Opakování

$3 < 7$ FPP – výrok (pravdivý)

$\forall x \in \mathbb{R}: x > -1 \Rightarrow |x| > 1$

$\exists x \in \mathbb{R}: x > -1 \wedge |x| \leq 1$

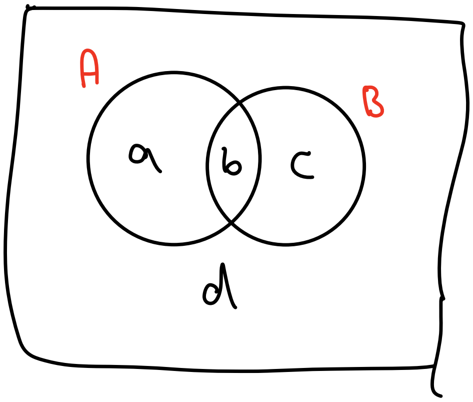
$A = \{x \in \mathbb{R}: (x > 7) \vee (x \leq -4)\} =$

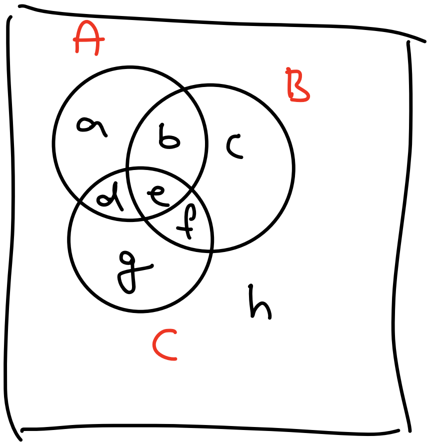
$= (7; \infty) \cup (-\infty; -4\rangle$

$\langle 3; 1\rangle = \{x \in \mathbb{R}: 3 \leq x \leq 1\} = \{\ \}$

$\langle 3; 3\rangle = \{3\}$

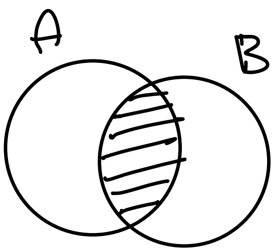
Vennove diagramy





Průnik množin $A \cap B$

Diagram:

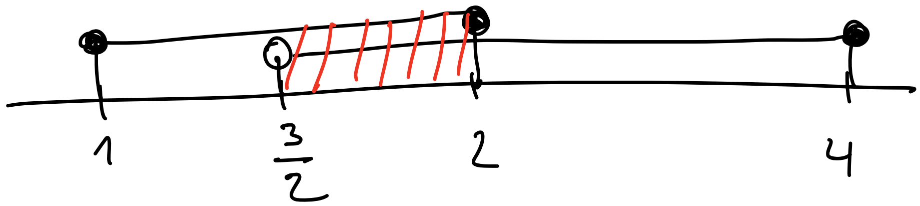


Příklad:

$A = \{1;2;3\}$ $\quad C = \langle 1;2 \rangle$

$B = \{2;4;8\}$ $\quad D = \left(\frac{3}{2};4\right\rangle$

$A \cap B = \{2\}$ $\quad C \cap D = \left(\frac{3}{2};2\right\rangle$



Definice:

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

$$(x \in A \cap B) \Leftrightarrow [(x \in A) \wedge (x \in B)]$$

$$(x \notin A \cap B) \Leftrightarrow [(x \notin A) \vee (x \notin B)]$$

Vlastnosti:

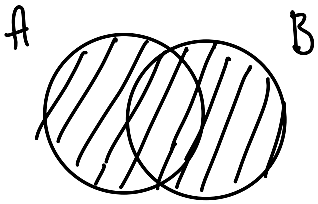
$$A \cap \emptyset = \emptyset$$

$$A \cap A = A$$

$$A \cap B = B \cap A$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Sjednocení množin $A \cup B$



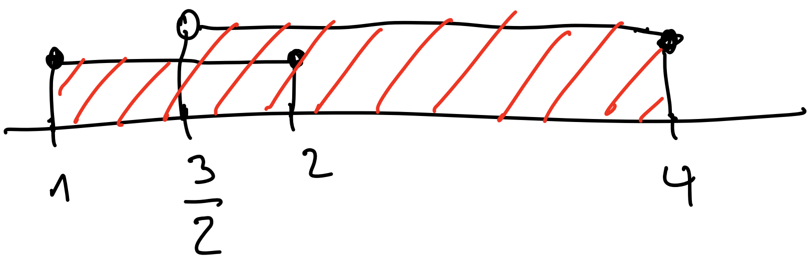
$A = \{1;2;3\}$

$B = \{2;7;8\}$

$A \cup B = \{1;2;3;7;8\}$

$C = \langle 1;2 \rangle$ $\qquad C \cup D = \langle 1;4 \rangle$

$D = (\frac{3}{2}; 4\rangle$



$A \cup B = \{x : x \in A \vee x \in B\}$

$(x \in A \cup B) \Leftrightarrow [(x \in A) \vee (x \in B)]$

$(x \notin A \cup B) \Leftrightarrow [(x \notin A) \wedge (x \notin B)]$

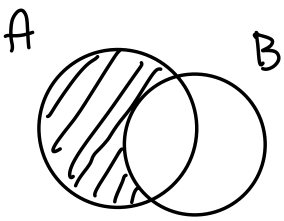
$A \cup \emptyset = A$

$A \cup A = A$

$A \cup B = B \cup A$ – komutativni zakon

$A \cup (B \cup C) = (A \cup B) \cup C$ – asociativni zakon

Rozdiel množín $A \setminus B$



$A = \{1; 2; 3\}$

$B = \{2; 4; 8\}$

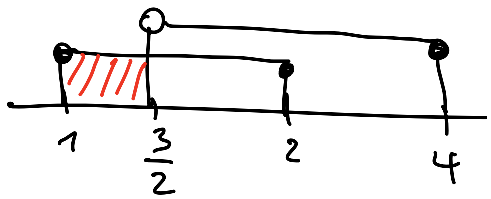
$A \setminus B = \{1; 3\}$

$B \setminus A = \{4; 8\}$

$C = \langle 1; 2 \rangle$

$D = (\frac{3}{2}; 4\rangle$

$C \setminus D = \langle 1; \frac{3}{2}\rangle$



$$A \setminus B = \{x : x \in A \wedge x \notin B\}$$

$$x \in A \setminus B \iff x \in A \wedge x \notin B$$

$$(x \notin A \setminus B) \iff [(x \notin A) \vee (x \in B)]$$

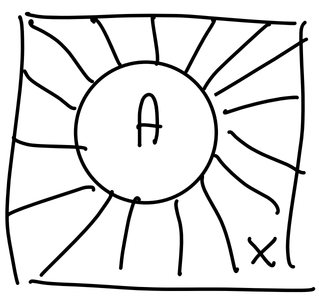
$$A \setminus \emptyset = A$$

$$A \setminus A = \emptyset$$

$$A \setminus B \neq B \setminus A$$

$$A \cup (B \setminus C) \neq (A \setminus B) \cup C$$

Doplněk množiny $\overline{A}_X$



$X = \mathbb{R}$

$A = \langle 1;2 \rangle$

$\overline{A}_{\mathbb{R}} = (-\infty;1) \cup (2;\infty)$

$\overline{A}_x = \{x \in X : x \in X \setminus A\}$

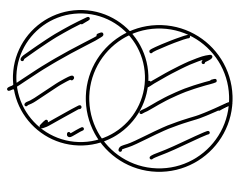
$\overline{A}_x = \{x \in X : x \in X \wedge x \notin A\}$

$x \in \overline{A}_x \iff x \in X \setminus A \iff x \in X \wedge x \notin A$

$x \notin \overline{A}$

⋮ (nestihl jsem)

symetrický rozdíl množin $A \Delta B$



$x \in A \Delta B \iff (x \in A \setminus B) \vee (x \in B \setminus A)$

$A \Delta B = A \setminus B \cup B \setminus A$

$A \Delta B = (A \cup B) \setminus (A \cap B)$

$A = \{1; 2; 3\}$

$B = \{2; 7; 8\}$ $\quad C = \{3; 5; 8\}$

$A \Delta B = \{1; 3\} \cup \{7; 8\} = \{1; 3; 7; 8\}$

$(A \Delta B) \Delta C = \{1; 3; 7; 8\} \Delta \{3; 5; 8\} =$

$= \{1; 7; 5\}$

$x \in A \Delta B \Leftrightarrow (x \in A \setminus B) \vee (x \in B \setminus A)$

$A \Delta B = A \setminus B \cup B \setminus A$

$A \Delta B = (A \cup B) \setminus (A \cap B)$

$A \Delta B = B \Delta A$

$(A \Delta B) \Delta C = A \Delta (B \Delta C)$

kartézský součin množin $A \times B$

$A = \{1; 2\}$

$B = \{\heartsuit; *\}$

$A \times B = \{(1, \heartsuit), (1, *), (2, \heartsuit), (2, *)\}$

$B \times A = \{(\heartsuit, 1), (\heartsuit, 2), (*, 1), (*, 2)\}$

$A \times B \neq B \times A$

$A \times B = \{(x, y) : x \in A \wedge y \in B\}$

$((x, y) \in A \times B) \Leftrightarrow [(x \in A) \wedge (y \in B)]$

$((x, y) \notin A \times B) \Leftrightarrow [(x \notin A) \vee (y \notin B)]$

$(x, y) = (a, b) \Leftrightarrow x = a \wedge y = b$

$\{x, y\}$ – Nezáleží na pořadí

$[x, y]$ – záleží na pořadí

(x, y) – záleží na pořadí

$|A| = m$

$|B| = n$

$|A \times B| = m \cdot n$

Přímý důkaz

Důkaz rovnosti množin

$\forall A, B:$

$$A,(A \setminus B) = A \cap B$$

$C = D \iff C \subseteq D \land D \subseteq C$

$(C \subseteq D) \iff (\forall x : x \in C \implies x \in D)$

$x \in A \setminus B \iff x \in A \land x \notin B$

$x \notin A \setminus B \iff x \notin A \lor x \in B$

1 $A \setminus (A \setminus B) \subseteq A \cap B$

$x \in A \setminus (A \setminus B) \Rightarrow \ldots\ldots\ldots \Rightarrow x \in A \cap B$

$x \in A \setminus (A \setminus B) \Rightarrow x \in A \wedge x \notin (A \setminus B) \Rightarrow$

$\Rightarrow x \in A \wedge (x \notin A \vee x \in B) \Rightarrow$

$\Rightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) \Rightarrow$

$\Rightarrow x \in A \wedge x \in B \Rightarrow x \in A \cap B$

2

$A \cap B \subseteq A \setminus (A \setminus B)$

$x \in A \cap B \Rightarrow x \in A \wedge x \in B \Rightarrow$

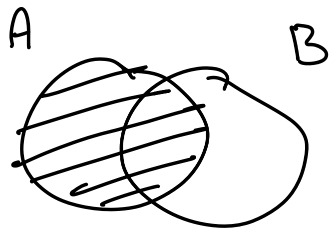
$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \notin A) \Rightarrow$

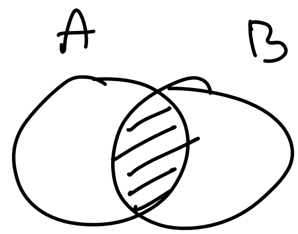
$\Rightarrow x \in A \wedge (x \in B \vee x \notin A) \Rightarrow$

$\Rightarrow x \in A \wedge x \notin A \setminus B \Rightarrow x \in A \setminus (A \setminus B)$

CBTD – co bylo třeba dokázat

$$A \setminus (B \setminus A) = A \cap B$$





$A = \{1,2\}$

$B = \{2;3;4\}$

$A \setminus (B \setminus A) = A \setminus \{3,4\} = \{1,2\} \setminus \{3,4\} =$
$= \{1,2\}$

$A \cap B = \{2\}$

Nerovnaí se

Přímý důkaz implikace

1) Dokažte, že součet dvou lichých čísel je sudé číslo

když m, n jsou liché $\Rightarrow$ $m + n$ je sudé

když m, n jsou liché čísla $\Rightarrow$

$\Rightarrow \exists k, l \in \mathbb{Z} : m = 2k+1,\ n = 2l+1 \Rightarrow$

$\Rightarrow m+n = (2l+1) + (2l+1) = 2k+2l$ ...

(nedokončeno)

2) Dokažte, že druhá mocnina celého čísla je liché číslo

imp: když n je liché $\Rightarrow$ n^2 je liché

Důkaz: když n je liché číslo $\Rightarrow$

$\Rightarrow \exists l \in \mathbb{Z} : n = 2l+1 \Rightarrow n^2 = (2l+1)^2 =$

$= 4k^2+4k+1 = 2\cdot(2k^2+2k)+1 \Rightarrow$

$\Rightarrow$ n^2 je liché

$$2 \nmid n \Rightarrow 2 \nmid n^2$$

$$2 \nmid n^2 \Rightarrow 2 \nmid n$$