POLITECNICO
MILANO 1863

Financial Engineering 2024/25

Report Assignment 4 RM

Dario Delmonaco, Andrea Catina, Tommaso Resta, Ahmet Sahil

Contents

1 Mark-to-Market (MtM) of the Portfolio 2

2 1-Year Forward Price if Rated IG 3

3 1-Year Forward Price if Rated HY 3

4 Effect of Downgrade Inclusion on VaR and Expected Loss 3

5 Effect of Correlation on VaR and Expected Shortfall 4

6 Discussion 4

7 Effect of Lower Recovery in Downgrade Events 4

Introduction

This lab focuses on the valuation and risk management of a portfolio consisting of 100 zero-coupon bonds (ZCBs), each with a 2-year maturity and a face value of €1,000,000, initially rated as Investment Grade (IG). We adopt a single-factor credit model that accounts for both default and credit migration (IG/HY), assuming a 40% recovery rate. The yearly transition matrix and the risk-free curve are used to compute forward prices, Mark-to-Market (MtM), and Value at Risk (VaR) over a one-year horizon, under a correlation assumption of $\rho = 15\%$. A Monte Carlo simulation (with at least 1,000,000 scenarios) is then carried out to estimate the expected number of defaults, downgrades, and the VaR under different correlation scenarios. The objective is to illustrate how the rating structure, recovery rate, and correlation influence the portfolio's credit risk measures.

1 Mark-to-Market (MtM) of the Portfolio

We compute the current value of each bond (initially rated IG), then multiply by the total number of ZCBs in the portfolio. After one year, each issuer may remain IG, be downgraded to HY, or default, with corresponding probabilities provided by the transition matrix.

The expected payoff is discounted based on each scenario:

- *Default after 1 year:* Payoff $= \mathrm{RR} \cdot \mathrm{FV} \cdot B(t_0, t_1)$
- *Remain IG or migrate to HY:* The bond will have 1 year remaining to maturity, so its 1-year forward price is computed and then discounted to t_0

Here, RR denotes the recovery rate (40%), FV is the face value, and $B(t_0, t)$ is the discount factor from t to today.

The formula used to compute the Mark-to-Market of each bond is:

$$\mathrm{MtM} = \pi_{11} \cdot B(t_0, t_1) \cdot \mathrm{FwdPrice}_{\mathrm{IG}} + \pi_{12} \cdot B(t_0, t_1) \cdot \mathrm{FwdPrice}_{\mathrm{HY}} + \pi_{13} \cdot B(t_0, t_1) \cdot \mathrm{FV} \cdot \mathrm{RR} \tag{1}$$

To calculate the Mark-to-Market value of the entire portfolio, we multiply the result obtained using the formula above by the total number of ZCBs. The value is reported in the following table:

MtM of the Portfolio
91,288,211 €

2 1-Year Forward Price if Rated IG

If the issuer is still rated IG at $t = 1_{year}$, then it remains a probability π_{13} of defaulting in the second year. The forward price at $t = 1_{year}$ reflects the expected payoff at $t = 2_{year}$, discounted only over the second year.

The formula used follows:

$$\text{FwdPriceIG} = FV \cdot \left[(1 - \pi 13) \cdot \frac{B(t_0, t_2)}{B(t_0, t_1)} + \text{RR} \cdot \pi_{13} \cdot \frac{B(t_0, t_{\text{def}})}{B(t_0, t_1)} \right] \tag{2}$$

The price obtained is shown in the table below:

Forward Price (IG) of a single ZCB
958,106 €

3 1-Year Forward Price if Rated HY

Similarly, if the issuer is downgraded to HY at $t = 1_{year}$, the probability of default during the second year is π_{12}. The forward price is computed as:

$$\text{FwdPriceHY} = FV \cdot \left[(1 - \pi_{12}) \cdot \frac{B(t_0, t_2)}{B(t_0, t_1)} + \text{RR} \cdot \pi_{12} \cdot \frac{B(t_0, t_{\text{def}})}{B(t_0, t_1)} \right] \tag{3}$$

The resulting price is reported in the following table:

Forward Price (HY) of a single ZCB
940,823 €

Comparing the results, we observe that the forward price under IG is higher than under HY, reflecting the lower default probability. Additionally, the current MtM is below the FV due to discounting and credit risk.

4 Effect of Downgrade Inclusion on VaR and Expected Loss

In this question we computed the one-year 99% Credit VaR using Monte Carlo simulation with 10^6 samples. We compared two scenarios: one considering only defaults, and the other including both defaults and downgrades. Since the loss given downgrade is higher than the loss under status quo, we expected a higher VaR and expected shortfall in the second case.

Table 1: Results for Question 4 ($\rho = 0.15$)

Scenario	**Expected Shortfall (€)**	**VaR 99% (€)**
Defaults only	7,108,572.96	5,006,134.14
Defaults + Downgrades	7,634,881.66	5,818,455.09

5 Effect of Correlation on VaR and Expected Shortfall

We repeated the simulation for two different correlation levels: $\rho = 0$ and $\rho = 0.3$. As expected, a higher correlation increases both the VaR and expected shortfall, due to a higher probability of simultaneous defaults and downgrades.

Table 2: Results with $\rho = 0$

Scenario	**Expected Shortfall (€)**	**VaR 99% (€)**
Defaults only	2,436,246.86	1,736,061.31
Defaults + Downgrades	2,499,324.97	2,150,863.50

Table 3: Results with $\rho = 0.3$

Scenario	**Expected Shortfall (€)**	**VaR 99% (€)**
Defaults only	12,370,294.10	8,276,206.96
Defaults + Downgrades	13,341,829.46	9,434,196.41

6 Discussion

From the results above, we can draw the following conclusions:

- **Migration risk vs. Default risk:** Including downgrades both VaR and expected shortfall consistently increase. This is because migration events are more frequent than defaults and result in mark-to-market losses due to the increased credit spread.
- **Correlation:** Increasing correlation amplifies systemic risk. With higher ρ, multiple defaults and downgrades become more likely to occur simultaneously, increasing the tail risk and therefore the VaR.

7 Effect of Lower Recovery in Downgrade Events

To capture additional risk, we introduced a recovery rate of 0.1 for downgrade events. We recomputed forward prices and MTM values, then recalculated VaR and expected shortfalls for different correlation values.

Table 4: Results with $\rho = 0$ (Recovery in downgrade = 0.1)

Scenario	**Expected shortfall (€)**	**VaR 99% (€)**
Defaults only	2,065,719.51	1,367,141.16
Defaults + Downgrades	2,509,549.53	2,136,106.69

Table 5: Results with $\rho = 0.15$ (Recovery in downgrade = 0.1)

Scenario	**Expected Shortfall (€)**	**VaR 99% (€)**
Defaults only	6,766,890.61	4,637,213.99
Defaults + Downgrades	8,026,303.44	6,143,433.04

Table 6: Results with $\rho = 0.3$ (Recovery in downgrade = 0.1)

Scenario	**Expected Shortfall (€)**	**VaR 99% (€)**
Defaults only	12,501,832.94	8,452,298.95
Defaults + Downgrades	13,852,308.05	10,086,350.71

Conclusion

As expected, the introduction of a lower recovery rate for downgrades increases both expected loss and VaR. This highlights the importance of accounting for not only migration frequency, but also the severity of credit quality deterioration.