Final Report – Portfolio Optimization

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1. Introduction

This report presents an analysis of twenty arbitrarily selected stocks from the EuroStoxx 600 index, each belonging to one of four economic sectors: Energy, Technology, Financials, and Real Estate. The selected stocks are as follows:

- Energy: SHELL, OMV, ORLEN, SUBSEA 7, REPSOL YPF
- Technology: AIXTRON (XET), ALTEN, SAGE GROUP, ASM INTERNATIONAL, SOITEC
- Financials: SWEDBANK A, AEGON, ALLIANZ (XET), ASSICURAZIONI GENERALI, AVIVA
- Real Estate: BRITISH LAND, CASTELLUM, COFINIMMO, COVIVIO, DERWENT LONDON

The analyses were performed across three different time periods to enable comparative study:

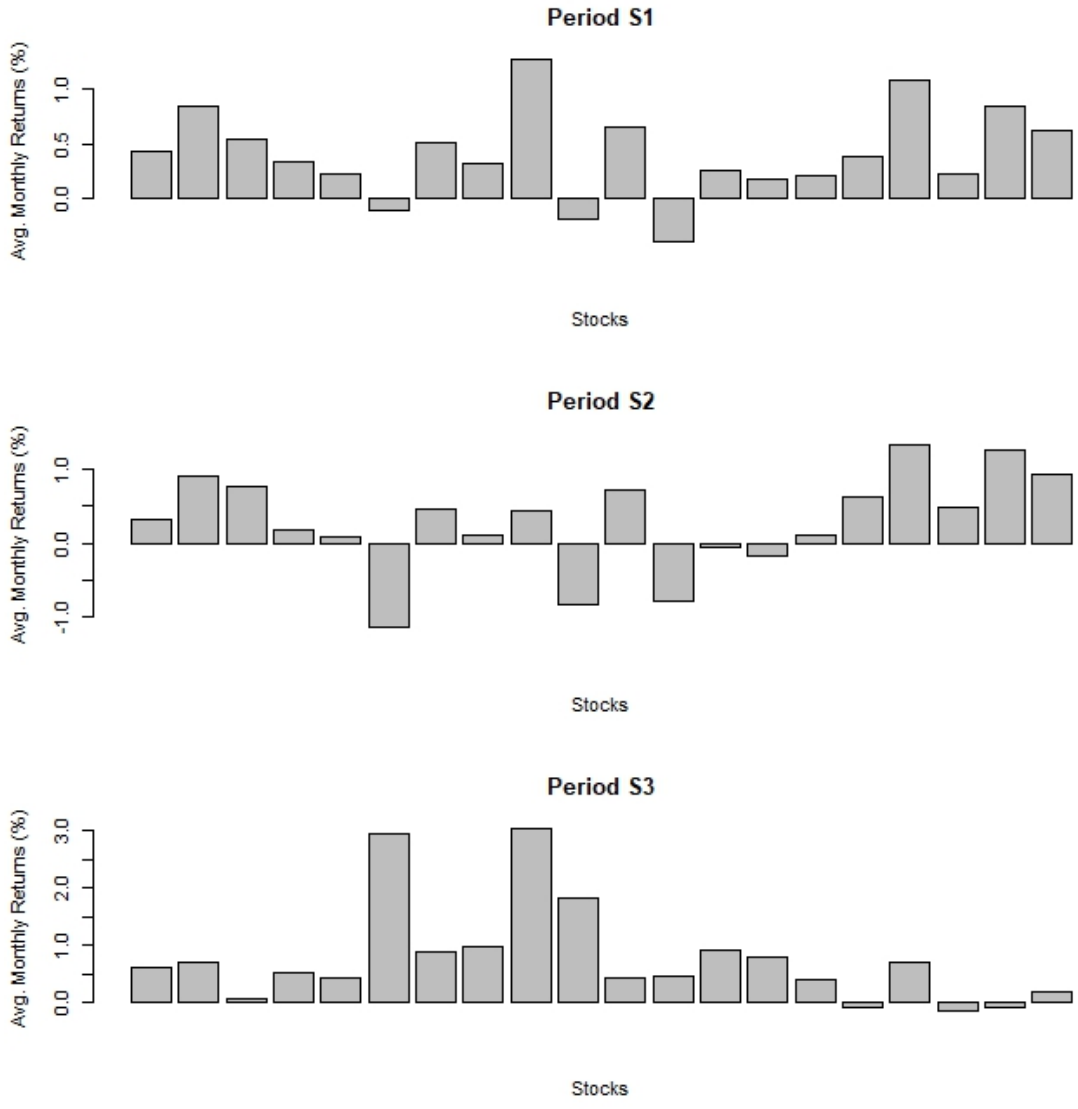
- S1: 31/12/1999 – 30/04/2024
- S2: 31/12/1999 – 31/12/2016
- S3: 31/12/2016 – 31/12/2023

2. Preliminary Statistical Analysis

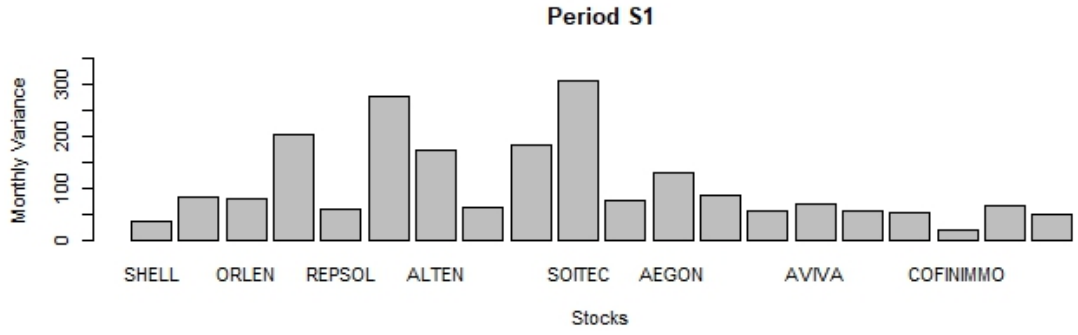
The bar chart of average monthly returns shows a clear difference between the pre-2017 period (S2) and the following years (S3).

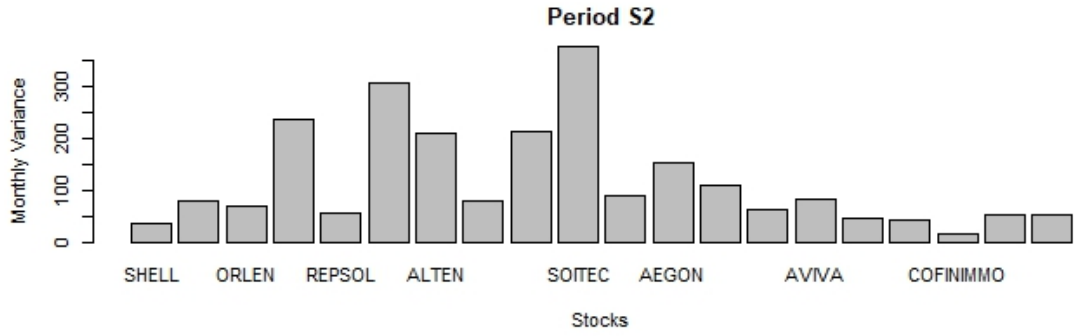
- In S2, the average monthly return of the analyzed stocks was approximately 0.2816%.
- In S3, this increased to 0.7874%.

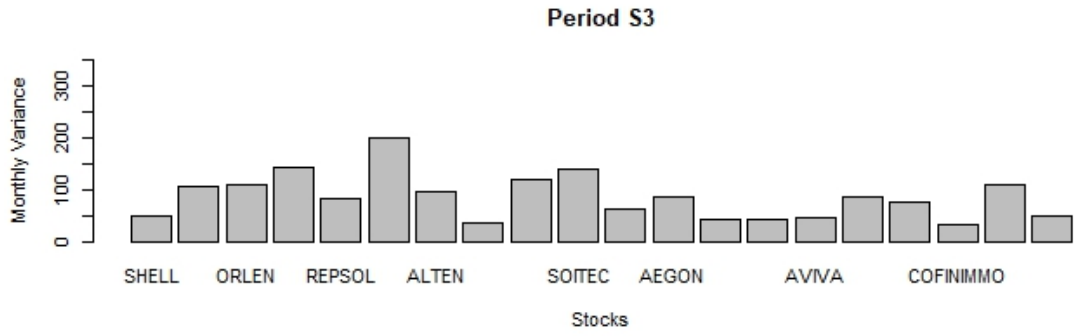
In S2, AIXTRON (XET), SOITEC, and AEGON had significantly negative returns. In S3, BRITISH LAND, COFINIMMO, and COVIVIO still had negative returns, but values were close to zero.



Volatility, as measured by variance, was generally higher in S2.

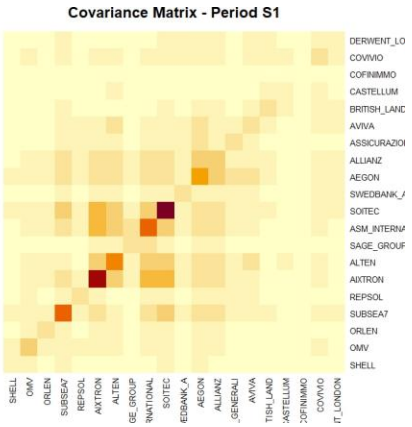


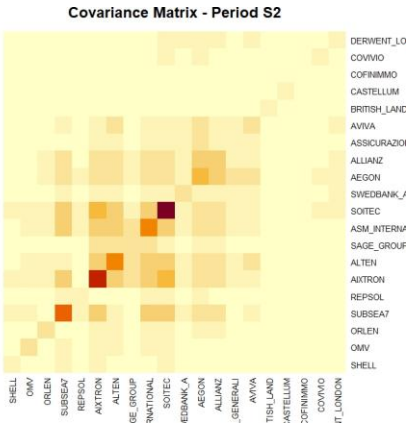


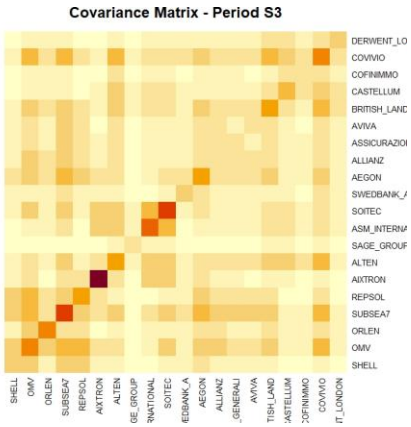


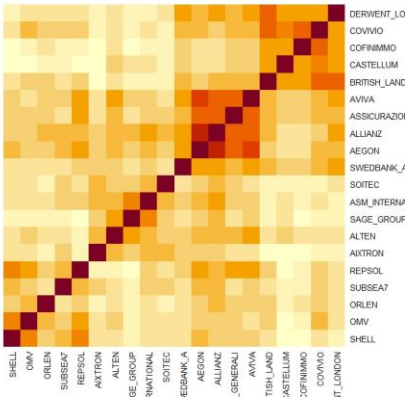
An analysis of the covariance and correlation matrices reveals strong intra-sector correlations in S3—evident in 5x5 diagonal blocks. Notably, Energy and Financials showed high mutual correlation.

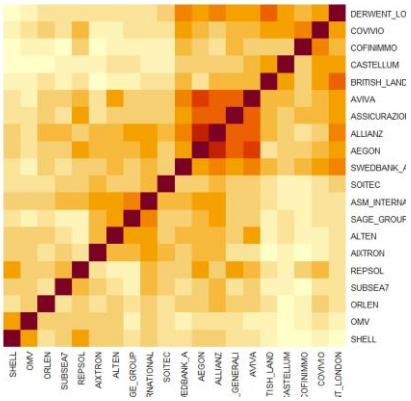
These patterns can likely be attributed to increasing integration and broader market influences, such as sustainability and geopolitical dynamics.

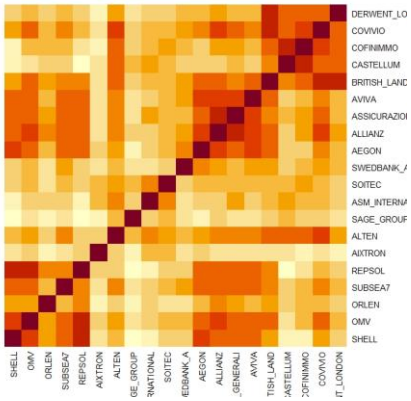

Covariance Matrix - Period S1
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL


Covariance Matrix - Period S2
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL


Covariance Matrix - Period S3
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL


Correlation Matrix - Period S1
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL

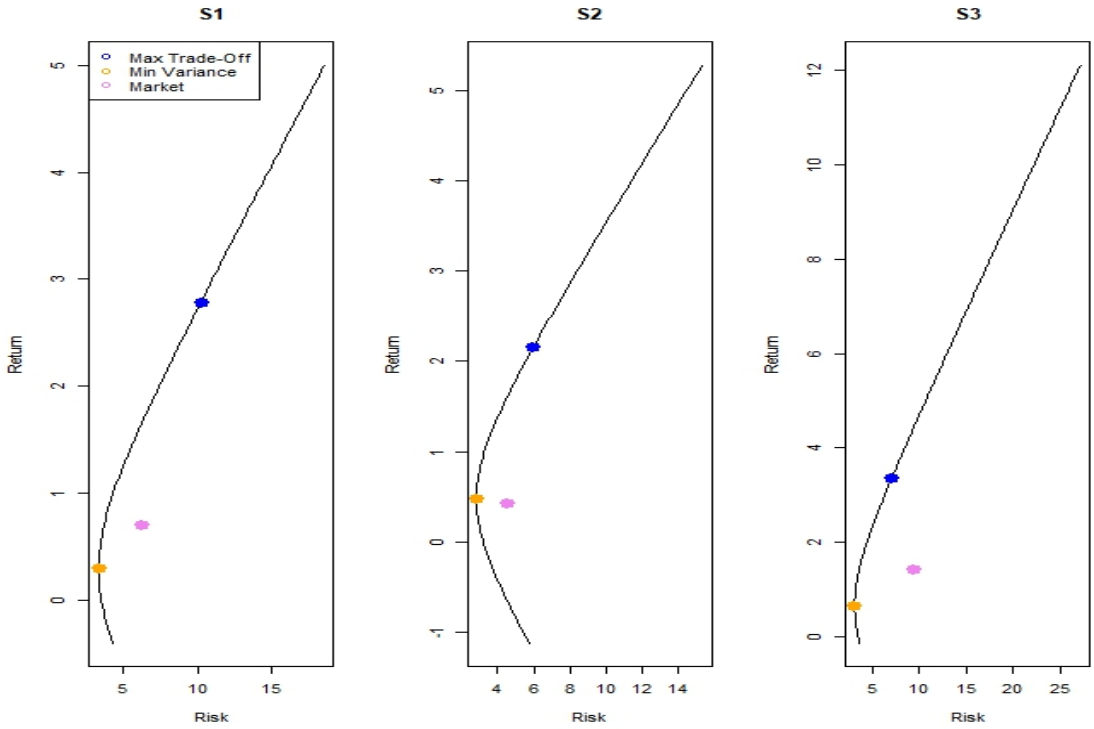

Correlation Matrix - Period S2
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL


Correlation Matrix - Period S3
DERWENT_LO
COVIVIO
COFINIMMO
CASTELLUM
BRITISH_LAND
AVIVA
ASSICURAZIOI
ALLIANZ
AEGON
SWEDBANK_A
SOITEC
ASM_INTERNA
SAGE_GROUP
ALTEN
AIXTRON
REPSOL
SUBSEA7
ORLEN
OMV
SHELL

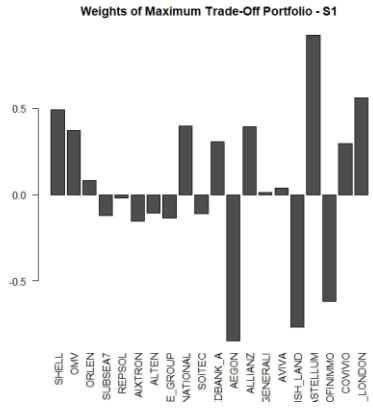
3. Efficient Frontier without Risk-Free Asset

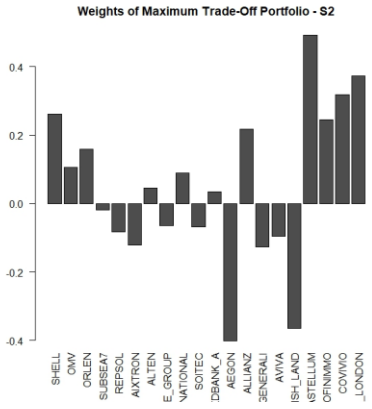
The efficient frontiers without risk-free assets were computed for all three periods, along with the Global Minimum Variance (GMV) and Maximum Trade-Off (Max-T) portfolios, and the market index.

In S3, all portfolios had higher risk and return values than in S1 and S2. In all cases, the market portfolio was inefficient, offering a lower return for a higher level of risk compared to the optimal portfolios.



In S3, Allianz had a weight >1 in Max-T, showing its strong performance. Shell, Sage Group, and Cofinimmo dominated the GMV portfolio across all periods. AEGON and COVIVIO often appeared with negative weights.


Weights of Maximum Trade-Off Portfolio - S1
0.5
0.0
-0.5
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
E_GROUP
VATIONAL
SOITEC
DBANK_A
AEGON
ALLIANZ
3ENERALI
AVIVA
ISH_LAND
NSTELLUM
OFINIMMO
COVIVIO
_LONDON


Weights of Maximum Trade-Off Portfolio - S2
0.4
0.2
0.0
-0.2
-0.4
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
E_GROUP
VATIONAL
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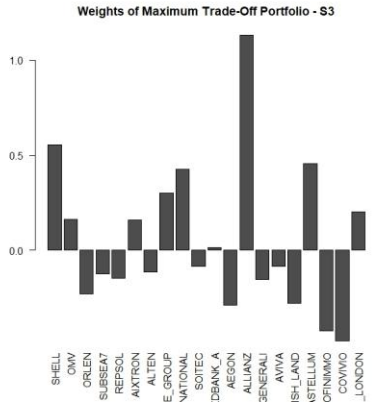
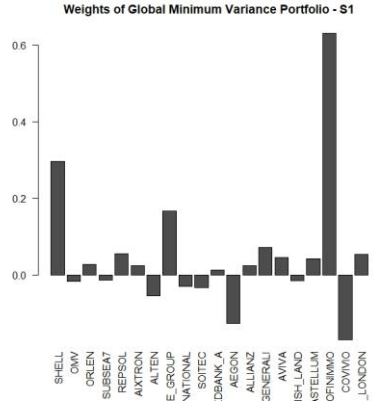
ISH_LAND
NSTELLUM
OFINIMMO
COVIVIO
_LONDON


Weights of Maximum Trade-Off Portfolio - S3
1.0
0.5
0.0
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
E_GROUP
VATIONAL
SOITEC
DBANK_A
AEGON
ALLIANZ
3ENERALI
AVIVA
ISH_LAND
NSTELLUM
OFINIMMO
COVIVIO
_LONDON


Weights of Global Minimum Variance Portfolio - S1
0.6
0.4
0.2
0.0
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
E_GROUP
VATIONAL
SOITEC
DBANK_A
AEGON
ALLIANZ
3ENERALI
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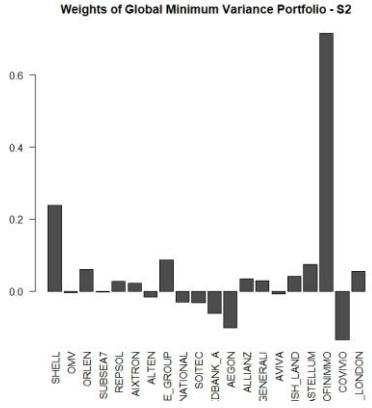
ISH_LAND
NSTELLUM
OFINIMMO
COVIVIO
_LONDON


Weights of Global Minimum Variance Portfolio - S2
0.6
0.4
0.2
0.0
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
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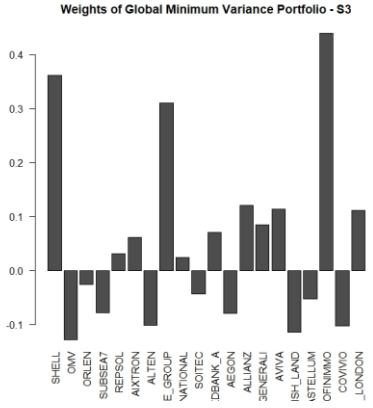
Weights of Global Minimum Variance Portfolio - S3
0.4
0.3
0.2
0.1
0.0
-0.1
SHELL
OMV
ORLEN
SUBSEA7
REPSOL
AIXTRON
ALTEN
E_GROUP
VATIONAL
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DBANK_A
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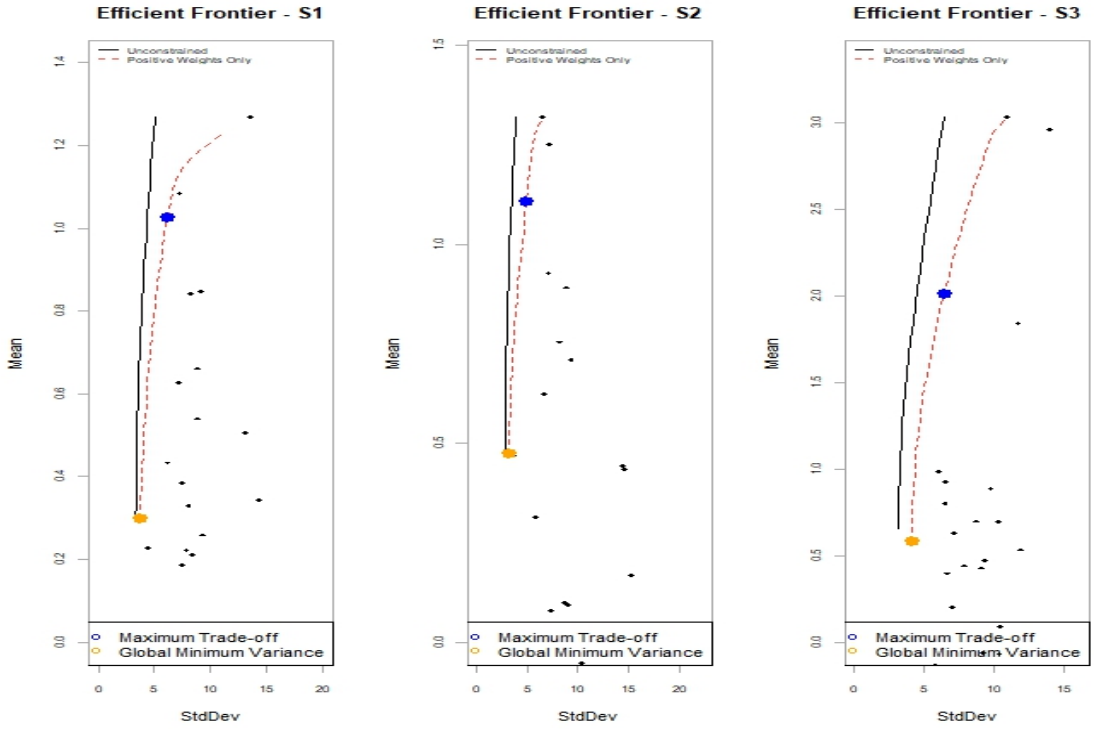
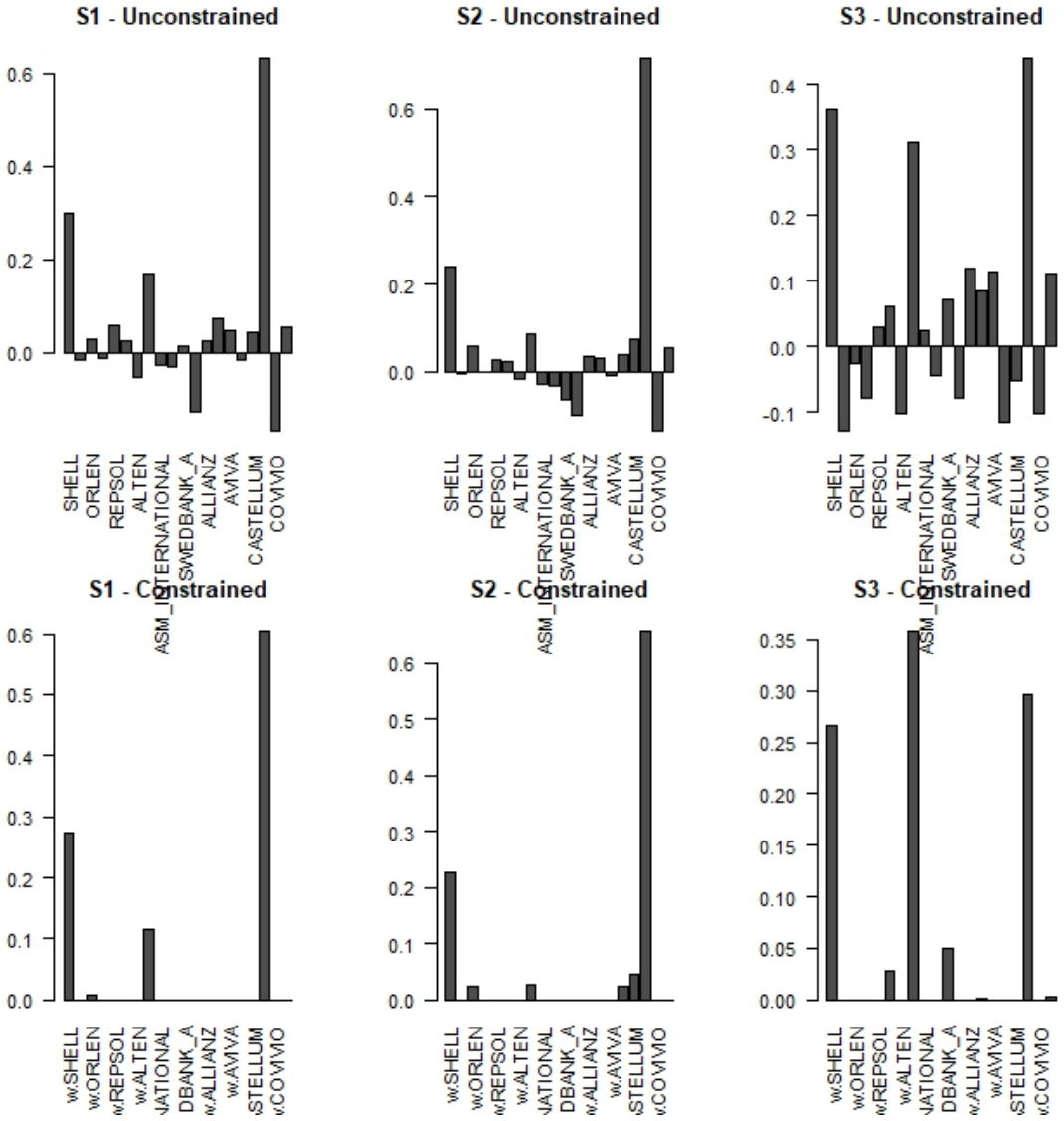
4. Efficient Frontier with Long-Only Constraint

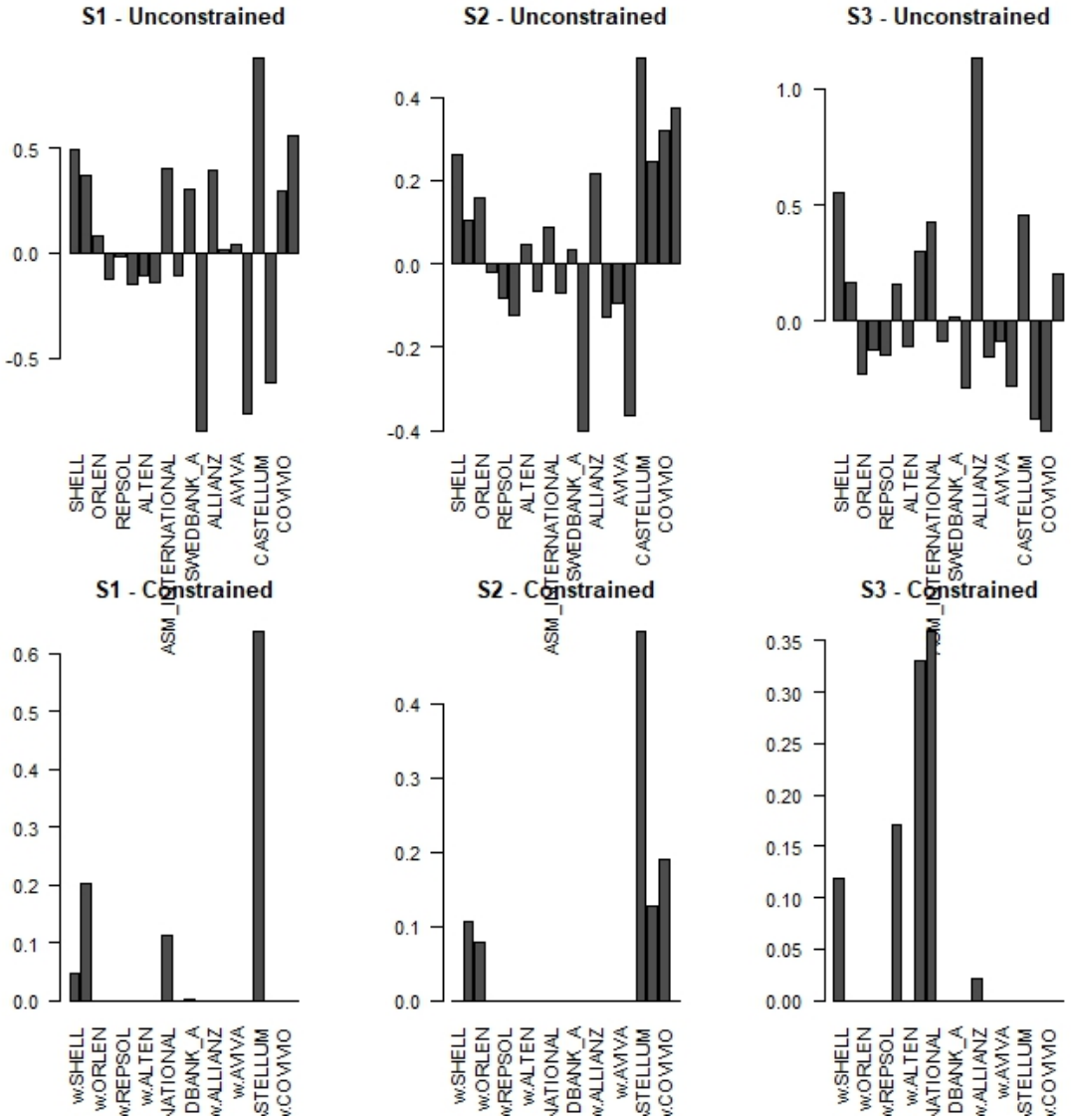
When non-negativity constraints are introduced, the efficient frontier shifts rightward. This reflects higher risk for a given return.



GMV portfolios under constraints continued to be dominated by Shell, Sage Group, and Cofinimmo. Cofinimmo had especially high weight in all the considered periods of time.

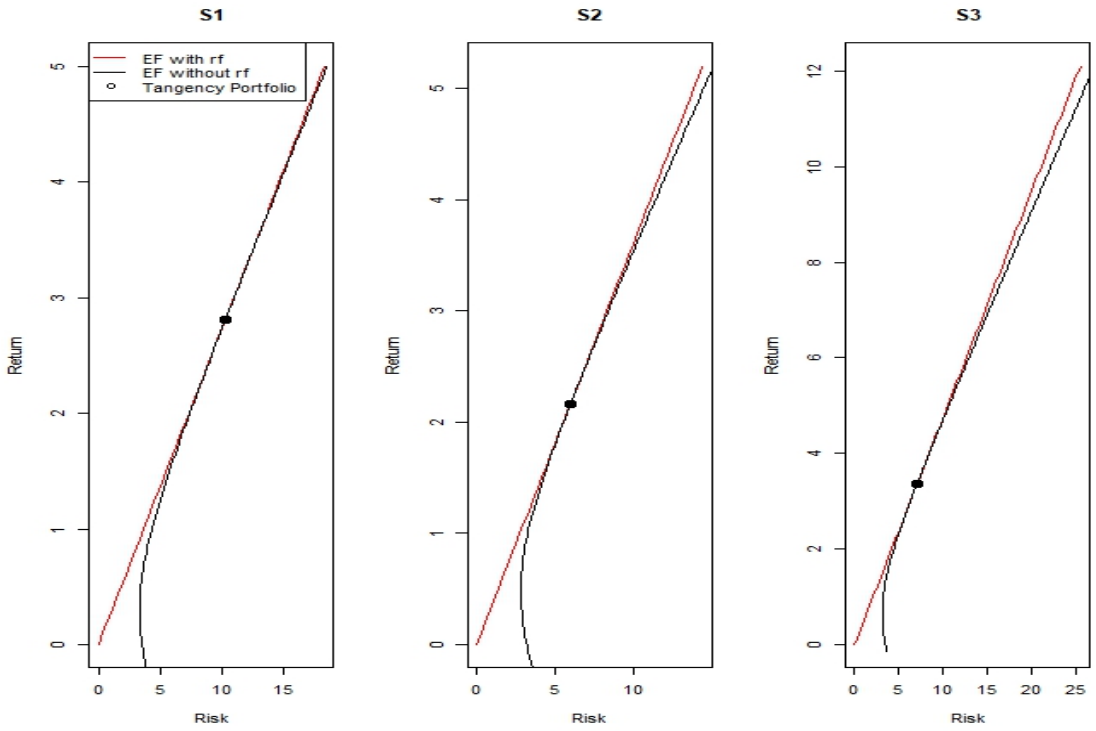


Max-T portfolios showed no consistency across periods. S1 emphasized Castellum, S2 featured Real Estate stocks, and S3 leaned heavily on Technology. Financials were largely absent.


S1 - Unconstrained
S2 - Unconstrained
S3 - Unconstrained
S1 - Constrained
S2 - Constrained
S3 - Constrained
SHELL
ORLEN
REPSOL
ALTEN
INTERNATIONAL
SWEDBANK_A
ALLIANZ
AVIVA
CASTELLUM
COVIVIO

5. Efficient Frontier with Risk-Free Asset

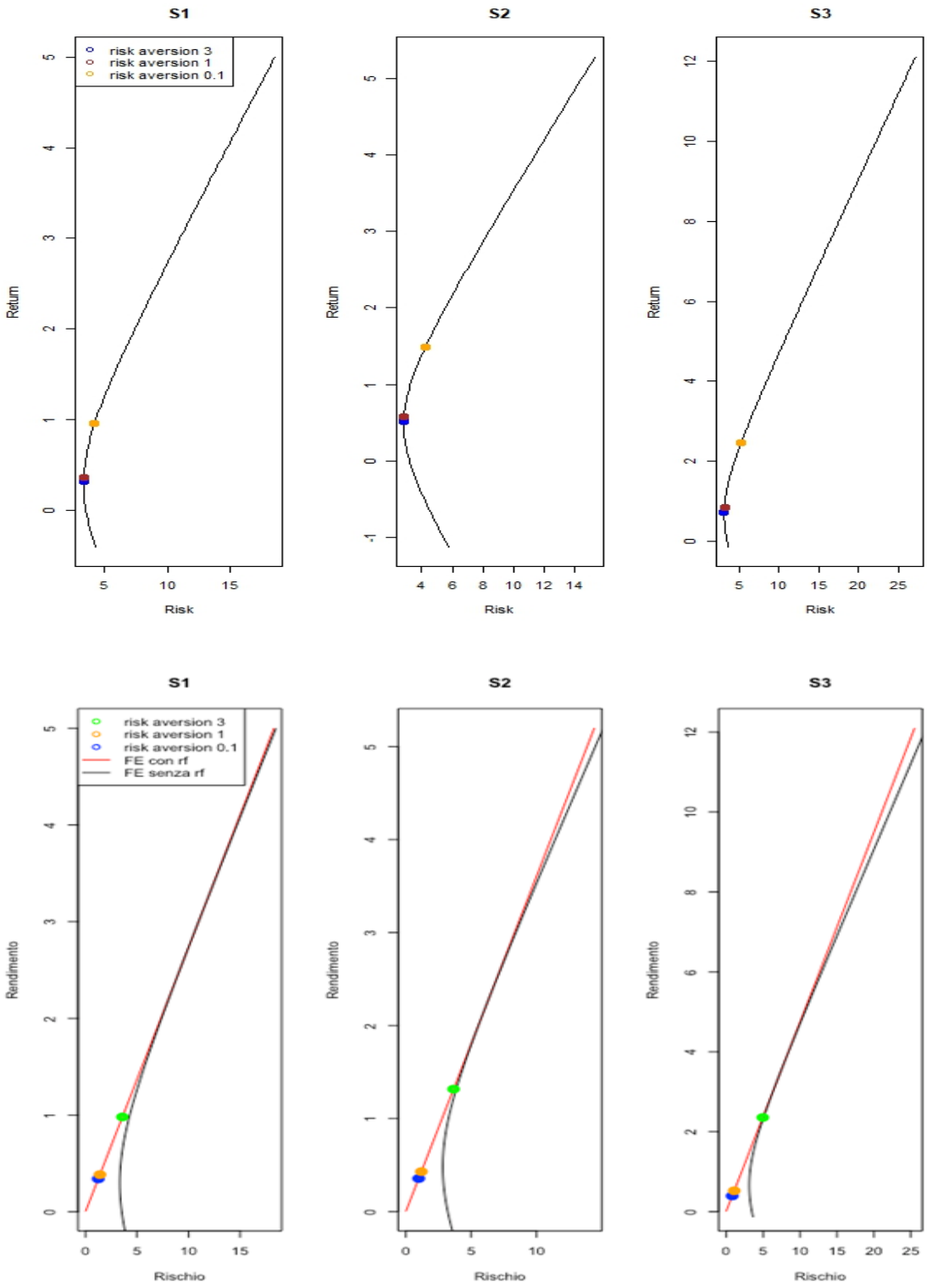
Introducing the risk-free asset results in a linear Capital Market Line tangent to the risky frontier. The tangency portfolio was calculated using the most recent Euribor for each period.



We examined three investor profiles:

- risk aversion = 3: conservative
- risk aversion = 1: moderate
- risk aversion = 0.1: aggressive

As expected, in both the absence and the presence of the risk-free asset, the optimal portfolio shifts from GMV (that in the second case corresponds to the risk-free asset) to Max-T as risk aversion decreases.



By the intersection of the two efficient frontiers we obtain the tangency portfolio, the only efficient combination of non-risk-free assets.

The statistical test carried out calculates the statistical significance of the Sharpe index of the tangency portfolio. We concluded that the tangency portfolio significantly outperforms the GMV portfolio in all periods ($t > 1.96$).

Tangency portfolio	S1	S2	S3
Test statistics	4.582275	4.990978	4.096729

However, computing the same test on the GMV portfolio, we can state that GMV only significantly outperforms the risk-free rate in S2. In S1 and S3, the Sharpe ratio was not statistically different from zero.

GMV portfolio	S1	S2	S3
Test statistics	1.503899	2.413446	1.887736

6. Sector Performance Analysis

It is also valuable to assess whether an economic sector is significantly better or worse than the others in the different time periods. As a criterion to establish it, we decided to compare the Sharpe ratio of the Maximum Trade-Off portfolio obtained with the assets of each one of the economic sectors. The choice was made considering that the Maximum Trade-Off portfolio represents the most effective combination of assets to consider the risk – expected benefits ratio.

The following statistical test statistics were obtained:

	Energy	Technology	Financials	Real Estate
S1	0.01300250	0.0043552922	0.011020112	0.010962155
S2	0.01435118	0.0003189084	0.006623746	0.037194734
S3	0.01147331	0.0403392360	0.020643656	-0.004435582

Concluding that the worst performing sector in period S1 and S2 was the Technological one, while from 2017 the worst one has become the Real Estate one.

Sometimes it might be also interesting to assess whether the exclusion of a specific economic sector or group of assets from the portfolio significantly worsens its expected performance. For example, some clients might be interested in evaluating financial products that exclude securities of companies operating in the defense sector.

Excluding the worst sector in each period showed no significant difference in portfolio performance ($p > 0.9999$). Hence, in this case, poor sectors had a negligible impact on overall efficiency.

Exclusion test	S1 – Technology	S2 - Technology	S3 – Real Estate
Test statistics	0.0139	0.0171	0.0504
p-value	1	1	1

7. Efficiency of Inverse-Variance Weighted Portfolio

Another interesting application is to consider some basic custom mathematical rule, that set the ratio between risk and expected return, to widen the portfolio offer among which a customer could choose.

A portfolio with weights inversely proportional to variance, thus assigning lower weights to higher risk assets, was tested for efficiency. The test compares the Sharpe ratio of the constructed portfolio to that of the efficient frontier.

	S1	S2	S3
Statistica test	0.06666921	0.1173026	0.1546972
p-value	1	1	1

It can be concluded that these portfolios are statistically efficient and provide a practical construction method.