Single Node general distribution theory

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October 19, 2018

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1   Introduction to Problem

We have some attack time distribution at a single node, X, with support, $\Pi = [0, \pi]$, and $B = \lceil \pi \rceil$. We also have the room cap size, b, we also have a cost c and arrival rate, λ.

This gives us the state space, (s, v) for $0 \leq s \leq B + 1$ and $0 \leq v \leq b$.

We will consider the Cost to Progress matrix, $\mathbf{C} = \mathbf{A} + \mathbf{O}$, where $\mathbf{A}$ is the cost to not act due to arrival process and $\mathbf{O}$ is the cost to not act due to the observed process.

Some Basic Properties Because the arrival process is not dependent on the observed number, $A_{i,j} = A_{i,1}$, that is $\mathbf{A}$ can be represented by a vector $\mathbf{a} = a_i = A_{i,1}$.

If $b = 0$, we have $\mathbf{O} = \mathbf{0}$, then $\mathbf{C} = \mathbf{A}$.

Because the observed process is dependent on s and v just multiples the answer by v. We can summarize the matrix $\mathbf{O}$ by $\mathbf{o} = O_{i,2}$. We can also note this is only possible if $b \geq 1$.

Then we can redefine, $C_{i,j} = a_i + j o_i$.

By our current definition $a_i = c\lambda \int_{i-1}^{i} F_X(x)dx$ and $o_i = c(F_X(i) - F_X(i-1))$.

So a_i is increasing.

If we want to decide when $C_{i,j}$ is increasing (non-decreasing) in i, j, by definition it is non-decreasing in j. so we will only look at it from the point of non-decreasing in i.

If o_i is non-decreasing then clearly it is non-decreasing.

However if o_i is decreasing then we must look closer. If once o_i becomes decreasing it has all $c_{i,j} \geq c\lambda$, then we can avoid the problem.

We could also restrict the problem to some $b = b^*$. For each decreasing row, we can restrict the column to only allow $c_{i,j} \geq c\lambda$.

We can define $b_i^+ = \left\lfloor \frac{a_i - c\lambda}{o_i} \right\rfloor$ and a set $\chi = \{i \mid o_i < 0\}$ and define our limit by $b^* = \min_{i \in \chi}\{b_i^+\}$

Let us consider the difference in $c_{i,j}$'s , $c_{i,j} - c_{i-1,j} = a_i - a_i + j(o_i - o_{i-1})$

We can define $c_{i,j} = c\int_{i-1}^{i} \lambda F_X(x) + j f_x(x)dx$. We can say this is non-decreasing if $\lambda F_X(x) + j f_x(x)$ is non-decreasing, i.e $\lambda f_X(x) + j f_X'(x) \geq 0$.

We can equivalently solve $\frac{\lambda}{j} f_X(x) \geq |f_X'(x)|$ for a function which is not strictly increasing. This is done by Gronwell's inequality theorem to get use $f(x_0)e^{-\frac{\lambda}{j}|x-x_0|} \leq$

$f(x) \leq f(x_0)e^{\frac{\lambda}{j}|x-x_0|}$ for some x_0 in the support of f_X. This is equivalent to say that the function is not increasing/decreasing exponentially.

If we only consider decreasing regions of f_X can we just check those, well lets say that in the regions $[a_i, b_i]$ for $i \in 1, ..., n$ for some n. The function is decreasing then we could apply the same idea of Gronwell's inequality and get $f(p)e^{-\frac{\lambda}{j}|x-p|} \leq f(x) \leq f(p)e^{\frac{\lambda}{j}|x-p|}$ for some chosen $p \in [a_i, b_i]$ (all choices are valid, easy choice is $p = a_i$) for all $i \in n$.

Then this allows us to say that in the decreasing regions of f_X the equation still holds, and in the other regions it is increasing so the equation holds. Hence it holds.

So we really only need the decreasing regions to be monitored for exponential decay, if they decrease too fast we can be in trouble.

Appendices

A   First appendix section