RZHP 1. domača naloga

Tomaž Hribernik

November 3, 2019

Naloga 1

a)

$$n(log_4(m+1)*n+2*1) = n^2*log_4(m+1)+2n \Rightarrow \\ \theta(n^2*log_4(m+1)) \tag{1}$$

b)

Najprej n definiramo kot razdaljo od e (end) in s (start)

$$n = j = e - s \tag{2}$$

Potem definiramo novi meji pri rekurzivnih klicih

$$n = (end) - (start) = (s + floor(j/6)) - (s) = \\ floor(j/6) = \lfloor\frac{j}{6}\rfloor = \lfloor\frac{n}{6}\rfloor \tag{3}$$

$$n = (end) - (start) = (e) - (s + floor(j/6) + 1) = \\ (e-s) - \lfloor\frac{j}{6}\rfloor + 1 = n - \lfloor\frac{n}{6}\rfloor + 1 = \lceil\frac{5n}{6}\rceil + 1 \tag{4}$$

+1, ki ostane, ostani zaradi tega, ker je med vključno dvema številoma (n1, n2): n2 - n1 + 1 števil. Poleg dveh rekurzivnih klicov moramo v vsakem klicu funkcije skozi for zanko veliko j = n. Dobimo končno enačbo:

$$T(n) = n + T(\lfloor\frac{n}{6}\rfloor) + T(\lceil\frac{5n}{6}\rceil) \tag{5}$$

Sedaj s pomočjo Akra Bazzi teorema dobimo tight bound

$$T(n) = T(\lceil\frac{5n}{6}\rceil) + T(\lfloor\frac{n}{6}\rfloor) + n = \\ T(\frac{5n}{6} + \lceil\frac{5n}{6}\rceil - \frac{5n}{6}) + T(\frac{n}{6} + \lfloor\frac{n}{6}\rfloor - \frac{n}{6}) + n \tag{6}$$

$$a_1 = 1 \qquad b_1 = \frac{5}{6} \qquad h_1 = \lceil\frac{5n}{6}\rceil - \frac{5n}{6} = O(\frac{n}{log^2 n}) \\ a_2 = 1 \qquad b_2 = \frac{5}{6} \qquad h_2 = \lfloor\frac{n}{6}\rfloor - \frac{n}{6} = O(\frac{n}{log^2 n}) \tag{7}$$

Dokaza za obe zgornji meji

$$\lceil\frac{5n}{6}\rceil - \frac{5n}{6} \leq c\frac{n}{log^2 n} \Rightarrow 1 \leq c\frac{n}{log^2 n} \\ \lfloor\frac{n}{6}\rfloor - \frac{n}{6} \leq c\frac{n}{log^2 n} \Rightarrow 0 \leq c\frac{n}{log^2 n} \tag{8}$$

Izračunamo p

$$1 * (\frac{5}{6})^p + 1 * (\frac{1}{6})^p = 1 \Rightarrow p = 1 \tag{9}$$

Končno

$$T(n) = \theta(n^p(1 + \int_1^n \frac{f(u)}{u^{p+1}}du)) = \theta(n^1(1 + \int_1^n \frac{u}{u^2}du)) = \\ \theta(n(1 + \int_1^n \frac{1}{u}du)) = \theta(n(1 + lnu|_1^n)) = \theta(n(1 + lnn - ln1)) = \\ \theta(nlogn) \tag{10}$$

c)

Funkcijo lahko zapišemo v obliki

$$T(n) = T(n-1) + T(n-1) + 1 = 2T(n-1) + 1, \quad T(0) = 1 \tag{11}$$

$$T(n) = 1 + 2 + 4 + ... + 2^n = \sum_{i=0}^{n} 2^i = 2^{n+1} \Rightarrow \theta(2^n) \tag{12}$$

Naloga 2

a)

$$T(n) = 4T(\frac{n}{9}) + \sqrt[2]{n}$$
$$a = 4 \qquad b = 9 \qquad d = \frac{1}{2} \tag{13}$$

Uporabimo Master theorem

$$a > b^d \Rightarrow 4 > 9^{\frac{1}{2}} = 3 \Rightarrow T(n) = \theta(n^{log_9 4}) \tag{14}$$

b)

$$T(n) = 5T(\frac{n}{4}) + log(n) \tag{15}$$

$$a = 5 \qquad b = 4 \qquad f(n) = log(n) \tag{16}$$

Uporabimo Master theorem

$$f(n) = O(n^{log_4 5 - \epsilon}) \Rightarrow T(n) = \theta(n^{log_4 5}) \tag{17}$$

Naloga 3

a)

$$T(n) = 2T(\frac{n}{4}) + 3T(\frac{n}{6}) + \theta(nlogn)$$
$$f(n) = nlogn \qquad k = 2 \qquad a_1 = 2 \qquad b_1 = \frac{1}{4} \tag{18}$$
$$a_2 = 3 \qquad b_2 = \frac{1}{6}$$

$$\sum_{i=1}^{k} a_i b_i^p = 1 \Rightarrow 2(\frac{1}{4})^p + 3(\frac{1}{6})^p = 1 \Rightarrow p = 1 \tag{19}$$

$$T(n) = \theta(n^p(1 + \int_1^n \frac{f(u)}{u^{p+1}}du)) = \theta(n^1(1 + \int_1^n \frac{ulogu}{u^2}du)) = \theta(n(1 + \int_1^n \frac{logu}{u}du)) \tag{20}$$

$$x = logu \Rightarrow dx = \frac{1}{u}du \Rightarrow du = udx \tag{21}$$

$$T(n) = \theta(n(1 + \int_1^n \frac{logu}{u}du)) = \theta(n(1 + \int_1^n \frac{x}{u}udx)) = \theta(n(1 + \frac{x^2}{2}|_1^n)) = \theta(n(1 + \frac{log^2u}{2}|_1^n)) = \theta(n(1 + \frac{log^2n}{2} - \frac{log^21}{2})) = \Rightarrow \theta(nlog^2n) \tag{22}$$

b)

$$\begin{aligned} T(n) = T(\frac{n}{9}) + T(\frac{n}{4}) + 2T(\frac{n}{36}) + \sqrt{n^3} \\ f(n) = \sqrt{n^3} \qquad a_1 = 1 \qquad b_1 = \frac{1}{9} \\ a_2 = 1 \qquad b_2 = \frac{1}{4} \\ a_3 = 2 \qquad b_3 = \frac{1}{36} \end{aligned} \tag{23}$$

Za izračun vrednosti spremenljivke p sem uporabil wolfram alpha

$$\sum_{i=1}^{k} a_i b_i^p = 1 \Rightarrow 1(\frac{1}{9})^p + 1(\frac{1}{4})^p + 2(\frac{1}{36})^p = 1 \Rightarrow p \approx 0.5695215 \tag{24}$$

$$\begin{aligned} T(n) = \theta(n^p(1 + \int_1^n \frac{f(u)}{u^{p+1}}du)) = \theta(n^p(1 + \int_1^n \frac{\sqrt{u^3}}{u^{p+1}}du)) = \\ \theta(n^p(1 + \int_1^n u^{\frac{3}{2}-(p+1)}du)) = \theta(n^p(1 + \int_1^n u^{-0.0695215}du)) = \\ \theta(n^p(1 + \frac{u^{0.9304785}}{0.9304785}|_1^n)) = \theta(n^p(1 + \frac{n^{0.9304785}}{0.9304785} - \frac{1^{0.9304785}}{0.9304785})) = \\ \theta(n^{0.5695215}\frac{n^{0.9304785}}{0.9304785}) = \theta(n^{1.5}) = \theta(\sqrt{n^3}) \end{aligned} \tag{25}$$

Naloga 4

a)

Seštejemo vsak nivo posebej, zgornja meja je podana z dolžino rekurzije, kjer se pri klicu deli z 2, saj je tam neuravnoteženo drevo najdaljše

$$T(n) = n^3 + 2n^3 + 4n^3 + ... + 2^{log_2 n} n^3 = \sum_{i=0}^{log_2 n} 2^i n^3 = n^3 \sum_{i=0}^{log_2 n} 2^i = n^4 \tag{26}$$

d)

$$\begin{aligned} T(n) = 8T(\lfloor \frac{n}{2} \rfloor) + 27T(\lfloor \frac{n}{3} \rfloor) + n^3 = \\ 8T(\frac{n}{2} + \lfloor \frac{n}{2} \rfloor - \frac{n}{2}) + 27T(\frac{n}{3} + \lfloor \frac{n}{3} \rfloor - \frac{n}{3}) + n^3 \\ a_1 = 8 \qquad b_1 = \frac{1}{2} \qquad h_1 = \lfloor \frac{n}{2} \rfloor - \frac{n}{2} = O(\frac{n}{log^2 n}) \\ a_2 = 27 \qquad b_2 = \frac{1}{3} \qquad h_2 = \lfloor \frac{n}{3} \rfloor - \frac{n}{3} = O(\frac{n}{log^2 n}) \end{aligned} \tag{27}$$

Dokaza za obe zgornji meji

$$\begin{aligned} \lfloor \frac{n}{2} \rfloor - \frac{n}{2} \leq c\frac{n}{log^2 n} \Rightarrow 0 \leq c\frac{n}{log^2 n} \\ \lfloor \frac{n}{3} \rfloor - \frac{n}{3} \leq c\frac{n}{log^2 n} \Rightarrow 0 \leq c\frac{n}{log^2 n} \end{aligned} \tag{28}$$

$$\sum_{i=1}^{k} a_i b_i^p = 1 \Rightarrow 8(\frac{1}{2})^p + 27(\frac{1}{3})^p = 1 \Rightarrow p = 3 \tag{29}$$

$$\begin{aligned} T(n) = \theta(n^p(1 + \int_1^n \frac{f(u)}{u^{p+1}} du)) = \theta(n^3(1 + \int_1^n \frac{u^3}{u^{3+1}} du)) = \\ \theta(n^3(1 + \int_1^n n^{-1} du)) = \theta(n^3(1 + lnu|_1^n)) = \theta(n^3(1 + lnn - ln1)) \\ \Rightarrow \theta(n^3 logn) \end{aligned} \tag{30}$$

Naloga 5

a)

$$\begin{aligned} T(n) &= T(n-1) + n^2 + n - 1 \\ T(n+1) - T(n) &= (n+1)^2 + (n+1) - 1 \\ (E-1)T(n) &= n^2 + 2n + 1 + n \\ (E-1)T(n) &= n^2 + 3n + 1 \\ \text{polinom na desni ustreza funkciji } & (a_0 + a_1 n + a_2 n^2)a^n, \\ \text{kjer je a = 1, anihilira pa jo } (E-1)^3 & \Rightarrow (E-1)^3(E-1)T(n) = 0 \end{aligned} \tag{31}$$

$$\begin{aligned} (E-1)^4 T(n) = 0 \qquad a = 1 \qquad d = 0 \\ T(n) = (\sum_{i=0}^{d-1} \alpha_i n^i) a^n = \alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 \\ T(0) = 1 = \alpha_0 + \alpha_1 0 + \alpha_2 0 + \alpha_3 0 \Rightarrow \alpha_0 = 1 \\ T(1) = T(0) + n^2 + n - 1 = 1 + 1^2 + 1 - 1 = 2 \\ T(2) = T(1) + n^2 + n - 1 = 2 + 2^2 + 2 - 1 = 7 \\ T(3) = T(2) + n^2 + n - 1 = 7 + 3^2 + 3 - 1 = 18 \end{aligned} \tag{32}$$

...

$$\begin{aligned} \alpha_0 = 1 \qquad \alpha_1 = -\frac{1}{3} \qquad \alpha_2 = 1 \qquad \alpha_3 = \frac{1}{3} \\ T(n) = \alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 = 1 - \frac{n}{3} + n^2 + \frac{n^3}{3} \end{aligned} \tag{33}$$