CS-E5740 Complex Networks, Answers to exercise set 4

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Problem 1

a) The expected number of nodes at d steps away (n_d), expressed as a function of $< k >$ and d and assuming tree-like behavior, was calculated starting from the value for $d = 1$ and progressing until an arbitrary value of d. Remembering that because of the friendship paradox the excess degree is:

$$< q >= \frac{< k^2 >}{< k >} - 1$$

And that in ER networks the variance and mean are equal:

$$< k >=< k^2 > - < k >^2$$

We can say that for $d = 1$, the number of nodes at 1 step away is the average degree of the network:

$$n_1 =< k >$$

For $d = 2$, we have that each of the nodes reached in $d = 1$ step will have on average $< q >$ other neighbors:

$$n_2 =< k >< q >=< k > (\frac{< k^2 >}{< k >} - 1) =< k >^2$$

And continuing this trend for an arbitrary d, we have:

$$n_d =< k >^{d-1}< q >=< k >^d$$

b/c) The n_d value and average fraction of loop edges were calculated empirically (and for the former compared with the theoretical results) for values of d between 0 and 15, network sizes $N = 10^4$ and $N = 10^5$ and average degrees $< k >= 0.5$, 1, and 2.
The resulting plots are reported in Figures 1 to 6.

For $< k >= 0.5$:

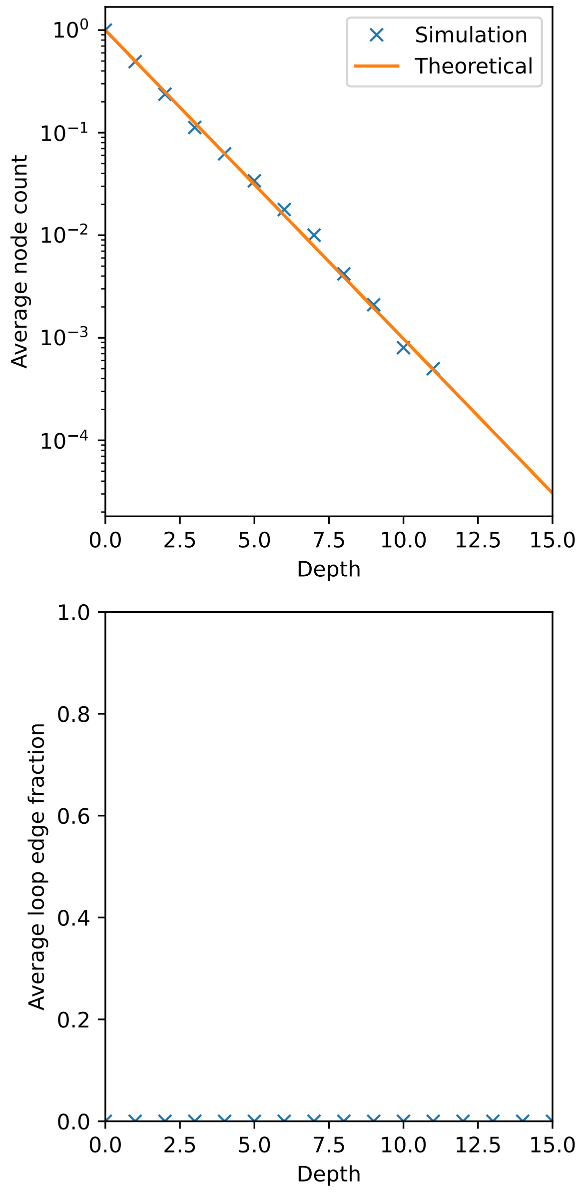

Figure 1: $< k >= 0.5$ and $N = 10^4$

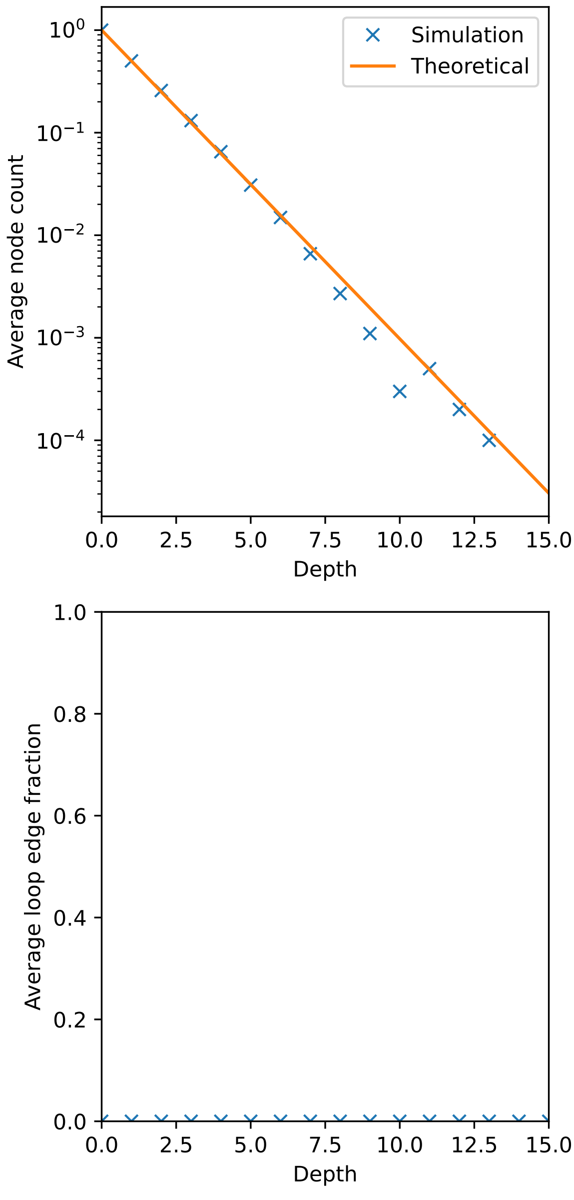

Figure 2: $< k >= 0.5$ and $N = 10^5$

For $< k >= 1$:

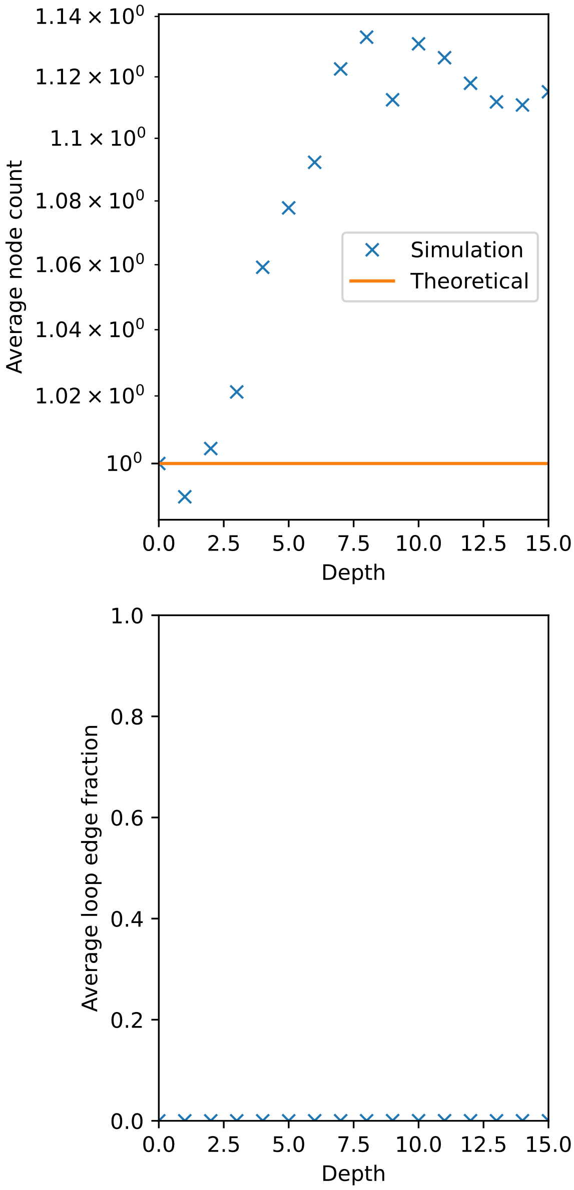

Figure 3: $< k >= 1$ and $N = 10^4$

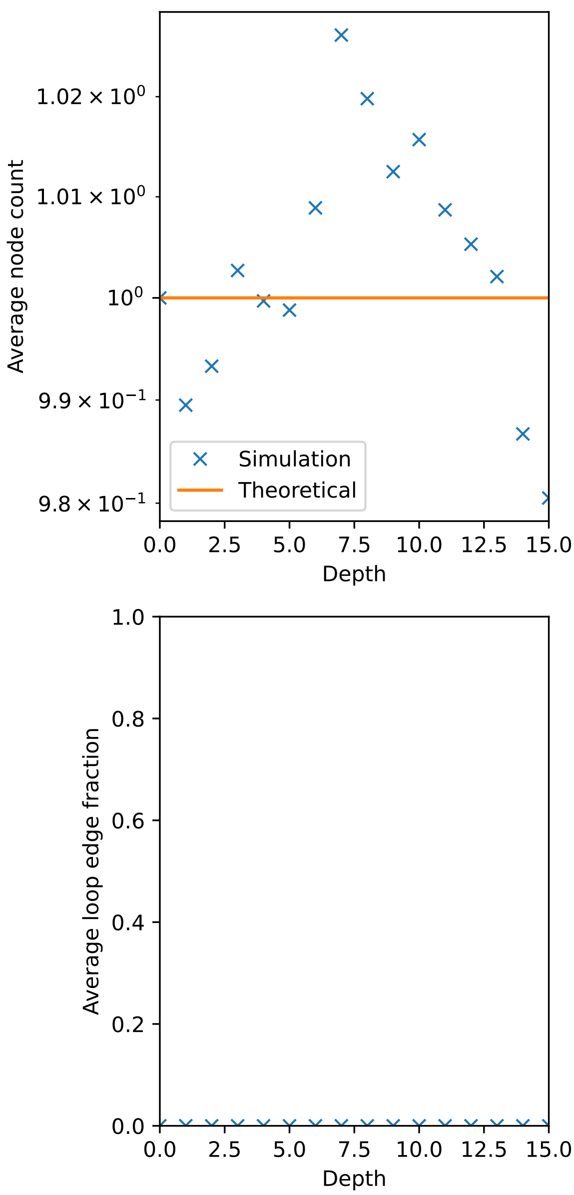

Figure 4: $< k >= 1$ and $N = 10^5$

For $< k >= 2$:

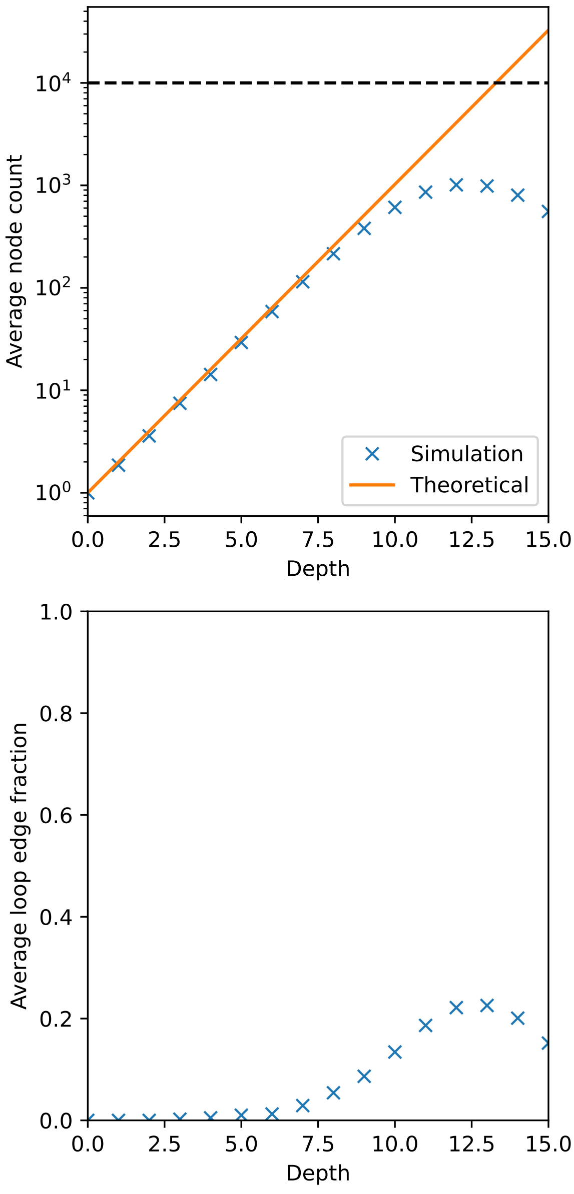

Figure 5: $< k >= 2$ and $N = 10^4$

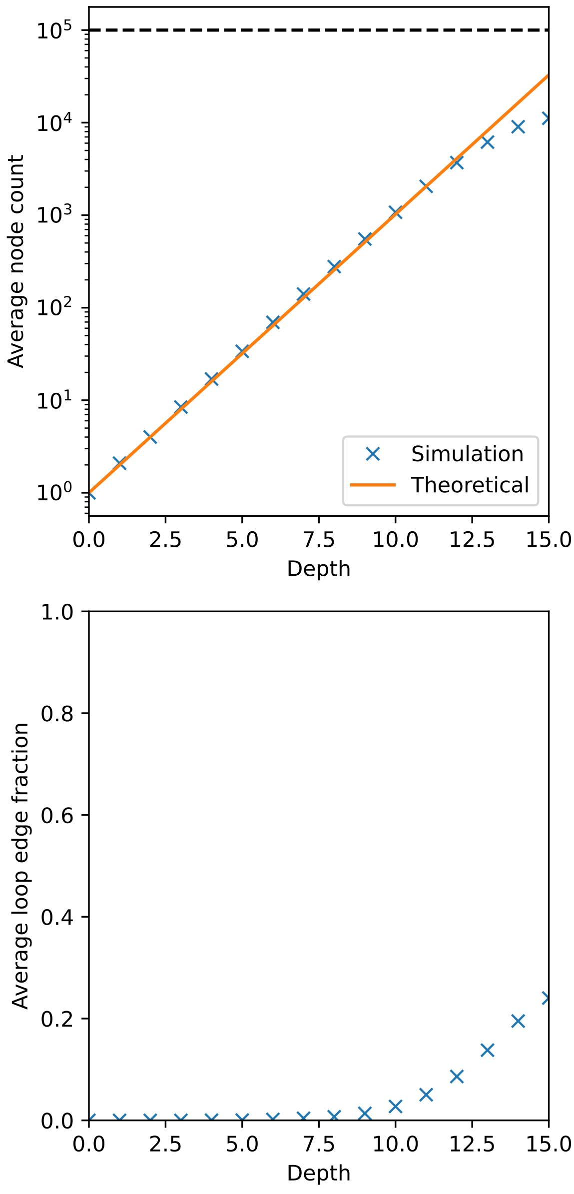

Figure 6: $< k >= 2$ and $N = 10^5$

d/e) The size of the largest connected component and the susceptibility were computed for networks of sizes $N = 10^4$ and $N = 10^5$, and for values of $< k >$ between 0 and 2.50 with step of size 0.05. The resulting plots are reported in Figure 8.

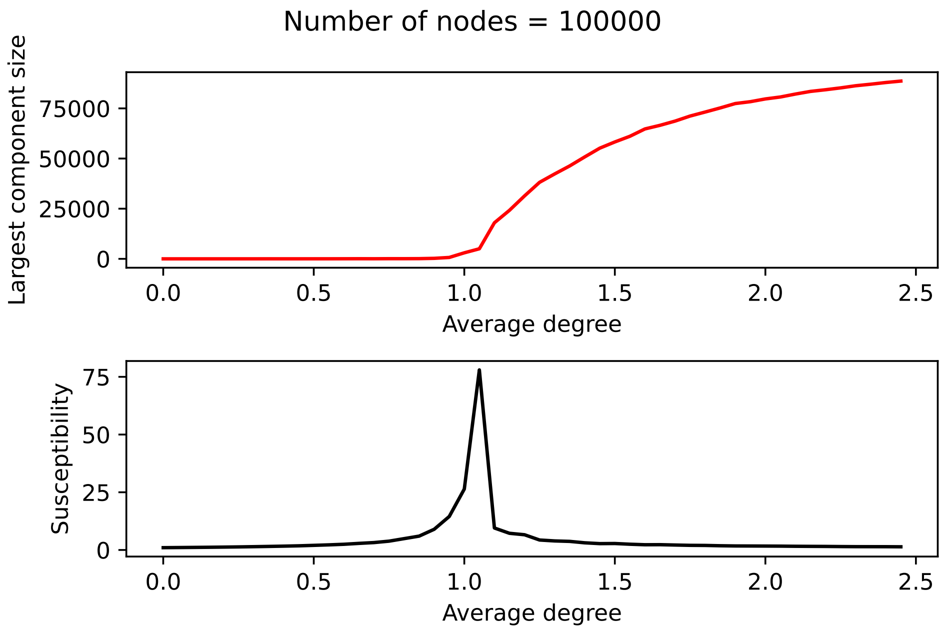

Figure 7: Size of largest component and susceptibility as functions of $< k >$

The curves indicate that once the average degree reaches a value of 1 the network, which was previously populated for the most part by connected components of small sizes, starts suddenly increasing as a large number of these coalesce into bigger subgraphs trough the formation of links between them.
Soon after, one main connected component starts to emerge by fusing with the other smaller subgraphs, as indicated by the sharp decline in susceptibility for values of $< k >$ beyond 1.

Problem 2

After generating the network described by the weighted edges in the `'OClinks_w_undir.edg'` file, its edges were progressively removed following a random order, an ascending link weight order and a descending link weight order, so as to simulate different kinds of attacks.
The size of the largest connected component was then plotted in relation to the fraction of removed links, as shown in Figure]**??**

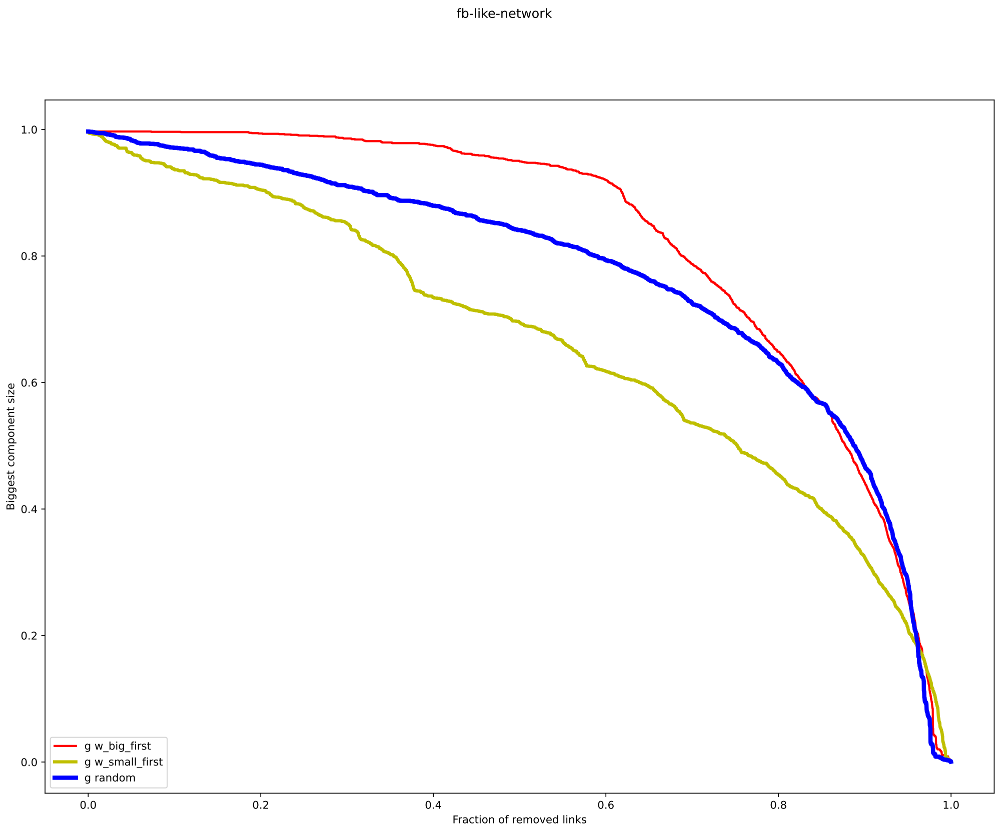

Figure 8: Largest connected component as a function of removed links for different attack patterns.

The decline in the largest component size can be observed to be sharper when starting from nodes with smaller weight, whereas starting from those with a higher weight appears to be the slowest way to reduce the maximum component size.