POLITECNICO
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Financial Engineering 2024/25

Report Assignment 3 RM

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Contents

Case study 1: 2

Case Study 2: 4

Case study 3: 5

Appendix: Code Corrections 6

Case study 1:

Gaussian parametric approach

To compute the VaR and the ES with a gaussian parametric approach we first computed the mean and the covariance matrix for the logarithmic returns of the companies. Then we computed the VaR and the expected shortfall.

Metodo	VaR (€)	ES (€)
Gaussian	415,319.48	520,494.47

Table 1: Results for Gaussian VaR and ES

Historical simulation

To compute the VaR and the ES with an historical simulation we sorted the losses and then selected as our VaR the one corresponding to our significance level; the ES was computed as the mean of the losses bigger than the VaR. We then did the same thing but with a statystical bootstrap, so sampling 300 random dates and then calculating our risk measures only on those dates; obviously the results with this method will be different considering various samples.

Metodo	VaR (€)	ES (€)
HS	315,086.53	466,044.00
Bootstrap	293,159.31	402,208.21

Table 2: Results HS and Bootstrap VaR and ES

Weighted historical simulation

To compute the VaR and the ES with a weighted historical simulation we assign exponentially decreasing weights to the losses, giving more importance to the more recent ones. Then we sort the losses and create a new vector of weights associating each weight with the corresponding loss.

Metodo	Valore (€)
WHS VaR	384,278.00
WHS ES	598,372.87

Table 3: Results for WHS VaR and ES

PCA

In this case we were considering a larger portfolio, made of 20 companies, and used PCA to reduce the dimensionality of the data, we varied the number of components taken into account and then computed VaR and ES as in point 1, obtaining the following results:

PCA(n)	VaR (€)	ES (€)
1	1,003,904.47	1,271,238.49
2	1,000,535.11	1,268,977.52
3	998,774.78	1,267,282.64
4	1,005,582.05	1,274,183.22
5	1,001,025.48	1,270,191.89
6	1,001,440.91	1,270,611.33
7	1,001,320.45	1,270,490.99
8	1,004,411.01	1,273,683.51
9	1,005,179.30	1,274,499.73
10	1,004,668.21	1,274,032.43
11	1,003,785.30	1,273,254.77
12	1,004,096.41	1,273,566.90
13	1,003,861.94	1,273,340.25
14	1,003,963.83	1,273,442.28
15	1,003,832.53	1,273,311.12
16	1,003,825.35	1,273,311.40
17	1,004,382.58	1,273,891.47
18	1,005,831.56	1,275,368.40
19	1,005,463.55	1,275,002.27
20	1,005,442.49	1,274,981.24

Table 4: VaR and ES for different values of PCA(n)

From Table 4 we can notice that varying the number of components considered does not have a big impact on the results, that are almost constant.

Plausibility check

We eventually performed a plausibility check on our VaR results from points 2,3 and 4 to check if they were reasonable. we can see that for points 2 and 4 the results we obtained are close to the ones obtained with the plausibility check, while for point 3 they differ significantly. This could be due to the fact that the plausibility check does not take into account the different weights of the WHS; to check it we considered a value of lambda closer and closer to 1, making the WHS a normal historical simulations and we saw that the results were much closer to the plausibility check.

Metodo	VaR (€)
Estimated VaR2	301,008.18
Estimated VaR3	459,431.92
Estimated VaR4	997,234.13

Table 5: Estimated VaR values

Case Study 2:

In order to evaluate the market risk of a portfolio composed, as of January 31, 2023, of Anheuser-Busch InBev shares for a notional value of €5,509,000 and an equal number of put options on the same stock (with April 5, 2023 expiry, a strike of €53, and an implied volatility of 18.5%), we compute the 10-day Value at Risk (VaR) at a 99% confidence level. The historical window for return estimation spans two years, and the underlying pays a 3% dividend. The option pricing is carried out via a Black-Scholes model whose risk-free rate is obtained from market data through a bootstrap procedure.

Historical Simulation via Monte Carlo

To quantify the potential loss, we used three different approaches. The first relies on a Monte Carlo simulation that uses the historical (logarithmic) returns of Anheuser-Busch shares. In each iteration, we draw (with replacement) ten daily returns, sum them, and obtain the 10-day simulated stock price. Based on this simulated price, we value the put option using a Black-Scholes model, adjusting the residual time to maturity and the risk-free rate. By summing the changes in stock and option values, we build a loss distribution from which we derive the 99% VaR. This methodology produces a VaR of approximately **€269,106.65**.

Historical Simulation via Delta Normal Approach

The Delta Normal Approach simplifies the analysis by focusing solely on the option's Delta, meaning that option price movements are assumed to be linear with respect to the stock price. The overall portfolio loss in each historical simulation is then computed by adding up the changes in stock and option values (taking Delta into account). After ranking the losses, the 99% VaR is found to be around **€450,190.75**, which is notably higher than the Monte Carlo result.

Historical Simulation via Delta-Gamma Approach

The third methodology, the Delta-Gamma Approach, adds an additional term to the linear approximation in order to account for the curvature (Gamma) of the option. This technique is more accurate when the underlying price changes significantly, since the option's sensitivity does not remain constant but varies with price and time. In practice, the portfolio loss is adjusted by a term proportional to the product of the Gamma and the square of the underlying price change, thus better capturing the non-linear impact on the option's value.

The results indicate that the Monte Carlo Simulation yields the lowest VaR ($\approx$ €269,106.65), likely reflecting the actual historical distribution of returns, which may contain fewer extreme events than those assumed by the linear models. By contrast, the Delta Normal Approach delivers a higher VaR ($\approx$ €450,190.75), as it ignores curvature and can in some cases overestimate potential losses. The Delta-Gamma method falls in an intermediate position **€283,541.08**, acknowledging some degree of non-linearity without achieving the full scope of a complete Monte Carlo simulation.

Overall, risk estimation is typically more accurate when the real shape of the return distribution is considered, as in Monte Carlo. However, simpler methods such as Delta Normal or Delta-Gamma may be preferred in contexts where computational speed or operational simplicity are paramount.

Case study 3:

We consider a 5-year cliquet option with annual reset dates. The underlying is a non-dividend-paying stock, trading at EUR 2.50, with a constant volatility of 25%. The notional of the trade is EUR 50 million. On January 31, 2023, at 10:45 C.E.T., Polimi Bank purchases this option from ISP. At each annual payment date t_i, the payoff is given by

$$[S(t_{i-1}) - S(t_i)]^+,$$

Since ISP is the seller, there is a non-zero probability that ISP might default before making the later payments. Let $\text{SurvivalProbability}_i$ denote the chance ISP survives until t_i. Then the *correct fair price* from Polimi Bank's perspective, often referred to as the CVA-adjusted price, is:

$$\text{Correct Price } = \sum_{i=1}^{5} \Big(\mathbb{E}^{\mathbb{Q}}\big[\max\{S(t_{i-1}) - S(t_i),\, 0\}\big] \times \text{DiscountFactor}_i \times \text{SurvivalProbability}_i \Big).$$

To compute $\mathbb{E}^{\mathbb{Q}}\big[\max\{S(t_{i-1}) - S(t_i), 0\}\big]$, we use a Monte Carlo scheme:

1. Simulate many stock-price paths under Geometric Brownian Motion (assuming a flat or term-structure-based risk-free rate r and volatility $\sigma = 25\%$).
2. For each path ω_j, compute the payoff $\max\{S_{\omega_j}(t_{i-1}) - S_{\omega_j}(t_i), 0\}$ at each year i.
3. Average over all paths to estimate $\mathbb{E}^{\mathbb{Q}}[\cdot]$.
4. Multiply each year's average payoff by the discount factor and the survival probability.
5. Sum over $i = 1 \ldots 5$.

From ISP's perspective, as the *seller* of the derivative, they would typically quote a higher price than the buyer's CVA-adjusted fair value. The main reasons include the cost of hedging (which references the risk-free market price), funding considerations, regulatory capital requirements, and any profit margin or dealing spreads. In practice, the price at which ISP is willing to sell the option often exceeds the buyer's correct price.

Table 6: Comparison of Cliquet Option Prices

Scenario	**Option Price (EUR)**
No Default Probabilities (Risk-Free)	1.053948259811
With Default Probabilities (Correct price)	1.028852998323

Appendix: Code Corrections

Wrong reference date. In the initial version, the reference date for the calculations was set incorrectly. We corrected it to

```
date = pd.Timestamp(2020, 1, 31)
```

so as to align the bond or option timeline with the available market data.

Correction of the `black_scholes_option_pricer` function. When computing d_1 and Gamma, some terms were set incorrectly. We replaced:

```
d1 = (np.log(S / K) + (r - d + sigma**2 / 2) * ttm) / (sigma * np.sqrt(ttm))
```

and, for Gamma:

```
np.exp(-d * ttm) * norm.pdf(d1) / (S * sigma * ttm ** (0.5))
```

Mismatch in the simulation number for the bootstrap approach. In the original version of the code, the explanatory comment mentioned 200 simulations, while the actual parameter in the code was set to 300. To ensure clarity and consistency, we corrected the parameter to:

```
simulations_num = 300
```