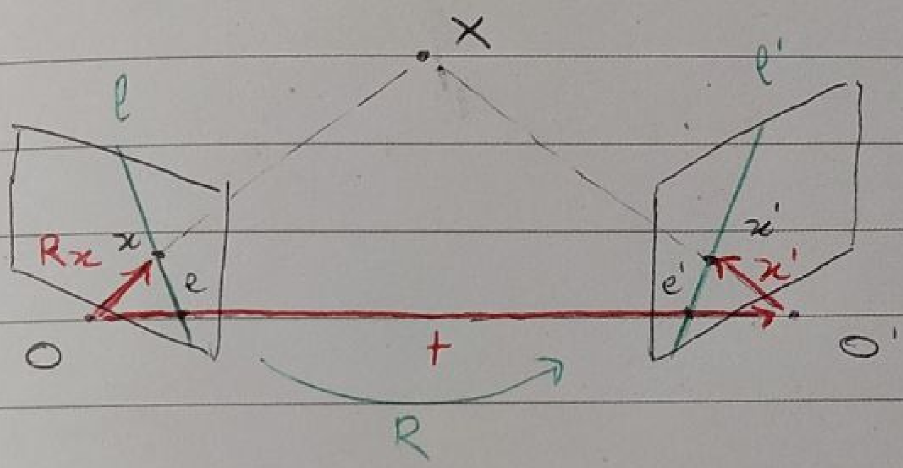
Computer Vision 12

Ex 2) Epipolar constraint:

$$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op} \right) = 0$$

From lesson 11, slide 19:

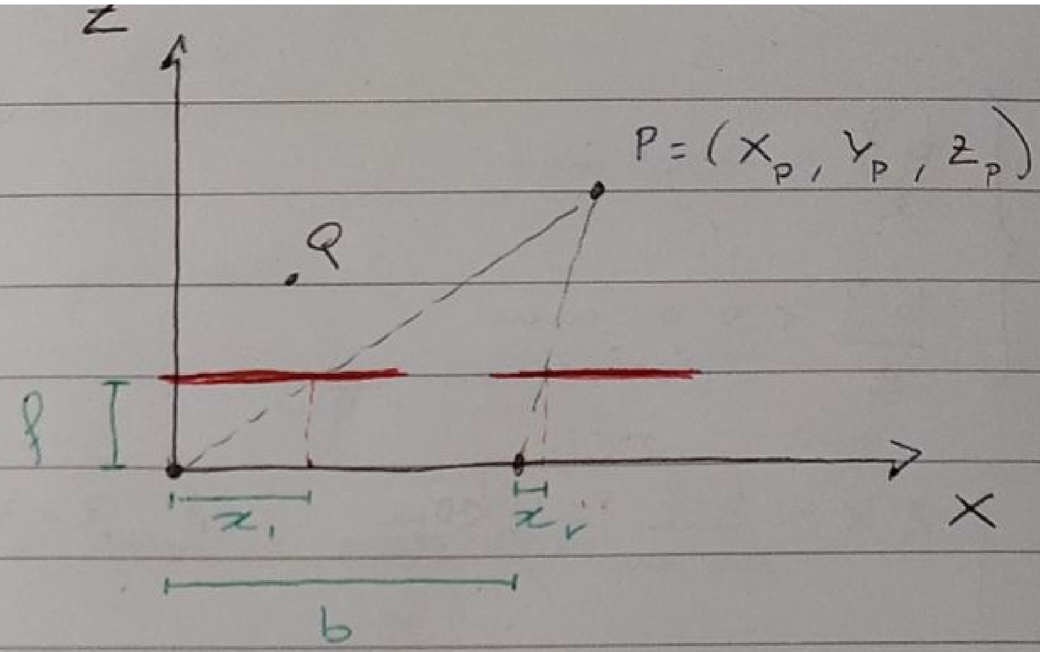


So we have that:

$$\left.\begin{aligned} \overrightarrow{O'p'} &= x' \\ \overrightarrow{Op} &= Rx \\ \overrightarrow{O'O} &= t \end{aligned}\right\} \Rightarrow \begin{aligned} &\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op} \right) = \\ &x'^T \cdot (t \times (Rx)) = \\ &x'^T \cdot (\underbrace{[t]_\times R}_{E} x) = \end{aligned}$$

$$x'^T E x = 0$$

Ex 3)



a) Knowing that:

$d = 1\text{cm} = |x_l - x_r|$

$b = 6\text{cm}$

$f = 1\text{cm}$

and that: $d = \dfrac{b \cdot f}{Z_p}$ and $Z_p > \alpha$

we can write:

$$Z_p = \frac{b \cdot f}{d} = 6\text{cm}$$

---

b) Knowing that:

$d_{min} = 1\ px$

$px\ width = 0.01\ mm$

The range of Z-coordinates for which the disparity is below 1px is:

$$-0.01\ mm < \frac{b \cdot f}{Z} < 0.01\ mm$$

$$\Rightarrow \begin{cases} \dfrac{b \cdot f}{Z} > -0.01 \text{ mm} \\ \dfrac{b \cdot f}{Z} < 0.01 \text{ mm} \end{cases}$$

$$\Rightarrow \begin{cases} Z < -\dfrac{b \cdot f}{0.01 \text{ mm}} = -60 \text{ m} & \text{if } Z < 0 \\ Z > \dfrac{b \cdot f}{0.01 \text{ mm}} = 60 \text{ m} & \text{if } Z > 0 \end{cases}$$

Thus: $Z < -60\text{ m} \vee Z > 60\text{ m}$

ⓒ Knowing that:

$Q = (3, 0, 3)$

We have that the image of Q on the left camera's image plane is:

$$Q_1 = f \cdot \begin{bmatrix} X_Q / Z_Q \\ Y_Q / Z_Q \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

And the corresponding epipolar line on the right camera's image plane is:

$$\ell_r = E\, Q_1 = [t]_\times R\, Q_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & +6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ +6 \\ 0 \end{bmatrix} \quad \text{or} \quad \ell_r : +6 \text{ cm} \cdot y = 0$$

exercise12

December 2, 2022

[3]:
```
# This cell is used for creating a button that hides/unhides code cells to␣
 ↪quickly look only the results.
# Works only with Jupyter Notebooks.

from IPython.display import HTML

HTML('''<script>
code_show=true;
function code_toggle() {
if (code_show){
$('div.input').hide();
} else {
$('div.input').show();
}
code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
<form action="javascript:code_toggle()"><input type="submit" value="Click here␣
 ↪to toggle on/off the raw code."></form>''')
```

[3]: <IPython.core.display.HTML object>

[4]:
```
# Description:
#   Exercise12 notebook.
#
# Copyright (C) 2018 Santiago Cortes, Juha Ylioinas
#
# This software is distributed under the GNU General Public
# Licence (version 2 or later); please refer to the file
# Licence.txt, included with the software, for details.

# Preparations
import os
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

```python
# Select data directory
if os.path.isdir('/coursedata'):
    # JupyterHub
    course_data_dir = '/coursedata'
elif os.path.isdir('../../../coursedata'):
    # Local installation
    course_data_dir = '../../../coursedata'
else:
    # Docker
    course_data_dir = '/home/jovyan/work/coursedata/'

print('The data directory is %s' % course_data_dir)
data_dir = os.path.join(course_data_dir, 'exercise-12-data')
print('Data stored in %s' % data_dir)
```

```
The data directory is /coursedata
Data stored in /coursedata/exercise-12-data
```

Fill your name and student number below.

0.0.1 Name: Tommaso Brumani

0.0.2 Student number: 100481325

1 CS-E4850 Computer Vision Exercise Round 12

The problems should be solved before the exercise session and solutions returned via MyCourses. Upload to MyCourses both: this Jupyter Notebook (.ipynb) file containing your solutions to the programming tasks and the exported pdf version of this Notebook file. If there are both programming and pen & paper tasks kindly combine the two pdf files (your scanned/LaTeX solutions and the exported Notebook) into a single pdf and submit that with the Notebook (.ipynb) file. Note that (1) you are not supposed to change anything in the utils.py and (2) you should be sure that everything that you need to implement should work with the pictures specified by the assignments of this exercise round.

1.0.1 Make sure to complete the pen and paper exercices in the PDF attached.

1.1 Fundamental matrix estimation.

a) Implement the eight-point algorithm as explained on slide 28 of Lecture 11. Note the skeleton function and follow the input output structure

b) Implement the normalized eight-point algorithm as explained on slide 31 of Lecture 11 (Algorithm 11.1. in Hartley & Zisserman).

The epipolar lines obtained with both F-matrix estimates should be close to those visualized by the example script.

[25]:

```python
def estimateF(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.

    # build x_tilde vector
    n = 0
    x_tilde = np.zeros([x1.shape[1], 9])
    for i in range(3):
        for j in range(3):
            x_tilde[:,n] = x2[i,:] * x1[j,:]
            n += 1

    # solve the homogeneous linear equation (eigenvector associated with
    # smallest eigenvalue of x_tilde)
    x_squared = np.dot(x_tilde.T, x_tilde)
    e_vals, e_vecs = np.linalg.eig(x_squared)
    e = e_vecs[:, np.argmin(e_vals)]

    # reshape into a 3x3 matrix
    E_est = e.reshape([3,3])

    # enforce the internal constraint (rank = 2)
    U, s, V = np.linalg.svd(E_est)

    # rebuild F
    s_constrained = np.diag([s[0], s[1], 0])
    F = U @ s_constrained @ V.T

    return F

def estimateFnorm(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix based␣
↪on normalized homogeneous points.

    # compute centroids
    centr_1 = x1.mean(axis=1)
    centr_2 = x2.mean(axis=1)

    # compute mean distance from centroids
```

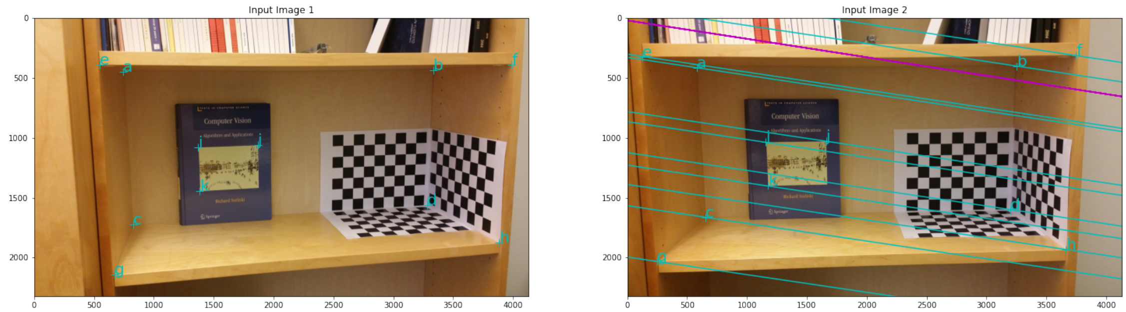
```
    mean_dist_1 = np.sqrt(np.sum(np.square(x1[0, :])) + np.sum(np.square(x1[1,:
↪])))
    mean_dist_2 = np.sqrt(np.sum(np.square(x2[0, :])) + np.sum(np.square(x2[1,:
↪])))

    t1 = np.sqrt(2 * x1.shape[1]) / mean_dist_1
    t2 = np.sqrt(2 * x2.shape[1]) / mean_dist_2

    # compute normalizing transformations
    T1 = np.array([
        [t1, 0, -t1 * centr_1[0]],
        [0, t1, -t1 * centr_1[0]],
        [0, 0, 1]
    ])
    T2 = np.array([
        [t2, 0, -t2 * centr_2[0]],
        [0, t2, -t2 * centr_2[0]],
        [0, 0, 1]
    ])

    # normalize x1 and x2
    x1_normalized = T1 @ x1
    x2_normalized = T2 @ x2

    # compute F from normalized points
    F_normalized = estimateF(x1_normalized, x2_normalized)

    # transform F back into original units
    F = T2.T @ F_normalized @ T1

    return F

def vgg_F_from_P(P1,P2):
    # Return the fundamental matrix F (3 by 3), based on two camera parameter␣
↪arrays.
    # Input: P1, P2 numpy ndarray (3 by 4) containing intrinsic and extrinsic␣
↪parameters.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    X=[]
    Y=[]
    X.append(P1[[1,2],:])
    X.append(P1[[2,0],:])
    X.append(P1[[0,1],:])
    Y.append(P2[[1,2],:])
    Y.append(P2[[2,0],:])
    Y.append(P2[[0,1],:])
    F=np.zeros([3,3])
```

```python
    for i in range(3):
        for j in range(3):
            M=np.concatenate([X[j],Y[i]])
            F[i,j]=np.linalg.det(M)
    return F
```

[26]:
```python
# Point locations
x1 = 1.0e+03*np.array([0.7435,3.3315,0.8275,3.2835,0.5475,3.9875,0.6715,3.
↪8835,1.3715,1.8675,1.3835])
y1 = 1.0e+03*np.array([0.4455,0.4335,1.7215,1.5615,0.3895,0.3895,2.1415,1.
↪8735,1.0775,1.0575,1.4415])
x2 = 1.0e+03*np.array([0.5835,3.2515,0.6515,3.1995,0.1275,3.7475,0.2475,3.
↪6635,1.1555,1.6595,1.1755])
y2 = 1.0e+03*np.array([0.4135,0.4015,1.6655,1.5975,0.3215,0.3135,2.0295,1.
↪9335,1.0335,1.0255,1.3975])

# Camera parameters
P1= np.row_stack([[-0.001162918366053,0.000102986385133,-0.000344703214391,0.
↪995200644722518],\
                  [-0.000019974831639,0.001106889654747,-0.000150591916681,0.
↪097841118173777],\
                  [-0.000000053632777,0.000000044849673,-0.000000270734766,0.
↪000249501614496]])

P2= np.row_stack([[-0.001272880601540, 0.000093061493378,-0.000574486218854, 0.
↪996457618133488],\
                  [-0.000002971652037, 0.001271207503106,-0.000200323351541, 0.
↪084074548573989],\
                  [-0.000000020226464, 0.000000043518811,-0.000000316928290, 0.
↪000265554210072]])

# Make homogenous representations of points
pts1=np.row_stack([x1,y1,np.ones_like(x1)])
pts2=np.row_stack([x2,y2,np.ones_like(x2)])

# Read images
# Read images
im1 = cv2.imread(data_dir+'/im1.jpg')
im2 = cv2.imread(data_dir+'/im2.jpg')

im1 = cv2.cvtColor(im1, cv2.COLOR_BGR2RGB)
im2 = cv2.cvtColor(im2, cv2.COLOR_BGR2RGB)



# Labels
```

```python
labels = ['a','b','c','d','e','f','g','h','i','j','k']

# Create figure
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(25,25))
ax = axes.ravel()
ax[0].imshow(im1)
ax[0].plot(x1, y1, 'c+', markersize=10)

# Put labels
for i in range(len(x1)):
    ax[0].annotate(labels[i], (x1[i], y1[i]), color='c', fontsize=20)
ax[0].set_title("Input Image 1")
ax[1].imshow(im2)
ax[1].plot(x2, y2, 'c+', markersize=10)
for i in range(len(x2)):
    ax[1].annotate(labels[i], (x2[i], y2[i]), color='c', fontsize=20)
ax[1].set_title("Input Image 2")

# Get ground truth fundamental matrix
F=vgg_F_from_P(P1,P2)

# Create lines
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)

#  Plot lines
px=np.array([0,np.shape(im2)[1]])
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'c-');

# Get fundamental matrix and draw epipolar lines
F=estimateF(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'m-');

# Get fundamental matrix from normalized algorithm and draw epipolar lines
F=estimateFnorm(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
```

```
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'y-');

ax[1].axes.set_xlim([0,np.shape(im2)[1]])
ax[1].axes.set_ylim([np.shape(im2)[0],0])
plt.show()
```



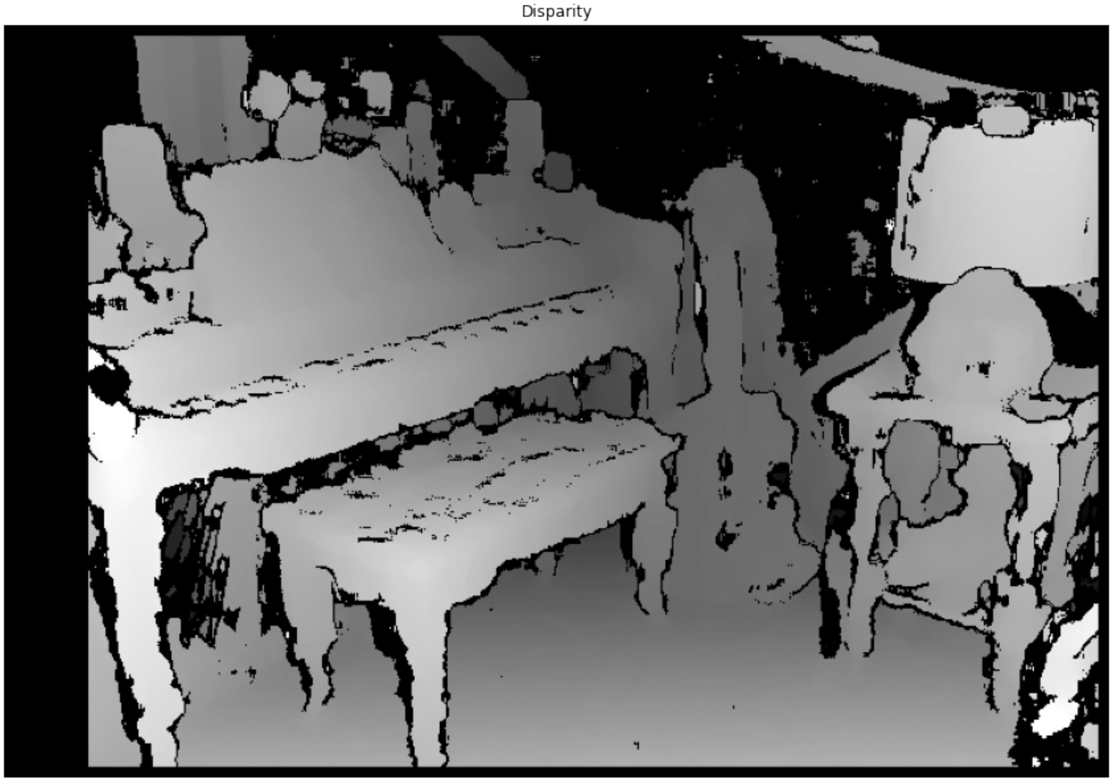
1.2 Demo. Stereo disparity computation. (Just a demo, no points given)

Run and study the opencv stereo disparity and depth estimation.

[27]:
```
# Import images
sc=0.25

imgL = cv2.resize(cv2.imread(data_dir+'/im0.png',0), (0,0), fx=sc, fy=sc)
imgR = cv2.resize(cv2.imread(data_dir+'/im1.png',0), (0,0), fx=sc, fy=sc)
imgL_col = cv2.resize(cv2.imread(data_dir+'/im0.png'), (0,0), fx=sc, fy=sc)
imgR_col = cv2.resize(cv2.imread(data_dir+'/im1.png'), (0,0), fx=sc, fy=sc)

# Show images
plt.figure(figsize=[15,15])
plt.subplot(121)
plt.imshow(imgL_col[:,:,[2,1,0]])
plt.axis('off')
plt.subplot(122)
plt.imshow(imgR_col[:,:,[2,1,0]])
plt.axis('off')

# Compute disparity
stereo = cv2.StereoBM_create(numDisparities=16*3, blockSize=15)
disparity = stereo.compute(imgL,imgR)
```

```python
# Show disparity
plt.figure(figsize=[15,15])
plt.imshow(disparity,'gray')
plt.axis('off')
plt.title('Disparity')
#ndistp=cv2.guidedFilter(imgL, disparity, 9, 4,0.1)

# Calibration data
baseline=17.8089 #cm
f_length=2826.171*sc #pixels
c_point=np.array([1415.97,965.806])*sc # pixels

# Get depth from disparity
point=np.zeros([np.count_nonzero(disparity>1),6])
ind=0
for i in range(np.shape(disparity)[0]):
    for j in range(np.shape(disparity)[1]):
        if disparity[i,j]>1:
            # Save point information into point cloud
            # [pixel_x,pixel_y,disparity,color]
            point[ind,0:3]=j,i,disparity[i,j]
            point[ind,3:6]=imgL_col[i,j]/255.0
            ind+=1
# Z=baseline*focal/disparity
# openCV disparity is (16*actual_disparity). This depends on the algorithm.
# It is in order to use signed shorts and keep good subpixel accuracy.
point[:,2]=baseline*f_length/(point[:,2]/16.0)
#X=Z*(pixel_u-center_u)/focal
point[:,0]=point[:,2]*(point[:,0]-c_point[0])/f_length
#Y=Z*(pixel_v-center_v)/focal
point[:,1]=-point[:,2]*(point[:,1]-c_point[1])/f_length

# Delete points on the far background
inl=(point[:,2]<2000)
point=point[inl,:]

plt.show()
```

Disparity



[28]:
```
def visualize_points(pts,R,img,f=1000,cp=[400,300]):
    #visualize colored points given a rotation matrix
    # rotate around the mean of the point cloud
    c=np.mean(point[:,0:3],0)
    #r_point=((point[:,0:3]-c)@R_y)+c
    r_point=np.dot((point[:,0:3]-c),R_y)+c

    #Project back to the same camera model
    K=np.float32([[f,0,cp[0]],[0,f,cp[1]],[0,0,1]])
```

```
    # Sort by depth (painter's algorithm)
    ind=np.argsort(r_point[:,2])
    r_point=r_point[np.flip(ind,0),:]

    #Project
    #uvk=K@r_point.T
    uvk=(np.dot(K,r_point.T))
    color=point[:,[5,4,3]]
    color=color[np.flip(ind,0),:]

    # Normalize homogeneous  coordinates
    uv=uvk[0:2,:]/(uvk[2,:])

    # Draw projected points
    plt.scatter(uv[0,:],uv[1,:],marker='.',s=10,c=color)
    plt.xlim([0,np.shape(imgL)[1]])
    plt.ylim([0,np.shape(imgL)[0]])
    plt.axis('off')



# Visualize points from two different angles
plt.figure(figsize=[30,15])
plt.subplot(121)
# Rotate around y axis to visualize
ang_y=-20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.subplot(122)
# Rotate around y axis to visualize
ang_y=20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.show()
```

[ ]:

```
```