Project

(Multi-name Credit Product)

Financial Engineering

On the 31st of January 2023 at 10:45 C.E.T., the reference portfolio of the SPV Cayman III is composed by I=500 mortgages, a portfolio that can be considered homogeneous with an average notional of 2 Mln Euro and an average recovery of 40% for each mortgage. Assume that mortgages provide a single payment at the end of the interest period T=4y; for the period T the default probability of each mortgage is 6%.

For this reference portfolio prices are known and market implied (Vasicek-like) correlations for the cumulated tranches (Kd 0% and Ku X%) are provided in the file in attach.

a. Calibrate the double t-Student [1] model with a real number of degrees of freedom (i.e. $\nu \in \mathfrak{R}^{+}, \nu \geq 2$) that best fits market data, considering valid the hypothesis of Large Homogeneous Portfolio.

b. Estimate the impact on pricing of the hypothesis of Large Homogeneous Portfolio on tranche prices (delimited by the set of subordinators 3%, 6% and 9%) for the double t-Student models (e.g. for the parameters obtained above). Show that the price of the Tranche in relative terms (as a percentage of tranche face value) is well described by equation (1) of the notes in the range I = (10, 10^4), where the LHP approximation holds. It is required to plot the price varying I (in log scale in the abscissa), with

- the exact solution (up to an I that your computer allows to obtain a price);

- the KL approximate solution;

- the LHP solution.

c. Calibrate the double t-student correlation and the real number of degrees of freedom of [1] considering the KL approximation. Comment the results.

d. Price the same trances of b. for I =500 with a Vasicek model with correlation ρ equal to the implied correlation of the equity tranche. Comment the results.

e. [Facultative] Price the same trances of b. for I =500 with a Li model and Gaussian copula with constant correlation ρ equal to the one in point b. Assume a constant intensity over the 4y. Comment the results.

Hints:

a. Given a value for v, find correlation parameter in (double) t-student model that equates equity tranche. Then among all possible v>2 find the one that best fits market implied correlations.

b. Pay attention that the Equity tranche in the Kullback-Leibler expansion should be treated differently.

[1] J. Hull and A. White (2004), Valuation of a CDO and an n-th to default CDS without a Monte carlo simulation, J. Derivatives 2, 8-23

Realize a library in Matlab. Optional Python.

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