POLITECNICO MILANO 1863

Financial Engineering 2024/25

Report Assignment 2 FE

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QUESTIONS:

Introduction

This assignment was about bootstrapping the discount curve and using it to perform calculations regarding IR products. Then, a 7y "I.B. coupon bond" was priced, and the NPV of monthly cash flows was computed using an AAGR and considering two different initial cash flows. Furthermore, the sensitivity indices of interest rate swaps were calculated, along with the Modified Duration of the coupon bond priced before.

1) Bootstrap

To bootstrap, we have used the first four quoted interbank deposit rates (up to the three months one), the first seven quoted interest rate futures and an incomplete set of swaps with maturities up to 50 years. The swap set was completed for the bootstrapping process.

Deposits:

In Deposit section, the discounted factors were calculated by the formula:

$$B(t_0, t_i) = \frac{1}{1 + \delta(t_0, t_i) \cdot L(t_0, t_i)}$$

where $L(t_0, t_i)$ is the mean of bid and ask for each deposit considered and $\delta(t_0, t_i)$ represents the year fraction between t_0 and t_i with the convention Act/360. Furthermore, we implemented the function:

$$\text{zRates} = \texttt{zeroRates}(dates, discounts)$$

in which we applied the formula:

$$y = \frac{-\ln B(t_0, t)}{\delta(t_0, t)}$$

in order to retrieve the zero rates given the discount factors.

Futures:

After the first four deposits we moved on the quoted ***Futures***, in particular we focused on the first seven since they are most liquid. From the futures prices we evaluated the forward rates $L(t_0; t_i, t_i + 1)$ and then we computed the discount rate at expiry considering the following relation:

$$B(t_0, t_{i+1}) = B(t_0; t_0, t_i) \cdot B(t_0; t_i, t_{i+1}) \quad B(t_0; t_i, t_{i+1}) = \frac{1}{1 + \delta(t_i, t_{i+1}) \cdot L(t_0; t_i, t_{i+1})}$$

Swaps:

The last instrument at our disposal were the ***Swaps***, for which the discount factors were calculated by the formula:

$$B(t_0, t_i) = \frac{1 - S(0, t_i) \sum_{n=1}^{i-1} \delta(t_{n-1}, t_n) B(0, t_n)}{1 + \delta(t_{i-1}, t_i) S(0, t_i)}$$

All these passages were implemented using a customized function:

$$[\text{dates}, \text{discounts}] = \texttt{bootstrap}(\mathit{datesSet}, \mathit{ratesSet})$$

Below there is the diagram of discount factors that correspond to the expiry dates of the interest rate products used in the bootstrap. The discount on 02/02/2073 is **0.398250**.

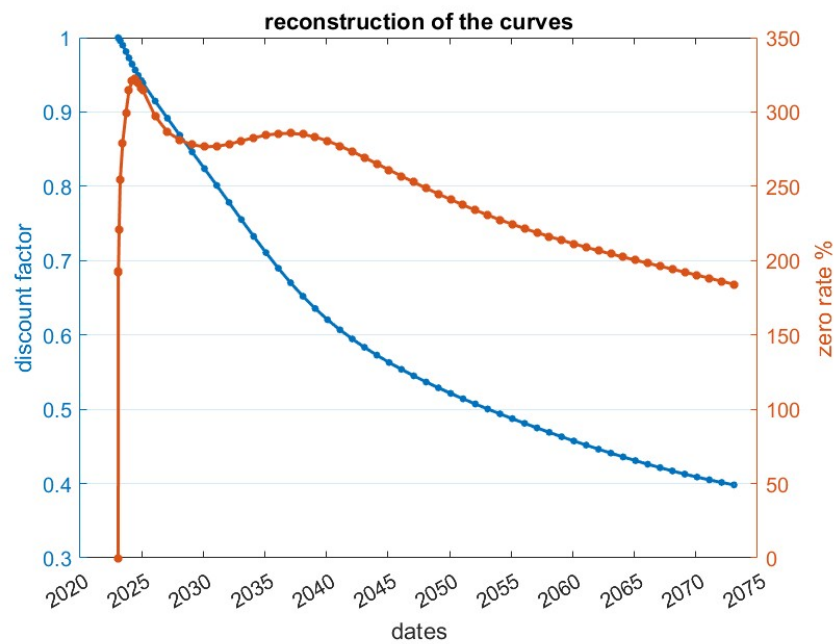

Figure 1: Bootstrap Diagram

Question:

Bootstrapping is a key technique for deriving discount factors from quoted market prices; even if alternative methods can achieve similar results, this approach is particularly valuable due to its ability to construct a stable yield curve, despite fluctuations in market data. This stability is obtained by interpolating between zero rates rather than discount factors. Additionally, constructing an optimal discount curve is crucial, not only for pricing purposes but also from a mathematical perspective. The reconstructed curve is used as the initial condition for the Cauchy Problem in the HJM framework, which models interest rate dynamics. This dual importance emphasizes the significance of using precise interpolation and ad hoc extrapolation techniques in order to obtain reliable discount factors

2) Pricing an "I.B. coupon bond"

The goal of this section is to price a 7y "I.B. coupon bond" issued on January 31 2023, with a coupon rate $S(0, t_7)$ equal to the corresponding mid-market 7-year swap rate. Furthermore, we assume a 30/360 European day count for the coupons and a face value of 100 million. We initially assess the cash flows, where $c = \delta(t_{i-1}, t_i) \cdot S(0, t_7) \cdot FV$ for $i = 1, 2, \ldots, 7$.

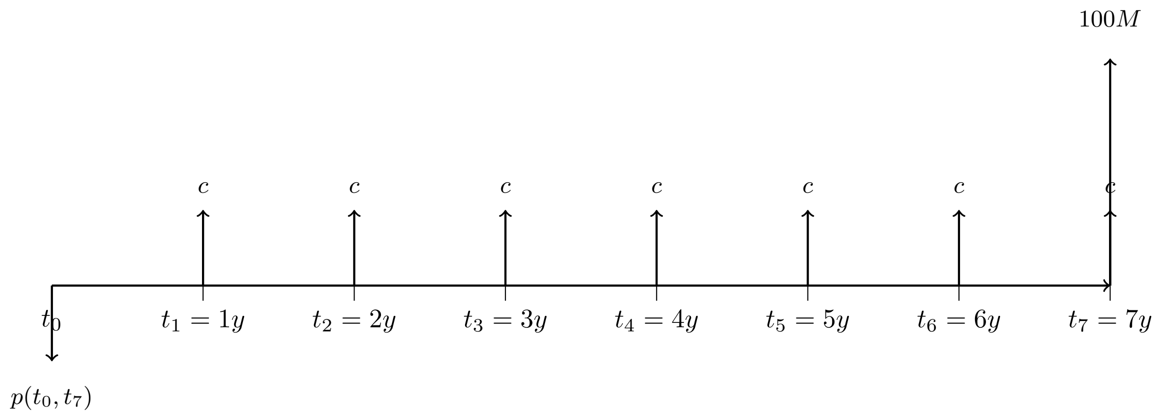

Figure 2: Cash flows of an "I.B. coupon bond" with maturity 7y

We price the "I.B coupon" by discounting the cash flows at time t_0 and imposing the Absence of Arbitrage, namely that the Net Present Value equals 0. Following this procedure we get the following formula:

$$P(t_0, t_7) = FV \cdot S(t_0, t_7) \cdot \sum_{i=1}^{7} \delta(t_{i-1}, t_i) \cdot B(t_0, t_i) + FV \cdot B(t_0, t_7) = FV \cdot \left(\frac{(1 - B(t_0, t_7))}{BPV_{07}} \cdot BPV_{07} + B(t_0, t_7) \right) = FV$$

where we have replaced the swap rate $S(t_0, t_7)$ with its explicit expression:

$$S(t_0, t_7) = \frac{1 - B(t_0, t_7)}{\sum_{i=1}^{7} \delta(t_{i-1}, t_i) \cdot B(t_0, t_i)} = \frac{(1 - B(t_0, t_7))}{BPV_{07}}$$

Thus, we can conclude that the price of the 7-year "I.B. coupon bond" matches its face value. Consistently, using MATLAB and the curve constructed above, we obtained that the price of the coupon bond under analysis is 1.000002×10^8.

3) Sensitivities for an IRS and Macaulay Duration for a coupon bond

The goal is to study the risk of a portfolio composed of a single swap, specifically a 7y plain vanilla interest rate swap against Euribor 3-months, with a fixed rate of 2.8175% and a notional amount of 100 million. The analysis is conducted by computing four parameters: DV01, BPV, $DV01_z$, and Macaulay Duration.

DV01:

The DV01 quantifies the dollar change in a bond's value resulting from a change in market interest rates. Thus, it measures the variation in the net present value (NPV) of a derivatives portfolio due to an increase in market rates within the 'ratesSet' structure (used in the bootstrap) for a parallel shift of 1 bp. In particular, DV01 is calculated as the absolute difference between the net present values before and after the shift:

$$DV01(t_0) = |NPV_{\text{shift}}(t_0) - NPV(t_0)|$$

Our result is $DV01(t_0)$=6.2548×10^{-4}, so that an increase (or decrease) of the rates of 1bp impact on the portfolio of 62548 €.

BPV:

BPV measures how much money the portfolio gain or lose as a consequence of 1bp parallel shift in the yield curve. The BPV is computed via the difference of the NPVs with the original fixed rate $S(t_0, t_7)$ and this fixed rate increased by 1bp. The result we get considering a single swap is $BPV = 6.2573 \times 10^{-4}$, so that an increase (or decrease) of rates of 1bp impacts on the portfolio of 62573 €.

DV01$_z$:

The $DV01_z$ is a measure of sensitivity similar to DV01, but simpler, as it is based on the difference in absolute values between the original NPV and the one obtained after a shift of 1 bp in the zero rates curve. This quantity is easier to compute compared to DV01 since it does not require reperforming the bootstrap. The result we get for a single swap is $DV01_z = 6.4603 \times 10^{-4}$ so that an increase (or decrease) of the zero rates of 1bp impacts the portfolio of 64603 €.

MD:

The Macaulay Duration is the variation in the value of an "I.B. coupon bond" for the corrisponding zero rate curve of 1bp normalized by the price of the bond. In particular it is defined as the present value of the bond's cash flows, weighted by the time until each cash flow is received, divided by the total present value of the cash flows; the formula we use is the following:

$$\text{MacD}(t_0) = \frac{\sum_{i=1}^{7} c_i \cdot (t_i - t_0) \cdot B(t_0, t_i)}{\sum_{i=1}^{7} c_i \cdot B(t_0, t_i)}$$

The result we get is $MacD(t_0) = 6.45$ anni.

Conclusion:

To sum up, we can use these parameters to adjust the exposure of our portfolio to interest rate risk. An integrated strategy that considers all three measures allows investors to assess their overall exposure to interest rate risk and to adopt the necessary corrective actions. In this way, it is possible to optimize portfolio performance and minimize the risk associated with changes in interest rates. In particular, if an increase in interest rates is expected, these measures of sensitivity will be decreased; conversely, if we expect rates to fall, these values will be increased.

4) Net Present Value computation

The Net Present Value (NPV) is a financial metric used to evaluate the profitability of a series of future cash flows discounted to their present value.

In this section, we focus on computing the NPV while considering an Annual Average Growth Rate (AAGR) of 5%, applied each year in September. The calculation follows the formula:

$$NPV(t_0) = \sum_{i=1}^{241} c_i \cdot B(t_0, t_i)$$

where $c_i = c_0 \cdot (1 + AAGR)^i$ represents the cash flow for the i-th month.

To determine the zero rates corresponding to the dates of the cash flows, we applied interpolation using the MATLAB function *interp1*. Then, we used the following formula:

$$B(t_0, t_i) = \exp(-\delta(t_0, t_i) \cdot y)$$

to compute the discount factors $B(t_0, t_i)$ from the interpolated zero rates.

Table 1 presents the NPVs obtained for two different initial cash flows. Notably, the percentage return remains the same in both cases. Specifically, the ratio between the two NPVs is exactly 4, which matches the ratio between the initial cash flows.

$C0$	NPV
1.5K €	399.3K €
6.0K €	1597.3K €

Table 1: NPV for different C0 with same starting date, expiry and AAGR