Social Navigation PyEnvs - Pedestrian Motion Models

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Abstract

This paper briefly describes the pedestrian motion models implemented in the Social Navigation PyEnvs framework [4]. First, the basic version of the Social Force Model (SFM) and two of its variants are introduced. Then, the Headed Social Force Model (HSFM), a more sophisticated motion model which accounts for the pedestrians heading, is described along with two of its modified versions. Finally, another version of the HSFM which employs a different torque force is introduced.

1 Social force model

Consider a system of n pedestrians moving in a 2D environment. Each pedestrian i, $i = 1, ..., n$, is modeled as a particle with mass m_i and radius r_i, and its position and velocity are expressed in the global reference frame and denoted as $\mathbf{p}_i = [x_i, y_i]^T$ and $\mathbf{v}_i = [v_{x_i}, v_{y_i}]^T$, respectively. The dynamics of the i-th human agent are given by the following equations:

$$\dot{\mathbf{p}}_i = \mathbf{v}_i, \tag{1}$$

$$\dot{\mathbf{v}}_i = \frac{1}{m_i}\mathbf{u}_i, \tag{2}$$

where $\mathbf{u}_i$ features the global force driving the i-th pedestrian. The expression of this force is the sum of other forces which depend on the motion model considered. However, in all cases, the global force is computed using the equation below.

$$\mathbf{u}_i = \mathbf{f}_i^0 + \sum_{j \neq i} \mathbf{f}_{ij}^p + \sum_w \mathbf{f}_{iw}^w, \tag{3}$$

where $\mathbf{f}_i^0$ represents the force attracting the i-th pedestrian towards its next goal, $\mathbf{f}_{ij}^p$ describes the repulsive force exerted from the j-th to the i-th pedestrian (modelling the tendency of humans to stay away from each other), and $\mathbf{f}_{iw}^w$ corresponds to the repulsive force of obstacle w (modelling the tendency of humans to stay away from obstacles).

In all the models presented below, the so-called desired force $\mathbf{f}_i^0$ is given by the following equation:

$$\mathbf{f}_i^0 = m_i \frac{\mathbf{v}_i^d - \mathbf{v}_i}{\tau_i}, \tag{4}$$

where $\mathbf{v}_i^d$ models the pedestrian's desired to move with a certain velocity towards the next goal, and $\tau_i > 0$ is a parameter which regulates the rate of change of the velocity. To determine the desired velocity vector, the following equation is used:

$$\mathbf{v}_i^d = v^d \mathbf{e}^d, \quad \text{with} \quad \mathbf{e}^d = \frac{\mathbf{p}_{goal} - \mathbf{p}_i}{||\mathbf{p}_{goal} - \mathbf{p}_i||}. \tag{5}$$

The parameter v^d corresponds to the preferred speed with which the pedestrian moves towards the goal.

1.1 Standard social force model

The basic version of the social force model was implemented following [2]. Considering the radius r_i of the i-th pedestrian, several elements can be defined:

$$r_{ij} = r_i + r_j, \tag{6}$$
$$d_{ij} = ||\mathbf{p}_i - \mathbf{p}_j||, \tag{7}$$
$$\mathbf{n}_{ij} = \frac{\mathbf{p}_i - \mathbf{p}_j}{d_{ij}} := [\mathbf{n}_{ij}(1), \mathbf{n}_{ij}(2)]^T, \tag{8}$$
$$\mathbf{t}_{ij} = [-\mathbf{n}_{ij}(2), \mathbf{n}_{ij}(1)]^T, \tag{9}$$
$$\Delta v_{ji}^{(t)} = (\mathbf{v}_j - \mathbf{v}_i)^T \mathbf{t}_{ij}. \tag{10}$$

In this case, the repulsive force $\mathbf{f}_{ij}^p$ exerted by the j-th to the i-th pedestrian is computed as:

$$\mathbf{f}_{ij}^p = \left[A_i e^{(r_{ij}-d_{ij})/B_i} + k_1 \max\{0, r_{ij} - d_{ij}\}\right] \mathbf{n}_{ij} + k_2 \max\{0, r_{ij} - d_{ij}\} \Delta v_{ji}^{(t)} \mathbf{t}_{ij}, \tag{11}$$

where A_i, B_i, k_1, and k_2 are constant parameters. The last two terms of the equation represent the compression and friction forces and come into play only when $d_{ij} < r_{ij}$.

The repulsive force $\mathbf{f}_{iw}^w$ of obstacle w is defined as:

$$\mathbf{f}_{iw}^w = \left[A_w e^{(r_i-d_{iw})/B_w} + k_1 \max\{0, r_i - d_{iw}\}\right] \mathbf{n}_{iw} - k_2 \max\{0, r_i - d_{iw}\} (\mathbf{v}_i^T \mathbf{t}_{iw}) \mathbf{t}_{iw}, \tag{12}$$

The elements d_{iw}, $\mathbf{n_{iw}}$, and $\mathbf{t_{iw}}$ are computed as described in (6)-(10) but replacing $\mathbf{p}_j$ with the position vector of the closest point of obstacle w to the i-th pedestrian.

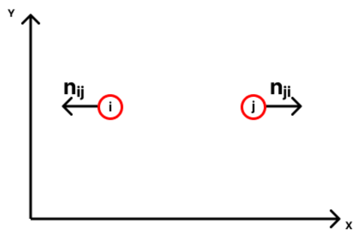

Figure 1: Example showing the direction of the unit vector $\mathbf{n}_{ij}$ involved in the computation of the repulsive forces exerted from other pedestrians for the standard SFM. The contribution along $\mathbf{t}_{ij}$ comes into play only if $d_{ij} < r_{ij}$.

The parameters used within the implementation of this model are the following: $\tau_i = 0.5s$, $A_i = A_w = 2 \cdot 10^3 N$, $B_i = B_w = 0.08m$, $k_1 = 1.2 \cdot 10^5 kg \cdot s^{-2}$, and $k_2 = 2.4 \cdot 10^5 kg \cdot m^{-1} \cdot s^{-1}$. While m_i, r_i and v^d can be customized for each particle.

1.2 Modified social force model with [1]

Even though the model previously described has demonstrated a good capacity to simulate human motion, in some cases the particles can get stuck because the resulting forces compensate each other. One way to avoid this problem is to add an additional sliding term to both the repulsive force of other pedestrians and the repulsive force of obstacles. Following the idea described in [1], the new equations of the two forces become:

$$\mathbf{f}_{ij}^p = \left[A_i e^{(r_{ij}-d_{ij})/B_i} + k_1 \max\{0, r_{ij} - d_{ij}\}\right] \mathbf{n}_{ij} + \left[C_i e^{(r_{ij}-d_{ij})/D_i} + k_2 \max\{0, r_{ij} - d_{ij}\} \Delta v_{ji}^{(t)}\right] \mathbf{t}_{ij}, \tag{13}$$

$$\mathbf{f}_{iw}^w = \left[A_w e^{(r_i-d_{iw})/B_w} + k_1 \max\{0, r_i - d_{iw}\}\right] \mathbf{n}_{iw} + \left[C_w e^{(r_i-d_{iw})/D_w} - k_2 \max\{0, r_i - d_{iw}\}\right] (\mathbf{v}_i^T \mathbf{t}_{iw}) \mathbf{t}_{iw}, \tag{14}$$

where C_i, D_i, C_w, and D_w are constant parameters. Notice that the two additional terms $C_i e^{(r_{ij}-d_{ij})/D_i}$ and $C_w e^{(r_i-d_{iw})/D_w}$ are multiplied by the vector tangent to the interaction vector between the two elements ($\mathbf{t}_{ij}$ and $\mathbf{t}_{iw}$, respectively). In this way, a new term working on the tangential direction is introduced for any value of d_{ij} and d_{iw}.

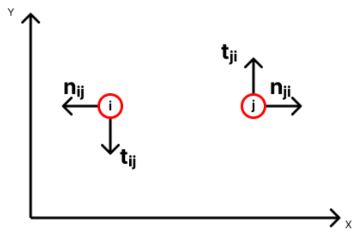

Figure 2: Example showing the direction of the unit vectors $\mathbf{n}_{ij}$ and $\mathbf{t}_{ij}$ involved in the computation of the repulsive forces exerted from other pedestrians for the modifed SFM with [1].

The parameters employed for this model are: $\tau_i = 0.5s$, $A_i = A_w = 2 \cdot 10^3 N$, $B_i = B_w = 0.08m$, $k_1 = 1.2 \cdot 10^5 kg \cdot s^{-2}$, $k_2 = 2.4 \cdot 10^5 kg \cdot m^{-1} \cdot s^{-1}$, $C_i = C_w = 1.2 \cdot 10^2 N$, and $D_i = D_w = 0.6m$. While $m_i = 80kg$ and r_i and v^d can be customized for each particle.

1.3 Modified social force model with [3]

A more sophisticated way of including a sliding term in the repulsive forces of the model is presented in [3]. In this case, the interaction vector is computed taking into account also the velocities of pedestrians. Let us denote $\mathbf{i}_{ij}$ ($\mathbf{t}_{ij}$ in [3]) as the interaction vector defined as:

$$\mathbf{i}_{ij} = \frac{\lambda(\mathbf{v}_i - \mathbf{v}_j) - \mathbf{n}_{ij}}{||\lambda(\mathbf{v}_i - \mathbf{v}_j) - \mathbf{n}_{ij}||} \tag{15}$$

where $\mathbf{n}_{ij}$ is computed as in (8), λ is a constant parameter, and $\mathbf{v}_i$ and $\mathbf{v}_j$ are the velocity vectors of the i-th and j-th pedestrians, respectively. Let us also define:

$$\theta_{ij} = \angle\mathbf{n}_{ij} - \angle\mathbf{i}_{ij} + \pi, \tag{16}$$

$$K_{ij} = \theta_{ij}/|\theta_{ij}|, \tag{17}$$

$$\mathbf{h}_{ij} = [-\mathbf{i}_{ij}(2), \mathbf{i}_{ij}(1)] \qquad (\mathbf{n}_{ij} \text{ in [3]}), \tag{18}$$

where $\theta_{ij} \in [-\pi, \pi]$ and the phase of each vector is computed using $atan2$. The repulsive force of other pedestrians can now be defined as:

$$\mathbf{f}^p_{ij} = -E_i e^{-d_{ij}/F_{ij}} \left[e^{-(n'_s F_{ij} \theta_{ij})^2} \mathbf{i}_{ij} + K_{ij} e^{-(n_s F_{ij} \theta_{ij})^2} \mathbf{h}_{ij} \right], \tag{19}$$

where E_i, n_s, and n'_s are constant parameters and $F_{ij} = \gamma||\lambda(\mathbf{v}_i - \mathbf{v}_j) - \mathbf{n}_{ij}||$. The first term of the equation describes the deceleration along the interaction direction $\mathbf{i}_{ij}$, while the second term accounts for the directional changes along the vector $\mathbf{h}_{ij}$. Note that the radius of pedestrians is not considered in (19).

In this case the repulsive force of other obstacles is the same presented in (12).

For this implementation, the parameters employed are: $\tau_i = 0.5s$, $A_w = 2 \cdot 10^3 N$, $B_w = 0.08m$, $k_1 = 1.2 \cdot 10^5 kg \cdot s^{-2}$, $k_2 = 2.4 \cdot 10^5 kg \cdot m^{-1} \cdot s^{-1}$, $E = 3.6 \cdot 10^2 N$, $\lambda = 2.0$, $\gamma = 0.35$, $n_s = 2.0$, and $n'_s = 3.0$. While m_i, r_i and v^d can be customized for each particle.

It is interesting to examine the difference between the two modified versions of the repulsive force of pedestrians. As shown in figure 4, the equation introduced in [3] generates a force which is essentially similar to the one introduced in the previous section, but rotated of an angle $\theta + \pi$ that depends on the pedestrians velocities.

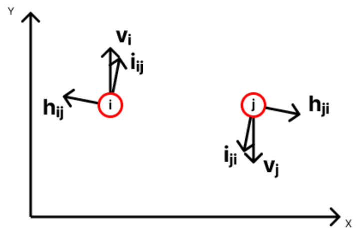

Figure 3: Example showing the direction of the unit vectors $\mathbf{i}_{ij}$ and $\mathbf{h}_{ij}$ (with $\lambda = 2.0$) involved in the computation of the repulsive forces of other pedestrians for the modifed SFM with [3].

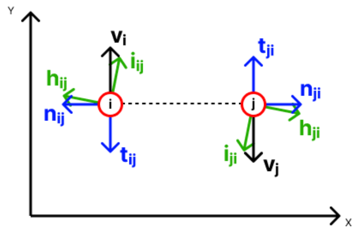

Figure 4: Example showing the difference between the unit vectors involved in the computation of the repulsive forces of pedestrians introduced in [1] (blue vectors) and [3](with $\lambda = 2.0$, green vectors).

2 Headed social force model

To improve the realism of the trajectories generated from the social force model, the HSFM explicitly includes the heading θ_i of each pedestrian i. For this aim, it is convenient to attach a body frame to each individual centered at the pedestrian's position and whose x-axis is directed towards the pedestrian's heading. Doing so allow us to define θ_i as the angle between the x-axis of the body-frame and the one of the global frame. Consider $\mathbf{q}_i = [\theta_i, \omega_i]^T$ to be the vector containing the agent's heading and angular velocity, respectively and denote $\mathbf{v}_i^B = [v_i^f, v_i^o]^T$ as the velocity vector expressed in the body frame. The velocity vector expressed in the global frame can be defined using the rotation matrix $\mathbf{R}(\theta_i)$, giving us $\mathbf{v}_i = \mathbf{R}(\theta_i)\mathbf{v}_i^B$, where:

$$\mathbf{R}(\theta_i) = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{bmatrix} := [\mathbf{r}_i^f, \mathbf{r}_i^o]. \tag{20}$$

The dynamics of the i-th human agent are now regulated by the three following equations:

$$\dot{\mathbf{p}}_i = \mathbf{R}(\theta_i)\mathbf{v}_i^B, \tag{21}$$

$$\dot{\mathbf{v}}_i^B = \frac{1}{m_i}\mathbf{u}_i^B, \tag{22}$$

$$\dot{\mathbf{q}}_i = \mathbf{A}\mathbf{q}_i + \mathbf{b}_i u_i^\theta, \tag{23}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{b}_i = \begin{bmatrix} 0 \\ \frac{1}{I_i} \end{bmatrix}. \tag{24}$$

I_i denotes the moment of inertia of the i-th pedestrian. Thus, the main forces involved in the dynamic system are the global force $\mathbf{u}_i^B = [u_i^f, u_i^o]^T$ and the torque force u_i^θ. The global force is given by the following equations:

$$u_i^f = (\mathbf{f}_i^0 + \sum_{j \neq i} \mathbf{f}_{ij}^p + \sum_w \mathbf{f}_{iw}^w)^T \mathbf{r}_i^f, \tag{25}$$

$$u_i^o = k^o(\sum_{j \neq i} \mathbf{f}_{ij}^p + \sum_w \mathbf{f}_{iw}^w)^T \mathbf{r}_i^o - k^d v_i^o, \tag{26}$$

where $\mathbf{f}_i^0$ is computed using (4), $\mathbf{f}_{ij}^p$ and $\mathbf{f}_{iw}^w$ are the repulsive forces of other pedestrians and obstacles and depend on the specific implementation of the model. The terms k^o and k^d are constant parameters. Note that while the forward component of the global force is essentially the sum of all the sub-forces carried in the forward direction of the body frame, the orthogonal component of the force just involves the repulsive forces and a damping term proportional to the sideward velocity v_i^o.

Due to the integration of the heading θ_i of the i-th pedestrian, also the torque force driving the angular acceleration must be specified:

$$u_i^\theta = -k^\theta(\theta_i - \theta_i^0) - k^\omega \omega_i, \tag{27}$$

where θ_i^0 is the phase of the desired force $\mathbf{f}_i^0$ in the global reference frame, ω_i is the current angular velocity of the pedestrian, and k^θ and k^ω are parameters computed as:

$$k^\theta = I_i k^\lambda f_i^0, \tag{28}$$

$$k^\omega = I_i(1+\alpha)\sqrt{\frac{k^\lambda f_i^0}{\alpha}}, \tag{29}$$

where f_i^0 is the magnitude of the desired force $\mathbf{f}_i^0$, and k^λ and α are constant parameters.

Using this as a base model, three different implementations were developed which differ on how the repulsive forces are computed: one using (11)-(12), another employing (13)-(14), and the last one using (19) and (12).

The additional parameters introduced to model the heading of each pedestrian were set as: $k^o = 1$, $k^d = 500kg \cdot s^{-1}$, $k^\lambda = 0.3N^{-1} \cdot s^{-2}$, $\alpha = 3$.

3 Modified headed social force model

The standard HSFM uses the desired force as a driving component for the computation of the torque force. Doing so, the orientation of the pedestrian ends up following the direction of the force that attracts him towards its goal. Since this is not always desired, especially when an obstacle is placed right between the human and its goal (in which case the HSFM drives the human to collide with the obstacle and to "bounce" on it until he eventually manages to avoid it), a new version of the HSFM is defined by modifying the torque force equation as described below.

Consider $\mathbf{f}_i^{tot}$ as the sum of the force attracting the pedestrian towards its goal and the repulsive forces exerted by obstacles and other humans:

$$\mathbf{f}_i^{tot} = \mathbf{f}_i^0 + \sum_{j \neq i} \mathbf{f}_{ij}^p + \sum_w \mathbf{f}_{iw}^w, \tag{30}$$

and let us denote as θ_i^{tot} and f_i^{tot} the phase and magnitude of this force, respectively. The new torque force will then be computed as below:

$$u_i^\theta = -k^\theta(\theta_i - \theta_i^{tot}) - k^\omega \omega_i, \tag{31}$$

where k^θ and k^ω can be derived by substituting f_i^{tot} to f_i^0 in equations (28) and (29).

Using this as a base model, three different implementations were developed which differ on how the repulsive forces are computed: one using (11)-(12), another employing (13)-(14), and the last one using (19) and (12).

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