The impact of taxation on the distribution of the wealth and income
from an agent-based model point of view

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Abstract

Exploring the problem of different taxation impact on wealth distribution from an agent-based model perspective, we apply different taxation on two representatives of the main models treated in literature. We discover that wealth taxation may lead a stronger impact than income taxation as progressive taxation may do it more than flat, under certain conditions. As this is consistent with others researches on these models we try to verify also from a more empirical point of view, but due to lack of real data we do comparison with survey based EUROMOD microsimulation. Comparison with two approaches show that there are similar conclusions but with some exceptions related to income taxation against wealth taxation. After summing up the results we also suggest how it would be possible in the future to refine these agent-based models linking wealth and income, to make possible a better empirical check of the results.

1 Introduction

Following the current development of the use of agent-based models in economics, this article aims to analyze how different forms of taxation can decrease the inequality in the distribution of wealth. The use of this methodology in the economic field is recent when compared to its use in the natural sciences but no less promising and increasingly taken into consideration. Starting from the Schelling's model in the late 1960s, this field of research has grown to have several applications in many different contexts of the study of economics (Turrell, 2016). In particular, the application developed in this paper refers to the line of inquiry brought to the forefront of the discipline by Thomas Piketty's Capital in the 21st Century, that is the distribution of welfare rather than the classical aim of research of the improvements in welfare (Kulp et al., 2019).

The application of agent-based models in this research field is certainly not new, but it has had a great development mainly thanks to small band of "econophisicist", who by applying principles of statistical mechanics have already achieved several interesting results; a review and catalog of different results achieved on different agent-based models, starting from this perspective, is reported in (Hayes, 2002), but a particular mention should be made of the link brought to light by (Drăgulescu and Yakovenko, 2002) between the probability distribution of energy and the distribution of wealth within a conservative economic system, i.e. where total wealth remains constant, as if even the distribution of wealth between agents respected the

Boltzmann-Gibbs distribution[1] where energy is replaced by money and temperature by the average amount of money per agent.

Various results have also been investigated and achieved regarding the effect of taxation on the distribution of wealth within agent-based models, mainly concurring on the difference between taxing the wealth of agents or transactions between agents. (Hayes, 2002), for example, show that imposing a tax on wealth prevents the implosion of the yard-sale economy[2], while (Kulp et al., 2019) exhibit that income taxes, both flat and progressive, that evenly redistributed taxed moneys do little to change the Gini coefficient from the Boltzmann-Gibbs distribution, while a very small wealth tax can lead to significant decreases in the Gini coefficient.

As before Piketty brought to the forefront of the discipline the problem of the distribution of wealth, taxing and redistributing wealth has always been a relevant political problem, at least since the birth of the welfare state. Claim the right to access the wealth of individual citizens in order to redistribute it in the form of services or subsidies, as guaranteeing a functional and efficient use of these resources is still a divisive issue within the public debate. For this reason, being able to account for the impact of different types of taxation becomes a relevant problem also from a policy-makers point of view.

Therefore, it is in our interest to investigate the different effect that different types of taxes have on the distribution of wealth. First analyzing the problem from the point of view of agent-based models, where we consider two representatives of the main sets of models taken into consideration by the literature and applying different types of taxes to see how the behavior changes. Secondly, by trying to check what is done so far in more reality prone models and review what we found in the previous section is consistent with latter results.

2 Simulate the distribution of wealth

The model used in these simulations was developed in Mesa, it is a modular framework for building, analyzing and visualizing agent-based models available for python. We have developed the necessary code for this work starting from some tutorials made available on the Mesa website.

2.1 The model

The construction of the model

As Hayes (2002) point out, the simulated economy in these models is based on the pure free market business. The exchange of goods is all that ever it happens here; there is no production of wealth and not even consumption. What remains is a closed system, where wealth is a conserved quantity, like energy or momentum, and because of that a person can only become richer if another becomes poorer.

But as all the agents will start with same amount of wealth, the gradual differentiation of wealth develops thanks to the rejection of an unrealistic assumption usually assumed in other models, that is that all transactions occur at precisely the right price. In practice, the assumption of perfect pricing seems a little unrealistic.

[1]Which states that the probability distribution function (PDF) of energy ϵ is $P(\epsilon) = Ce^{-\epsilon T}$, where T is the temperature, and C is a normalizing constant.

[2]One of the main models identified that has the peculiarity of converging in a state where only an agent accumulates all the available wealth, summarized in the formula "the winner takes all".

Some buyers are more discerning than others, and some sellers are more persuasive. So, all the models we will consider are based on the fact that one of the agents included in a transaction will lose part of his wealth to the benefit of the other participant in the transaction, note that only transactions between a maximum of two agents will be allowed at a time.

Several models have been identified in the literature that determine the amount of wealth respectively lost or gained, but it has already been shown how they can all be divided into two classes, which determine their final behavior: those that break time-reversal symmetry and those that don't.

The first type do not allow that given a transaction between two agents it is always possible to return with a subsequent transaction to the initial situation, an example is what is called by Hayes (2002) the Yard-sale model, where the amount of wealth exchanged is randomly drawn between zero and the total wealth of the less wealthy agent of the two. For example imagine that an agent starts with three of wealth and the second with five, the first might initially give two to the second agent but the latter might give back only one in the next transaction. This type of models after a sufficient number of steps always collapse into a system where a single agent acquires all the wealth, as you can see in Figure 4.

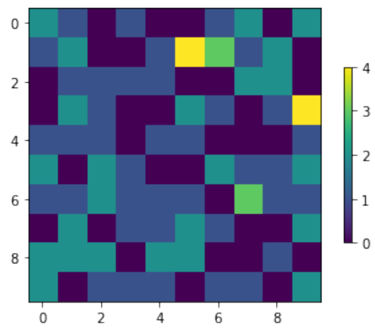
In the second class of models instead it is always possible to rebalance a transaction with a subsequent transaction, we take as representative of this set the model called by Hayes (2002) Theft and fraud, where the amount of the transaction is a random number between zero and the total wealth of the person who will have to lose wealth in the transaction. These types of models instead lead to a distribution of wealth that follows the Boltzmann-Gibbs distribution, as you can see in Figure 5. For our research we will apply both of these two models to see how and if the results change according to the class of models treated.

In addition to specifying how the amount of the transaction between agents will be determined, there is a need to specify all the assumptions that determine the exact unfolding of the model. Any agent will be situated in a cell inside a grid, that will represent the space, all with the same initial wealth and they will activate one at a time and will perform these actions in order:

1. It will move in a random box among those adjacent[3] to the one in which they were.

2. If in this new cell there is at least one other agent, it will randomly choose one with which to entertain a transaction and it will lost part of its wealth according to the transaction model chosen for the simulation.

Figure 1: Example of final distribution of population in the grid



It is also necessary to underline that the order of activation of the agents will change randomly every turn, this choice that could seem obvious is actually decisive because, starting from what Comer (2014) said, the activation order can have a decisive impact on this model.

[3]it was chosen to take into consideration the Moore neighborhood, where the eight surrounding cells are considered close, while the grid is of the toroidal type so as not to disadvantage any cell that would otherwise be peripheral

For example, maintaining a fixed activation order would have benefited early agents.

Free market behavior analysis

Before introducing taxes it is necessary to take a look at the behavior of the models per se. First we evaluate how it evolves the rankink between angents over time in the two different models by fixing the number of agents at twenty in a five by five grid, with initial amount of wealth per agent equal to one.

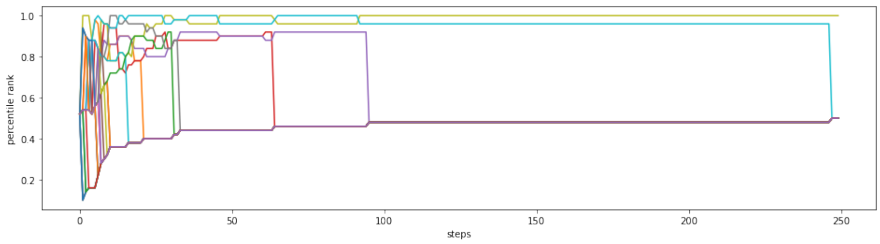


Figure 2: Rank story with Yard-Sale model

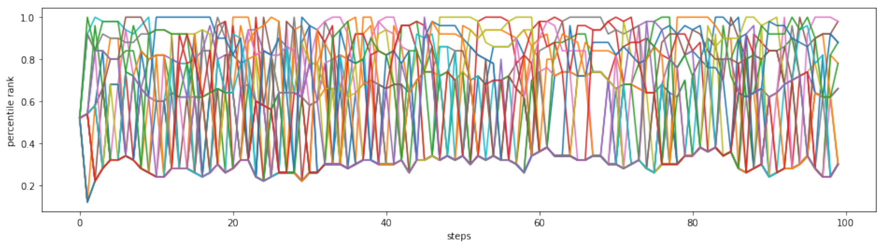


Figure 3: Rank story with Theft-Fraud model

As you can see in the Figure 2, after enough steps remains only one agent at the top of the rank while all the others collapse into a tied second place; while in Figure 3 the rank continues to change with the ascents and descents of the agents.

We pass to evaluate the final distributions, furthermore we increase the number of agents in the second model to make the relationship with the Boltzmann-Gibbs distribution meaningful.

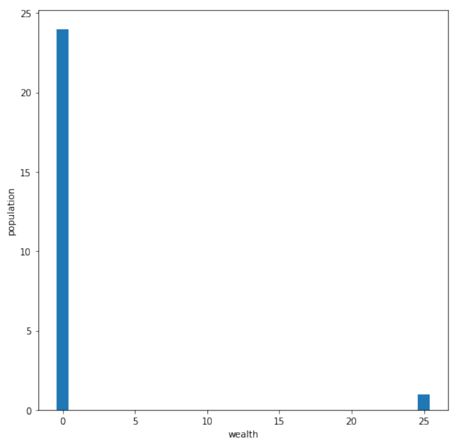


Figure 4: Final wealth distribution for Yard-Sale model

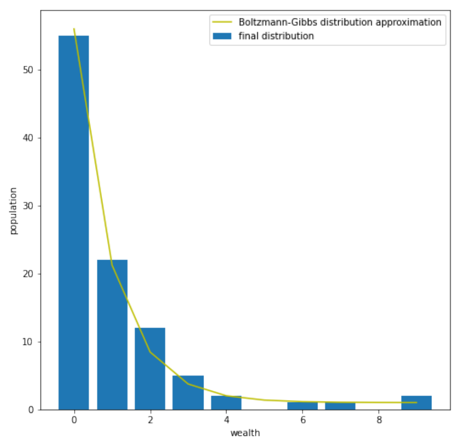


Figure 5: Final wealth distribution for Theft-Fraud model

It is clearly visible how in one case all the wealth ends up in a single agent while in the other the distribution is well approximated by the law of thermodynamics.

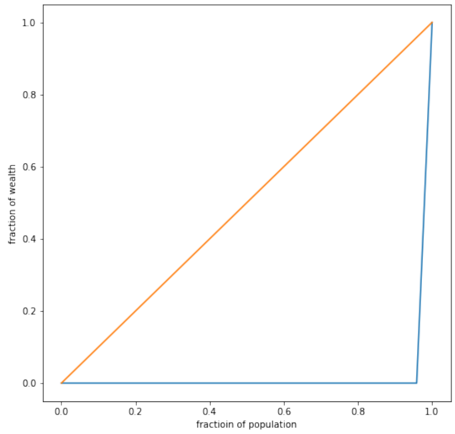


Figure 6: Final Lorenz curve for Yard-Sale model

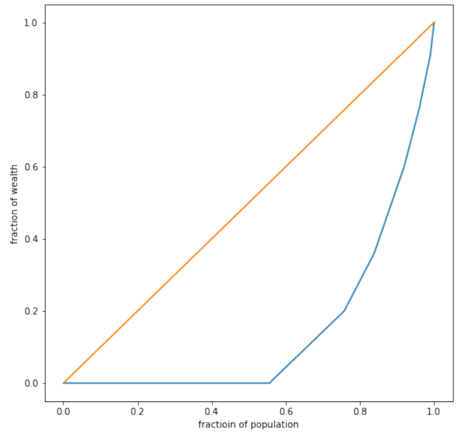


Figure 7: Final Lorenz curve for Theft-Fraud model

The Lorenz curve indicates how much wealth is respectively accumulated in fractions of the population at the end of the simulation and allows us to introduce what will be the main indicator of our experiments, that is the Gini index. It is worth zero if the distribution is perfectly equitable and is worth one when a single agent possesses all the wealth and varies with the variation of the area between the Lorenz curve and the line of perfect redistribution.

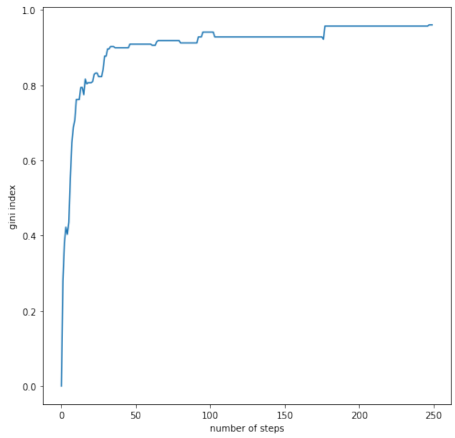


Figure 8: Trend of the Gini index during the simulation for the Yard-Sale model

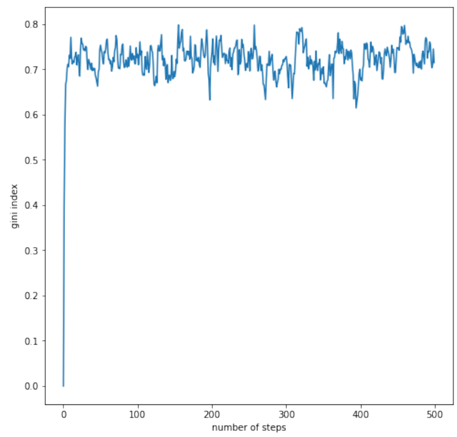


Figure 9: Trend of the gini index during the simulation for the Theft-Fraud model

Looking at the trend of Gini index it is possible to notice how, starting from zero, in one case it reaches 1, while in the other it oscillates around 0.7 more or less.

Adding taxation

As anticipated, it is our interest to study the effect of two types of taxes on the distribution of wealth: those on the wealth of agents themselves and those on their income. As regards the first type, we will introduce a polynomial function that will allow us to differentiate and combine different types of taxes while for the second we will limit ourselves to adding a tax rate, which we will denote with α_{income} or in abbreviation α_i.

Since in this research we have decided to focus on the study of taxation, the social expenditure will not vary along the simulations. We assume that all the resources collected are redistributed and each time an agent finds itself, in its turn, below the initial wealth threshold, if there are enough resources he will be given a unit of wealth. In this way we will simulate the redistribution according to subsidies or services for people in poverty.

We will combine three different types of taxes on wealth that will be calculated from the difference between an agent's current wealth and initial wealth. In this way our population is divided into those who are below the initial threshold and therefore will be provided with a subsidy, those who are still at the initial threshold and therefore will not be taken into consideration and those who have become enriched will be able to see their accumulated wealth taxed.

By indicating with w_i the wealth of the agent i, T the initial threshold[4], tax_i the tax to be imposed and $tax_{i\%}$ the percentage taxation; we define the amount due as follows:

$$\delta_i = w_i - T$$

$$tax_{i\%} = \alpha + \beta\delta_i$$

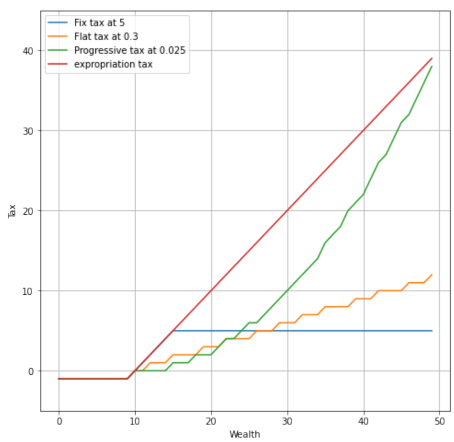
$$tax_i = \kappa + tax_{i\%}\delta_i = \kappa + \alpha\delta_i + \beta\delta_i^2$$

In this way κ indicates a hypothetical fixed tax, the percentage tax on the difference in wealth instead has a fixed rate defined by α, while β indicates the growth of the percentage tax for each unit of accumulated wealth, where α and β are limited between zero and one. If the total tax exceeds delta, then it will simply collect the itself. Since we want to keep wealth at discrete values, the actual tax value will be rounded off to units. We will call: fixed tax the tax obtain from having just κ different from zero, flat tax or proportional tax the one obtain having just α different from zero and progressive tax the one obtain having just β different from zero.

In figure 10 we can see how taxes change according to wealth for each type of tax, where negative values associated with wealth below T indicate the subsidy. It is significant to note that by arbitrarily increasing the parameters, each type of tax collapses into what we will call the expropriation tax, where all the difference between an agent's wealth and initial wealth is taxed.

Even if this method of taxation may seem unnecessarily complicated rather than a simple taxation by bands, for this research it was essential to parameterize the taxes in order to evaluate, in the next section, how the gradual increase of each parameter affects the behavior of the system. Finally, to regulate the frequency of taxation, we will add a parameter P, which will determine how many steps will pass between one taxation and the next.

Figure 10: Taxes plot with T=10



[4]The choice of T is also because the initial wealth takes the part of the temperature in the isomorphism done with thermodynamics

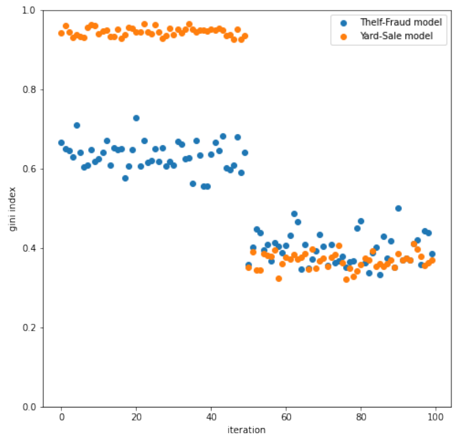
In the face of such a differentiated proposal of taxes on wealth, we justify the lack of as much diversity on income taxes with the fact that in this model, while accumulated wealth can grow a lot, incomes are always rather limited, preventing the difference between various types of taxation to be meaningful.

2.2 Multiple runs experiments

To find the overall distribution of our model, including taxes, we need to do multiple runs, with fixed parameters to find the overall distributions the model generates, and with varying parameters to analyze how they drive the model's outputs and behaviors. First of all, let's fix some values of our model in order to have a have a sufficiently complex and balanced system but which at the same time does not increase complexity and computational time too much, making unfeasible to do a large number of runs. For this purpose we set: N = 100 number of agents in a 10 by 10 grid, an initial wealth T = 5, P = 5 taxation period, 100 as the number of steps for each single run and 10 as the number of iterations for each model with fixed parameters.

As a first experiment we can now verify the final value of the Gini index in the two extreme situations that will be used as a benchmark for the following experiments, namely the tax-free model and the one with the expropriation tax. We obtain the output of figure 11, from 100 model runs, for each transaction model but passing from a totally free market to one with expropriation tax at the fiftieth run. It is noted that there are some differences depending on the transaction model, the yard-sale model is on average more extreme, especially as regards the free-market: average free-market yard-sale Gini index is 0.945 against 0.633 of the Theft-fraud model, while in the expropriation taxation the average values are respectively 0.368 and 0.397. In addition to this it seems that the final value of the Gini index is much more variable in the second model. Both of these conclusions are consistent with what was observed in the preliminary analysis made in the previous section, as seen in Figure 8 and Figure 9.

Figure 11: Comparison between free-market and expropriation tax



We can finally begin to evaluate the impact of various types of taxes on the Gini index as their parameters vary. In order to do that, focusing on one tax at a time, we repeat a simulation where all the parameters of the taxes not taken in consideration are fixed to zero; then increasing the tax of interest parameter by a little amount, from the free-tax market to the expropriation tax market. As before, we exhibit the results for both the two exchange models. Remember that κ indicates the fixed tax on excess wealth, α the percentage rate on excess wealth, β the percentage rate on excess wealth while α_i the proportional rate on income.

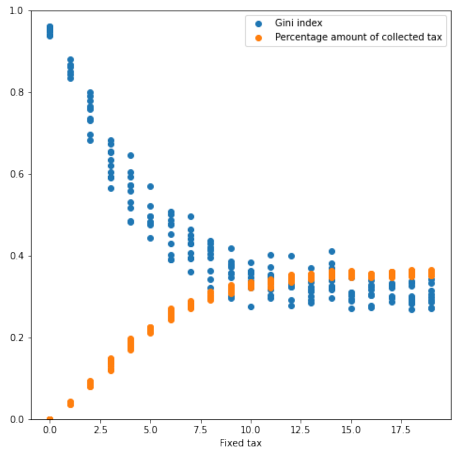


Figure 12: Gini index against increasing of κ under the Yard-Sale model

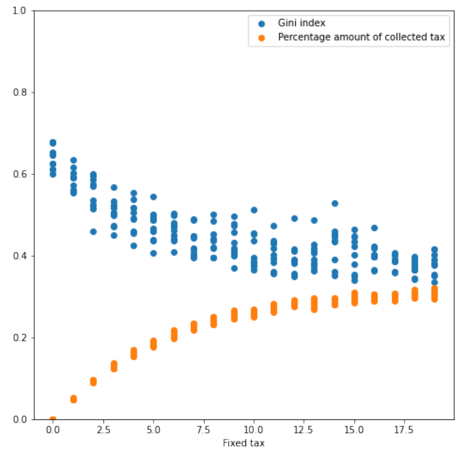


Figure 13: Gini index against increasing of κ under the Theft-Fraud model

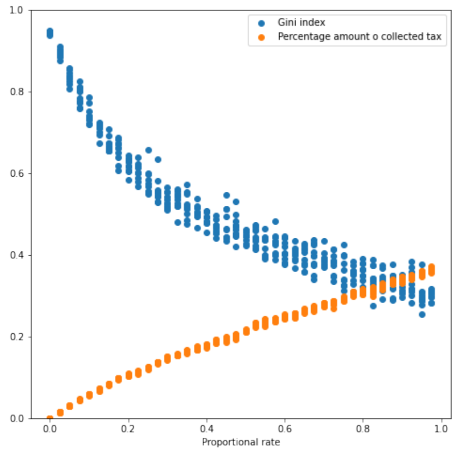


Figure 14: Gini index against increasing of under the Yard-Sale model

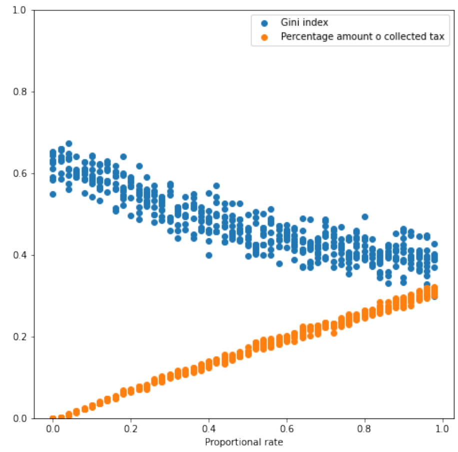


Figure 15: Gini index against increasing of α under the Theft-Fraud model

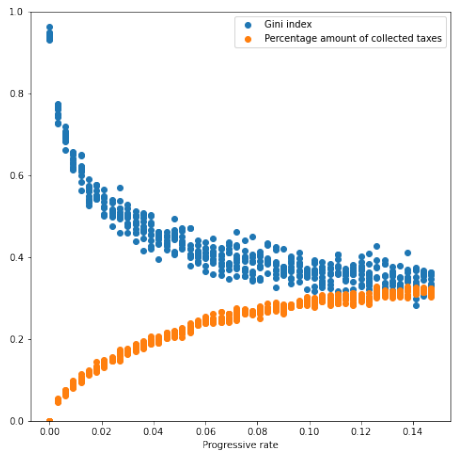


Figure 16: Gini index against increasing of β under the Yard-Sale model

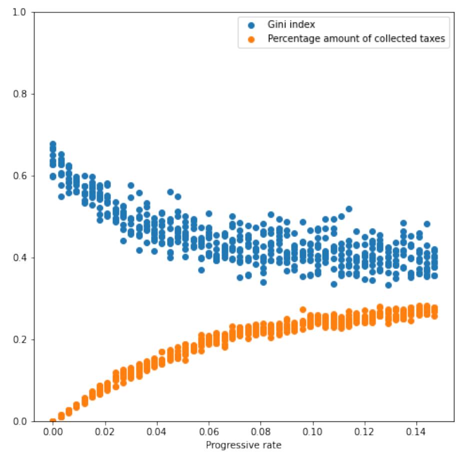


Figure 17: Gini index against increasing of β under the Theft-Fraud model

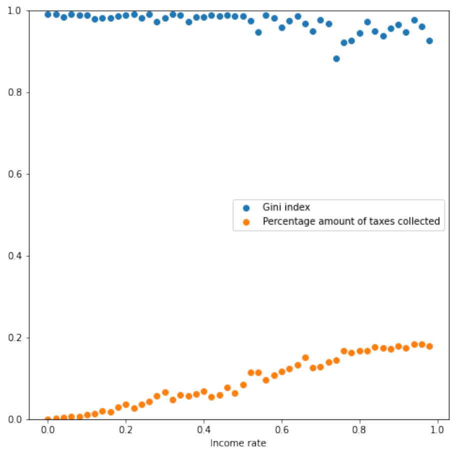


Figure 18: Gini index against increasing of α_i under the Yard-Sale model

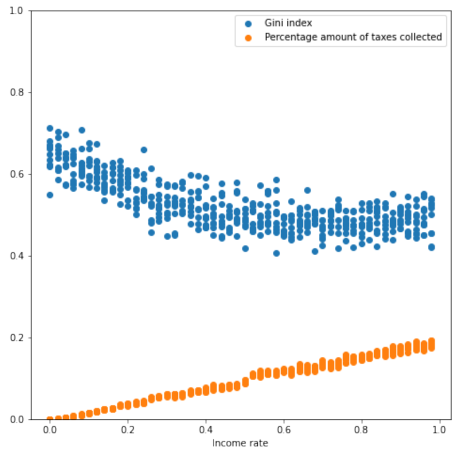


Figure 19: Gini index against increasing of α_i under the Theft-Fraud model

A first general conclusion, from Figure 12 to Figure 19, is that an increase in taxes, with these conditions always leads to a situation of less inequality. In fact, in all the graphs the Gini index is negatively correlated with the increase in the taxation parameter.

A first important qualitative difference emerges regarding taxation on wealth and taxation on income. All forms of wealth taxation bring the value of the Gini index gradually from the free market average to that of expropriation tax regardless of the transaction model, while income taxation leads to very different results depending on the transaction model. In the case of the Yard-Sale model, the taxation of income hardly lowers the value of the Gini index. We assume that this depends on the fact that in this trading model the amount exchanged is always less than the wealth of the less wealthy of the two agents, thus leading to a lower average transaction value and therefore less income per transaction. Just think that only in this case to obtain a significant amount of taxes collected we had to double the number of steps for each simulation. As regards the Theft-fraud model, a behavior is much more similar to what has been seen in wealth taxation emerges. The initial value of the Gini index is gradually lowered until a threshold is reached beyond which taxes do not bring down the Gini index.

To better understand the different impacts that taxation has on the behavior of the model, it is advisable to plot them together. Note that, the percentage of the amount of taxes collected is calculated by dividing the absolute value of taxes collected by the maximum amount possible to be collected[5], if the model taxes the wealth, and for total income into the income taxation model. For this reason we must compare them starting from the absolute value of the taxes collected. In order not to completely immerse the relationship in the noise, we take the average value of every ten simulations with fixed parameters.

From what we get we can see in Figure 20 and Figure 21 how even the different taxes on wealth do not have the same impact on the Gini index.

In the yard-sale model the progressive tax causes the Gini index to fall more efficiently than the proportional tax compared to the amount of collected taxes, in turn more efficient than the fixed tax. These two results are confirmed by the one

[5]Calculate as follow: how many would have been collected if the wealth had always ended up in the hands of only one agent before the tax round with an expropriation tax

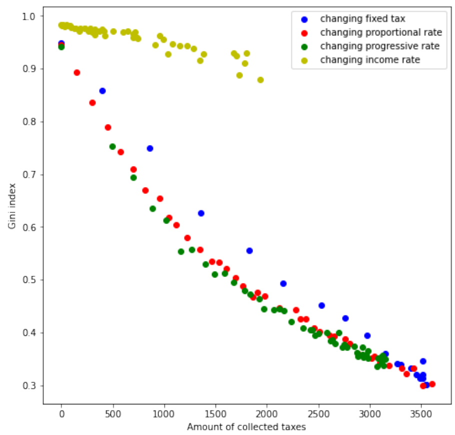


Figure 20: Scatter plot of average Gini Index against amount of taxes for each form of taxation, under Yard-Sale model

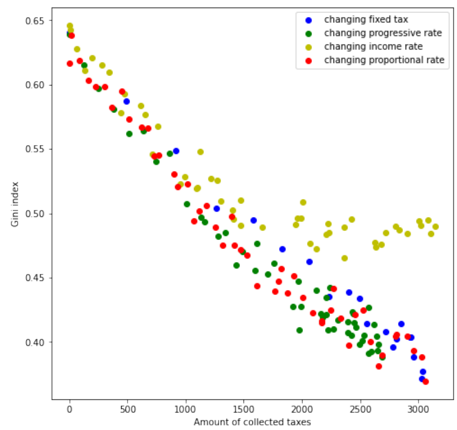


Figure 21: Scatter plot of average Gini Index against amount of taxes for each form of taxation, under Theft-Fraud model

sample t-test made on the null hypothesis that the difference between the Gini index value is equal to zero, against the hypothesis that it is greater than zero. As regards the difference between fixed tax and proportional tax the t Statistic is 8.35, really larger than the threshold value; while for the difference between proportional and progressive tax the t Statistic is 4.92, significantly higher than the threshold value too. So we can reject the null hypothesis and confirm that they reduce the Gini index more efficiently in this order: progressive taxes, proportional taxes, fixed tax and finally income tax.

Regarding the Theft-fraud model the situation seems to be much more confused, probably due to the greater variability of the model itself. The three different wealth taxes might maintain their relationship of greater efficiency in combating inequality with rising taxes, but remaking the one sample t-test on the difference between relative Gini index values. The difference between fixed and proportional tax remains significantly different from zero with a t-test equal to 3.71 and a p-value equal to 0.0001, the difference between proportional tax and progressive tax is no longer significantly different from zero with a t-test equal to -0.19 and a p-value equal to 0.4237. The most interesting result, however, concerns the income tax which initially causes the Gini index to drop in a similar way to taxes on wealth and then halts the decline to a value higher than the minimum value achievable with other taxes. Despite the similar slope in the first part, the income tax remains less efficient than the others, as confirmed by the one sample t-test on the difference with the proportional tax, where the t statistic is 5.53 and the p-value is 0. If in the case of taxes on wealth it was easy to understand why at some point the growth of the parameter did not lead to a decrease in inequality, once all the wealth gained is taxed, no more can be done, the same cannot be said of the income tax. From Figure 19 it looks like that when α_i exceeds 0.6 the increase in taxes collected does not lead to a decrease in inequality. This could depend on the fact that, by not taxing the agents all the steps, at a certain point it becomes irrelevant for the distribution of wealth what happens in the taxation turn where the exchanges are reduced by taxes compared to what happens in the remaining turns that are not affected from taxes.

3 Literature countercheck

3.1 Obstacles in analysis and scarcity of wealth and income data

Taxing wealth is one of the big topics, especially in years before or after big crises and events, e.g. First and Second World Wars, the global financial crisis of 2008/09, presidential elections in USA 2019/20, pandemics. It is one of the ways that governments put on the table to decrease national debt resulted from crises, and they usually do one-off wealth taxation. Not only in years of crises, taxation and wealth are of relevant topics, because there is extreme wealth and income inequality, but the former is even worse (Suisse, 2019).

Despite the renewed and growing interest in wealth taxes, empirical researches on their budgetary, redistributive, and motivational consequences are still rare, which is understandable considering current datasets for wealth. OECD and Eurostat datasets are lacking data about wealth, especially for the purpose to make a comparison between our simulation and real-world data. While governments tax income, it does not tax wealth, mainly. They rarely keep data on citizen's individual wealth, and it is difficult to assign an objective value to many forms of wealth, such as art collection and family businesses. We can find a better situation regarding income data. For example, at Eurostat (2017) dataset there is a calculated Gini coefficient pre and post-tax and after redistribution and we will use it to compare the impact of proportional and progressive taxes on inequality with other sources.

Microsimulation modeling is an intriguing approach to address these problems. Despite the growing research on the impact of personal income taxation using microsimulation approaches, there is a significant gap in the knowledge on wealth taxation. This is partly owing to the fact that microsimulation models' policy scope is heavily reliant on the underlying dataset. Administrative tax data generally do not cover all the information necessary because most countries do not have a general wealth register and most asset types are tax exempt such that they should not be reported to the tax authorities. For a long time survey data also hardly covered any information on savings and asset accumulations.

In the next two subchapters, we will briefly review some researches that had the same questions as we have "How diferent types of taxes impact wealth and income inequality?". They considered their empirical research using the EUROMOD[6] platform, which is also a simulation, but more complex than ours and relies on the survey's dataset. Considering the complexity and usage of a real dataset, we think that this is the closer approach that can match reality, even though it is also simulation and has some limitations. Considering all the boundaries, despite our first intention, it is not feasible to do an empirical countercheck but just a literature one.

3.2 Flat versus progressive tax

In this section, we will discuss a more empirical approach on how proportional and progressive tax impact the Gini index. Our models in section 2.2 have shown that proportional tax reduces inequality less than progressive one, and similar conclusions

[6]EUROMOD is a tax-benefit microsimulation model for the European Union and UK that enables researchers and policy analysts to calculate, in a comparable manner, the effects of taxes and benefits on household incomes and work incentives for the population of each country and for the EU as a whole.

were got in a technical report about progressive tax reforms in flat tax countries in the EU using EUROMOD Barrios et al. (2018).

Many developing countries in Central and Eastern Europe changed progressive tax systems with simpler proportional tax with fewer tax brackets and overall easier administrative jobs. These countries were firstly Estonia, Latvia, Lithuania and after that Russia, Romania, and Hungary, all changes were in the mid-90s to the beginning of the 2000s. Overall, all countries that made this shift got a positive impact on growth in tax revenue, due to higher compliance and reporting. Russia especially made a great benefit from this switch (Ivanova et al., 2005).

In the paper "Progressive tax reforms in flat tax countries" (Barrios et al., 2018) introduce three scenarios for switching from proportional tax to progressive tax, but the first one about switching from proportional to the pure progressive tax is the most similar to our model so we will concentrate on that scenario. Firstly we will show a copy of the table from this paper, which would be important for the EUROMOD part. As it is shown in the table, three countries that introduced proportional tax first had used the upper tax rate from the previous progressive system and the next three countries used bottom tax rate or even lower rates for newly introduced proportional tax rate. Lithuania, Latvia, and Estonia decreased their firstly introduced proportional tax rates, because the they were too high and in Table 1, we can see the last and current rates indicated for 2017.

Country	Lithuania	Latvia	Estonia	Romania	Bulgaria	Hungary
Year	1994	1997	1994	2005	2008	2011
Before introduction	18% - 33%	10% - 25%	16% - 35%	18% - 40%	10% - 24%	17% - 32%
After introduction	33%	25%	26%	16%	10%	16%
2017	15%	23%	20%	16%	10%	16%

Table 1: Adapted Table from Barrios, Ivaškaitė-Tamošiūnė et al (2018, p.6).

Despite the fact that these countries implemented a proportional tax, they all included tax breaks and credits, implying a certain level of progressivity (so this is not pure proportional tax, but still it is not progressive as the previous progressive one in these countries). According to the paper, countries with proportional tax systems redistribute income significantly less than countries with progressive tax systems.

Below are two tables, first one is from the Barrios et al. (2018), and the second one is our table and represent the data we collected. We did this comparison just to see if there are any different conclusions and differences between theirs EUROMOD sources, and our Eurostat data.

Country	Gini (pre tax)	Gini(post tax)	Redistributive effect	Ranking in EU28
Hungary	0.499	0.289	0.210	14
Romania	0.543	0.365	0.179	21
Lithuania	0539	0.371	0.168	24
Estonia	0.494	0.330	0.164	25
Latvia	0.498	0.350	0.148	27
Bulgaria	0.502	0.359	0.144	28
EU-28	0.505	0.295	0.210	-

Table 2: Adapted Table from Barrios, Ivaškaitė-Tamošiūnė et al (2018, p.8).

Country	Gini (pre tax)	Gini(post tax)	Redistributive effect	Ranking in EU28
Hungary	0.507	0.281	0.226	8
Romania	0.516	0.331	0.185	18
Lithuania	0.520	0.376	0.144	26
Estonia	0.457	0.316	0.141	27
Latvia	0.472	0.345	0.127	28
Bulgaria	0.552	0.402	0.150	25
EU-28	0.491	0.296	0.195	-

Table 3: Eurostat data for 2017. year.

We got similar results. Countries with a proportional tax system have less redistributive effect compared to the EU-28 average. The difference is in rankings, they got that Bulgaria had the least redistributive effect but in our case it is Latvia. The second difference is that in their data, Hungary had the same redistributive effect as the EU-28 average, we got that it is higher than the EU average. Despite that difference, it is clear that the other six countries have a less redistributive effects than the EU average.

After it is shown that proportional taxes have a lower redistributive effect we can consider if progressive taxes can reduce inequality and lower the Gini index for disposable income more than propotional ones, or make a bigger redistributive gap for these countries. Now on the scene goes EUROMOD model and testing what are effects of the new policy changes. It was kept the existing personal income tax proportional rate as the second bracket of the new progressive tax. For the first bracket, they reduced it by 5 percentage points and for the top bracket, they increased it by 7 percentage points. It is done the same for all six countries for better comparability of the results. Below is a table that summarizes three brackets of the new tax system.

Bracket (tax rates)	Lithuania	Latvia	Estonia	Romania	Bulgaria	Hungary
1st bracket	10%	18%	15%	11%	5%	11%
2nd bracket (existing)	15%	23%	20%	16%	10%	16%
3rd bracket	22%	30%	27%	23%	17%	23%

Table 4: *EUROMOD estimate from Barrios, Ivaškaitė-Tamošiūnė et al (2018, p.6).

After simulation, a progressive personal income tax increased tax revenues and reduced the average disposable income of the richest households. The impact on tax revenues was positive in all countries, from 6.2% in Latvia to 13.8% in Hungary. Most importantly, all countries experience a fall in the Gini coefficient in a range from -0.77 pp in Romania to -1.34 pp in Hungary.

Another interesting approach to see if proportional or progressive taxes are better for lowering the Gini index is by measuring the Kakwani index[7]. Kakwani index is the difference between tax concentration coefficient, the measure of income group's shares of total taxes, and the Gini coefficient before-tax income. Index can vary between -1 to 1, the larger the index is, the more progressive is the tax. For example, if an individual who earns 5% of the income pays 5% of total taxes, the Kakwani

[7]The index was developed by Nanak C. Kakwani, "Measurement of Tax Progressivity: An International Comparison," 87 Econ. J. 71 (1976))

index equals zero and tax does not have progressivity. On the other side, if high-income individuals pay a greater share of total taxes than their share of total income before tax, the Kakwani index is positive and that tax is progressive, but if high-income individuals pay a lesser share of total taxes than their share of total income before tax, the index is negative and tax is regressive one.

In the paper Taxes, Transfers, Progressivity, And Redistribution(Slavov and Viard, 2016), it is shown that tax progressivity and redistribution are positively correlated. The United States and Ireland had the most progressive taxes comparing to other OECD countries in 2013., their Kakwani index was around 0.14 with high redistribution. Nordic countries had a negative Kakwani index around -0.1 (regressive taxes), but still had high redistribution effects. The regression in (Slavov and Viard, 2016) shows that there is a positive tax progressivity impact on redistribution, but it is also important how the collected tax amount is distributed later on direct cash transfers, subsidies, and spending on education and health(Slavov and Viard, 2016).

3.3 Wealth versus income tax – impact on inequality

Wealth inequality is much more severe than inequality based only on income. As we mentioned before, wealth tax is one of the major topics, especially in the US which has never had a pure net tax like some of the European countries. It is logical to think that, if there is high inequality, the government can try to tax the wealthy, increase annual tax revenue and transfer to unwealthy ones and decrease the inequality, but in reality, it is not that simple. Some EU countries had wealth tax such as Austria, Denmark, Germany, Netherlands, Iceland, Finland, etc., but abolished wealth tax due to high administrative costs, negative impact on economic growth, risk-taking and entrepreneurship. France and Switzerland still have wealth tax and rates are usually from 0.3% to 1% (Iara, 2015).

There is a lot of arguments in favor and against taxing wealth. Some arguments in favor are: can decrease inequality, equality of opportunity, decreasing public debt. Arguments against could be: double taxation, liquidity constraints, cost of tax administration, tax avoidance & evasion.

In this section, we will review the results of the paper EWIGE 2 – Update and Extension of the EUROMOD Wealth Taxation Project(Bone et al., 2019). Here is also used EUROMOD simulation and households survey data. In (Bone et al., 2019), there are 17 EU countries for which are calculated Gini coefficient before and after income taxes and after wealth taxes. The results are that in all countries, income taxes reduce inequality more than wealth taxes do, and it is fascinating how small the inequality impact of wealth taxes is. Income tax reduced inequality between 1.5% in Poland to 22% in Austria. On the other side, wealth tax has no impact at all on inequality, or even increases it. For example, in Belgium, inequality increased around 5% after wealth tax. In all 17 countries, the average tax rate of wealth taxes is significantly lower than income taxes. The average wealth taxes was between 0.1% (Hungary, Slovakia) and 3.5% (Estonia), and income taxes were between 10% (Spain) and 35% (Belgium). Also shown in (Bone et al., 2019), the progressivity of wealth taxes is much lower than income one. In the majority of countries, wealth taxes are regressive (negative Kakwani index), or proportional (close to zero). So, the very low redistributive effect can be explained by the low level and regressivity of wealth taxes. In our Agent-based model, we got that taxing the wealth can decrease the Gini index significantly more than taxing the income. This is due to fact that we used quite high numbers for the wealth tax rate (same numbers as for income tax rate).

Another interesting point could be shifting tax from labor to wealth (to unburden the income tax). In EUROMOD it was done by representing a budgetary loss in the revenue from social insurance contribution (part of income taxes) and compensated that loss by introducing a new wealth tax. The result was that with a very small wealth tax rate,0.0191% in Cyprus and 0.1693% in Slovakia, a substantial amount of revenue could be raised.

4 Conclusion

From what we came up with, there are many conclusions about the impact that different types of taxation may have on the distribution of wealth.

In our simulation, the outputs validate what others agent-based models discovered on the difference between taxing wealth instead of income: where taxes on wealth can generally impact more the Gini index than income taxes. But in our results, this relation is quantitatively really sensitive to which transaction model is chosen. We found that in models that use EUROMOD, wealth taxation does not have always a great impact on redistribution and decreasing inequality, mainly due to the low nature of wealth tax rates taken into consideration. Besides that, wealth taxes can be more efficient than income taxes for increasing or maintaining the tax budget revenue of the state.

About the difference between progressive and flat tax, we discovered that progressive should have a greater impact on the Gini index, but it is again sensitive on which model of transaction is chosen. It is important to note that it is not always the case that every time we run the multiple runs experiments on Theft-fraud model we didn't find a significant difference, so it is also determined by chance because of the greater variability of this model. On the other side, EUROMOD and Kakwani index gave similar results. Progressive taxes could increase more the state's tax budget and may have a greater impact on lowering the Gini, but after collecting taxes the most important thing is if the state transfers the money and subsidies to people with lower income or use different social expenditure.

Exploring the research question with this method also brought us some consideration on the potential and the limits of agent-based model in this context.

Considering the lack of data and empirical researches, agent-based models and other simulations like EUROMOD are useful for deriving insights, even they are the basic ones, with knowing all the possible limits of our models and simulations of others.

On the other hand, to deeply study these problems with an agent-based perspective and try to evaluate the results with real data a great step forward is needed. We refer to the fact that to see how taxation on income differs from taxation on wealth we need models that, in addition to bringing out interesting wealth distribution, are consistent with real-world income distribution. In all the agent-based models that we found, income of an agent in one step is at most poorly correlated with incomes of previous steps. This is the reason why, although wealth remains from time to time in the hands of a few agents in Theft-Fraud model, after many steps most of the agents are part of the richest section at least for few steps. This is not what happens in the real world and it is a big limit if we want to calculate the Gini index of income and compare it with real data.

In conclusion, we suggest that for future research will be useful to define other agent-based models where also the income distribution will be unfair and maybe due to the introduction of some sort of correlation between income trend per agent.

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