Exercise5

October 6, 2022

```
[1]: # This cell is used for creating a button that hides/unhides code cells to␣
     ↪quickly look only the results.
     # Works only with Jupyter Notebooks.

     from IPython.display import HTML

     HTML('''<script>
     code_show=true;
     function code_toggle() {
     if (code_show){
     $('div.input').hide();
     } else {
     $('div.input').show();
     }
     code_show = !code_show
     }
     $( document ).ready(code_toggle);
     </script>
     <form action="javascript:code_toggle()"><input type="submit" value="Click here␣
     ↪to toggle on/off the raw code."></form>''')
```

```
[1]: <IPython.core.display.HTML object>
```

```
[2]: # Description:
     #   Exercise5 notebook.
     #
     # Copyright (C) 2018 Santiago Cortes, Juha Ylioinas
     #
     # This software is distributed under the GNU General Public
     # Licence (version 2 or later); please refer to the file
     # Licence.txt, included with the software, for details.

     # Preparations
     import os
     import numpy as np
     import matplotlib.pyplot as plt
     import cv2
```

```
# Select data directory
if os.path.isdir('/coursedata'):
    # JupyterHub
    course_data_dir = '/coursedata'
elif os.path.isdir('../../../coursedata'):
    # Local installation
    course_data_dir = '../../../coursedata'
else:
    # Docker
    course_data_dir = '/home/jovyan/work/coursedata/'

print('The data directory is %s' % course_data_dir)
data_dir = os.path.join(course_data_dir, 'exercise-05-data/')
print('Data stored in %s' % data_dir)
```

```
The data directory is /coursedata
Data stored in /coursedata/exercise-05-data/
```

1 CS-E4850 Computer Vision Exercise Round 5

Remember to do the pen and paper assignments given in Exercise05task1.pdf.

The problems should be solved before the exercise session and solutions returned via MyCourses. Upload to MyCourses both: this Jupyter Notebook (.ipynb) file containing your solutions to the programming tasks and the exported pdf version of this Notebook file. If there are both programming and pen & paper tasks kindly combine the two pdf files (your scanned/LaTeX solutions and the exported Notebook) into a single pdf and submit that with the Notebook (.ipynb) file. Note that (1) you are not supposed to change anything in the utils.py and (2) you should be sure that everything that you need to implement should work with the pictures specified by the assignments of this exercise round.

1.1 Robust line fitting using RANSAC.

Run the example script robustLineFitting, which plots a set of points $(x_i, y_i), i = 1, \ldots, n$, and estimate a line that best fits to these points by implementing a RANSAC approach as explained in the slides of Lecture 4:

Repeat the following steps N times (set N large enough according to the guidelines given in the lecture):

- Draw 2 points uniformly at random from set (x_i, y_i).

- Fit a line to these 2 points.

- Determine the inliers to this line among the remaining points (i.e. points whose distance to the line is less than a suitably set threshold t).

Take the line with most inliers from previous stage and refit it using total least squares fitting to all inliers. Plot the estimated line and all the points (x_i, y_i) to the same figure and report the

estimated values of the line's coefficients.

```
[3]: # Load and plot points
     data = np.load(data_dir+'points.npy')
     x, y = data[0,:], data[1,:]
     plt.plot(x, y, 'kx')
     plt.title('Point distribution')
     plt.axis('equal')
     plt.show()
```


Point distribution
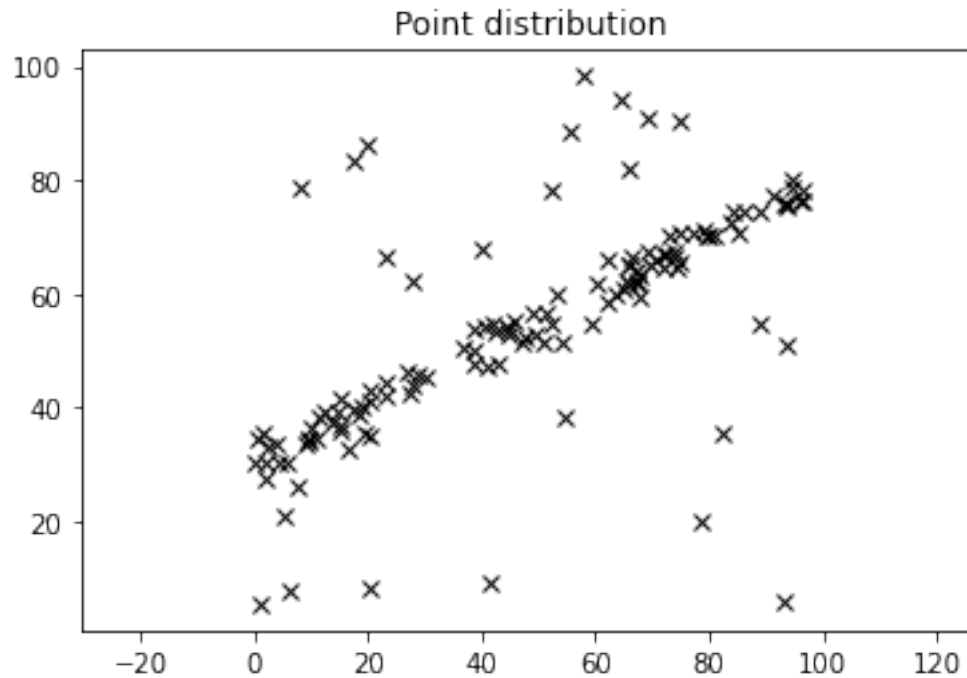
```
[26]: ## Robust line fitting
      ##--your-code-starts-here--##
      # parameters
      N = 1000
      s = 2
      t = 10
      p = 0.95

      # algorithm
      i = 0
      best_inliers = []
      while i < N:
          # draw 2 points at random
          row_idx = np.random.choice(data.shape[1], size=s, replace=False)
          points = data[:, row_idx].T
```

```python
    # find fitting line parameters
    A = np.vstack([points[:,0],np.ones(len(points[:,0]))]).T
    m, c = np.linalg.lstsq(A, points[:,1])[0]

    # identify inliers
    inliers = []
    # for all points in dataset
    for p in range(data.shape[1]):

        # calculate distance
        dist = np.abs(data[1,p] - (m*data[0,p]+c)) / np.sqrt(1+m**2)

        # add to inliers if distance below threshold
        if dist < t:
            inliers.append(data[:,p])

    # check if line with most inliers
    if len(inliers) > len(best_inliers):
        best_inliers = np.array(inliers)

        # update N
        outlier_ratio = 1 - len(inliers)/len(data[0])
        N = np.log(1-p) / np.log(1 - ((1 - outlier_ratio)**2))

    # increment counter
    i += 1

print(best_inliers.shape)

# fit line to best inliers
A_best = np.vstack([best_inliers[:,0], np.ones(len(best_inliers[:,0]))]).T
m_best, c_best = np.linalg.lstsq(A_best,best_inliers[:,1])[0]

# print parameters
print("Best parameters: m = {}, c = {}".format(m_best,c_best))

# plot
plt.plot(x, y, 'kx')
axes = plt.gca()
x_line = np.array(axes.get_xlim())
y_line = c_best + m_best * x_line
plt.plot(x_line, y_line, '--')
plt.title('Best fit')
plt.axis('equal')
plt.show()
```

```
##--your-code-ends-here--##
```

```
/opt/conda/lib/python3.7/site-packages/ipykernel_launcher.py:19: FutureWarning:
`rcond` parameter will change to the default of machine precision times ``max(M,
N)`` where M and N are the input matrix dimensions.
To use the future default and silence this warning we advise to pass
`rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
/opt/conda/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning:
invalid value encountered in log
/opt/conda/lib/python3.7/site-packages/ipykernel_launcher.py:48: FutureWarning:
`rcond` parameter will change to the default of machine precision times ``max(M,
N)`` where M and N are the input matrix dimensions.
To use the future default and silence this warning we advise to pass
`rcond=None`, to keep using the old, explicitly pass `rcond=-1`.
```
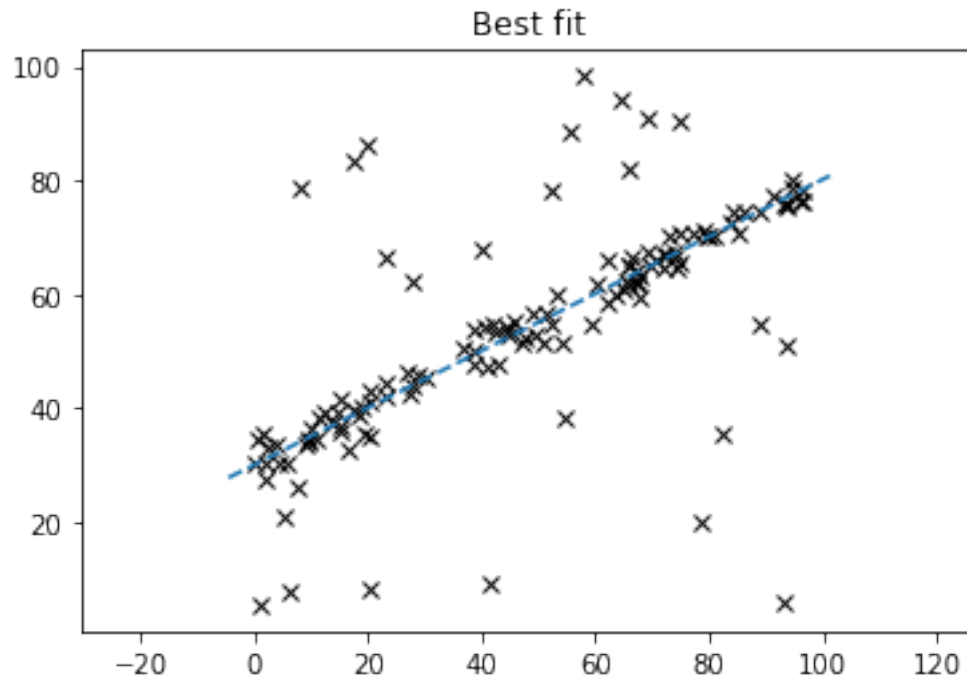
```
(106, 2)
Best parameters: m = 0.5016389403367025, c = 29.995086013408482
```


1.2 Line detection by Hough transform. (Just a demo, no points given)

Run the example cell below, which illustrates line detection by Hough transform using opencv built-in functions.

```python
[27]: #DEMO CELL
      # Logistic sigmoid function
```

```python
def sigm(x):
    return 1 / (1 + np.exp(-x))

# This demo detects the Canny edges for the input image,
# calculates the Hough transform for the Canny edge image,
# displays the Hough votes in an acculumator array
# and finally draws the detected lines

# Read image
I = cv2.imread(data_dir+'board.png', 0)
r, c = I.shape

plt.figure(1)
plt.imshow(I, cmap='bone')
plt.title('Original image')
plt.axis('off')
# Find Canny edges. The input image for cv2.HoughLines should be
# a binary image, so a Canny edge image will do just fine.
# The Canny edge detector uses hysteresis thresholding, where
# there are two different threshold levels.
edges = cv2.Canny(I, 80, 130)
plt.figure(2)
plt.imshow(edges, cmap='gray')
plt.title('Canny edges')
plt.axis('off')
# Compute the Hough transform for the binary image returned by cv2.Canny
# cv2.HoughLines returns 2-element vectors containing (rho, theta)
# cv2.HoughLines(input image, radius resolution(pixels), angular resolution
 ↪(radians),treshold )
H = cv2.HoughLines(edges, 0.5, np.pi/180, 5)

# Display the transform
theta = H[:,0,1].ravel()
rho = H[:,0,0].ravel()

# Create an acculumator array and the bin coordinates for voting
x_coord = np.arange(0, np.pi, np.pi/180)
y_coord = np.arange(np.amin(rho), np.amax(rho)+1, (np.amax(rho)+1)/50)

acc = np.zeros([np.size(y_coord),np.size(x_coord)])

# Perform the voting
for i in range(np.size(theta)):
    x_id = np.argmin(np.abs(x_coord-theta[i]))
    y_id = np.argmin(np.abs(y_coord-rho[i]))
    acc[y_id, x_id] +=  1
```

```python
# Pass the values through a logistic sigmoid function and normalize
# (only for the purpose of better visualization)
#acc = sigm(acc)
acc /= np.amax(acc)

plt.figure(3)
plt.imshow(acc,cmap='bone')
plt.axis('off')

plt.title('Hough transform space')

# Compute the Hough transform with higher threshold
# for displaying ~30 strongest peaks in the transform space
H2 = cv2.HoughLines(edges, 1, np.pi/180, 150)

x2 = H2[:,:,1].ravel()
y2 = H2[:,:,0].ravel()

# Superimpose a plot on the image of the transform that identifies the peaks
plt.figure(3)
for i in range(np.size(x2)):
    x_id = np.argmin(abs(x_coord-x2[i]))
    y_id = np.argmin(abs(y_coord-y2[i]))
    plt.plot(x_id, y_id, 'xr','Linewidth',0.1)

# Visualize detected lines on top of the Canny edges.
plt.figure(4)
plt.imshow(I, cmap='bone')
plt.title('Detected lines')
plt.axis('off')


for ind in range(0,len(H2)):
    line=H2[ind,0,:]
    rho=line[0]
    theta=line[1]
    a = np.cos(theta)
    b = np.sin(theta)
    x0 = a*rho
    y0 = b*rho
    x1 = int(x0 + 1000*(-b))
    y1 = int(y0 + 1000*(a))
    x2 = int(x0 - 1000*(-b))
    y2 = int(y0 - 1000*(a))

    plt.plot((x1,x2),(y1,y2))
```

```
#plt.plot(xk, yk, 'm-')
plt.xlim([0,np.size(I,1)])
plt.ylim([0,np.size(I,0)])
plt.show()
```
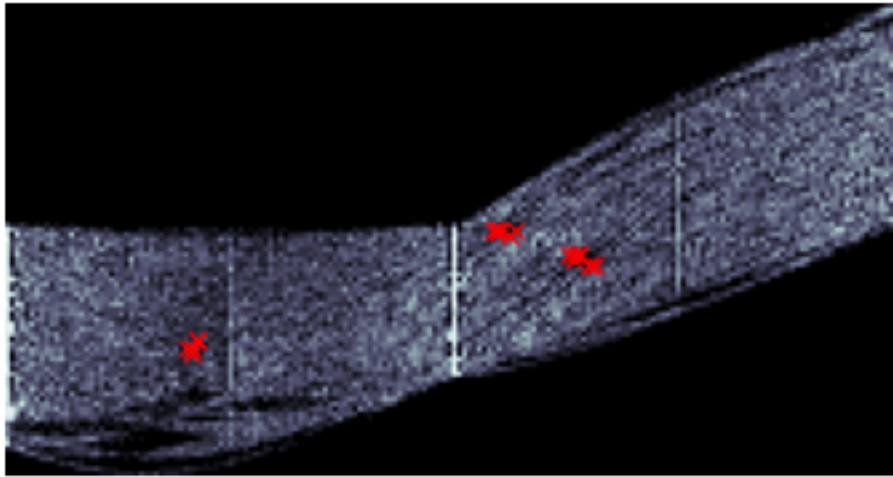
Original image

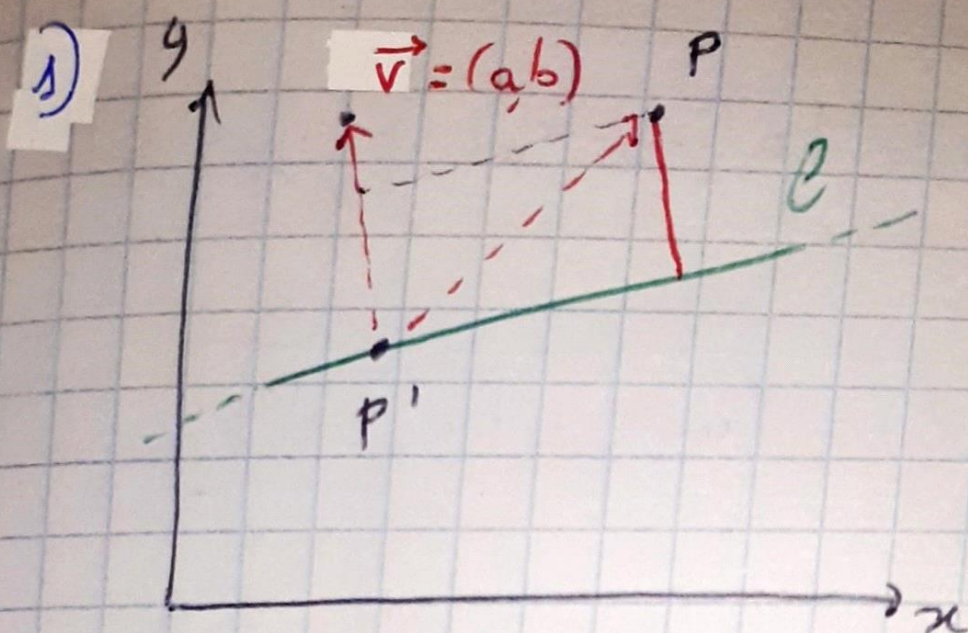Canny edges

Hough transform space

Detected lines

[ ]:

COMPUTER VISION Ex 5

1)

we have:

$$\ell: ax + by - d = 0$$

with $a^2 + b^2 = 1$

$$P: (x_P, y_P)$$

$$P': (x'_P, y'_P)$$

Let P' be any point on line ℓ and $\vec{v}$ be the the vector (a,b) starting at point P' and perpendicular to ℓ: we have that the distance between P and ℓ is the orthogonal projection of $\overrightarrow{P'P}$ on $\vec{v}$.

The length of this projection is:

$$dist(P, \ell) = \frac{|\overrightarrow{P'P} \cdot \vec{v}|}{\|\vec{v}\|}$$

We have that:

$$\overrightarrow{P'P} = (x_P - x'_P, y_P - y'_P)$$

$$\hookrightarrow \overrightarrow{P'P} \cdot \vec{v} = a(x_P - x'_P) + b(y_P - y'_P)$$

We also have that: $\|\vec{v}\| = \sqrt{a^2+b^2} = 1$

Thus:

$$dist(P,\ell) = |a(x_p - x'_p) + b(y_p - y'_p)|$$

Since $p' \in \ell$:

$$\boxed{dist(P,\ell) = |ax_p + by_p - d|}$$

---

② Sum of squared distances between points (x_i, y_i)

and line ℓ :

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

To find minimum:

$$\frac{\delta E}{\delta d} = \sum_{i=1}^{n} \frac{\delta}{\delta d}\left[(ax_i + by_i - d)^2\right]$$

$$= \sum_{i=1}^{n} (+2)(ax_i + by_i - d)(-1)$$

$$= -2\sum_{i=1}^{n}(ax_i + by_i - d) = 0$$

$$\longrightarrow \sum_{i=1}^{n}(ax_i + by_i) = nd$$

$$\longrightarrow \boxed{d = \frac{1}{n}\sum_{i=1}^{n}(ax_i + by_i)}$$

③ Substituting d in E we have:

$$E = \sum_{i=1}^{n} \left[ax_i + by_i - \frac{1}{n}\sum_{j=1}^{n}(ax_j + by_j) \right]^2$$

$$= \sum_{i=1}^{n} \left[ax_i - \frac{1}{n}\sum_{j=1}^{n} ax_j + by_i - \frac{1}{n}\sum_{j=1}^{n} by_j \right]^2$$

$$= \sum_{i=1}^{n} \left[a\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right) + b\left(y_i - \frac{1}{n}\sum_{j=1}^{n} y_j\right) \right]^2$$

as a vector product

$$= \sum_{i=1}^{n} \left[\left[\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right) \quad \left(y_i - \frac{1}{n}\sum_{j=1}^{n} y_j\right) \right] \cdot \begin{bmatrix} a \\ b \end{bmatrix} \right]^2$$

defining matrix

$$U = \begin{bmatrix} x_1 - \frac{1}{n}\sum_{j=1}^{n} x_j & y_1 - \frac{1}{n}\sum_{j=1}^{n} y_j \\ & \vdots & \\ x_n - \frac{1}{n}\sum_{j=1}^{n} x_j & y_n - \frac{1}{n}\sum_{j=1}^{n} y_j \end{bmatrix}$$

we can write

$$E = \left(U \cdot [a\ b]^T \right)^2 = \left(U \cdot [a\ b]^T \right)^T \cdot \left(U \cdot [a\ b]^T \right)$$

$$= \boxed{[a\ b]\, U^T\, U\, [a\ b]^T}$$

(