Exercise 8

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Exercise 1

c) The reason why most of the features are not tracked for very long is likely the low resolution of the recording: probably because of limitations of the recording device, as well as the faster movement in the latter part of the video, the image appears more blurred and thus the movement the features becomes harder to keep track of. This results in the loss of the majority of the tracked pixels.

d) Using more advanced techniques such as Shi-Tomasi feature tracker might yield improved results, as it should deal better with larger displacements. Another option is to increase window size, in order to reduce sensitivity to noise, or employ Gaussian pyramids.

Exercise 2

The calculations for the exercise are reported in the attached images.

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2. The equation in the paper is:

$$\Delta p = H^{-1} \sum_x \left[\nabla I \frac{\delta W}{\delta p} \right]^T \left[T(x) - I(W(x;p)) \right]$$

where

$$H = \sum_x \left[\nabla I \frac{\delta W}{\delta p} \right]^T \left[\nabla I \frac{\delta W}{\delta p} \right]$$

$\nabla I = [I_x, I_y]$ (assuming 2 coordinates)

$T(x)$: template (extracted sub-region of image at time $t-1$

$I(x)$: image at time t

$W(x;p)$: parametrized set of all possible warps

$p = [p_1, \ldots, p_n]^T$ is a vector of parameters

$I(W(x;p))$: is the image I warped back onto the coordinate frame of T

Hence, using slides notation:

$$T(x) \equiv I(x, y, t-1)$$

$$I(W(x;p)) \equiv I(x, y, t)$$

And thus:

$$T(x) - I(W(x;p)) \equiv -I_t$$

Since $\frac{\delta W}{\delta p}$ is the Jacobian of the warp, if W is a translation we have:

$$P = \begin{bmatrix} u \\ v \end{bmatrix} \qquad \frac{\delta W}{\delta p} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Then:

$$H = \sum_x \left[[I_x, I_y] \cdot I_2 \right]^T \left[[I_x, I_y] \cdot I_2 \right]$$

$$= \sum_x [I_x, I_y]^T [I_x, I_y]$$

$$= \sum_x \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

And:

$$\Delta p = H^{-1} \sum_x \left[[I_x, I_y] I_2 \right]^T (-I_t)$$

It follows that:

$$H \Delta p = -I_t \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$

$$\downarrow$$

$$\sum_x \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\sum_x \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix}$$

Note: we are assuming $p_0 = [0, 0]^T$