Single Node general distribution theory

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1 Introduction to Problem

We have some attack time distribution at a single node, X, with support, $\Pi = [0, \pi]$, and $B = \lceil \pi \rceil$.We also have the room cap size, b, we also have a cost c and arrival rate, λ.

This gives us the state space, (s, v) for $0 \leq s \leq B + 1$ and $0 \leq v \leq b$.

We will consider the Cost to Progress matrix, $\mathbf{C} = \mathbf{A} + \mathbf{O}$, where $\mathbf{A}$ is the cost to not act due to arrival process and $\mathbf{O}$ is the cost to not act due to the observed process.

Some Basic Properties Because the arrival process is not dependent on the observed number, $A_{i,j} = A_{i,1}$, that is $\mathbf{A}$ can be represented by a vector $\mathbf{a} = a_i = A_{i,1}$.

If $b = 0$,we have $\mathbf{O} = \mathbf{0}$, then $\mathbf{C} = \mathbf{A}$.

Because the observed process is dependent on s and v just multiples the answer by v. We can summarize the matrix $\mathbf{O}$ by $\mathbf{o} = O_{i,2}$. We can also note this is only possible if $b \geq 1$.

Then we can redefine, $C_{i,j} = a_i + j o_i$.

By our current definition $a_i = c\lambda \int_{i-1}^{i} F_X(x)dx$ and $o_i = c(F_X(i) - F_X(i-1))$.

So a_i is increasing.

If we want to decide when $C_{i,j}$ is increasing (non-decreasing) in i, j, by definition it is non-decreasing in j. so we will only look at it from the point of non-decreasing in i.

If o_i is non-decreasing then clearly it is non-decreasing.

However if o_i is decreasing then we must look closer. If once o_i becomes decreasing it has all $c_{i,j} \geq c\lambda$, then we can avoid the problem.

We could also restrict the problem to some $b = b^*$. For each decreasing row, we can restrict the column to only allow $c_{i,j} \geq c\lambda$.

We can define $b_i^+ = \left\lfloor \frac{a_i - c\lambda}{o_i} \right\rfloor$ and a set $\chi = \{i \,|\, o_i < 0\}$ and define our limit by $b^* = \min_{i \in \chi}\{b_i^+\}$

Let us consider the difference in $c_{i,j}$'s , $c_{i,j} - c_{i-1,j} = a_i - a_i + j(o_i - o_{i-1})$

We can define $c_{i,j} = c\int_{i-1}^{i} \lambda F_X(x) + j f_x(x)dx$. We can say this is non-decreasing if $\lambda F_X(x) + j f_x(x)$ is non-decreasing, i.e $\lambda f_X(x) + j f_X^{'}(x) \geq 0$.

so trying to solve $\frac{\lambda}{j} f(x) \geq -f^{'}(x)$. In regions where the function is non-decreasing, this is most definitely true as $f^{'}(x) \geq 0$. In a region of x, say

$R = [x_1, x_2]$ where it is decreasing then we need to solve the inequality, $\frac{\lambda}{j} f(x) \geq |f^{'}(x)|$.

Using Gronwell's inequality theorem we can get that the answer must be $f(x_3)e^{-\frac{\lambda}{j}|x-x_3|} \leq f(x) \leq f(x_3)e^{\frac{\lambda}{j}|x-x_3|}$ where x_3 is some point in the region R. As $f(x)$ is decreasing in these regions $f()$ So in these regions we must check this condition

As we are dealing with a function of $f_X(x)$ which is not non-decreasing.

Appendices

A First appendix section