CS-E5740 Complex Networks,
Answers to exercise set 2

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Problem 1

Answers to this task are provided in the scans attached at the end of this file.

Problem 2

a) In ER networks, the average clustering coefficient $\langle c \rangle$ equals p because the expected value of c for one node is the ratio between the number of edges between the node's neighbors, and the total number of edges that can exist between them.
By definition, in ER networks the probability of nodes being connected is p, so we can expect that number to correspond on average to the aforementioned ratio.

b) If $N \to \infty$ with $\langle k \rangle$ bounded, we can expect $\langle c \rangle \to 0$ as the likelihood of two neighbors of a node being connected with one another decreases, since the average degree of nodes remains bounded while the number of total nodes to which they could be connected increases towards infinity.

c) The calculations are reported in the attached Jupyter notebook, and the plots are presented in Figure 1 and 2.

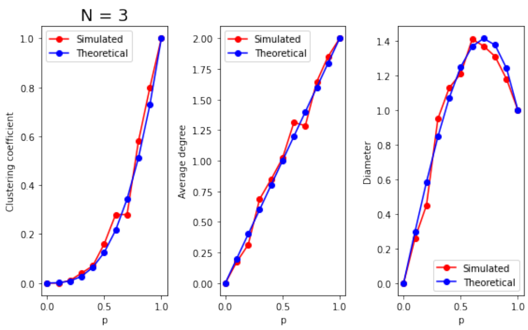


Fig. 1: properties for N=3

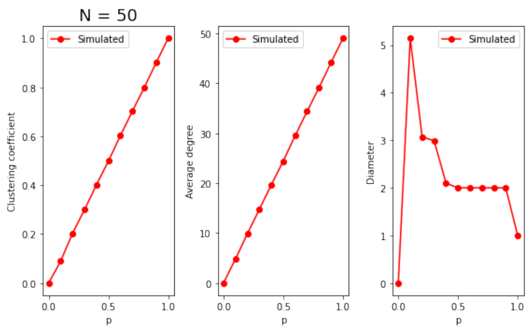


Fig. 2: properties for N=50

Problem 3

- a) The calculations are reported in the attached Jupyter notebook, and the plots are presented in Figure 3 and 4.

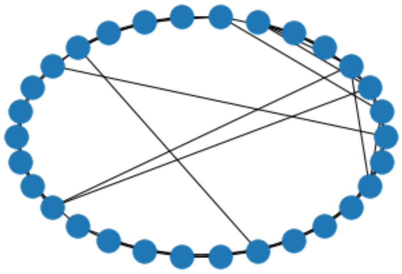


Fig. 3: N=30 , m=2, p=0.2

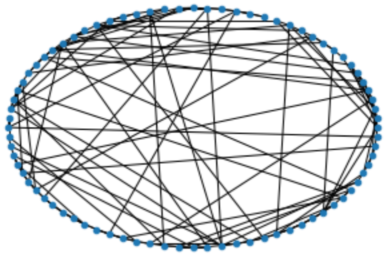


Fig. 4: N=80 , m=2, p=0.4

- b) The calculations are reported in the attached Jupyter notebook, and the plots, which are in line with those in the lecture slides, are presented in Figure 5.

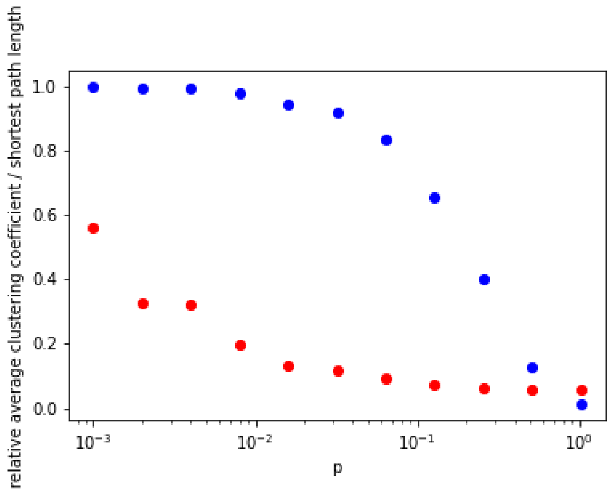
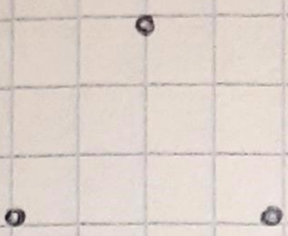


Fig. 5: relative average coefficient and shortest path length

The relative clustering coefficient decreases as the probability increases because as p approaches 1 the randomness in the network increases: while for $p = 0$ a node's neighbors will for the most part be connected to one another because of the network's regular structure, in a more random network this clustering will decrease, reducing the relative clustering.
The relative path length also decreases as p increases (albeit more rapidly) because the rewiring procedure produces random "shortcuts" that reduce the shortest paths.

Ensemble for $G(3, p)$:

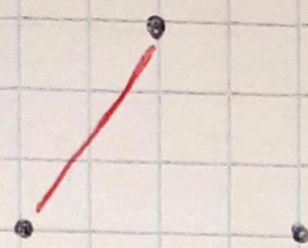


$\pi_1 = (1-p)^3$

$k(G_1) = 0$

$c(G_1) = 0$

$\delta^*(G_1) = 0$

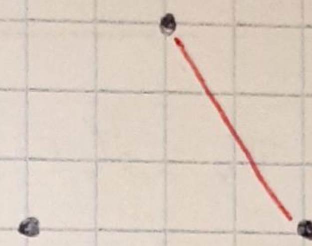


$\pi_2 = p(1-p)^2$

$k(G_2) = 2/3$

$c(G_2) = 0$

$\delta^*(G_2) = 1$

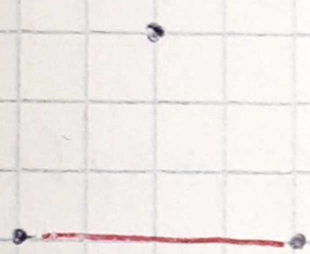


$\pi_3 = p(1-p)^2$

$k(G_3) = 2/3$

$c(G_3) = 0$

$\delta^*(G_3) = 1$

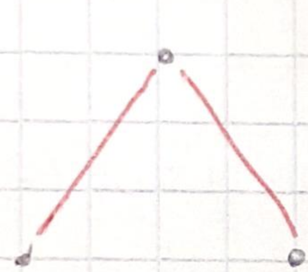


$\pi_4 = p(1-p)^2$

$k(G_4) = 2/3$

$c(G_4) = 0$

$\delta^*(G_4) = 1$

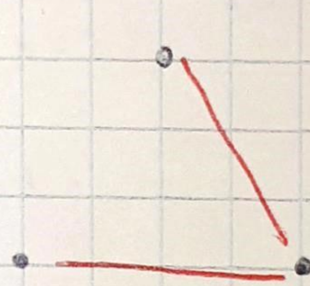


$\pi_5 = p^2(1-p)$

$k(G_5) = 4/3$

$c(G_5) = 0$

$\delta^*(G_5) = 2$

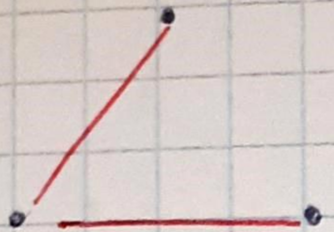


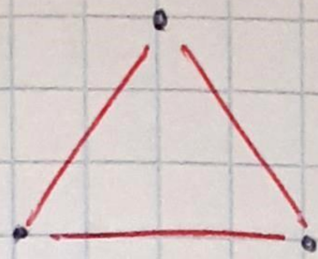
$\pi_6 = p^2(1-p)$

$k(G_6) = 4/3$

$c(G_6) = 0$

$\delta^*(G_6) = 2$





$\pi_7 = p^2(1-p)$ $\qquad$ $\pi_8 = p^3$

$\kappa(G_7) = 4/3$ $\qquad$ $\kappa(G_8) = 2$

$c(G_7) = 1$ $\qquad$ $c(G_8) = 1$

$\delta^*(G_7) = 2$ $\qquad$ $\delta^*(G_8) = 1$

$$\langle \kappa \rangle = \sum_{i=1}^{8} \pi_i \, \kappa(G_i)$$

$$= \cancel{(1-p)^3 \cdot 0} + \cancel{3}p(1-p)^2 \cdot 2/\cancel{3} + \cancel{3}p^2(1-p)\,4/\cancel{3} + p^3 \cdot 2$$

$$= 2p\left[(1-p)^2 + 2p(1-p) + p^2\right]$$

$$= 2p\left[\underbrace{(1-p) + p}_{1}\right]^2$$

$$= 2p$$

$$\langle c \rangle = \sum_{i=1}^{8} \pi_i \, c(G_i)$$

$$= (1-p)^3 \cdot 0 + 3p(1-p)^2 \cdot 0 + 3p^2(1-p) \cdot 0$$

$$+ p^3 \cdot 1$$

$$= p^3$$

$$\langle d^* \rangle = \sum_{i=1}^{8} \pi_i \, d^*(G_i)$$

$$= \cancel{(1-p)^3 \cdot 0} + 3p(1-p)^2 \cdot 1 + 3p^2(1-p) \cdot 2 +$$

$$p^3 \cdot 1$$

$$= p\left[3(1-p)^2 + 6p(1-p) + p^2\right]$$

$$= p\left[3 - \cancel{6p} + 3p^2 + \cancel{6p} - 6p^2 + p^2\right]$$

$$= p\left[3 - 2p^2\right]$$