POLITECNICO
MILANO 1863

Financial Engineering 2024/25

Report Assignment 2 RM

Leonardo Pascotto, Alberto Pellegrini, Tommaso Resta, Dalila Rosati

Contents

QUESTIONS:

Introduction

The main goal of this case study is to analyze the credit risk of Beta, an IG corporate bond issuer, by examining the default probabilities implied by market prices and historical data. Firstly, default intensities are estimated under the assumption of a constant hazard rate. Using these intensities, we derive the default probabilities over one and two years and then we compute the bonds' Z-spreads. Secondly, we consider a piecewise constant intensity and evaluate alternative bootstrapping methods. Lastly, we present a comparison between these market-implied probabilities and historical default probabilities obtained from a given rating transition matrix.

Question 1

We have modified the following in the code:

- We have changed the value of the variable *maturity2* from 3 to 2 years, according to the assignment
- We have set the dirty prices respectively equal to 1.00 and 1.02 in order to reflect the default notional of 1.0; for the required calculations it was useless to consider the notional
- In the function `bond_cash_flows`, the `while` loop used to determine *cash_flows_dates* has been modified to start from the reference date instead of the expiry date. This change ensures that the correct number of cash flows (four instead of five) is obtained. Additionally, in the same function, *yearfrac* is used to compute the cash flows instead of dividing by the coupon frequency.
- Instead of writing a function to compute the dirty price given the intensity and then using `fsolve` in the main, we have implemented a function, `constant_intensity_from_bond`, that directly calls `fsolve`. This approach was chosen to avoid redundant computations, as the calculation needed to be repeated multiple times throughout the assignment.

Maturity	**Coupon**	**Dirty Price**	**Intensity**
1Y	5%	€100.00	2.438963%
2Y	6%	€102.00	2.433770%

Table 1: Constant Intensities

In Table 1, we report the intensities for the two bonds analyzed. The results are quite similar, as the effects of the longer maturity and higher coupon are balanced by the higher dirty price.

Intensity is closely linked to maturity, coupon and dirty price, in fact, fixing the other parameters, the following happens:

- As maturity increases, intensity increases
- As coupon increases, intensity increases because it means the investment is riskier
- As dirty price increases, intensity decreases because it means the investment is less risky

Question 2

Assuming the default intensities (constant over time) reported in table 1, the probability of default for a bond is given by:

$$P_{\text{default}} = 1 - P_{\text{survival}}, \quad \text{where} \quad P_{\text{survival}} = e^{-\lambda(t_N - t_0)}$$

Here, λ represents the constant intensity, t_0 is the initial time (today), and t_N is the time horizon considered (bond's maturity). This formula follows from the assumption that defaults occur as a Poisson process with intensity λ, meaning that the probability of survival decays exponentially over time. The longer the time horizon, the lower the survival probability, and consequently, the higher the probability of default. In particular, from table 2, we can notice that the default probability obtained for the two-years maturity bond is approximately twice the default probability of the one-year maturity bond.

Maturity	**Default Probability**
1Y	2.409461%
2Y	4.763675%

Table 2: Default probabilities at different maturities

Question 3

The Z-spread represents the additional yield that, when added to risk-free interest rates, ensures that the present value of a bond's cash flows equals its market price, namely the dirty price.

To address this question, we develop the function `defaultable_bond_dirty_price_from_z_spread`, which, as the name suggests, computes the dirty price $\bar{C}(t)$. In particular, each cash flow involved in this computation is calculated by multiplying the bond's periodic payment by the discount factor, which accounts for both the risk-free rate and the credit risk premium. The final payment at maturity is also discounted accordingly. The Z-spreads are then obtained by solving for z the equation.

The results are shown in table 3.

Maturity	Z-Spread
1Y	1.721444%
2Y	1.727300%

Table 3: Z-Spreads at different maturities

As expected, we have obtained a slightly higher Z-spread for the bond with 2 years maturity. The Z-spread represents the credit risk premium, which compensates investors for the possibility of default. Since the probability of default increases over time, the bond with two-years maturity has an higher credit risk than the one-year bond, so having a slightly higher Z-spread for the 2-years bond is consistent. Furthermore, the relatively small difference suggests that the market perceives only a modest increase in credit risk between the two maturities, which indicate a relatively stable issuer credit profile.

Question 4

Maturity	Intensity	Default Probability
1Y	2.438963%	2.409461%
2Y	2.429167%	4.764214%

Table 4: Piecewise constant intensities and default probabilities

Using the 1-year bond we have computed the intensity and the default probability on the first year. As we expected they are the same result of the 1-year bond obtained at point 1 and 2.
For the second year we obtain slightly lower results than with constant intensity. Overall the results are consistent with point one, therefore the constant intensity hypothesis might produce in general satisfactory results, but the piecewise constant intensity gives a better approximation of creditworthiness because it captures changes in default risk over time.

Question 5

To derive the transition matrix at 2-years it is necessary to compute the square of the given one-year transition matrix, namely:

$$Q^{(2)} = Q^{(1)} \times Q^{(1)}$$

This follows from the property of Markov chains, where the n-year transition matrix is obtained by iterating the one-period transition matrix.

By applying this approach, we compute the two-years transition matrix and then we determine the historical default probabilities for the firm Beta. The results obtained are shown in table 5.

Maturity	Historical Default Probability
1Y	2.00%
2Y	4.71%

Table 5: Historical default probabilities at different maturities

As expected, the historical default probability at 2-years is larger than that at 1-year. Moreover, comparing these values to the estimated default probabilities reported in Tables 2 and 4, we observe that the historical default probability is lower at both the 1-year and 2-year horizons.

Question 6

By decreasing the dirty price of the 2-year bond, the intensity of the first year remains unchanged, while that of the second year increases, as reported in Table 6. Consequently, the 2y default probability also increases. Since the one-year bond with which we calculate the intensity in the first year remains unchanged, it is correct that the 1y piecewise constant intensity remains unchanged. A decrease in bond prices signals a deterioration in the issuer's creditworthiness, making default more likely, which naturally leads to an increase in default intensity and probability: the values reported in Table 6 result to be consistent.

Maturity	Intensity	Default Probability
1Y	2.438963%	2.409461%
2Y	6.221046%	8.326880%

Table 6: Piecewise constant shock intensities and default probabilities

Question 7

By increasing the dirty prices of bonds, the piecewise constant intensity and default probability decrease. This result aligns with financial intuition: an higher market price reflects an improved creditworthiness of the issuer, leading to lower expected default risk. In particular, we obtain the following results:

Maturity	**Default Probability**
1Y	1.00%
2Y	2.90%

Table 7: Comparison of Conditional Default Probabilities

The increase in dirty prices implies that investors are willing to pay more for these bonds, signaling a reduction in perceived credit risk. Compared to the original scenario, where the credit risk was higher, the lower default intensities in Scenario 2 suggest that the market anticipates a stronger financial position for Beta.

Question 8

To check the consistency of the default probabilities between the first and second year under Scenario 1 with the equivalent historical probabilities derived from the transition matrix, we compare the conditional default probabilities. To compute the conditional default probabilities, we used the classical formula for conditional probability, namely:

$$P_{\text{default}|\text{no default in 1Y}} = \frac{P_{\text{default in 2Y}} - P_{\text{default in 1Y}}}{1 - P_{\text{default in 1Y}}}$$

This formula accounts for the fact that we are conditioning on the survival of the bond issuer beyond the first year.

Type	**Default Probability**
Historical	2.765306%
Scenario 1	6.0363517%

Table 8: Comparison of Conditional Default Probabilities

These results indicate a significant discrepancy between the two results. The conditional default probability under Scenario 1 is more than twice the historical probability. This difference suggests that the intensities estimated in Scenario 1 imply an higher risk of default in the second year, conditional on survival in the first year, compared to what is observed in historical data. This comes from the fact that scenario 1 supposes that the dirty price of the 2-years bond drops to 97, indicating a riskier situation.

The discrepancy arises because the transition matrix approach reflects past realized defaults may underestimate the immediate risk perceived by current market participants, while Scenario 1 is based on market-implied intensities. Additionally, the assumption of a constant default intensity in Scenario 1 may not accurately capture how default probabilities change over time due to economic conditions and firm-specific factors.

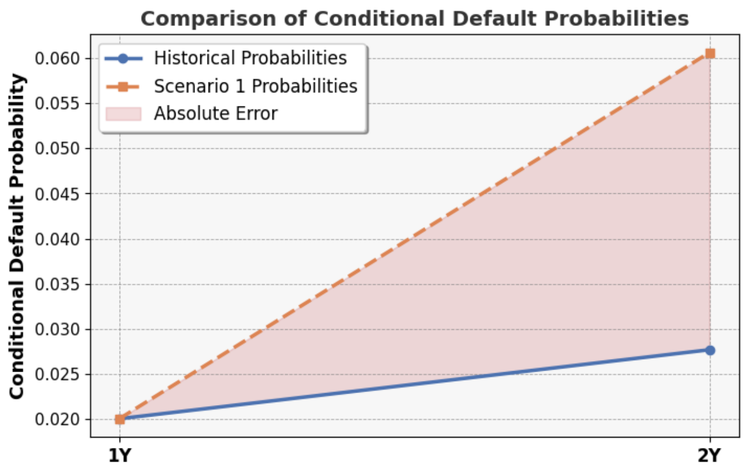

Figure 1: Comparison of Conditional Default Probabilities