Assignment 3.

1. <u>Exercise:</u> Asset Swap

Given the discounting curve vs Euribor 3m on the 31st of January 2023 at 10:45 C.E.T. and knowing that the bond with maturity 31st of March 2028 clean price for an issuer YY is 101 (101% of the *face value*) with an annual coupon equal to 4.8% (*annual bond*) with coupons paid on the same swap dates, compute the *Asset Swap Spread Over Euribor3m*. The bond has been issued on the 31st of March 2022.

2. Case Study: CDS Bootstrap

Given values for discounts on the case-study on curve bootstrap on the 31st of January 2023 at 10:45 C.E.T. consider the obligor ENI with a recovery π equal to 40% and CDS spreads (annual bond): 1y 40 bps, 2y 44 bps, 3y 47 bps, 4y 49 bps, 5y 51 bps, 6y 52 bps.

a. Build $\lambda(t)$ piecewise constant for the issuer, neglecting the "accrual" term.
b. Which is the impact of the "accrual" term? Show that this term is really negligible.
c. Consider Jarrow-Turnbull approximation (a constant λ and continuously paid CDS spread) and compare the result with the one previously obtained.

3. Exercise: Credit Simulation

Given *survival probability* at time t up to T (30 years): $P(t,T) = e^{-\int_t^T \lambda_s\, ds}$;
with $\lambda_s = \lambda_1\, 1_{s\le\theta} + \lambda_2\, 1_{s>\theta}$ and $\lambda_1 = 5$ bps, $\lambda_2 = 9$ bps, $\theta = 4\ years$.

a. Simulate the *default time* τ.
b. Fit the survival probability.

Fit the distribution of default times using a sample of M=10^5 points. Provide Estimator and Confidence Interval (CI) on the Estimator (only for λ_1 and λ_2, not for θ).
Hint: A possibility (not binding) is to consider a plot of the "experimental" survival probability and the one obtained from the fit (in a loglinear scale) with a CI. Another one is to consider the ML estimator and its CI.

4. Case Study: MBS Pricing

On the 31st of January 2023, the reference portfolio of the SPV *Cayman II* has a total notional of € 1 bn: this portfolio can be considered homogeneous. Assume for simplicity that mortgages provide a single payment at the end of the interest period T equal to three years and defaults are independent from interest rates; for the period T the default probability of each mortgage is p=5%, correlation is 40% and an average recovery of 20% for each mortgage.

a. For the mezzanine Tranche (with subordinations -detachment points- K_d 5% and K_u 9%), estimate the impact of the hypothesis of Large Homogeneous Portfolio for I = 400 for the Vasicek model. Show that the price of the Tranche in relative terms (as a percentage of tranche face value) is well described by the Kullback-Leibler (KL) approximation in the range I = (10, 2 10^4), where the LHP approximation holds. It is required to plot the price varying I (in log scale in the abscissa), with

i) the exact solution (up to an I that your computer allows to obtain a price);
ii) the approximate solution;
iii) the LHP solution.

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[Hint: Use a grid for I similar to the one in the document and plot prices in % in log-lin scale]

b. [facultative] Price the Equity tranche with detachment points K_u 5% varying I in the same range considered at the previous point. Is the KL approximation adequate? How can you modify it?

Name of some Matlab functions:

nchoosek(I,n):	Newton Binomial coefficient
norminv(alpha):	Inverse normal CDF
normpdf(x):	pdf normal
quadgk:	for numerical integration

Function signatures
[datesCDS, survProbs, intensities] =
bootstrapCDS(datesDF, discounts, datesCDS, spreadsCDS, flag, recovery).

<u>dates and discounts are the same outputs of bootstrap function; datesCDS include the settlement date; function outputs are vectors with the same length; flag = 1 (approx), 2 (exact) or 3 (JT).</u>

<u>All vectors are column vectors.</u>

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