

Master Computer Science

Applying Counterfactual Explanations and Multivariate Forecasting to Medical Prediction Tasks

Name: Tomke Meyer

Student ID: s2231086

Date: dd/mm/2025

Specialisation: Bioinformatics

1st supervisor: Jan van Rijn

2nd supervisor: Panagiotis Papapetrou

Master's Thesis in Computer Science

Leiden Institute of Advanced Computer Science (LI-ACS)

Leiden University Niels Bohrweg 1

2333 CA Leiden

The Netherlands

Abstract

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1 Introduction

The goal of this thesis is to gain insights into the application of multivariate forecasting and counterfactual explanations to medical prediction tasks.

The increasing availability of large and complex healthcare datasets has accelerated the adoption of machine learning (ML) and deep learning (DL) techniques for clinical decision support [ASFWHY⁺25, MPS19]. In particular, time series forecasting plays a vital role in monitoring patient states and predicting clinical outcomes based on continuously recorded data, such as vital signs and laboratory results [KCS⁺20, JPS⁺16]. However, despite the progress in predictive accuracy, many DL models remain black boxes, limiting their interpretability and acceptance in critical healthcare environments.

Counterfactual explanations (CEs) have emerged as a promising approach to enhance model interpretability by identifying minimal and actionable changes in input features that would lead to a more desirable prediction [VDH21, Gui24]. While CEs have been widely studied in static tabular data contexts, their extension to time series forecasting—especially in multivariate medical data—is still underexplored. Existing methods for counterfactual time series mainly focus on altering historical data points, which is impractical in real-world scenarios where the past is fixed [WSMP24].

This thesis aims to address this knowledge gap by developing a novel counterfactual forecasting framework that proposes actionable changes to exogenous variables during the forecast horizon, enabling dynamic and interpretable decision-making in healthcare. We leverage and integrate classical statistical models and state-of-the-art deep learning architectures to learn the relationships between target variables and exogenous inputs, facilitating constrained forecasting that aligns with clinical goals.

Our main contributions are summarized as follows:

- We introduce a new counterfactual time series forecasting method that modifies exogenous variables in the forecast horizon to achieve desired constrained outcomes.
- We incorporate various forecasting models—including SARIMAX, OLS, GRU, and NBEATSx—to learn interpretable relationships between exogenous inputs and target variables, ensuring actionable insights.
- We validate our approach on two healthcare applications, glucose level prediction in diabetes management and heart failure with preserved ejection fraction (HFpEF), demonstrating its practical utility.

[TODO: more thesis introduction, what is the goal, what is the motivation] The global population is both growing and ageing, which is resulting in a rise in chronic diseases. This is resulting in a demand for more efficient, personalised, and proactive healthcare solutions [HSL+23]. Previously, treatment was only provided once symptoms had appeared, and treatment plans were very general. Nowadays, this approach is no longer sufficient, as patients are in need of a more personalised approach to their care. Technological advancements in the field of healthcare have had a significant impact on healthcare providers and patients, with statistical methods being used to predict outcomes. This approach struggles with the complex nature of clinical, demographic and molecular factors that influence the disease progression, leading to Machine Learning (ML), Deep Learning (DL), Artificial Intelligence (AI), and Big Data Analytics becoming increasingly popular fields within the medical and health science domains [ASFWHY+25]. Recent healthcare has been characterised by an increased need for data-driven approaches, with the care process being driven by the flow of data between patients and doctors, and the sharing of decisions, instructions

and information amongst care providers. The role that data and information play in decision making and provision of healthcare, has only increased with the growing digitisation of healthcare. This results in great amounts of data, which enables the implementation of advanced analytical methods, including ML and AI, to derive valuable and actionable insights. These insights are essential in supporting decision-making processes, ensuring high quality patient care, responding to real-time situations and ultimately reducing mortality. Especially ML becomes more and more relevant in healthcare applications, including predictive analysis, treatment optimisation, and patient monitoring [MPS19]. ML algorithms can potentially be used to improve diagnostic accuracy, as well as support early disease detection and prediction. Other applications include, analysing medical imaging data, such as X-rays and MRIs, to detect signs of cancer or neurological disorders, which allows for early diagnosis, as well as a personalised treatment plan for each patient. These uses show the potential ML has for both research and clinical trials, and support the improvement of healthcare overall [RNZ17], [JHS+22].

Time series analysis uses an ordered sequence of data points recorded over time, usually at regular intervals, to understand patterns, trends, and relationships within the data over time. This time series data also allows for forecasting or predicting future values, based on historical observations. Time series forecasting plays a crucial role in a number of different applications. Some applications include its usage in finance, for predicting stock prices and market trends [MSG14], in meteorology for weather and climate forecasting [KS20], in transportation, for effective traffic flow forecasting [LBF13] and especially recently in healthcare. In healthcare, time series forecasting is increasingly relevant, as patient data such as heart rate, blood pressure, glucose levels, and laboratory results are being collected continuously or in regular intervals. Precise forecasting of these medical time series can allow for early detection of negative outcomes, support the personalisation of treatment plans, and optimise healthcare management overall [KCS⁺20], [JPS⁺16].

[TODO: Extend, add citations] Generally, time series forecasting consists of finding temporal dependencies and trends in the data. Depending on the nature of the time series, such as for example linear or non-linear, stationary or non-stationary, and univariate or multivariate, requires different forecasting methods. These different forecasting methods can be categorised into three subgroups, statistical models, such as ARIMA and ETS, ML models, such as Linear Regression and SVRs, and DL models, such as RNNs, LSTMs, GRUs as well as transformer-based models Autoformer or DLinear.

Especially the latter models have shown promising results in recent years, achieving very accurate predictions. Despite these recent developments, many deep-learning based forecasting systems are considered "black-box"-models, as it is challenging to interpret and understand both the modelling process and the forecasting outcome. This is particularly problematic in healthcare applications, where especially interpretability is very important, as clinicians need to understand the reasoning behind the predictions to make informed decisions. One approach to tackle this problem is working with counterfactual explanations, which aim to identify small changes in the input variables needed to change a model's forecast to a more desirable outcome.

In healthcare, time series forecasting has been applied to a number of different tasks, especially the analysis of electronic health records (EHRs). These contain information about a patient's health over time, allowing predictions about the disease progression,

treatment efficacy or patient mortality. For example, forecasting glucose levels in diabetic patients or predicting heart failure with preserved ejection fraction (HFpEF) can aid early interventions and optimise treatment plans.

Traditional time series forecasting focuses mainly on predicting future values given historical observations, but modifying past values is not feasible in real-world settings, especially in healthcare applications. Instead, a more practical approach is to explore how changing exogenous variables during the forecast horizon could lead to a desired outcome. This approach allows continuous monitoring of patients, making it possible to dynamically adjust treatment plans to lead towards more optimal results.

1.1 Diabetes

Diabetes is one of the most prevalent chronic diseases in the world, with it being a leading cause of death and disability. According, to the World Health Organisation (WHO), around 830 million people worldwide suffer from diabetes, and it is the direct cause of 1.5 million deaths a year. Nowadays, it affects around 14% of adults, this number has doubled since 1990, making it a major public health problem. Diabetes is characterised by elevated levels of blood glucose, which over time seriously damages the heart, blood vessels, eyes, kidneys and nerves. Type 2 diabetes is much more common and occurs when the body either becomes resistant to insulin or does not make enough insulin. It usually has a later onset and its development can be attributed to factors such as being overweight, not getting enough exercise and genetic. Type 1 Diabetes is a chronic condition, where the pancreas only produces little to no insulin by itself. It is caused by the autoimmune destruction of pancreatic β -cells and affects 5-10% of the diabetes patients [Dia], [EMA+24a]. For patients with conditions such as type 1 diabetes mellitus (T1DM), closely tracking their glucose levels is a necessity. To reduce the risk of complications such as hyperglycaemia or hypoglycaemia, these patients rely on continuous glucose monitoring (CGM) devices and automated insulin delivery. Using machine learning (ML) to model CGM, can help to gain a better understanding of predicting abnormal glucose events and help with insulin dosage planning. By also incorporating variables such as insulin intake, carbohydrate consumption, and physical activity, a predictive model can allow timely interventions through the generation of actionable recommendations for patients or healthcare providers. This allows for more dynamic treatment based on these forecasted expected glucose trends, which can reduce the long-term risk of diabetes-related complications [EMA⁺24b].

1.2 Heart failure with preserved ejection fraction

Heart failure with preserved ejection fraction (HFpEF) is a prevalent and severe cardio-vascular condition [BP10]. Heart failure (HF) is classified based on the left ventricular ejection fraction (LVEF) and can be split into three subcategories. Heart failure with reduced ejection fraction (HFrEF) with LVEF $\leq 40\%$, heart failure with mildly reduced ejection fraction (HFmrEF) with LVEF = 41-49%, and heart failure with preserved ejection fraction (HFpEF) with LVEF $\geq 50\%$ [MMA+21]. Studies have shown that the mortality within one year is around 29%, [OHH+06], [SBA+24], with increased mortality for patients with previous heart failure hospitalisations and other comorbidities [MGL+20]. This makes HFpEF a very serious condition, where early diagnosis is key. So another

possible application of medical time series forecasting could be to try identifying early warning signs of heart failure or more specific HFpEF to suggest either lifestyle or treatment changes. Using the vital signs, as for example the heart rate and blood pressure, of a patient as well as other factors like gender and possible comorbidities, allows specific and personal monitoring of disease progression. This way, early warning signs of worsening heart condition can be identified and personalised modifications to lifestyle or medication can be suggested. Such proactive monitoring can help reduce hospital readmissions and improve patient outcomes.

1.3 Counterfactual Explanations

Counterfactual explanations (CE) are an emerging technique with the potential to improve the interpretability and explainability of ML models. The objective is to identify minimal changes to the input features that would result in a different, and typically more desirable, outcome. More specifically, they provide information such as if an input data point would be X' instead of X, then the trained ML models prediction would be Y' instead of Y, assuming outcome Y' would be more favourable [VDH21]. So counterfactuals are a type of explanation method in ML, that might help users to understand the model predictions and decisions better. This is supported by counterfactual explanations being actionable, since they suggest specific changes to alter the outcome, and intuitive for humans, since they align with human reasoning about cause and effect. Often CEs are also model-agnostic, meaning they can be applied to a variety of black-box models [Gui24]. A more specific example application of counterfactuals would be a suggestion that slightly reducing a patients systolic blood pressure could have led to a prediction of a lower cardiovascular risk. So these kind of insight can not only help clinicians understand a models decisions better, but also provide help for intervention and treatment planning.

1.4 Main Contributions

[TODO: Check, rewrite?] Time series forecasting can be quite useful in healthcare by predicting how well a treatment works or predicting the risk of complications, relapse or mortality. Recent studies, such as COMET [WSMP24], have been focusing on counterfactuals in time series forecasting, but these methods work by altering historical observations. Our work aims to bridge this gap by developing a counterfactual forecasting mechanism that identifies optimal changes in exogenous variables during the forecast horizon to achieve desirable outcomes. More specifically, we propose a method that learns the relationship between the forecasted targets and exogenous variables, which leads to a more effective and interpretable decision-making in healthcare.

The main contributions of this thesis can be summarised as follows:

- We propose a new model for counterfactual time series forecasting to achieve a desired constrained forecast by modifying exogenous variables within the forecast horizon.
- We incorporate existing forecasting models, such as SARIMAX, OLS, GRU and NBEATSx, for learning the relationship between exogenous variables and a target variable to ensure actionable and interpretable predictions.

• We evaluate the models on two applications in healthcare, specifically for glucose level prediction and HFpEF management, demonstrating the practical utility of our approach.

1.5 Problem Formulation

1) Let $X := (x_i)_{i \in \{1,...,n\}}$ be a time series of length n (back horizon), with each $x_i \in \mathbb{R}^{m+1}$ composed of the target variable $y_i \in \mathbb{R}$ and the exogenous variables

$$z_i = \begin{pmatrix} z_{1,i} \\ \vdots \\ z_{m,i} \end{pmatrix} \in \mathbb{R}^m.$$

Then X can be denoted as the combined matrix of the target vector $y \in \mathbb{R}^{1 \times n}$ and the exogenous matrix $Z \in \mathbb{R}^{m \times n}$:

$$m{X} = egin{pmatrix} m{y} \ m{Z} \end{pmatrix} := egin{pmatrix} y_1 & \dots & y_n \ z_{1,1} & \dots & z_{1,n} \ dots & \ddots & dots \ z_{m,1} & \dots & z_{m,n} \end{pmatrix}.$$

The relationship between y and Z can be described by the function:

$$r: \mathbb{R}^{m \times n} \to \mathbb{R}^{1 \times n}$$

which calculates the target vector from the exogenous matrix:

$$r(\boldsymbol{Z}) = \boldsymbol{y}.$$

2) Given a time series forecasting model f that predicts the next t values (forecasting horizon) of X, we define the forecast as:

$$f(\mathbf{X}) = \mathbf{X}' := (x_{n+i})_{i \in \{1,\dots,t\}}.$$

Additionally, consider a lower and upper bound set of constraints for each time step in the forecasting horizon, denoted as:

$$\boldsymbol{\alpha} = (\alpha_{n+i})_{i \in \{1,\dots,t\}}, \boldsymbol{\beta} = (\beta_{n+i})_{i \in \{1,\dots,t\}}.$$

The objective is to generate a counterfactual time series sample Z^* , such that $y^* = r(Z^*)$ satisfies the given bounds:

$$\alpha_i \leq y_i^* \leq \beta_i, \forall y_i^* \in \boldsymbol{y}^*, i \in \{n+1, \dots, n+t\}.$$

Summarized research objective: Given a target vector y affected by the exogenous matrix Z, a forecast horizon t, the original forecasted vector y' and the original forecasted exogenous matrix Z', the goal is to modify Z' to Z^* such that the corresponding target vector y^* is within constraints α and β .

2 Related Work

[TODO: check, add citations, rewrite]

2.1 Related Forecasting Models

Recent research has explored various deep learning (DL) models for time series forecasting, including recurrent neural network (RNN)-based models such as gated recurrent units (GRU) and long short-term memory (LSTM), as well as attention-based architectures like transformers. Transformer-based models, including Autoformer and Informer, have demonstrated strong performance in both univariate and multivariate forecasting tasks by capturing long-range dependencies more effectively than traditional RNN approaches. In the clinical domain, deep learning models have been applied extensively to glucose forecasting. For instance, Deep Multi-Output Forecasting [FAJ+18] introduced a multi-step forecasting framework that explicitly models the distribution of future glucose values over a prediction horizon using a multi-output deep architecture. Similarly, WaveNet has been adapted for glucose forecasting by leveraging dilated convolutional neural networks (CNNs) to model long-term dependencies [ZLH+18]. In addition, transfer learning techniques have been employed to enhance predictive performance by fine-tuning pre-trained models on patient-specific data while incorporating exogenous covariates such as insulin dosage and carbohydrate intake [MB20].

Beyond predictive performance, explainability remains a critical challenge in deep learning-based forecasting models. Traditional statistical models, such as ARIMAX and VARIMAX, are able to quantify relationships between exogenous factors and the target variable, but their forecasting accuracy is often outperformed by DL approaches. Recent research has focused on integrating explainability into forecasting models to combine the strengths of both interpretability and predictive performance. For example, NBEATSx extends the NBEATS framework by incorporating exogenous variables into its deep architecture, enabling a more structured decomposition of trend and seasonality. However, its interpretability remains static and does not fully capture the dynamic nature of forecasting outcomes.

To address the need for explainability, counterfactual explanations have gained traction in time series analysis. Initial efforts focused on time series classification, where counterfactuals were generated through instance-based modifications and gradient-based perturbations [AALC20]. This was done by introducing a framework for generating counterfactual explanations for multivariate time series classification, identifying minimal input modifications needed to alter the model's decision, providing interpretability for high-dimensional time series models.

More recently, counterfactual explanations have been extended to time series forecasting. ForecastCF [WMSP23] proposed a deep learning-based method for generating counterfactuals in time series forecasting by identifying minimal input changes required to achieve desired prediction outcomes. Building on this, COMET [WSMP24] extended counterfactual explanations to multivariate time series forecasting, focusing on modifying exogenous variables (e.g., insulin, carbohydrates, and exercise) to generate actionable recommendations for glucose management.

In addition to counterfactual forecasting, diagnostic tools like TimeTuner [HSYZ23] have been developed to analyse how time representations influence forecasting models. By

employing counterfactual explanations, TimeTuner enables the evaluation of multivariate time series representations and their impact on model predictions.

Despite these advances, counterfactual explanations for multivariate time series analysis remain an emerging research area. While existing methods demonstrate the feasibility of generating counterfactuals for univariate forecasts, their generalisation to multivariate forecasting and real-world clinical applications remains limited. This work aims to extend on these existing methods by integrating counterfactual reasoning with multivariate forecasting models, focusing on modifying exogenous variables within the prediction horizon to provide actionable and interpretable interventions.

2.2 Related Work in Glucose Forecasting

Forecasting physiological indicators such as blood glucose levels, is crucial for managing diabetes. Time series forecasting and model explainability are becoming increasingly important in this field of medical prediction tasks, as in many others. Recent contributions to this area of research highlight the growing use of multivariate machine learning models to improve predictive accuracy and personalisation. This shows the growing need for interpretable and actionable insights, which aligns closely with the goals of this thesis. Recent research highlights the growing development of multivariate and deep learningbased models for predicting glucose levels. For example, Kalita and Mirza [KM25] proposed a model that combines multi-head attention layers with neural basis expansion networks, capturing complex temporal and cross-feature dependencies in glucose data. Similarly, Benaida et al. [BAI25] demonstrated the effectiveness of deep learning architectures for both single- and multi-step glucose forecasting, emphasising the importance of longer-term prediction capabilities in real-world applications. These multivariate models are consistent with the focus of this thesis on leveraging multiple signals, such as past glucose levels, physiological parameters, and contextual variables, for accurate forecasting. Personalisation has emerged as a key factor in clinical forecasting settings, as patient diversity affects model performance. Shen and Kleinberg [SK25a] addressed this issue by using incrementally retrained LSTM networks that adapt to each individual's glucose dynamics. This improves performance, even when the CGM data is limited. Lara-Abelenda et al. [LCP+25] introduced large language models (LLMs) to model personal glucose trends, highlighting the capacity of foundation models to generalise across individuals while retaining patient-specific nuances. These methods emphasise the importance of adaptive and context-aware forecasting.

Several works have also incorporated physiological signals beyond glucose levels to support multivariate forecasting. For example, Giancotti et al. [GBV⁺24] explored the utility of heart rate as a predictor of forecasting glucose levels in patients with type 1 diabetes, which demonstrates that multimodal data can significantly enhance predictive accuracy. Similarly, Rodríguez-Rodríguez et al. [RCR23] utilised data, such as physical activity and diet logs, to enable more holistic and personalised glycaemic forecasting.

Interpretability remains a major challenge for deep learning-based forecasting models, especially in critical fields such as medicine. In response to this, Sun and Kosmas [SK25b] combined Bayesian forecasting with expert medical knowledge to model CGM values in type 2 diabetes patients. Their framework improves both uncertainty quantification and clinician interpretability, which is an essential consideration in healthcare AI. The need for model transparency directly motivates the use of counterfactual explanations to improve

the explainability and actionability of predictive models in medical applications. While many current studies emphasise predictive accuracy, fewer address how predictions can be explained and acted upon by clinicians or patients.

Taken together, these studies reflect a shift towards data-driven, multivariate, and personalised models for medical forecasting. However, there remains a clear gap in integrating these powerful models with robust, interpretable explanations. This thesis aims to bridge this gap by combining multivariate forecasting approaches with counterfactual reasoning, to provide accurate predictions and actionable, understandable explanations, which are essential components for supporting medical decision-making and patient self-management.

2.3 Related Work in Mortality Prediction of Patients with HFpEF

Heart Failure with Preserved Ejection Fraction (HFpEF) is a complex and heterogeneous condition, characterised by diagnostic and prognostic uncertainty. This makes it a compelling use case for machine learning (ML) in clinical decision support. Recent research has applied various ML techniques to improve diagnosis, predict outcomes such as hospitalisation and mortality, and guide individualised management strategies. These efforts emphasise the increasing relevance of multivariate forecasting and the growing demand for explainable models, which are central to the objectives of this thesis.

A significant amount of research has focused on prognostic modelling using structured clinical data. For example, Hu et al. [HMH⁺25] developed and validated a machine learning model to predict the risk of readmission within one year for HFpEF patients, demonstrating the utility of routinely collected electronic health records (EHRs) in anticipating adverse outcomes. Similarly, McDowell et al. [MKT⁺24] constructed models for predicting both mortality and morbidity in HFpEF patients, showing that complex risk factors, including comorbidities and laboratory values, can be effectively integrated into predictive models. These studies emphasise the importance of leveraging multivariate data sources to forecast long-term patient outcomes.

Short-term outcome prediction has also been explored, particularly in the context of the early identification of high-risk patients. The study "Predicting 30-Day and 1-Year Mortality in HFpEF" [SBA+24] used machine learning to predict short-term mortality, which is essential for planning acute care. Other models have focused on hospitalisation prediction, using historical patient trajectories to anticipate future events. These forecasting tasks not only require accurate time series modelling but also benefit from interpretability to inform clinical decisions.

The diagnosis of HFpEF remains a challenging area due to its symptomatic overlap with other heart failure subtypes. Kavas et al. [KBKB23] developed an ML-based decision support system using photoplethysmography (PPG) signals to differentiate between HFpEF and HFrEF (Heart Failure with reserverd Ejection Fracion), demonstrating the potential of non-invasive, sensor-based diagnostics. Liao and Hung [LH⁺24] further extended this approach by incorporating data from a wearable patch device to enhance diagnostic precision. These works highlight the growing role of physiological signal data in heart failure classification, which directly supports multivariate modelling approaches by introducing continuous and high-frequency signals into prediction tasks.

Genomic and molecular data have also been used to support precision medicine approaches in HFpEF. Zhou et al. [ZGW⁺21] utilised gene expression profiles to build ML models ca-

pable of risk stratification in HFpEF patients, adding a layer of biological interpretability to purely clinical models. Although these models are powerful, they are often perceived as "black boxes," emphasising the need for explainability techniques such as counterfactual explanations to bridge the gap between model prediction and clinical insight.

Across these studies, however, the challenge of model transparency and interpretability remains largely unaddressed. Most existing models prioritise predictive performance without offering sufficient explanations for individual predictions, which is a critical issue in medical contexts where understanding why a prediction was made is often as important as the prediction itself. This thesis aims to bridge this gap by integrating counterfactual reasoning into multivariate forecasting models, offering clinicians not just a forecast, but a clear explanation of the factors driving the prediction and the minimal changes that could alter an adverse outcome.

In summary, the current research in HFpEF prediction demonstrates the power of machine learning to handle complex, multivariate data across diagnostic and prognostic applications. However, a lack of interpretability limits clinical adoption. This thesis tries to contribute to the field by combining accurate time series forecasting with interpretable, counterfactual explanations, thereby supporting more transparent and actionable decision-making in heart failure care.

3 Methodology

3.1 Algorithm

[TODO: update, check] The proposed algorithm is designed to generate counterfactual exogenous inputs that lead to a desired change in the forecasted target variable within a multivariate time series context. It operates by integrating a differentiable forecasting model and a counterfactual optimization process constrained by both feasibility and historical plausibility. The algorithm takes multivariate time series data as input, which is split into a target variable t (for example blood glucose level) and associated exogenous variables E (for example insulin dosage, carbohydrate intake, activity level). Several hyperparameters are defined: the learning rate η , target bounds (α, β) , a clipping range (ρ, ϕ) , and the maximum number of optimization iterations max_iter. Additionally, differentiable models for forecasting $f(\cdot)$ and counterfactual generation $c(\cdot)$ are provided, where f predicts the target and c estimates the outcome, given perturbed inputs. The goal is to identify a perturbed exogenous input E' that results in a forecasted target t' within the desired bounds. The algorithm consists of different stages:

- Multivariate Forecasting: The forecasting model f is applied to the input sequence to obtain predicted targets \hat{t}^* and exogenous outputs \hat{E}^* . These serve as a reference for evaluating the effect of the perturbations.
- Sample and Bound Selection: A representative subset S of the test data is selected to compute statistical bounds (α, β) on the desired target outcome. These bounds guide the optimization process by defining what is considered a successful counterfactual prediction
- Loss Initialization: A composite loss function L is initialized. It incorporates forecasting error, a regularization term to penalize unrealistic perturbations, and a historical

similarity constraint.

- Iterative Counterfactual Optimization: Counterfactuals are generated through gradient-based optimization, with the Adam optimizer. In each iteration the loss is back propagated with respect to \hat{E}^* , and updates are applied. Then a clipping function ensures that the perturbed values stay within predefined limits (ρ, ϕ) and a historical similarity constraint \mathcal{G} encourages solutions to be close to real historical values, to enhance realism and interpretability.
- Stopping criteria: The loop terminates once the forecasted target falls within the desired bounds or when the maximum number of iterations is reached.
- Return Output: The final optimized exogenous input E' and its associated target t' are returned as the counterfactual explanation.

This algorithmic approach provides an adjustable and interpretable method for generating actionable interventions in time series forecasting tasks. It ensures that the generated counterfactuals are both effective, as they achieve the target, and feasible, as they are within realistic operational constraints, making it a suitable method for the delicate field of healthcare. Algorithm 1 shows the pseudocode for the described method.

3.1.1 Constraints

[TODO: check, rewrite, improve] To ensure that the generated counterfactuals are both feasible and realistic, the optimization process is guided by two key constraint mechanisms: clipping constraints and historical value constraints. These are embedded directly within the optimization loop and play a crucial role in generating interpretable and valid exogenous inputs.

3.1.1.1 Clipping Constraint

The clipping constraint enforces that the perturbed exogenous variables \hat{E}^* remain within a predefined, domain-specific value range. These bounds (ρ, ϕ) are selected based on physical limits, clinical safety, or operational feasibility (e.g., insulin dosages must be non-negative and within a patient-safe maximum range).

After each optimization step, every element in \hat{E}^* is clipped such that:

$$\hat{E}_*^{i,j,k} = \min(\phi_k, \max(\rho_k, \hat{E}_*^{i,j,k}))$$

for each sample i, time step j, and feature k. This ensures that the generated counterfactuals remain within a trusted domain of possible values.

3.1.1.2 Historical Value Constraint

The historical value constraint is introduced to promote plausibility by encouraging counterfactual inputs that are statistically similar to observed historical inputs. This constraint acts as a regularization mechanism, penalizing counterfactuals that deviate too far from past observed patterns. It leverages a historical dataset \mathcal{G} to compute a similarity or distance metric—often based on Euclidean or Mahalanobis distance.

Formally, a constraint term C is computed based on the deviation of \hat{E}^* from the nearest historical samples:

$$C(\hat{E}^*) = \lambda \cdot \min_{E \in \mathcal{G}} \|\hat{E}^* - E\|_2$$

where λ is a hyperparameter that controls the strength of this constraint. The closer \hat{E}^* is to a real historical sample, the lower the penalty.

This mechanism ensures that the generated exogenous sequences are not only valid within a static range but also dynamically aligned with real-world behavior.

3.1.1.3 Combined Loss Function

The total loss function L used for optimization combines multiple components:

- Forecast deviation loss: Penalizes predictions that fall outside the desired target bounds (α, β) .
- Clipping violation penalty: Implicitly enforced via projection, not explicitly penalized.
- Historical deviation penalty: Measures distance from historical patterns

Overall, the total loss is defined as:

$$L = \text{ForecastLoss}(\hat{t}^*, \alpha, \beta) + \lambda_{\text{hist}} \cdot C(\hat{E}^*)$$

where λ_{hist} controls the importance of staying historically realistic.

Algorithm 1 Counterfactual Hybrid Forecasting

```
1: Input: Time series data: target t, exogenous variables E, learning rate \eta, desired bounds (\alpha, \beta), clipping range (\rho, \phi), max iterations max_iter, differentiable forecaster f(\cdot) differentiable counterfactual generator c(\cdot), w, \mathcal{G}
```

```
2: Output: Counterfactual E' with desired outcome t'
```

```
3: (t^*, E^*) \leftarrow (t, E)
```

4:
$$(\hat{t}^*, \hat{E}^*) \leftarrow f(t^*, E^*)$$

5:
$$S \leftarrow \text{SelectTestSamples}$$

6:
$$(\alpha, \beta) \leftarrow \text{GenerateBounds}(S)$$

7:
$$loss \leftarrow L((t^*, E^*), \alpha, \beta, (t, E))$$

8: time $\leftarrow 0$

9: while
$$(\hat{t}^* > \beta \text{ or } \hat{t}^* < \alpha) \land (\text{time} < \text{max_iter}) \text{ do}$$

10: $\hat{E}^* \leftarrow \text{AdamOptimize}(\hat{E}^*, loss, \eta)$

11:
$$\hat{E}^* \leftarrow \text{Clip}(\hat{E}^*, \rho, \phi)$$

12:
$$\hat{t}^* \leftarrow c(\hat{E}^*)$$

13:
$$C \leftarrow \text{HistValueConstraint}(\hat{E}^*, \mathcal{G})$$

14:
$$loss \leftarrow L(\hat{E}^*, w, \alpha, \beta, X^*)$$

- 15: $time \leftarrow time + 1$
- 16: end while
- 17: $(t', E') \leftarrow (\hat{t}^*, \hat{E}^*)$
- 18: **return** (t', E')

3.2 Data

While the proposed model has broad applicability across domains beyond healthcare, this study focuses exclusively on medical use cases. In particular, we investigate the models usefulness in optimizing treatment plans for two conditions: type 1 diabetes and heart failure with preserved ejection fraction (HFpEF). The datasets used in these experiments contain physiological measurements and treatment-related variables, allowing for personalized forecasts and counterfactual intervention generation. To prepare the data for modelling, several preprocessing steps have been applied. First, the data is split into training, validation and test sets and normalized using min-max scaling to ensure stable and consistent model training. The data is then separated into target variables and exogenous inputs. Then, a sequence generator is employed to segment the time series into overlapping windows comprising a back horizon (historical input) and a prediction horizon (target output). This is done both for the target variable alone and for sequences that include both the target and exogenous features. Any sequences containing missing values are discarded to ensure data integrity.

3.2.1 SimGlucose

The SimGlucose dataset is generated using the FDA-approved UVA/PADOVA type 1 diabetes simulator [Xie18], a Python-based tool that models the physiological responses of individuals with type 1 diabetes. The simulator includes 30 virtual patients, comprising of 10 adults, 10 adolescents, and 10 children and produces continuous glucose monitoring (CGM) measurements along with insulin dosages and carbohydrate intake events. The dataset is generated with a predefined CGM sampling frequency and insulin pump settings based on the algorithm developed in [DMML+14]. For this study, simulated data was generated for ten adult patients over a one-week period. The blood glucose (BG) levels serve as the primary target variable, while the insulin dosage and carbohydrate intake are used as exogenous variables influencing the glucose levels. After generation, the data undergoes preprocessing steps as described above to prepare it for the forecasting and counterfactual generation. An example of the generated data is shown in Figure 1. In this example, BG denotes blood glucose levels, CHO indicates carbohydrate intake, and Insulin reflects the administered dosage. The risk index illustrates periods of hyperglycaemic or hypoglycaemic risk. The green band in the BG trace highlights the target glucose range (70–180 mg/dL), while red regions denote values that fall outside this range, corresponding to hypo- or hyperglycaemia.

3.2.2 OhioT1DM:

The OhioT1DM dataset [MB20] contains real-world glucose monitoring data collected from 12 individuals with type 1 diabetes over an eight-week period by Ohio University. Following prior research [CHN+21], [WSMP24], we extracted the most clinically relevant features for forecasting: continuous glucose monitoring (CGM) measurements, basal insulin, bolus insulin, carbohydrate intake, and physical activity. Compared to the SimGlucose dataset, OhioT1DM includes a more varied set of exogenous variables, particularly basal and bolus insulin administration, dietary intake, and physical activity data. As a

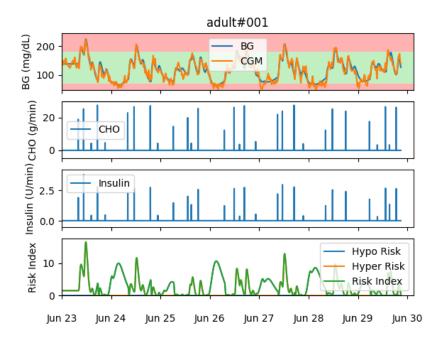


Figure 1: Simulation of an adult patient over the span of one week.

real-world dataset, it presents additional challenges such as missing values and irregular sampling intervals. These are addressed using interpolation and resampling techniques to ensure consistency in the input sequences. The inclusion of diverse exogenous variables enables the development of more nuanced counterfactual interventions and supports improved forecasting performance. Figure 2 illustrates a 24-hour time window for one patient, showing the temporal relationship between glucose levels, insulin administration, and carbohydrate consumption. The blue dotted line represents CGM-based blood glucose levels, while the black line represents the basal insulin, reflecting its slow-acting, sustained delivery throughout the day. Orange dots indicate the amount of bolus insulin, while small blue boxes indicate meals. Spikes in blood glucose often correspond to meal times (carbohydrate intake), followed by bolus insulin doses that aim to bring glucose back into the target range. This visualization exemplifies the complex dynamics and temporal dependencies that the model must capture to enable accurate and personalized glucose forecasting.

3.2.3 MIMIC-IV:

The proposed model aims to generalize beyond diabetes forecasting to other medical applications, such as predicting disease progression in HFpEF patients. The MIMIC-III dataset [JPS+16] contains de-identified electronic health records (EHRs) of ICU patients, including vital signs (heart rate, blood pressure, oxygen saturation, etc.), medication records, and laboratory results. For this study, a subset of MIMIC-IV focusing on cardiovascular patients is utilized. The target variable, mortality risk, was divided into two groups: death within 30 days and death within one year. The exogenous variables include vital signs and laboratory values, while gender and comorbidities are used to split the data into multiple cohorts. By analysing the data in different cohorts, it is possible to get more specific and

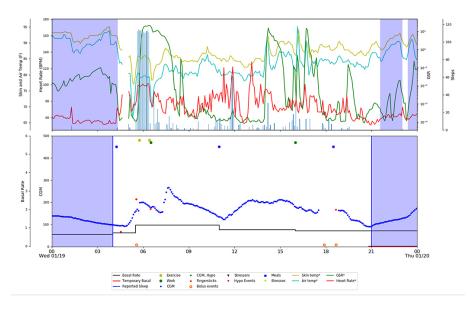


Figure 2: 24-hour measurements of one patient from the OhioT1DM Viewer [MB20].

accurate counterfactual interventions.

3.2.3.1 Preprocessing

Following a preceding study [SBA⁺24], the International Classification of Diseases (ICD) codes were used for the initial preprocessing. The ICD-codes are a standardized international classification system used for the categorisation and encoding of diseases, symptoms, and associated health-related conditions. By using the appropriate ICD-9 and ICD-10 codes, as outlined in Table 1, the hospital admissions involving patients aged > 18 with HFpEF as a primary diagnosis have been identified. Given that the diagnosis was based on ICD codes, the clinical notes were analysed in order to validate the selection of patients. This was achieved through filtering the clinical notes on mentions of the left ventricular ejection fraction (LVEF) value, with a value of 50 and above being counted as a normal LVEF value. Some clinical notes only mentioned a normal or preserved LVEF value with out a measured LVEF. These were also counted as normal LVEF values. Table 1 also shows the number of hospital admissions per diagnosis. The study sample consisted of 16122 individual hospitalizations with a suspected diagnosis of HFpEF. We had access to clinical notes for 11720 (72.7%) hospital admissions of which 4458 (38%) had an LVEF measurement reported. Of these, 3798 (85.2%) had an LVEF value $\geq 50\%$, and 400 (9%) had an LVEF < 50%. An additional 260 (5.8%) admissions had mention of a normal or preserved LVEF. For these 4058 admissions, vital signs and laboratory values were available for 2432 (60%). It should be noted that in this instance, only the most recent admission of a patient who had previously been admitted with a similar diagnosis was taken into consideration. Prior admissions were incorporated into the analysis as a comorbidity, and after adding all laboratory values, vital signs and comorbidities, the resulting number of unique hospital admissions is 2113 with 1845 unique patients.

The extracted features are listed in table 2, split into four categories. These are the targets (Death within 30 days, Death within 1 year) as well as vital signs and laboratory values, comorbidities, and gender. Prior admission was included in the list of comorbidities, since

| Diagnosis | ICD Code | Frequency |
|---|----------|-----------|
| Unspecified diastolic (congestive) heart failure | I5030 | 38 |
| Diastolic heart failure, unspecified | 42830 | 69 |
| Acute diastolic (congestive) heart failure | I5031 | 84 |
| Acute diastolic heart failure | 42381 | 180 |
| Chronic diastolic (congestive) heart failure | I5032 | 256 |
| Chronic diastolic heart failure | 42382 | 453 |
| Acute on chronic diastolic (congestive) heart failure | I5033 | 426 |
| Acute on chronic diastolic heart failure | 42833 | 623 |

Table 1: The corresponding Diagnosis and ICD-9 and ICD-10 codes for HFpEF.

it is here used as a comorbidity for the clustering and not the forecasting.

3.2.3.2 Clustering

The MIMIC data does not only include a wide range of features, but also a great number of comorbidities. Since these comorbidities can influence the chances of HFpEF, it is important to include these in the analysis. In this study, the comorbidities were used to cluster the patients, to get more specific counterfactuals. For this, the data was split according to patients having similar comorbidities to get factual cohorts. Another split was made by dividing the data on gender, since HFpEF is more prevalent in female patients and might need different treatment. Figure 3 shows the prevalence of different comorbidities of the MIMIC patients in the clusters, divided into four sub figures depending on the clustering coefficient and the gender. Figure 4 shows the Principal Component Analysis (PCA) of the different comorbidities, again divided into the same subgroups.

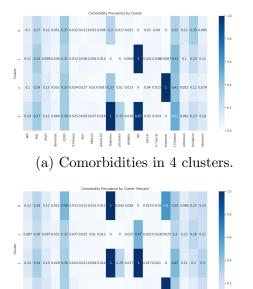
3.3 Experiments

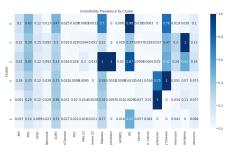
3.3.1 Experimental Setup

The model can be split into two main parts. Initially a multivariate forecasting model is used to make a first forecast for both the target variable and the exogenous variables. This forecast is then used for the second part, where different regression models are used to change the exogenous and target variable to get the desired outcome. For the forecasting part we used GRU and NBEATS and for the second part we used four different models. Here the focus lies on using very different kinds of models, like a statistical (SARIMAX), a regression based (OLS), and two different deep learning based (GRU and NBEATSx) models.

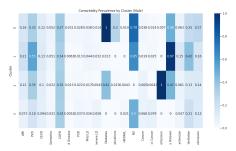
3.3.1.1 Multivariate Forecasting:

For the multivariate forecasting task, two deep learning architectures were implemented and evaluated: (1) a 2-layer Gated Recurrent Unit (GRU) model, and (2) a 4-layer Neural Basis Expansion Analysis for Time Series (N-BEATS) model. The GRU model consisted of two stacked layers, each comprising 200 hidden units, followed by a linear output layer





(b) Comorbidities in 6 clusters.



- (c) Comorbidities for female patients in 4 clusters.
- (d) Comorbidities for male patients in 4 clusters.

Figure 3: Prevalence of different comorbidities of the MIMIC patients in the clusters. There are 4 subgroups, clustering by comorbidity with k = 4, clustering by comorbidity with k = 6, clustering only the female patients with k = 4 and clustering only the male patients with k = 4.

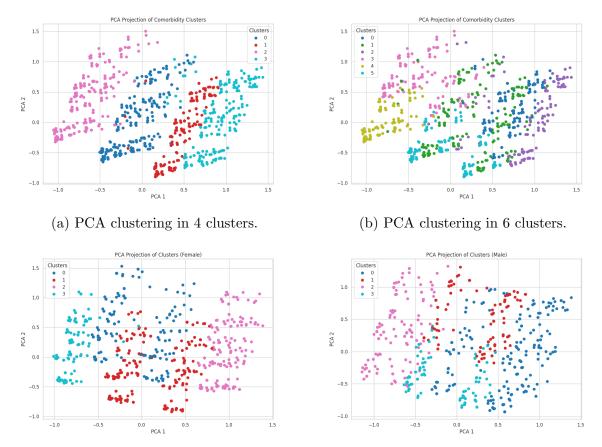
to produce the forecast. The N-BEATS model was configured with four fully connected layers, integrating both backcast and forecast blocks, and concluded with a linear output head to reconstruct future values. This design allows the model to capture both short-term patterns and longer-range temporal dependencies effectively.

To prevent overfitting, early stopping was employed in both architectures with a patience of 10 epochs, and training was conducted using a fixed learning rate of 0.0001. The models were trained on varying backward input horizons and forecast windows, allowing a thorough investigation of how different historical contexts influenced future prediction accuracy.

Model performance was evaluated using two standard error metrics: symmetric Mean Absolute Percentage Error (sMAPE) and Root Mean Squared Error (RMSE). These metrics were selected to balance sensitivity to outliers (RMSE) with scale-invariant accuracy (sMAPE), ensuring a robust evaluation of predictive performance. Lower values for both metrics indicated better forecasting quality.

Following comprehensive evaluation across multiple configurations, the model yielding the lowest average error across all validation sets was selected as the base model for subsequent counterfactual analysis. This ensured that counterfactual explanations were generated from the most reliable forecasting baseline.

[TODO: Extend on method]



(c) PCA clustering for female patients in 4 clusters. (d) PCA clustering for male patients in 4 clusters.

Figure 4: Principal component analysis (PCA) clustering of different comorbidities of the MIMIC patients. There are 4 subgroups, clustering by comorbidity with k = 4, clustering by comorbidity with k = 6, clustering only the female patients with k = 4 and clustering only the male patients with k = 4.

3.3.1.2 Counterfactual Generation:

For the counterfactual generation four different models have been used, all following a similar implementation. First two subsets of test samples were selected using two thresholds. This way, the samples were divided into a hyperglycemia set (blood glucose levels $\geq 180 \text{ mg/dL}$) and a hypoglycemia set (blood glucose levels $\leq 70 \text{ mg/dL}$) and then the following procedure was used:

- Bound Generation: Polynomial-based upper and lower bounds were generated for each test sample's predicted output, targeting desired glucose thresholds within a given window.
- Gradient Perturbation: For each forecasting model, the model was inverted by optimizing the exogenous inputs via gradient descent to steer the predicted output within the generated bounds.
- Loss Function: A custom loss function was used to penalize deviation from the

bounds, and gradients were approximated using automatic differentiation or finite differences depending on model compatibility.

The models used for the counterfactual generation span both traditional statistical methods and deep learning architectures. [TODO: Extend on method, add more about mimic]

SARIMAX: The Seasonal Autoregressive Integrated Moving Average with Exogenous Variables (SARIMAX) model was employed to account for seasonality and external factors. Model parameters, including the order of autoregression (p), differencing (d), and moving average (q), were selected based on the Akaike Information Criterion (AIC). Exogenous inputs such as insulin dosage and carbohydrate intake were incorporated as regressors, and model parameters were optimized using maximum likelihood estimation.

OLS: Ordinary Least Squares (OLS) regression was used to examine linear relationships between exogenous variables and the target series. Feature engineering involved the creation of time-lagged variables and interaction terms between insulin and carbohydrate intake. To mitigate overfitting, regularization was applied. Additionally, linear constraints on exogenous variables were enforced post-hoc to facilitate counterfactual generation.

GRU: A Gated Recurrent Unit (GRU) network was implemented. The GRU architecture consisted of two layers, each with 128 hidden units, and dropout regularization set at 0.2 to prevent overfitting. The network was trained using the Adam optimizer with an initial learning rate of 0.001, which was reduced by a factor of 0.1 if the validation loss plateaued over five epochs. Counterfactual explanations were generated by computing gradients of the output with respect to exogenous inputs, enabling identification of minimal interventions.

NBEATSx: The NBEATSx model was utilized, extending on the standard N-BEATS architecture by incorporating exogenous variables into its forecasting blocks. The model employed trend and seasonality decomposition through fully connected layers, and included exogenous inputs such as carbohydrate intake, insulin dosage, and physical activity. Quantile loss was used during training to capture distributional uncertainty, and feature importance analyses were performed to assess the contribution of exogenous variables to forecast accuracy.

3.3.2 Evaluation Metrics

[TODO: check, rewrite? unsure] To evaluate the quality of the counterfactual interventions, multiple evaluation metrics were implemented. First, traditional forecasting performance was measured using Root Mean Squared Error (RMSE) and Symmetric Mean Absolute Percentage Error (sMAPE), giving a quantitative assessment of prediction accuracy across the forecast horizon. For the evaluation of the generated counterfactuals, several additional metrics were introduced. These have been applied to ensure that the newly generated data for the exogenous variables is realistic, plausible and applicable.

RMSE, sMAPE scores sum of MSE scores, normalize for horizon?

Average value of change The average value of change quantifies the magnitude of adjustments made to the exogenous variables in order to achieve the desired forecasted outcome. For this the averaged original results of the multivariate forecast were compared to the averaged results of the counterfactual generation. euclidean distance?

The fraction of values to change captures the proportion of exogenous variable entries that required modification, offering insight into the sparsity of the interventions. By evaluating for how many of the timesteps in the time series the data changes, it is possible to analyse the differences that the counterfactuals

Severity of change To further characterize the nature of the interventions, the severity of change metric measures the relative intensity of the modifications compared to the original values. outlier score, LOF-score compare original exog to changed prediction

Fitting of predictions in bounds Additionally, the fitting of predictions within bounds was evaluated, reflecting the extent to which the counterfactual predictions adhered to the predefined upper and lower target constraints. This was done using the euclidean distance of the predicted target to the bounds. how good is the prediction, one in the band better than others?

Compare exogenous variables to those of a healthy patient Finally, to ensure the plausibility of the counterfactual scenarios, the modified exogenous variable profiles were compared to reference profiles from healthy patients. This comparison provides a qualitative and quantitative check on the biological or clinical realism of the generated counterfactual forecasts.

what are the differences in exog? changes from healthy patient

4 Results

4.1 Multivariate Forecasting

[TODO: explain chosen horizon, back horzion] To evaluate the performance of the multivariate forecasting and to ensure a realistic basis for the counterfactual generation, both the GRU and the N-BEATS model were tested across multiple datasets and forecasting configurations. The forecasting quality was assessed using two standard metrics: Symmetric Mean Absolute Percentage Error (sMAPE) and Root Mean Squared Error (RMSE). Models were trained on historical segments of the data and evaluated on subsequent future segments, allowing for a robust comparison between predicted and actual values. Table 3 summarizes the results across different datasets, prediction horizons, and back horizons. For the OhioT1DM dataset, N-BEATS consistently outperformed the GRU model in both sMAPE and RMSE across all configurations. Notably, with a back horizon of 12 and a forecast horizon of 6, N-BEATS achieved a sMAPE of 6.03 compared to GRU's 6.28, and a lower RMSE of 14.15 versus 14.85 for GRU. The same pattern was observed for back horizon 6 and 24, with N-BEATS slightly outperforming GRU in all cases. For the SimGlucose dataset, N-BEATS again generally showed improved forecasting performance. While GRU slightly outperformed N-BEATS in sMAPE for the 20-step back, 5-step horizon configuration (0.3792 vs. 0.3867), N-BEATS achieved a substantially lower RMSE (0.8271 vs. 1.6770), indicating more accurate absolute predictions. This performance gap widened in the 40-back, 10-horizon configuration, where N-BEATS clearly surpassed GRU on both metrics. [TODO: add mimic] Overall, these results demonstrate that the N-BEATS model generally offers superior performance across datasets and configurations, especially in terms of RMSE, making it a strong candidate for generating accurate multivariate forecasts and serving as a foundation for counterfactual analysis.

Table 3: Multivariate forecasting training metrics.

| Dataset | Back Horizon | Horizon | Model | sMAPE | RMSE |
|------------|--------------|---------|--------|--------|---------|
| OhioT1DM | 12 | 6 | GRU | 6.2816 | 14.8471 |
| | | | NBEATS | 6.0328 | 14.1500 |
| | 24 | 6 | GRU | 6.1983 | 14.6856 |
| | | | NBEATS | 5.8003 | 13.7677 |
| SimGlucose | 20 | 5 | GRU | 0.3792 | 1.6770 |
| | | | NBEATS | 0.3867 | 0.8271 |
| | 40 | 10 | GRU | 1.5423 | 5.5572 |
| | | | NBEATS | 1.4303 | 3.0461 |
| MIMIC | | | GRU | | |
| | | | NBEATS | | |

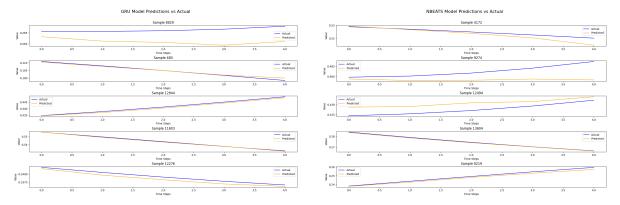
[TODO: rerun, recalculate results] To complement the quantitative evaluation of the multivariate forecasting models, several visualizations were created to illustrate how well the N-BEATS and GRU models track actual future values. These plots compare the predicted time series against the true values for randomly selected test samples from the datasets. Each subplot presents a single test sample, with time steps on the x-axis and the corresponding variable value on the y-axis. The actual values are shown in blue, while the predicted values are shown in orange. These plots offer a qualitative insight into the temporal alignment and amplitude accuracy of the forecasts, beyond what is captured by metrics like sMAPE or RMSE.

4.1.1 SimGlucose

For a back horizon of 20 timesteps and a horizon of 5 timesteps, both the GRU and N-BEATS models operate on a relatively short forecast horizon, making it easier to capture the trend with high fidelity. As shown in Figure 5, N-BEATS predictions closely align with the actual values across multiple samples, reflecting low forecasting error. Minor deviations appear mostly at the end of the forecast window. While the trend direction for the GRU predictions is usually correct, the model exhibits slightly larger prediction drift compared to N-BEATS, especially in sharper transitions. Still, GRU maintains reasonable short-term accuracy. For a back horizon of 40 timesteps and a horizon of 10 timesteps, the forecast horizon is increased, making the prediction task more difficult. Figure 6 shows the N-BEATS model's ability to handle long-range dependencies. Although small prediction lags and amplitude mismatches occur, the model captures the overall progression well, preserving directionality in most sequences. When looking at the GRU predictions under the same setup, the model struggles a bit more with extended forecasts, often showing divergence from actual trajectories, particularly toward the final time steps. These deviations are consistent with the slightly higher RMSE reported for this configuration.

4.1.2 OhioT1DM

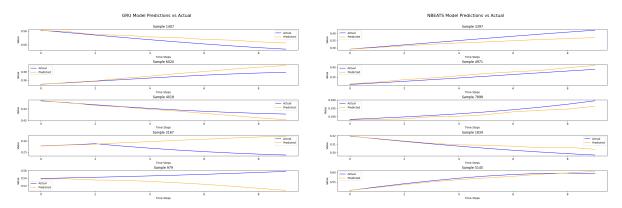
Figure 7 compare N-BEATS and GRU predictions for a back horizon of 12 timesteps and 6 future timesteps. The N-BEATS model tracks the general shape and scale of the actual values effectively. Although slight under- or over-predictions occur, especially in rising or falling trends, the model remains closely aligned overall. In contrast, the GRU predictions



(a) Results of the multivariate forecasting using (b) Results of the multivariate forecasting using GRU.

NBEATS.

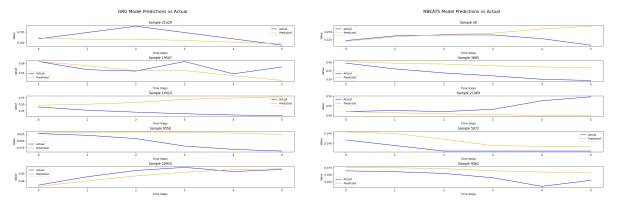
Figure 5: Results of the multivariate forecasting for the SimGlucose dataset with back horizon = 20 and forecast horizon = 5, showing the accuracy of the forecasting.



(a) Results of the multivariate forecasting using (b) Results of the multivariate forecasting using GRU. NBEATS.

Figure 6: Results of the multivariate forecasting for the SimGlucose dataset with back horizon = 40 and forecast horizon = 10, showing the accuracy of the forecasting.

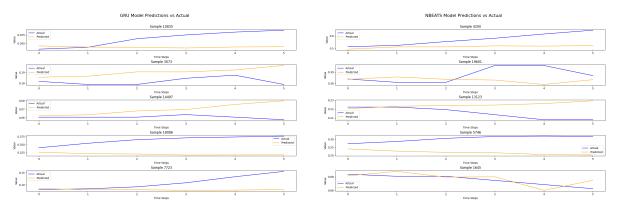
tend to diverge slightly more in the later steps of the forecast horizon. GRU often preserves the direction of the trend but struggles with amplitude and slope consistency, leading to larger deviations as the prediction window progresses. With a back horizon of 24 timesteps and the same future timesteps, figure 8 depict the performance of N-BEATS and GRU under a longer input sequence. The figure illustrates that N-BEATS maintains robust trend prediction, with only modest lag or dampening effects in some samples. The model adapts well to both upward and downward trajectories. The GRU model performs more variably under this longer back horizon. While the general trajectory is still often captured, the forecasts occasionally overreact or smooth out variations, resulting in lower precision at the end of the forecast horizon.



(a) Results of the multivariate forecasting using (b) Results of the multivariate forecasting using GRU.

NBEATS.

Figure 7: Results of the multivariate forecasting for the OhioT1DM dataset with back horizon = 12 and forecast horizon = 6, showing the accuracy of the forecasting.



(a) Results of the multivariate forecasting using (b) Results of the multivariate forecasting using GRU. NBEATS.

Figure 8: Results of the multivariate forecasting for the OhioT1DM dataset with back horizon = 24 and forecast horizon = 6, showing the accuracy of the forecasting.

4.1.3 MIMIC

4.2 Counterfactuals

4.2.1 OhioT1DM

Figure 9 presents an example of generated counterfactuals for the OhioT1DM dataset. It shows a short sequence of 6 timesteps where the predicted blood glucose (BG) level is approximately 155 mg/dL—lower than the original range of 180–190 mg/dL. This new BG level falls within the predefined desired bounds, indicating a plausible improvement. Notably, the generated exogenous variables—such as insulin doses, carbohydrate intake, and exercise intensity—differ considerably from the original values. These differences suggest actionable changes that may help achieve improved glycemic control.

To evaluate the realism of the generated exogenous variables, we searched for pairs of samples in the dataset where both the original and counterfactual blood glucose levels were similar. By comparing their exogenous features, we can assess whether the gener-

ated values are plausible within the real data distribution, ensuring that counterfactual recommendations are actionable and clinically meaningful.

For this analysis, we identified two samples of 6 timesteps each, with a minimal Euclidean distance of 2.0295 between their blood glucose target sequences. Their exogenous variables are compared in Figure 10. The observed differences highlight the variability in exogenous features that can still lead to similar blood glucose outcomes.

Additionally, Table 4 summarizes the Euclidean distances between exogenous features for the five closest pairs of samples, using both actual and normalized values. This quantitative measure helps to contextualize the variability seen in the visual comparison and supports the feasibility of the generated counterfactuals within real-world data ranges.

4.2.2 SimGlucose

4.2.3 MIMIC

4.3 Measurements

4.3.1 Mean Absolute Change

Figure 12 presents the mean absolute change between the original and generated counterfactual time series for all four features from the OhioT1DM dataset. This metric quantifies the average magnitude of alteration introduced by the counterfactual generation process. A relatively stable decline in change over timesteps can be observed across features, indicating temporal consistency in the perturbation patterns.

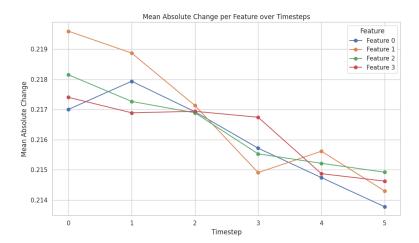


Figure 12: Mean absolute change per feature over timesteps, comparing original and counterfactual sequences.

4.3.2 Severity of Change

To better understand the extent and nature of modifications across features, Figure 13 visualizes two key indicators. The top panel shows the fraction of changed values per feature over time, revealing that modifications are sparse and localized. The bottom panel reiterates the mean absolute change, highlighting that although the fraction of altered values is low, the individual changes can still be substantial. Together, these plots provide a detailed view of how features were selectively modified.

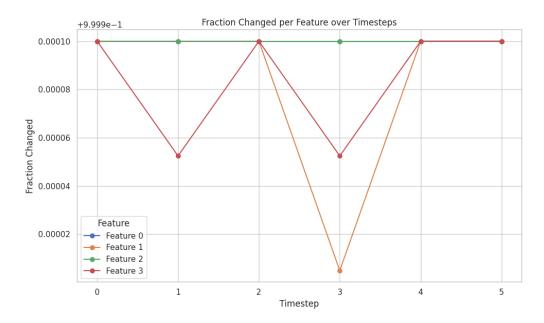


Figure 13: Top: Fraction of features changed over timesteps. Bottom: Mean absolute change per feature.

4.3.3 Visualization of Feature-wise Changes

Figure 14 shows the normalized time series for each feature, comparing the original inputs with the generated counterfactuals. The dotted line represents the absolute difference between the two series at each timestep. These visualizations help pinpoint specific regions where interventions have occurred and how significantly each feature has been affected. This is particularly useful for interpreting which variables are being targeted by the counterfactual model.

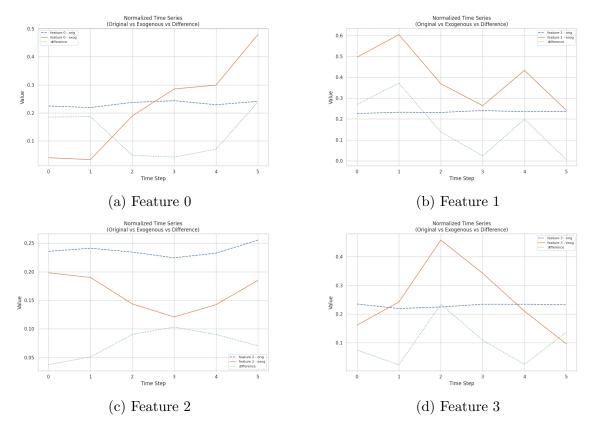


Figure 14: Comparison of original and counterfactual time series for each feature, showing normalized values and their absolute differences.

4.3.4 Coefficient of Determination

The coefficient of determination (R^2 score) is used to evaluate how well the counterfactual time series preserves the original temporal dynamics. A high R^2 score indicates that the generated data maintains the structure of the original inputs while introducing targeted modifications. This metric supports assessing the realism and fidelity of counterfactual sequences.

4.3.5 Target Prediction within Bounds

In addition to matching the temporal patterns, an important constraint for counterfactual validity is that the prediction for the target variable remains within clinically acceptable bounds. This ensures that generated scenarios do not produce unrealistic or medically implausible outcomes. Here, we verify that counterfactual predictions remain within the same classification range (e.g., hyperglycemia vs. normoglycemia) as the original data or follow a desired clinical transition.

4.3.6 Comparison to Healthy Patient

Finally, we compare the generated counterfactuals with healthy patient profiles to evaluate whether the model is steering inputs toward realistic, health-aligned trajectories. This analysis allows us to assess whether counterfactual edits reflect desirable or therapeutically meaningful interventions, rather than arbitrary perturbations that may reduce prediction error but lack interpretability.

5 Discussion

5.1 Results

5.1.1 Multivariate Forecasting

[TODO: überarbeiten, controllieren] The multivariate forecasting experiments highlight key insights into the performance dynamics of sequence models across different datasets and configurations. Overall, the N-BEATS model demonstrated stronger forecasting capabilities compared to the GRU, as reflected in both the quantitative metrics (sMAPE and RMSE) and qualitative visualizations.

Across datasets such as OhioT1DM and SimGlucose, the N-BEATS model consistently outperformed GRU in RMSE, indicating improved absolute accuracy in forecasting. Even in configurations where GRU achieved slightly lower sMAPE scores (e.g., SimGlucose 20-back, 5-horizon), the N-BEATS model's RMSE was substantially better, pointing to more reliable magnitude predictions even when percentage error might appear marginally higher. This is particularly relevant in domains like blood glucose forecasting, where the scale of the error directly impacts clinical decisions. The GRU model, on the other hand, often captured general trends and directions effectively but struggled with capturing exact amplitudes, particularly in extended horizon forecasts. Its recurrent structure may limit its ability to retain long-range dependencies, especially when faced with sparse or irregular temporal dynamics.

The impact of forecast horizon length is clearly observable: as the prediction window increases (e.g., from 5 to 10 steps), both models encounter greater difficulty maintaining accuracy, but N-BEATS shows better resilience to degradation. The model's architectural design, which decomposes the forecasting problem into backcast and forecast components, seems better suited for extrapolating patterns over longer horizons. The choice of back horizon, so how far back in time the model looks, also influences performance. Increasing the back horizon generally benefits N-BEATS more than GRU, likely due to its ability to effectively process longer sequences through its feedforward blocks. For example, in the OhioT1DM dataset, moving from a back horizon of 12 to 24 improved both sMAPE and RMSE in N-BEATS, whereas GRU showed more modest gains.

Dataset complexity plays a role in forecasting difficulty. The SimGlucose dataset, which is synthetic and well-structured, yields much lower errors compared to the real-world OhioT1DM data. This highlights the challenge of modeling patient-specific behaviors and noisy real-world measurements. Nevertheless, the relative performance gains of N-BEATS persist across both simulated and real datasets, reinforcing the robustness of its design. Visualizations confirmed the metric-based assessments: N-BEATS forecasts track ground truth values more closely across time, particularly in maintaining amplitude and responding to abrupt trend changes. GRU's forecasts tend to exhibit smoother transitions and sometimes dampened responses to sharp changes. These patterns match the sMAPE and RMSE trends, suggesting the chosen metrics are representative of the qualitative forecasting quality. Given that accurate forecasting is foundational to generating meaningful counterfactuals, these results justify the selection of the N-BEATS model as the primary forecasting component in the counterfactual pipeline. Its superior RMSE performance ensures that forward projections are grounded in realistic, data-consistent trajectories, which is crucial when optimizing exogenous variables to achieve desired outcomes. Additionally, its architectural stability over longer input sequences allows for greater flexibility in back horizon design.

- 5.2 Limitations
- 5.3 Future Work
- 6 Conclusion

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| Feature types | Specifics | Occurrences |
|-----------------------------------|-------------------------|-------------|
| Targets | | _ |
| | Death within 30 days | 136 |
| | Death within 1 year | 150 |
| Vital signs and laboratory values | | |
| | BMI | 1845 |
| | Heart Rate | 1845 |
| | SpO2 | 1845 |
| | Diastolic BP | 1837 |
| | Systolic BP | 1837 |
| | Temperature | 1829 |
| | Creatinine | 1825 |
| | Sodium | 1825 |
| | Bicarbonate | 1823 |
| | Hemoglobin | 1814 |
| | WBC Count | 1814 |
| | Platelet Count | 1812 |
| | Troponin | 958 |
| | NT-proBNP | 76 |
| Comorbidities | | |
| | Diabetes | 798 |
| | RD | 768 |
| | Coronary Artery Disease | 761 |
| | COPD | 658 |
| | Hypertension | 599 |
| | PVD | 577 |
| | Atrial Fibrillation | 424 |
| | AMI | 230 |
| | CEVD | 218 |
| | Prior Admission | 188 |
| | Pulmonary Hypertension | 170 |
| | Diabetes + Complica- | 156 |
| | tions | |
| | Dementia | 75 |
| | Cancer | 65 |
| | Metastatic Cancer | 50 |
| | Rheumatoid Disease | 50 |
| | Mild LD | 46 |
| | PUD | 40 |
| | Moderate/Severe LD | 30 |
| | HP/PAPL | 27 |
| Gender | | |
| | Male | 773 |
| | Female | 1072 |

Table 2: All features of the MIMIC data divided by feature types: Target, Vital signs and laboratory values, Comorbidities, and Gender.

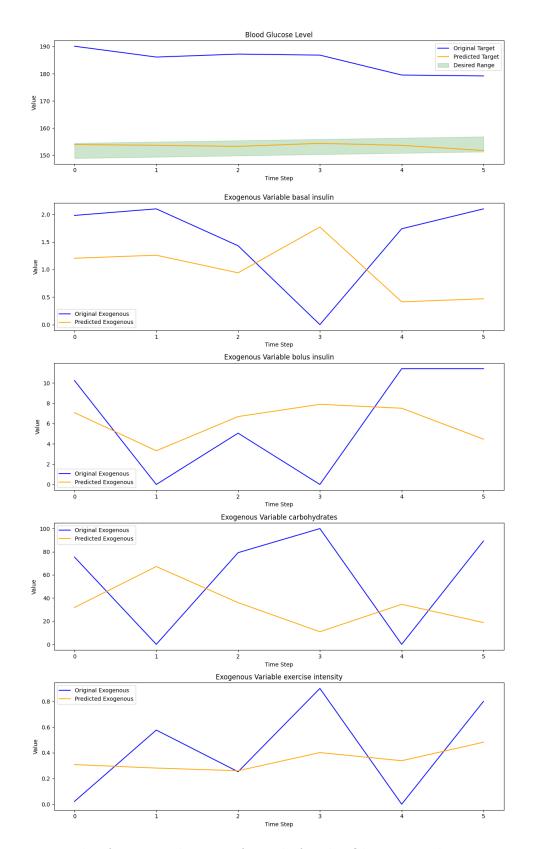


Figure 9: Example of generated counterfactuals for the OhioT1DM dataset. Top: Blood glucose levels for the original and counterfactual samples. Bottom: Comparison of exogenous variables (original in blue, counterfactual in yellow, bounds in green).

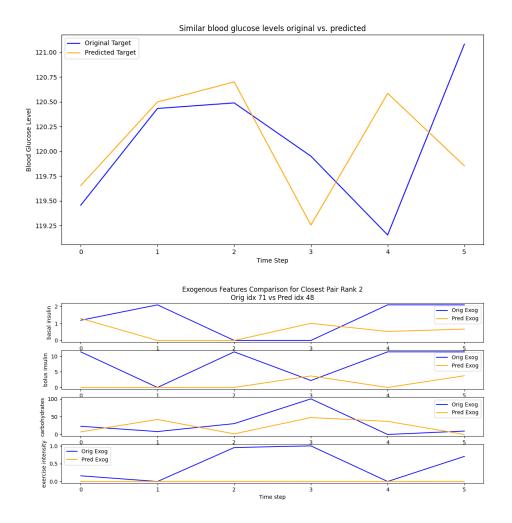


Figure 10: Comparison of two samples from the OhioT1DM dataset with similar blood glucose levels. Top: Blood glucose targets. Bottom: Exogenous variables for both samples (original in blue, comparison sample in yellow).

| E | Pair 71, 48 | | Pair 27, 82 | | Pair 27, 11 | | Pair 68, 31 | | Pair 74, 75 | |
|--------------------|-------------|--------|-------------|--------|-------------|--------|-------------|--------|-------------|--------|
| Feature | Act | Norm |
| Basal Insulin | 2.1575 | 0.5678 | 3.1525 | 0.6280 | 2.0002 | 0.2891 | 2.7798 | 0.3388 | 2.9895 | 0.5143 |
| Bolus Insulin | 19.4057 | 0.5012 | 21.2238 | 0.8798 | 19.8393 | 0.4116 | 10.8953 | 0.2978 | 15.0757 | 0.3838 |
| Carbohydrates | 100.5660 | 0.4907 | 80.1914 | 0.5888 | 125.0327 | 0.4353 | 31.7565 | 0.4943 | 143.2101 | 0.5476 |
| Exercise Intensity | 1.1705 | 0.5956 | 1.5558 | 0.9469 | 1.1187 | 0.5106 | 0.9448 | 0.2775 | 0.8671 | 0.7661 |

Table 4: Euclidean distances between exogenous features for the five closest target pairs in the OhioT1DM dataset, showing both actual and normalized values.

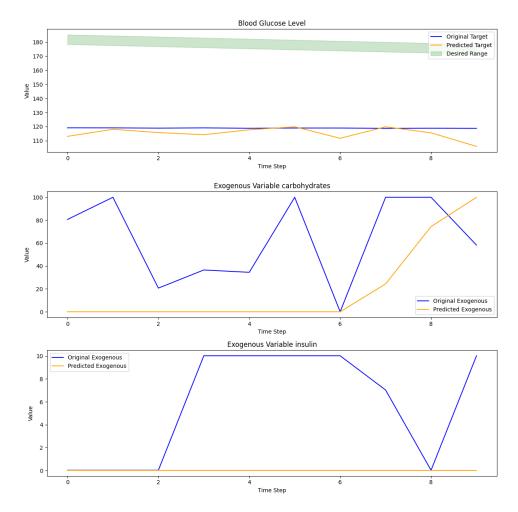


Figure 11: Example of the counterfactuals generated for the SimGlucose dataset, with the blood glucose level within the desired bounds. The different exogenous variables, with the original values in blue and the predicted in yellow.