Patrolling Games

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Introduction to Patrolling Games

A Patrolling game, G is made of 3 major components

- A Graph, $Q = (N, E)$, made of nodes, N, and a set of edges, E.
- An attack time, m.
- A time horizon, T.

The game involves two players, the attacker and the patroller.

- The patroller's strategy is a walk on the graph, $W : \mathcal{T} \to N$.
- The attacker's strategy is a node, i and starting time, τ.

Example of Theorem Applied to joined extended star graphs

As the values of the star graphs joined together are

- $V(S_3^2) = \frac{m}{10}$ when $m \geq 6$
- $V(S_4) = \frac{m}{8}$ when $m \geq 2$
- $V(S_3^{(2,1)}) = \frac{m}{12}$ when $m \geq 6$

To join these together by the centres we will require that $6 \leq m \leq 8$ (hence none of the individual extended star graphs values are invalid), then we will get a value of $V = \frac{m}{30}$. This is achieved by the attacker attacking as they would on individual graphs and the patroller playing on these 3 graphs with the probabilities $\frac{10}{30}, \frac{8}{30}, \frac{12}{30}$ respectively.

Example of a game

Example of problem

Consider Q, made by joining the centres of two copies S_2. Then we know by the theorem that $V(Q) = \frac{m}{8}$ when $2 \leq m \leq 4$, but when $m = 5$ say then the problem is that we know that playing in either of them with probability $\frac{1}{2}$ is no longer the best decision. It might be best to count from the starting star's centre