CS-E5740 Complex Networks, Answers to exercise set 6

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Problem 1

a) Below are reported the computed centralities for the network reported in Figure 1:

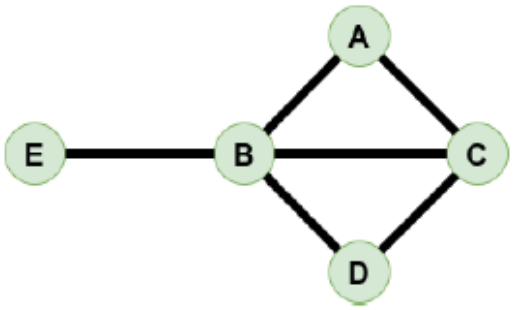

Figure 1: Small undirected network.

i) Betweenness centrality of node B:

$$\begin{aligned} bc(B) &= \frac{1}{(N-1)(N-2)} \sum_{s \neq B} \sum_{t \neq B} \frac{\sigma_{sBt}}{\sigma_{st}} \\ &= \frac{1}{12} * \left(\left(\frac{0}{1} + \frac{1}{2} + \frac{1}{1} \right) + \left(\frac{0}{1} + \frac{0}{1} + \frac{1}{1} \right) + \left(\frac{1}{2} + \frac{0}{1} + \frac{1}{1} \right) + \left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1} \right) \right) \\ &= 0.583 \end{aligned}$$

ii) Closeness centrality of node B:

$$C(B) = \frac{N-1}{\sum_{v \neq B} d(B,v)} = \frac{4}{1+1+1+1} = 1$$

iii) K-shell centrality of all nodes:

$$k_s(A) = 2 \quad k_s(B) = 2 \quad k_s(C) = 2 \quad k_s(D) = 2 \quad k_s(E) = 1$$

b) The centrality measures of **betweenness, closeness, k-shell,** and **eigenvector centrality** were calculated for the networks reported in Figure 2, as well as for the **Zachary Karate Club** network.

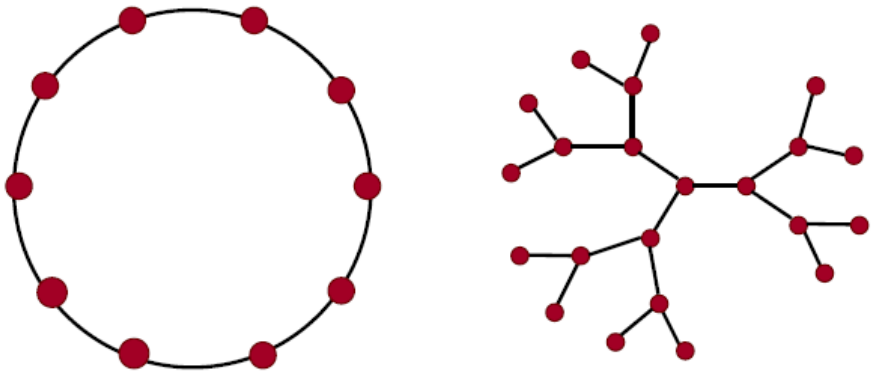

Figure 2: Ring 10 and Cayley Edge networks.

Their value was plotted as a function of degree in the scatterplots provided in Figures 3, 4, and 5.

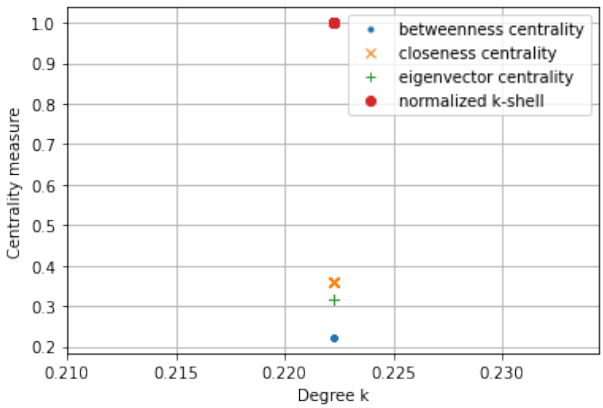

Figure 3: Scatterplot of centrality measures for the Ring 10 network.

c) For each network, 5 visualizations were plotted, each time coloring the nodes based on their value according to a different centrality measure. The results are provided in Figures 6, 7, and 8.

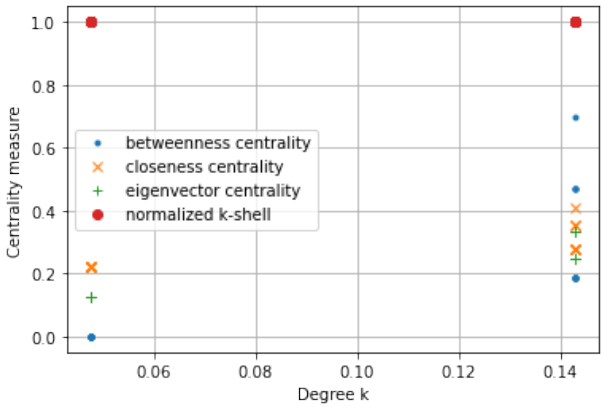

Figure 4: Scatterplot of centrality measures for the Cayley Edge network.

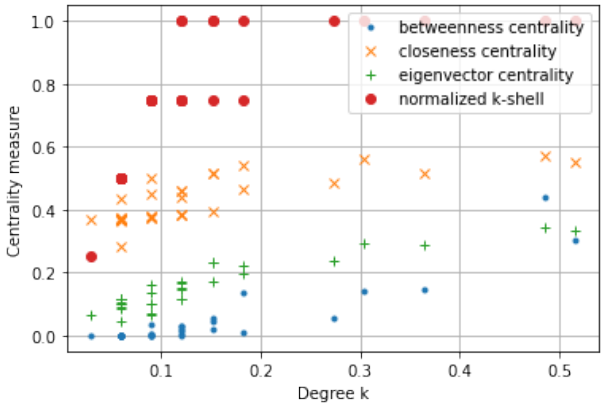

Figure 5: Scatterplot of centrality measures for the Karate Club network.

d) While node 20 in the Karate Club network does have a low degree, it also happens to be directly connected to the two nodes bearing the highest degree in the network. This means that its distance from most nodes in the network is still quite low, as it can quickly reach them by passing through these 'hubs', contributing to a high closeness centrality value for that node.

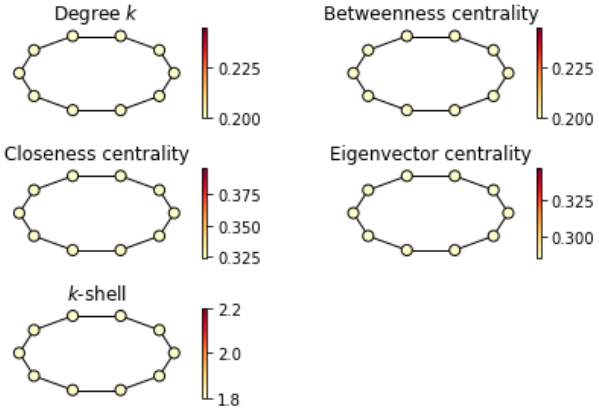

Figure 6: Visualization of centrality measures by node for the Ring 10 network.

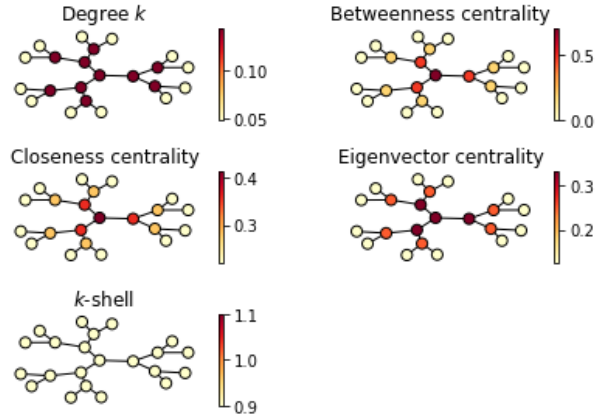

Figure 7: Visualization of centrality measures by node for the Cayley Edge network.

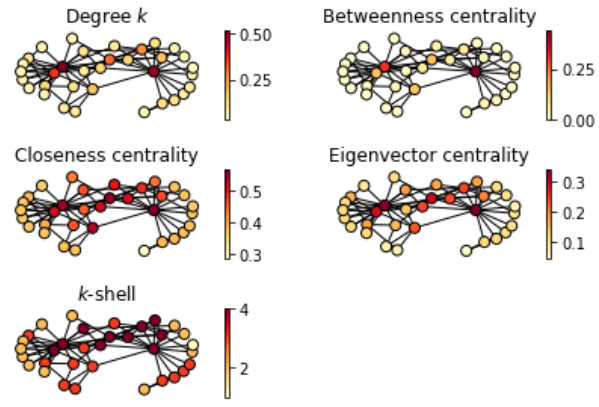

Figure 8: Visualization of centrality measures by node for the Karate Club network.

Problem 2

A number of analyses were conducted on the **Zachary Karate Club** and the **Snowball-Sampled Facebook Friendships** networks.

a) The scatterplots of the degrees of nodes connected in pairs were calculated for both networks, and are reported in Figures 9 and 10.

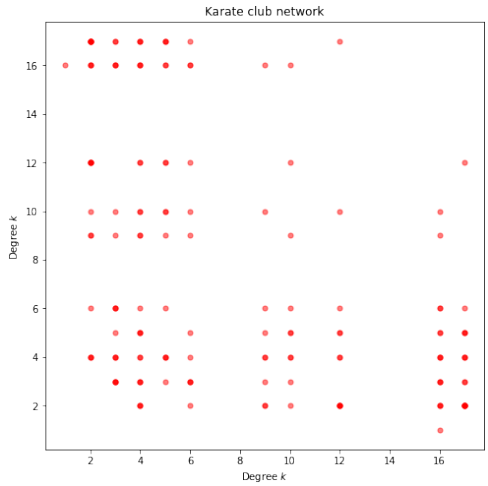

Figure 9: Scatterplot of node pairs' degrees for the Karate Club network.

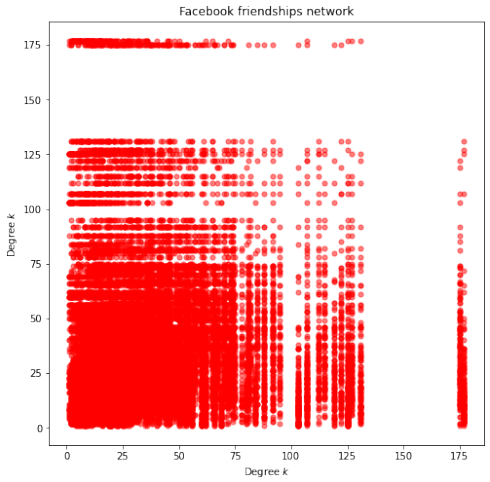

Figure 10: Scatterplot of node pairs' degrees for the Facebook Friendships network.

b) A heatmap was produced for the degrees of connected node pairs in the Facebook Friendships network, which is reported in Figure 11. By using a heatmap instead of a scatter plot, it is easier to see that most of the connected pairs in this network are made up of nodes with a small degree

c) The assortativity coefficient was calculated for both networks, both by applying the function **scipy.stats.pearsonr** on the degrees of node pairs, and by using the function **networkx.degree_assortativity_coefficient**. The comparison between the results obtained is reported in Figure 12.

As can be observed, the Karate Club network is actually disassortative: this is because the network has very few 'hubs' which are not directly connected, resulting in an averagely decreasing nearest neighbor degree.

d) The average nearest neighbor degree was computed and plotted alongside a scatterplot of nearest neighbor degrees as a function of node degrees. The result is provided in Figure 13.

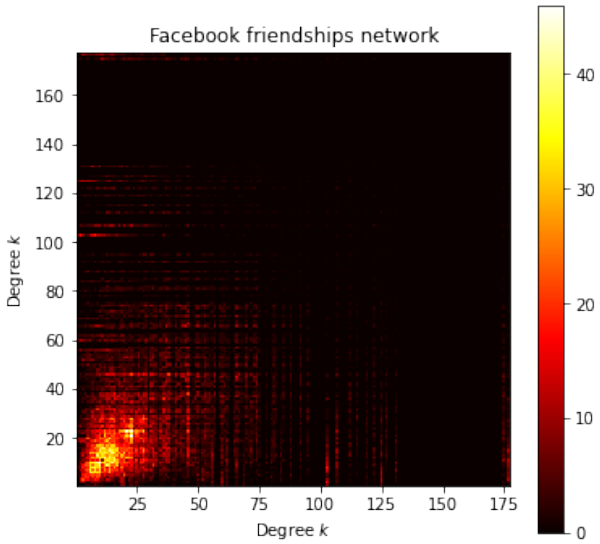

Figure 11: Heat map of node pairs' degrees for the Facebook Friendships network.

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Own assortativity for Karate club network: (-0.47561309768461385, 3.509446967037993e-10)
NetworkX assortativity for Karate club network: -0.47561309768461413
Own assortativity for Facebook friendships network: (0.055984784765930155, 6.574678019602541e-24)
NetworkX assortativity for Facebook friendships network: 0.055984784765930475
```

Figure 12: Assortativity of the networks calculated with Scipy and NetworkX.

As can be observed, the nearest neighbor degree does not have a monotonic behavior as a function of node degree, but instead decreases for node's with a low k, increases again for degrees between 10 and 100, and finally assumes an irregular(although on average decreasing) behavior for degrees beyond 100.

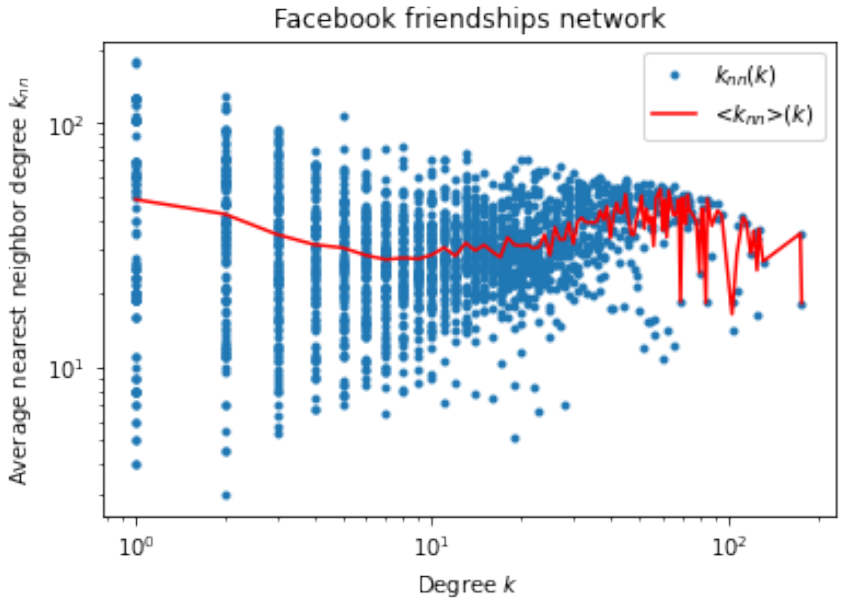

Figure 13: Average nearest neighbor degree for the Facebook Friendship network.

Problem 3

Considering the bipartite network reported in Figure 14.

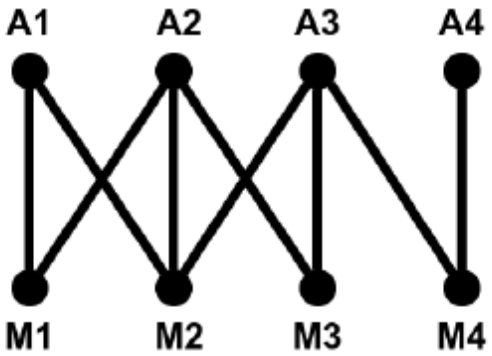

Figure 14: Bipartite actors and movies network.

a) Two unipartite projections of the network were constructed, and are reported in Figures 15 and 16.

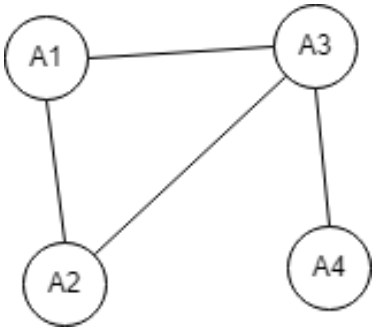

Figure 15: Unipartite actors network.

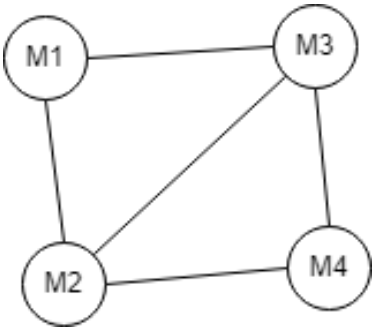

Figure 16: Unipartite movies network.

b) To prove that it is not possible to uniquely reconstruct a bipartite network from its two unipartite projections, a counterexample network bearing the same unipartite projections as the one in Figure 14 but different from it was constructed. The network is reported in Figure 17.

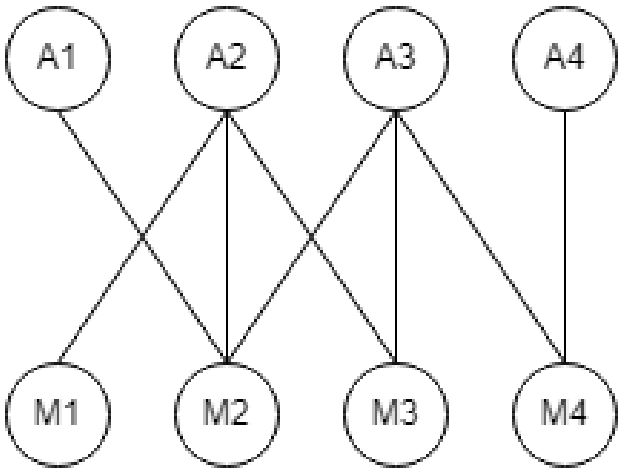

Figure 17: Counterexample network.