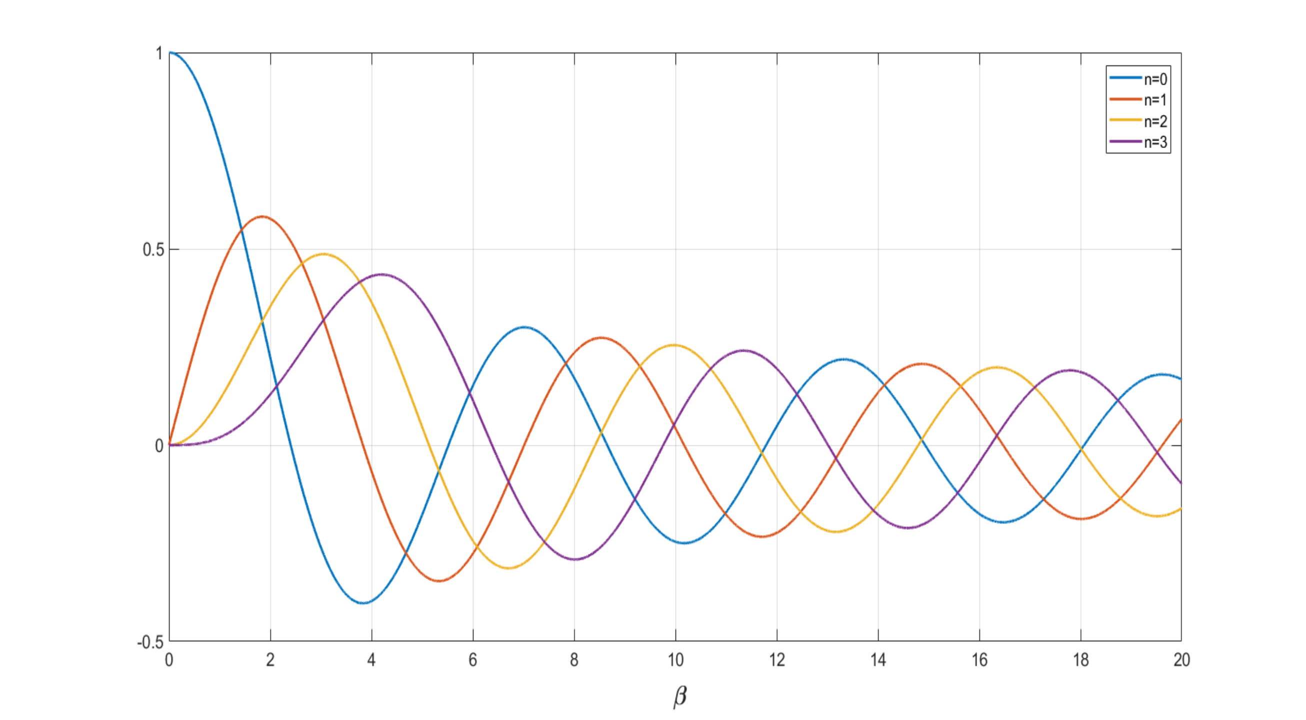
$g(t) = e^{j\beta\cos(2\pi f_m t)}$ → Periódica de período $1/f_m$ → SF

$$g(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{j2\pi n f_m t}$$

$$C_n = f_m\, G_p(f)\Big|_{f = nf_m\ ,\ n \in \mathbb{Z}}$$

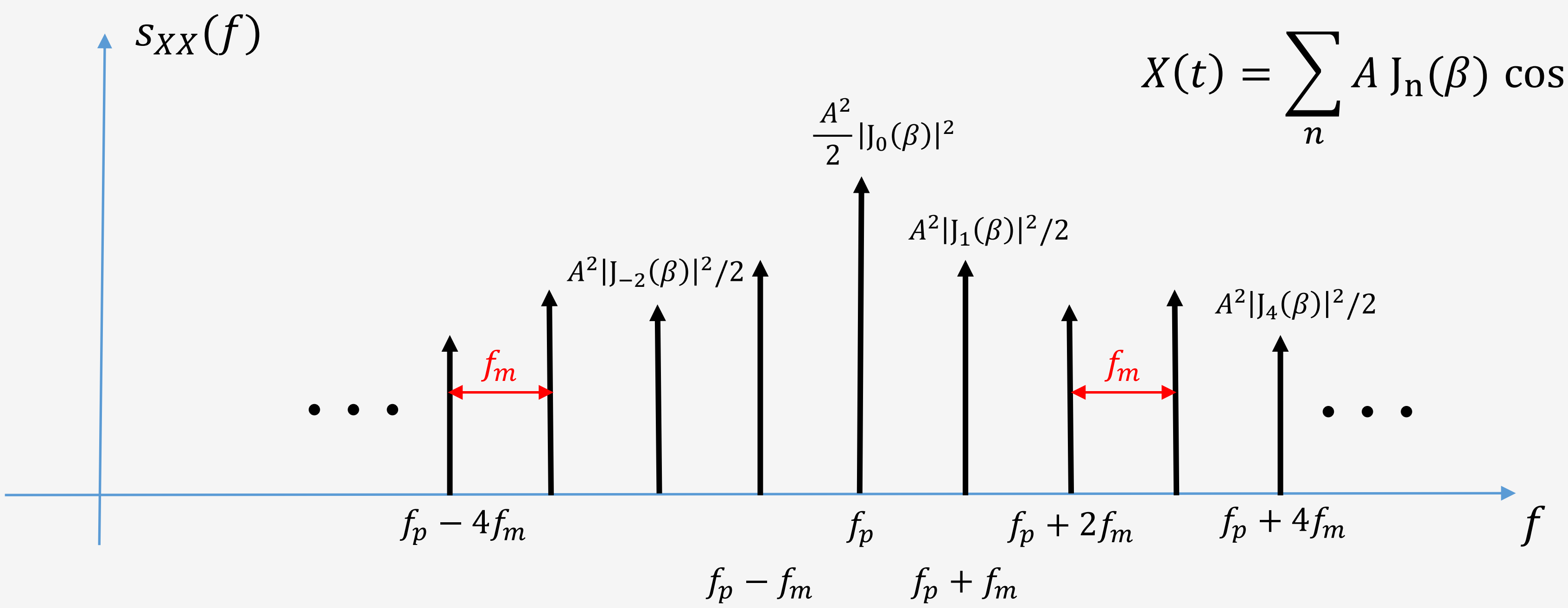
$C_n = \mathrm{J_n}(\beta)$ Funciones de Bessel de 1ra. especie y de orden n

$\mathrm{J_{-n}}(\beta) = (-1)^n\, \mathrm{J_n}(\beta)$ $\lim_{n\to\infty} \mathrm{J_n}(\beta) = 0$



Ancho de Banda teórico es infinito

DEP. Ancho de Banda de Carson



$$X(t) = \sum_n A\, J_n(\beta)\cos 2\pi(f_p + nf_m)t$$

(unilateral)

Limitamos el espectro de la señal modulada a partir del k-ésimo armónico / la señal filtrada $X_f(t)$ contenga el 98% de la potencia total.

$$\frac{P_{X_f}}{P_X} = \frac{(A^2/2)\sum_{n=-k}^{k}|J_n(\beta)|^2}{A^2/2} = 98\% \longrightarrow k = \beta + 1$$

Criterio de CARSON

$$BW_C = 2kf_m \cong 2(\beta+1)f_m \longleftarrow$$ Ancho de Banda de Carson

$$BW_C \cong 2(\Delta f + f_m)$$

BW. Modulación arbitraria. DEP.

Para modulación arbitraria definimos el índice de desviación: $D = \Delta f / W$

$$BW_C = 2(D+1)W$$ ← Ancho de Banda de Carson

Ej: Radiodifusión en FM Comercial

$$\left.\begin{aligned} W &= 15\text{kHz} \\ \Delta f &= 75\text{kHz} \end{aligned}\right\} \quad D = 5 \longrightarrow BW_C = 2(D+1)W = 180\text{kHz}$$

DEP: Teorema de Woodward

Sea $M(t)$ un PAESA, id con fdp de amplitudes $f_M(m)$

$$X(t) = A\cos\left(2\pi f_p t + \lambda \int_{-\infty}^{t} M(\sigma)d\sigma\right)$$

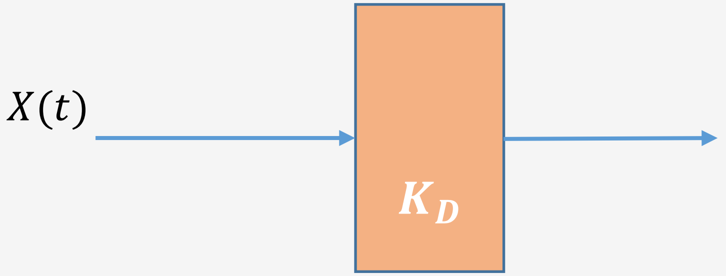
Para λ grandes:

$$S_{XX}(f) = \frac{A^2}{4\lambda}\left[f_M\left(\frac{f+f_p}{\lambda}\right) + f_M\left(\frac{f-f_p}{\lambda}\right)\right]$$

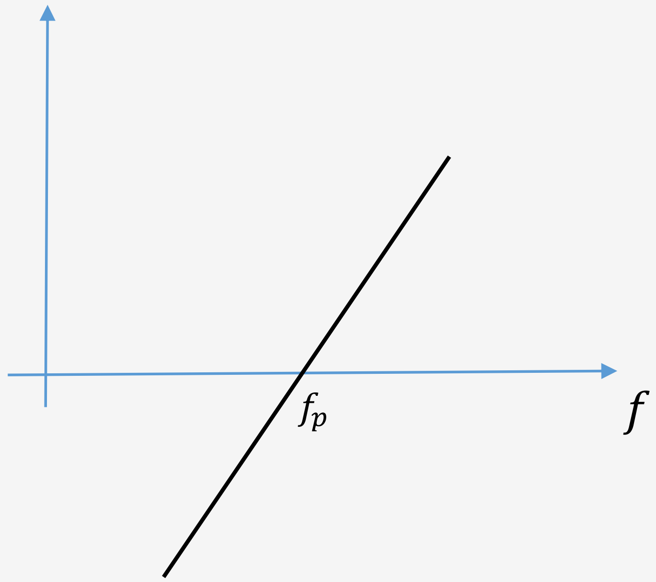
Demodulación

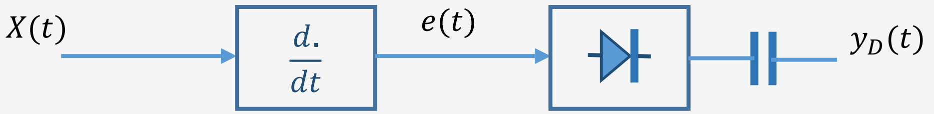
$$X(t) = A\cos(2\pi f_p t + \phi(t))$$

Discriminador en frecuencia ideal



$$y_D(t) = \frac{1}{2\pi}\, k_D \frac{d\phi}{dt}$$



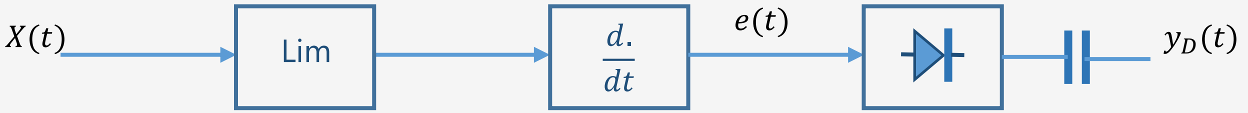


$$e(t) = -A\left(2\pi f_p + \frac{d\phi}{dt}\right) sen\left(2\pi f_p t + \phi(t)\right)$$

$$y_D(t) = A\frac{d\phi}{dt}$$

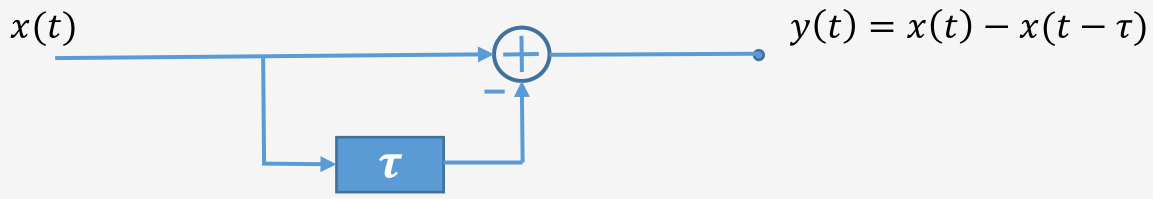
En el caso de FM: $y_D(t) = A\; 2\pi k_f M(t)$

$$k_D = 2\pi A$$



Demodulación

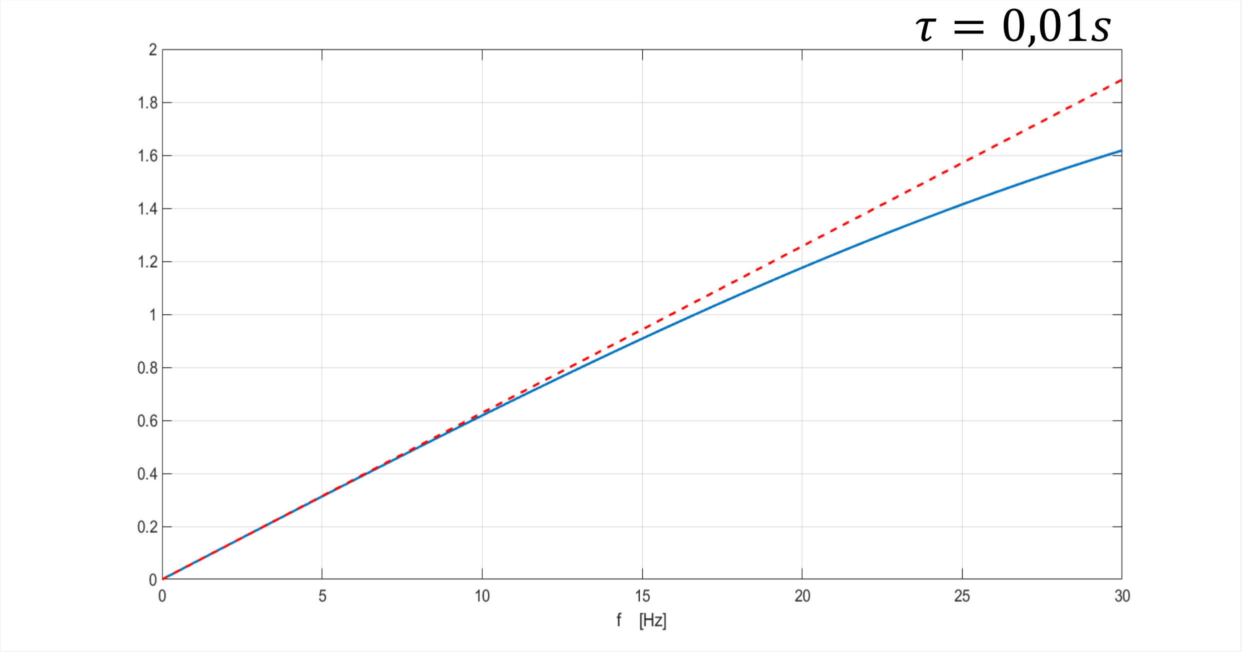
derivador

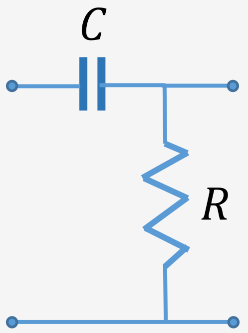


$$H(f) = 1 - e^{-j2\pi f\tau} = 1 - \cos(2\pi f\tau) + \mathrm{j}\,\mathrm{sen}(2\pi f\tau)$$

$$|H(f)| \approx 2\pi\,\tau\,f$$

$\tau \ll 1/2\pi f$



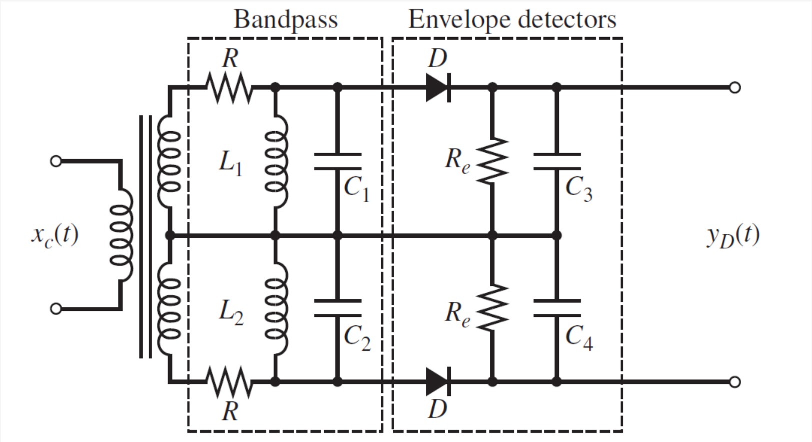


$$H(s) = \frac{SCR}{1 + SCR}$$

$$|H(f)| \approx 2\pi RCf$$

RC ≪ 1

Demodulación



$$f_i = \frac{1}{2\pi\sqrt{L_i C_i}}$$

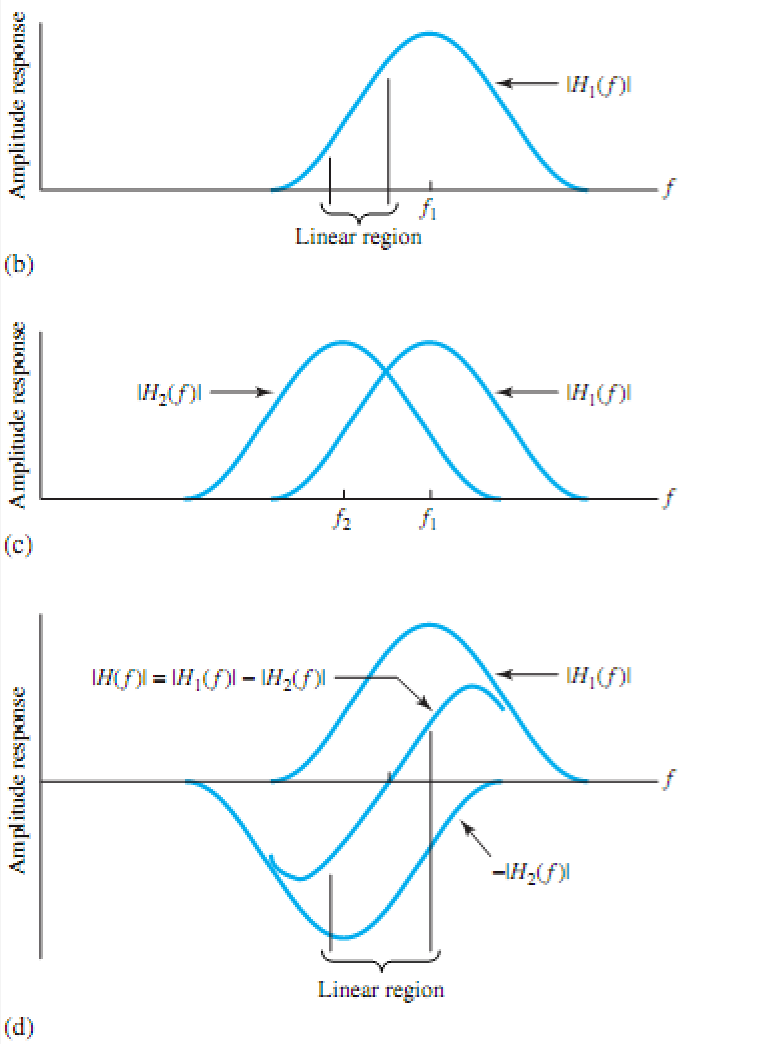
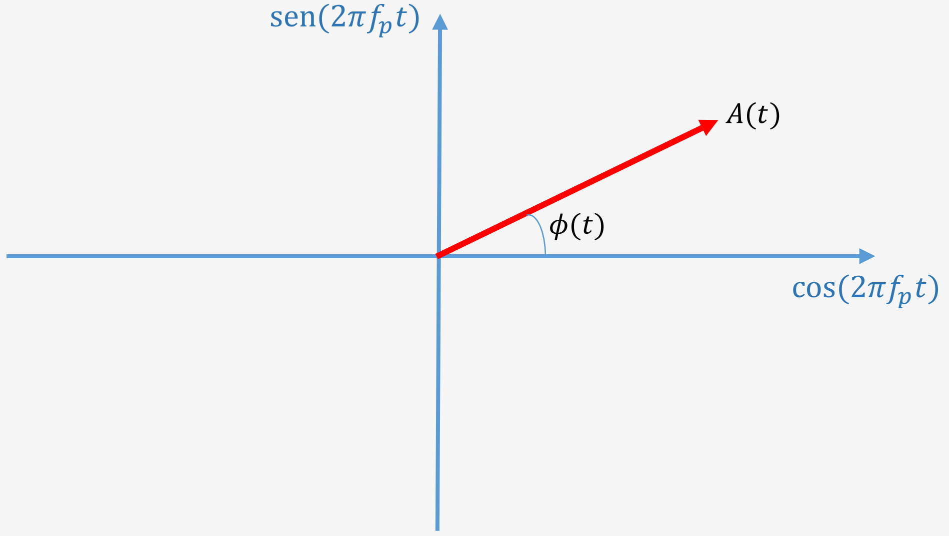


Figure Balanced discriminator with the corresponding frequency

Demodulación

Matlab: fmdemod

diff(unwrap(angle(yq)))



Fuentes:

- Principles of Communications, 5/E by Rodger Ziemer and William Tranter, John Wiley & Sons. Inc.
- Sitio de CONAE

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