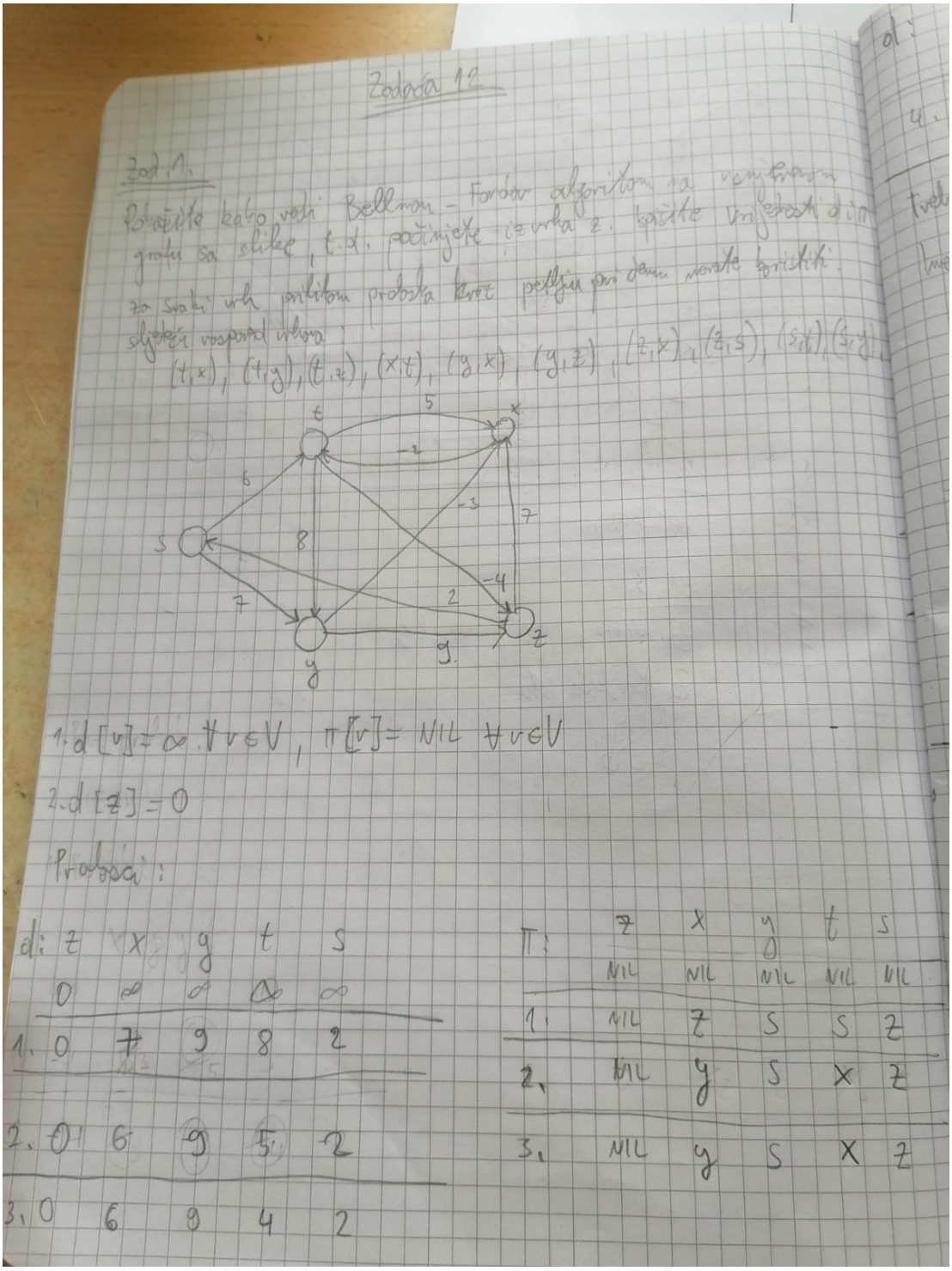
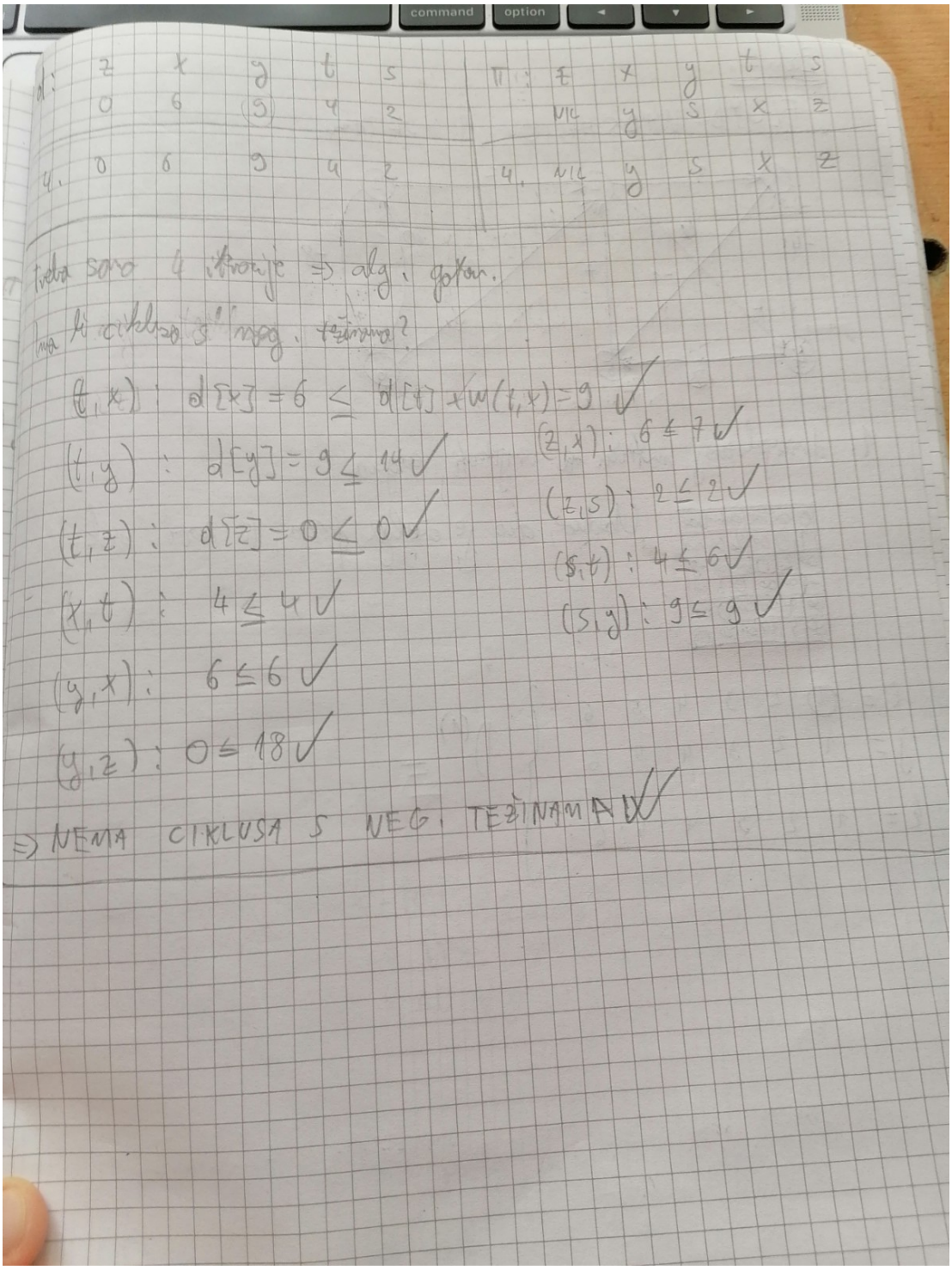
Zadaca 12

Zadatak 1





Zadatak 2

D0:

0 INF INF INF -1 INF
1 INF INF 2 INF INF
INF 2 0 INF INF -8
-4 INF INF 0 3 INF
INF 7 INF INF 0 INF
INF 5 10 INF INF 0

D1:

0 INF INF INF -1 INF
1 INF INF 2 0 INF
INF 2 0 INF INF -8
-4 INF INF 0 -5 INF
INF 7 INF INF 0 INF
INF 5 10 INF INF 0

D2:

0 INF INF INF -1 INF
1 INF INF 2 0 INF
3 2 0 4 2 -8
-4 INF INF 0 -5 INF
8 7 INF 9 0 INF
6 5 10 7 5 0

D3:

0 INF INF INF -1 INF
1 INF INF 2 0 INF
3 2 0 4 2 -8
-4 INF INF 0 -5 INF
8 7 INF 9 0 INF
6 5 10 7 5 0

D4:

0 INF INF INF -1 INF
-2 INF INF 2 -3 INF
0 2 0 4 -1 -8
-4 INF INF 0 -5 INF
5 7 INF 9 0 INF
3 5 10 7 2 0

D5:

0 6 INF 8 -1 INF
-2 4 INF 2 -3 INF
0 2 0 4 -1 -8
-4 2 INF 0 -5 INF
5 7 INF 9 0 INF
3 5 10 7 2 0

Zadatak 3

In [ ]:
```
# a)
from queue import PriorityQueue
INF = 1e9
```

```
def dijkstra(graph, start, end, parent, prob):
    prob[start] = 1 # vjerojatnost da ce se moci doci do starta je maksim
    pq = PriorityQueue()
    # -1, jer trebam extraxt-max umjesto extract-min operaciju
    pq.put((-1, start))
    while(not pq.empty()):
        curr = pq.get()[1]
        for edge in graph.edges.whereStart(curr):
            if(prob[edge.end] < prob[curr] * edge.prob):
                prob[edge.end] = prob[curr] * edge.prob
                pq.put((-prob[edge.end], edge.end))



def shortestPath(graph, start, end):
    for edge in graph.edges:
        if edge.prob == 1:
            graph.removeEdge(edge) # makni ove s vjerojatnoscu 1 jer sigu
    parent = [None] * graph.vertices
    prob =  [0] * graph.vertices
    dijkstra(graph, start, end, parent, prob)
    path = []
    curr = end
    while(curr != None):
        path.append(curr)
        curr = parent[curr]
    path.reverse()
    return path
```

In [ ]:
```
# b)
# 1)
def heaviestTruckInit(graph, start, end, weight):
    dp = [-1 for i in range(graph.size)]
    dp[0] = INF
    heaviestTruck(graph, start, end, dp)
    return dp

def heaviestTruck(graph, start, end, dp):
    max_weight = -1
    prev = None
    for edge in graph.edges.whereEnd(end):
        if(dp[edge.start][end] == -1):
            dp[edge.start] = heaviestTruck(graph, start, edge.start, dp)
        max_weight = max(max_weight, min(dp[edge.start], edge.weight))
    dp[end] = max_weight
    return dp[end]

# objasnjenje:
# Do svakog vrha v iz V moze se doci sa maksimalnom tezinom kamiona w. Ta
# Npr, ako od osijeka do Belog manastira mogu otputovati kamionom tezine
```

In [ ]:
```
# 2)
# Posto su tezine sigurno pozitivne (ceste ne mogu imati negativnu duzinu
def dijkstra(graph, start, end, truckWeight):
    dp = heaviestTruckInit(graph, start, end, truckWeight)
    for vertex in graph.vertices:
        if(dp[vertex] < truckWeight):
            for edge in graph.edges.whereStart(vertex):
                graph.removeEdge(edge)
```

```
    dist = [INF for i in range(graph.size)]
    dist[start] = 0
    pq = PriorityQueue()
    pq.put((0, start))
    while(not pq.empty()):
        curr = pq.get()[1]
        for edge in graph.edges.whereStart(curr):
            if(dist[edge.end] > dist[curr] + edge.weight):
                dist[edge.end] = dist[curr] + edge.weight
                pq.put((dist[edge.end], edge.end))
    return dist[end]
```

In [ ]:

```
# c)
# vertexi predstavljaju gradove
# Istocno bi znacilo da od odredista moze ici samo prema istoku, tj. da n
# takoder, ne smije ici juzno/sjeverno, sto znaci da ne smije 2x posjetit

# IDEJA: augmentirati graf tako da svakom vertexu u grafu nadodam vrijedn

from queue import Queue
def modified_bfs(graph, start, end):
    level = [-1 for i in range(graph.size)]
    level[start] = 0
    q = Queue()
    q.put(start)
    while(not q.empty()):
        curr = q.get()
        for edge in graph.edges.whereStart(curr):
            if(level[edge.end] == -1):
                level[edge.end] = level[curr] + 1
                q.put(edge.end)

def dijkstra(graph, start, end, truckWeight):
    dp = heaviestTruckInit(graph, start, end, truckWeight)
    # O(V)
    for vertex in graph.vertices:
        if(dp[vertex] < truckWeight):
            for edge in graph.edges.whereStart(vertex):
                graph.removeEdge(edge)
    # O(V + E)
    modified_bfs(graph, start, end) # ovo augmentira graf s levelima

    # O(E * lg V)
    dist = [INF for i in range(graph.size)]
    dist[start] = 0
    pq = PriorityQueue()
    pq.put((0, start))
    while(not pq.empty()):
        curr = pq.get()[1]
        for edge in graph.edges.whereStart(curr):
            if(dist[edge.end] > dist[curr] + edge.weight):
                dist[edge.end] = dist[curr] + edge.weight
                pq.put((dist[edge.end], edge.end))
    return dist[end]
```

Zadatak 4

Zad. 4.

a) fj: (1) $\forall i,j \in \{1,2,\dots,n\}: x_i + A[i,j] \leq x_j$

(2) $\forall i,j \in \{1,2,\dots,n\}: x_i + B[i,j] \geq x_j$

Premetanjem prve jednadžbe, imamo:

$x_i - x_j \leq -A[i,j]$ (4)

Premetanjem druge jednadžbe, imamo:

$x_i + B[i,j] \geq x_j$

$x_i - x_j \geq -B[i,j]$

$x_j - x_i \leq B[i,j] \iff x_i - x_j \leq B[j,i]$ (5)

$x_i - x_j \leq -A[i,j]$ i $x_i - x_j \leq B[j,i] \iff x_i - x_j \leq \min(-A[i,j], B[j,i])$

$\Rightarrow C[i,j] = \min(-A[i,j], B[j,i])$

b) Pokažite da Bellman-Ford algo, kad se pokrene na grafu s ograničenjima jednadžbe (3), minimizira izraz $(\max\{x_i\} - \min\{x_i\})$ podložan jedn. (3) i ograničenje $x_i \leq 0 \; \forall x_i$.

Rj:

Kad se Bellman-Ford pokrene na grafu s ograničenjima, $w_{i,j} = C[i,j]$. Također, moramo dodati jedan vertex u graf, s, da ga možemo, t.d. je sa svakim čvorom povezan s težinom 0. To je jer od početka nismo do sada imali nema konvergencija.

3)

- [illegible] [illegible] slijedi s ng. ukljuceno je rješa kompatibilni s B-F alg.

Lema: Znamo: $x_i \leq 0$. Pokažimo: $\max\{x_i\} = 0$.

Neka je $p = \langle s, v_1, \dots, v_k \rangle$ najkraći put $s - v_k \Rightarrow (s, v_1)$ najkraći put od s do v_1. Po konstrukciji $\langle s, v_1 \rangle = 0$, a svaki $x_i \leq 0 \Rightarrow \max\{x_i\} = 0$ ∎

Tm. Kad se B-F pokrene na grafu s ograničenjima, minimizira $\{\max\{x_i\} - \min\{x_i\}\}$

Neka je p put $\langle v_0, v_1, \dots, v_k \rangle$ i $v_0 = s$.

$$w(p) = \sum_{i=0}^{k-1} w_{i(i+1)} = \underbrace{w(\langle v_0, v_1 \rangle)}_{0} + \sum_{i=1}^{k-1} C[i, i+1] =$$

$\max\{x_i\} = 0 \Rightarrow \max\{x_i\} - \min\{x_i\} = -\min\{x_i\}$

$\Rightarrow$ Umanjuje $-\min\{x_i\} \Leftrightarrow$ uvećava $\min\{x_i\}$

$$= \sum_{i=1}^{k-1} C[i, i+1]$$

$$\left.\begin{aligned} x_1 - x_0 &\leq 0 \\ x_2 - x_1 &\leq C[1,2] \\ x_3 - x_2 &\leq C[2,3] \\ &\dots \\ x_k - x_{k-1} &\leq C[k-1,k] \end{aligned}\right\} (+) \quad x_0 + x_k = x_k \leq \sum_{i=1}^{k-1} C[i, i+1] = w(p) \ ∎$$

c) Algoritam:

1) Konstruiraj graf s ograničenjima u jednadžbi (3). Ako $C[i,j] = \infty$, nemoj dodati put (v_j, v_i) u graf

2) Pokreni Bellman-Forda na grafu s ograničenjima da dobiješ rješenje $\{x_1, x_2, \dots, x_n\}$

3) Neka je $w = \min\{x_i\}$. Izračunaj [illegible] $x_i' = x_i - w, \forall x_i$.

Ispiši $\{x_1', x_2', \dots, x_n'\}$