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Report Assignment 1 RM

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QUESTIONS:

Introduction

This report analyzes a portfolio composed of a long receiver swaption and a vanilla 10-year IRS. The swaption (1-month expiry, 10-year maturity, 5-year underlying, ATM) has a notional of 700 million € while the IRS (fixed rate payer) has a notional of 600 million €. Our objective is to determine the portfolio's market value (MtM), evaluate its DV01 using both a parallel shift and an analytical approximation, and design delta-hedging strategies. In particular, we emphasize a coarse-grained bucket DV01 analysis, where the yield curve is split into distinct maturity segments, and then propose hedging strategies with one or two IRS instruments. Throughout the discussion, theoretical issues from the lab notes (e.g., differences between an IRS and an IB bond, completeness of the swaption delta formula, the need for DV01 based on zero rates, etc.) are seamlessly integrated into the narrative.

1 Portfolio Valuation (MtM)

The portfolio is valued as the sum of the swaption price (scaled by 700 million €) and the IRS price (scaled by 600 million €). Since the IRS is entered at par, its MtM is essentially zero. The swaption, being ATM, is priced with its strike equal to the forward swap rate. Our calculations yield a swaption price of approximately 0.08152 per unit of notional, resulting in a portfolio MtM of about 57,062,717 € (i.e., $0.08152 \times 700\,\mathrm{M}$). Note that, conceptually, a par-rate IRS closely resembles an interbank (IB) fixed coupon bond with an equivalent coupon; both have an NPV of zero when priced at par, though the IRS exchanges net cash flows while the bond pays full coupons and principal at maturity.

2 Portfolio DV01 – Parallel Shift

We then assess the portfolio's interest rate sensitivity by applying a parallel 1 bp shift to all market rates, re-bootstrapping the discount curve, and computing the new NPV. The DV01 is given by:

$$\mathrm{DV01} = \mathrm{NPV}_{\mathrm{shifted}} - \mathrm{NPV}_{\mathrm{base}}.$$

Our numerical results indicate a DV01 of approximately 417,416 €. It is important to emphasize that a naive swaption delta formula such as

$$\Delta(t_0) = B(t_0, t_\alpha)\, \mathrm{BPV}(t_0)\, N(ds1)$$

is incomplete because it does not capture how the forward swap rate depends on the entire yield curve—only the direct sensitivity to the forward rate is considered.

3 Analytical DV01 Approximation

An alternative method approximates DV01 by combining the sensitivities of the swaption and the IRS. Specifically, we use:

$$\mathrm{DV01}_{\mathrm{approx}} = \Big(\text{Swaption Notional} \times \Delta_{\mathrm{swaption}} + \text{IRS Notional} \times \text{Duration}\Big) \times 10^{-4}.$$

Here, the swaption delta is calculated using a Black model and the IRS duration is analogous to the Macaulay duration for a par fixed coupon bond (for which, note, DV01 obtained by shifting the zero-rate curve is equal to minus the duration). Our analytical DV01 comes out to be about 505,706 €—approximately 88,290 € higher than the numerical result—since the analytical method neglects higher-order effects and cross-derivatives.

4 Delta-Hedging with a Single IRS

To offset the portfolio's DV01, we compute the unitary DV01 of a 10-year IRS (i.e., the change in its MtM per unit notional for a 1 bp shift) and determine the required hedge notional. Based on the numerical DV01, the hedge notional is approximately -572 million € (a short, or payer, position), whereas using the analytical DV01 gives about -503 million €. In both cases, the sign indicates that a short IRS position is needed to hedge the long receiver swaption.

5 Portfolio Coarse-Grained Bucket DV01

A parallel shift is a simplification; in reality, yield curve movements are non-parallel. We therefore decompose the curve into macro-buckets to capture different maturity sensitivities. In our analysis, we define two buckets:

For the 10-year bucket:

- Maturities below 11 years receive a full +1 bp shock.
- Maturities between 11 and 15 years experience a linear taper from +1 bp down to 0.
- Maturities above 15 years are unshifted.

For the 15-year bucket:

- Maturities below 11 years are not shifted.
- Between 11 and 15 years, the shock increases linearly from 0 to +1 bp.
- For maturities above 15 years, a full +1 bp shock is applied.

For instance, if the board specifies buckets at 10 years and 20 years, then for the 10-year bucket the weights are 1 for maturities up to 10 years, tapering to 0 between 10 and 15 years; for the 20-year bucket, weights are 0 up to 15 years, then increasing linearly to 1 by 20 years, and remaining 1 thereafter.

After re-bootstrapping the curve with these bucket-specific shifts, the portfolio DV01s are calculated as the differences between the shifted and the base NPVs. We obtain:

- **10-year Bucket DV01:** Approximately 558,828 €.
- **15-year Bucket DV01:** Approximately -141,000 €.

This indicates that the portfolio's risk is primarily concentrated in the 10-year bucket, while the swaption's tail exposure contributes an opposite (negative) sensitivity in the 15-year bucket.

6 Delta-Hedging with Two Liquid IRS

Since a single IRS hedge may not capture non-parallel shifts adequately, we propose hedging using two instruments:

- A 10-year IRS to cover the positive DV01 in the 10-year bucket.
- A 15-year IRS (or a suitably liquid instrument representing the mid-to-long end) to offset the negative DV01 in the 15-year bucket.

By matching the DV01 of each bucket to the corresponding unitary DV01 of the hedging IRS, our calculations indicate that approximately -651 million € (short) is required for the 10-year bucket and about +117 million € (long) for the 15-year bucket. This two-instrument strategy is more granular and, in theory, should neutralize the portfolio's sensitivity across different maturity segments more effectively than a single instrument.

7 Delta-Hedging with Two Buckets and P&L Analysis

To further refine our hedging strategy, we consider a decomposition of the swaption's sensitivity into two buckets: one at 10 years and the other at 15 years. We then hedge these exposures using a 10-year swap and a 15-year swap, respectively.

In our simulation, the results obtained are:

- The difference in portfolio value with the single (parallel) hedge (i.e., using one IRS) is

  $$\text{ptf\_bucket}[0] - \text{ptf\_initial}[0] \approx -8{,}525 \text{ EUR},$$

  indicating a nearly neutralized portfolio.

- In contrast, with the two-bucket hedge (using a 10y swap for the 10-year bucket and a 15y swap for the 15-year bucket), the portfolio P&L difference is

  $$\text{ptf\_bucket2} - \text{ptf\_initial2} \approx -400{,}359 \text{ EUR}.$$

This substantial difference implies that the two-instrument hedge, in this particular simulation, results in a larger adverse P&L compared to the single IRS hedge. The interpretation is that when hedging the swaption using two buckets, the sensitivity decomposition may not align perfectly with the actual risk profile of the swaption. The 10-year swap, while effective in hedging short-term movements, fails to capture the mid-to-long term exposures adequately. Consequently, the residual risk in the 15-year bucket is not offset sufficiently, leading to a significant net loss.

This outcome underscores the challenge in implementing a bucket-based hedge: while it provides a more detailed view of the risk distribution, the calibration of hedge ratios is critical. Imperfect matching between the bucket DV01 and the chosen hedging instruments can result in over- or under-hedging, as illustrated by the large P&L swing in the two-bucket approach. In practice, additional refinements, continuous rebalancing, or even the use of more sophisticated derivatives may be necessary to achieve a truly neutral hedge.