Joining Extended Star Graphs

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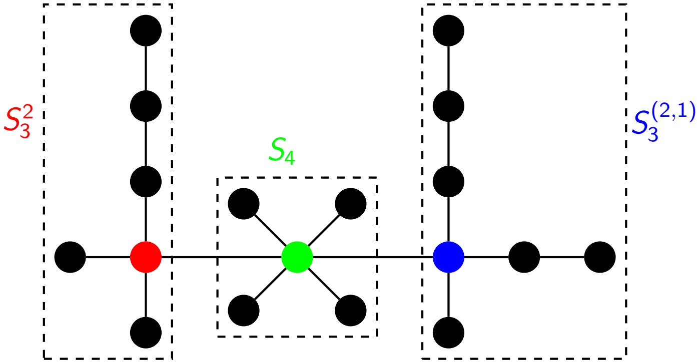
Example of Theorem Applied to joined extended star graphs

3 extended star graphs joined linked by the centers.



Example of Theorem Applied to joined extended star graphs

As the values of the star graphs joined together are

$$V(S_3^2) = \frac{m}{10} \text{ when } m \geq 6$$
$$V(S_4) = \frac{m}{8} \text{ when } m \geq 2$$
$$V(S_3^{(2,1)}) = \frac{m}{12} \text{ when } m \geq 6$$

To join these together by the centres we will require that $6 \leq m \leq 8$ (hence none of the individual extended star graphs values are invalid), then we will get a value of $V = \frac{m}{30}$. This is achieved by the attacker attacking as they would on individual graphs and the patroller playing on these 3 graphs with the probabilities $\frac{10}{30}, \frac{8}{30}, \frac{12}{30}$ respectively.

Problem with joining

The problem with this type of joining is the fact of requiring

$$m \leq 2\left(n_i + \sum_j (V_j)_i\right).$$

This is required as otherwise, time is wasted by the patroller when they can be certain that the attacker is not there. Consider the fact that they are m away from the end and they then check all points, then they can move away from the graph and possibly catch some other attacks on other joined extended star graphs.

Example of problem

Consider Q, made by joining the centres of two copies S_2. Then we know by the theorem that $V(Q) = \frac{m}{8}$ when $2 \leq m \leq 4$, but when $m = 5$ say then the problem is that we know that playing in either of them with probability $\frac{1}{2}$ is no longer the best decision. It might be best to count from the starting star's centre