GARCH Analysis Report

Table of Contents

1. Introduction

2. Preliminary Statistical Analysis

3. GARCH(1,1) Model Estimation

4. Comparison of GARCH Model Specifications

5. One-Step-Ahead Forecasting and Diebold-Mariano Test

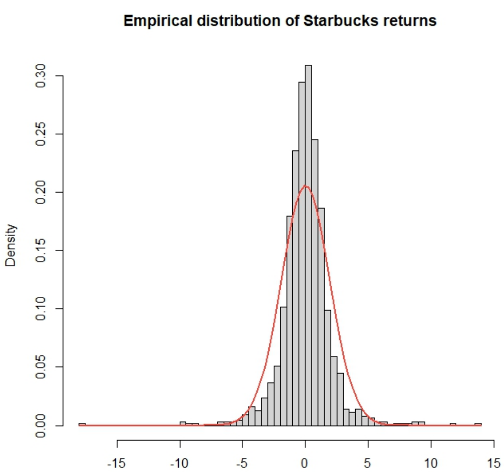
6. Rolling Correlation of Standardized Residuals

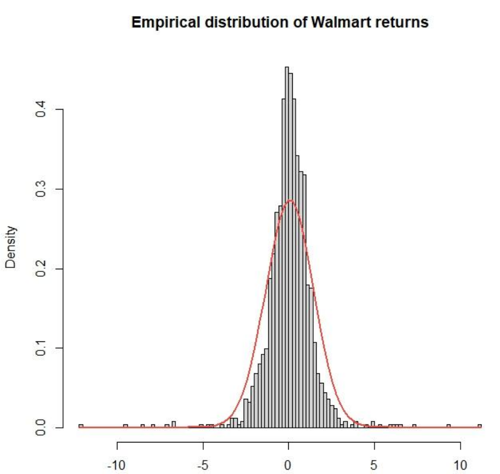
1. Introduction

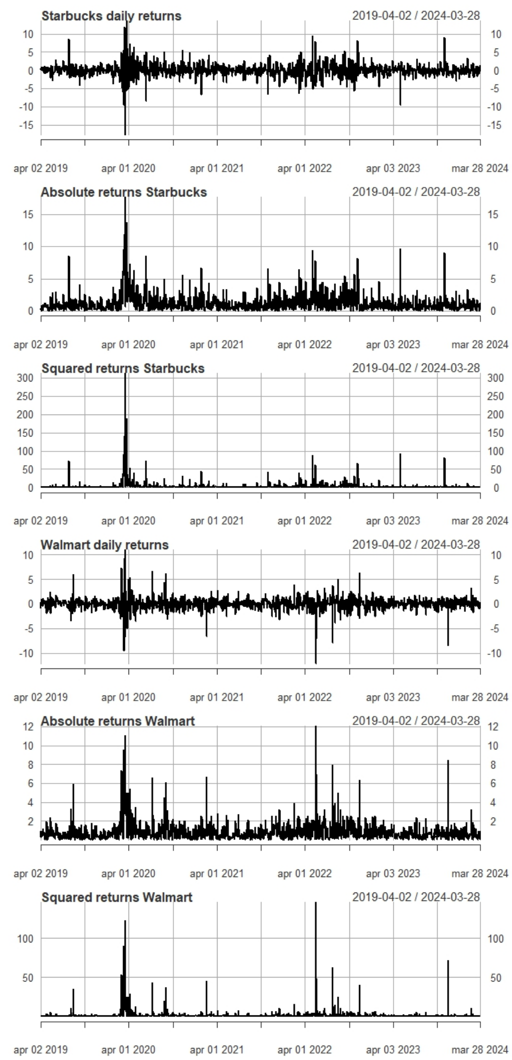
This report provides an analysis of volatility modeling using GARCH-type models for two financial instruments: Starbucks Corporation (SBUX.O) and Walmart Inc. (WMT). The objective is to examine the presence of heteroskedasticity in the return series, fit appropriate GARCH models, compare their predictive performance, and explore the temporal correlation structure of the standardized residuals.

2. Preliminary Statistical Analysis

The return series of both Starbucks and Walmart exhibit characteristics typical of financial time series: non-normality, volatility clustering, and serial correlation in squared returns.







These are confirmed through graphical analysis, the Jarque-Bera test for normality, the LM-ARCH test for heteroskedasticity, and the Ljung-Box test for serial correlation. The results indicate strong evidence of conditional heteroskedasticity, justifying the use of GARCH models.

3. GARCH(1,1) Model Estimation

A GARCH(1,1) model with normally distributed innovations was estimated using data up to March 2023.

SBUX: GARCH(1,1) – norm

Optimal Parameters

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.075257	0.050955	1.4769	0.139693
omega	0.296889	0.087543	3.3913	0.000696
alpha1	0.133737	0.030724	4.3529	0.000013
beta1	0.782598	0.048583	16.1085	0.000000

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.25760	0.7968	
Negative Sign Bias	0.61225	0.5405	
Positive Sign Bias	0.08919	0.9289	
Joint Effect	0.81946	0.8448	

WMT: GARCH(1,1) - norm

Optimal Parameters

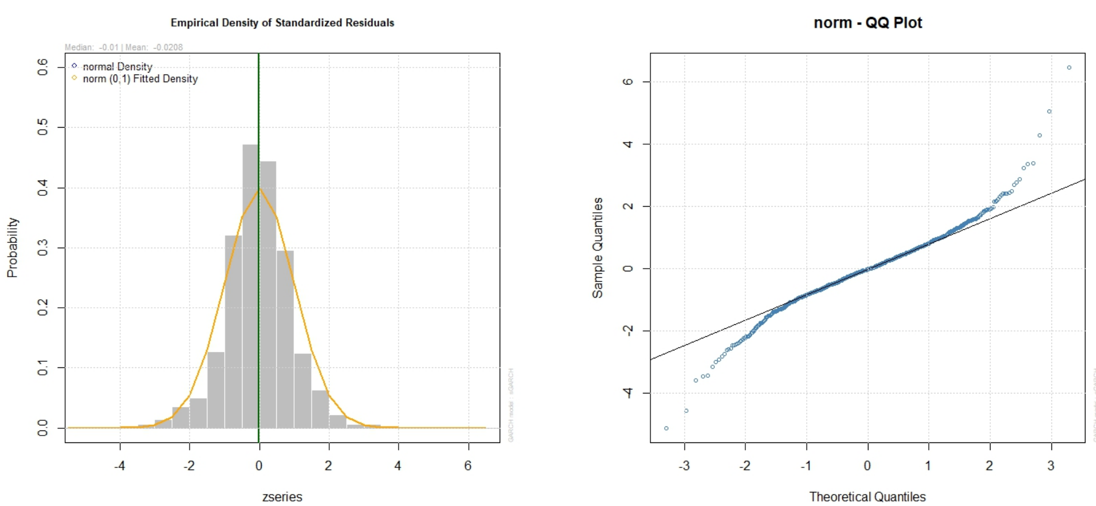
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.044673	0.037708	1.1847	0.236133
omega	0.118304	0.039837	2.9697	0.002981
alpha1	0.104144	0.019421	5.3624	0.000000
beta1	0.843521	0.030703	27.4739	0.000000

Sign Bias Test

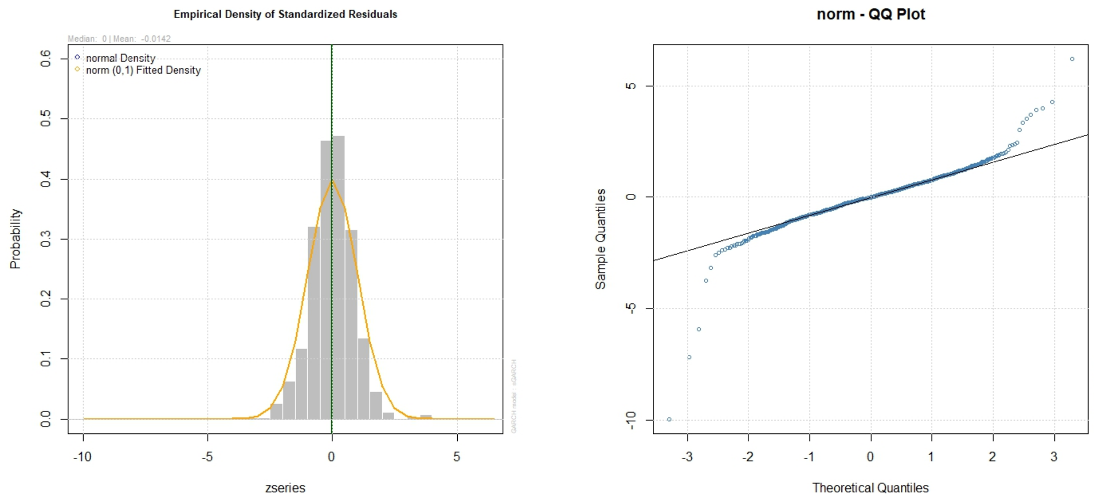
	t-value	prob	sig
Sign Bias	0.7074	0.4795	
Negative Sign Bias	1.1891	0.2347	
Positive Sign Bias	1.0851	0.2782	
Joint Effect	2.7723	0.4281	

For both assets, the estimated models showed the sum of alpha and beta parameters below 1, indicating stationarity and the non-significance of mu. Moreover, the sign bias tests showed that there is no asymmetry in residuals. However, the standardized residuals exhibited non-normality, suggesting the normal distribution is not adequate for the innovations.

SBUX: GARCH(1,1) – norm



WMT: GARCH(1,1) - norm



Further diagnostic checks showed no significant ARCH effects remaining in the residuals, indicating that the GARCH(1,1) model captured the volatility clustering effectively.

4. Comparison of GARCH Model Specifications

To improve the fit, alternative GARCH-type models were estimated, including EGARCH, GJR-GARCH, and APARCH, with both normal and Student-t innovations. Model comparison was based on information criteria (AIC, BIC, etc.) and graphical diagnostics.

	GARCH-norm	EGARCH-norm	GJR-norm	APARCH-norm	GARCH-std	EGARCH-std	GJR-std	APARCH-std
Akaike	4973.143	4966.522	4955.575	4948.558	4822.854	4815.803	4815.828	4809.253
Bayes	4997.664	4997.172	4986.225	4985.338	4853.504	4852.583	4852.608	4852.163
Shibata	4973.104	4966.461	4955.514	4948.469	4822.792	4815.715	4815.739	4809.133
Hannan-Quinn	4982.459	4978.167	4967.220	4962.531	4834.498	4829.777	4829.801	4825.556

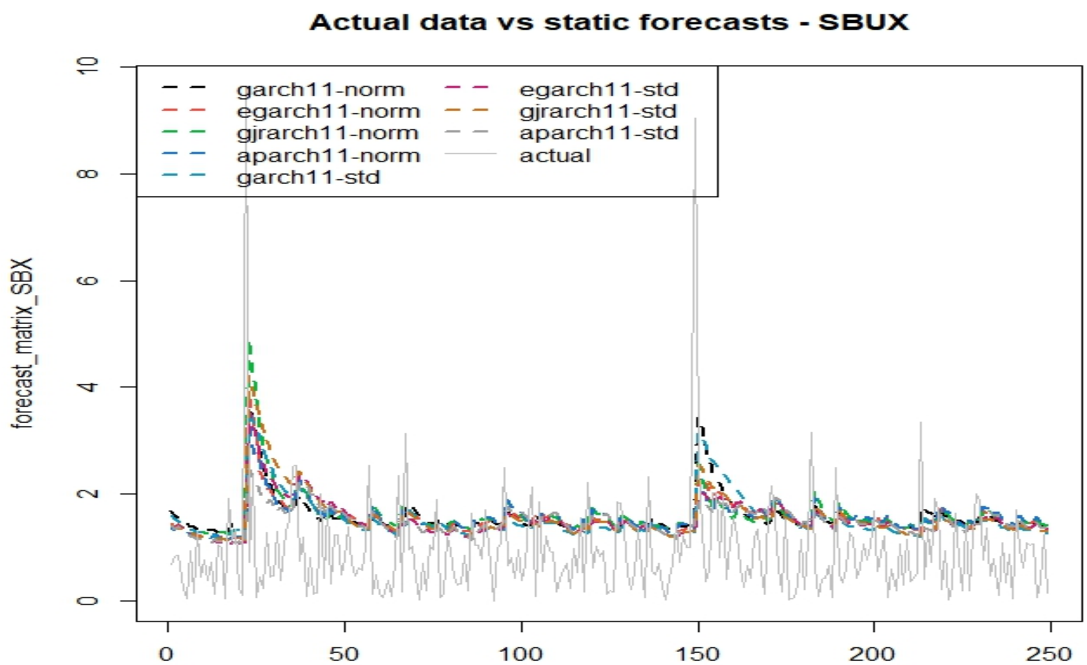
For Starbucks, the APARCH(1,1) model with Student-t innovations provided the best fit across all information criteria.

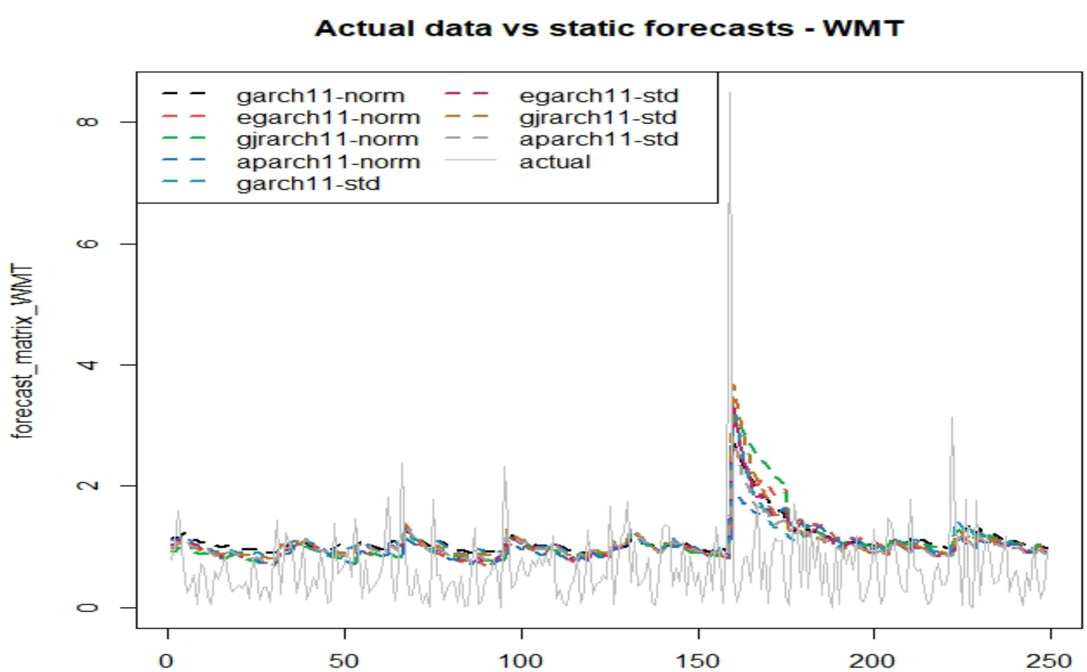
	GARCH-norm	EGARCH-norm	GJR-norm	APARCH-norm	GARCH-std	EGARCH-std	GJR-std	APARCH-std
Akaike	4255.324	4209.156	4229.174	4206.767	3955.648	3950.022	3952.253	3949.952
Bayes	4279.844	4239.806	4259.825	4243.547	3986.298	3986.802	3989.034	3992.862
Shibata	4255.284	4209.094	4229.113	4206.678	3955.586	3949.934	3952.165	3949.832
Hannan-Quinn	4264.640	4220.801	4240.819	4220.740	3967.292	3963.996	3966.227	3966.254

For Walmart, the best model varied slightly depending on the criterion, but the APARCH(1,1) model with normal innovations was generally among the top-performing models.

5. One-Step-Ahead Forecasting and Diebold-Mariano Test

Static one-step-ahead volatility forecasts from April 2023 to March 2024 were generated using all fitted models.





The performance of the models was assessed using the Diebold-Mariano test, which compares predictive accuracy between model pairs.

We obtain the results from a table constructed so that each cell contains the DM test statistic, calculated as the difference between the loss function of the model in the corresponding row and that of the model in the corresponding column.
Therefore, a positive value indicates that the loss function of the model in the row is greater than that of the model in the column.
Since the DM test statistic asymptotically follows a N(0,1) distribution, we consider values with an absolute magnitude greater than 1.64 to be significant.
If the value is less than -1.64, the model in the row has significantly better predictive performance than the model in the column. Conversely, if the value is greater than 1.64, the model in the column has significantly better predictive performance than the model in the row.

	garch11_norm	egarch11_norm	gjrgarch11_norm	aparch11_norm	garch11_std	egarch11_std	gjrgarch11_std	aparch11_std
garch11_norm	NA	1.03250040	-0.8731069	0.7157661	1.19261809	1.5707007	0.008068564	2.216233
egarch11_norm	-1.032500402	NA	-1.7770057	-0.1647667	-0.06151333	1.8087997	-0.693751431	2.618747
gjrgarch11_norm	0.873106935	1.77700571	NA	1.3323392	1.36712752	2.4372529	1.375200991	2.255507
aparch11_norm	-0.715766051	0.16476668	-1.3323392	NA	0.01269230	1.6299310	-0.512276595	4.354742
garch11_std	-1.192618091	0.06151333	-1.3671275	-0.0126923	NA	0.9191129	-0.740730614	1.513298
egarch11_std	-1.570700693	-1.80879971	-2.4372529	-1.6299310	-0.91911289	NA	-1.740890667	1.570203
gjrgarch11_std	-0.008068564	0.69375143	-1.3752010	0.5122766	0.74073061	1.7408907	NA	1.692789
aparch11_std	-2.216233336	-2.61874738	-2.2555074	-4.3547416	-1.51329842	-1.5702026	-1.692788831	NA

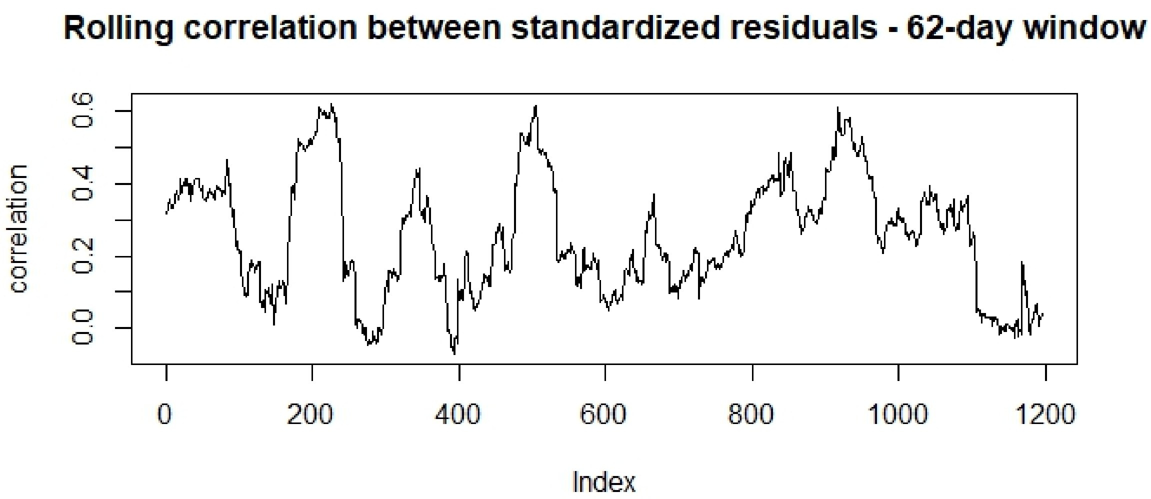
For Starbucks, the APARCH(1,1) model with Student-t innovations significantly outperformed all models with normal innovations and the GJR-GARCH(1,1) with Student-t innovations.

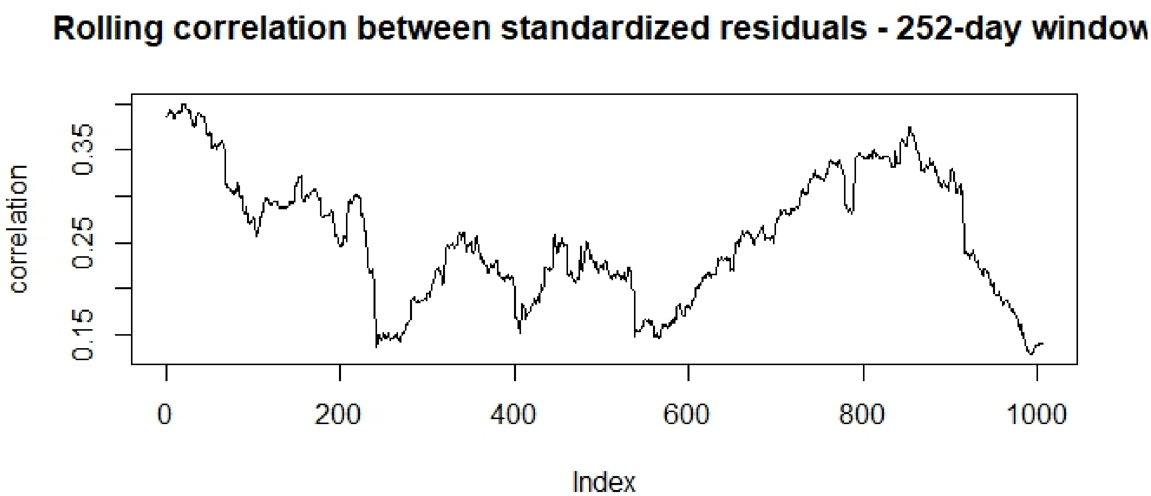
	garch11_norm	egarch11_norm	gjrgarch11_norm	aparch11_norm	garch11_std	egarch11_std	gjrgarch11_std	aparch11_std
garch11_norm	NA	3.17193487	-0.2020811	4.479841	2.1837610	3.29172725	0.5466890	6.394624
egarch11_norm	-3.1719349	NA	-1.9188373	2.221251	-0.5098833	0.01949479	-1.0308645	1.454533
gjrgarch11_norm	0.2020811	1.91883727	NA	2.082475	1.0312835	1.66895106	0.8695902	1.942453
aparch11_norm	-4.4798410	-2.22125149	-2.0824751	NA	-2.1799708	-1.85838244	-1.8237196	-1.478110
garch11_std	-2.1837610	0.50988333	-1.0312835	2.179971	NA	1.19521739	-0.8449644	2.924441
egarch11_std	-3.2917272	-0.01949479	-1.6689511	1.858382	-1.1952174	NA	-1.6832784	2.098394
gjrgarch11_std	-0.5466890	1.03086452	-0.8695902	1.823720	0.8449644	1.68327841	NA	1.933028
aparch11_std	-6.3946241	-1.45453311	-1.9424531	1.478110	-2.9244411	-2.09839423	-1.9330280	NA

For Walmart, the APARCH(1,1)-normal model showed competitive predictive accuracy, although no clear dominance was observed across all model pairs.

6. Rolling Correlation of Standardized Residuals

Using the best-performing models, standardized residuals were obtained for both assets. Rolling correlations were computed using two window sizes: 62 and 252 observations. The average rolling correlation over the shorter window was 0.2553 and over the longer window was 0.2550. Although the averages are similar, graphical analysis reveals fluctuations in correlation over time, indicating that the dependency structure is not constant.


Rolling correlation between standardized residuals - 62-day window
correlation
0.0
0.2
0.4
0.6
0
200
400
600
800
1000
1200
Index


Rolling correlation between standardized residuals - 252-day window
correlation
0.15
0.25
0.35
0
200
400
600
800
1000
Index