One-pass (relations not sorted)

Operator	# buffer frames available	Page accesses (Disk I/O)
selection, projection	M = 2	B
duplicate elimination, grouping	M = B + 1	B
Bag union	M = 1	B(R) + B(S)
union, intersection, difference, cartesian product, join	M = min {B(R), B(S)} + 2	B(R) + B(S)

One-pass (relations sorted on search key)

Operator	# buffer frames available	Page accesses (Disk I/O)
selection	M = 2	$\log_2 B$
duplicate elimination	M = 2 (if all attributes form the search key), B+1 (worst case)	B
grouping	M = 2	B
union, intersection, difference	M = 3	B(R) + B(S)
cartesian product, join	M = min {B(R), B(S)} + 2	B(R) + B(S)

Nested Loop

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination	M = 3	B × (B + 1) / 2
union, intersection, difference, cartesian product, join	M = 3	B(S) + B(R) × B(S)

Block Nested Loop

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination	M = 3 (With exactly three available it's equal to the nested loop)	B * (3/2 + B/M) + M
union, intersection, difference, cartesian product, join	M = 3	B(S) + B(R) x (1 + B(S)/M)

Two pass based on sorting

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination, grouping	Ceil (B(R) / M) ≤ M - 1	3 B(R)
union, intersection, difference	Ceil(B(R)/M) + Ceil (B(S)/M) ≤ M-1	3 (B(R) + B(S))
Simple-sort join	B(R) ≤ M×(M-1), B(S) ≤ M×(M-1) assuming no value y of joining value Y appearing in both R and S, the pages with the tuples of R and S with value y in Y occupy more than M-1 buffer frames	5 (B(R) + (B(S))
Merge-sort join	ceil(B(R)/M) + ceil(B(S)/M) ≤ M-1 assuming no value y of joining value Y appearing in both R and S, the pages with the tuples of R and S with value y in Y occupy more than M - (ceil(B(R)/M) + ceil(B(S)/M)) buffer frames	3 (B(R) + B(S))

Two pass based on hashing

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination	$B(R) \leq M(M-1)$	3 B(R)
Grouping and aggregation	$B(R) \leq M(M-1)A$ where A is the average of tuples per bucket	3 B(R)
union, intersection, difference	$\min(B(R),B(S)) \leq (M-1)(M-2)$	3 (B(R) + B(S))
Join	$\min(B(R),B(S)) \leq (M-1)(M-2)$	3 (B(R) + B(S))

Multipass sort-based (K is the number of passes)

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination, grouping	$B(R) \leq (M-1)^{k-1} \times M$	(2K-1) B(R)
union, intersection, difference, join	$B(R) + B(S) \leq (M-1)^{k-1} \times M$	(2K-1) (B(R) + B(S))

Multipass hash-based (K is the number of passes)

Operator	# buffer frames available	Page accesses (Disk I/O)
duplicate elimination, grouping	$B(R) \leq (M-1)^{k-1} \times M$	(2K-1) B(R)
union, intersection, difference, join	$\min(B(R) + B(S)) \leq (M-1)^{k}$	(2K-1) (B(R) + B(S))