Bocconi

BOCCONI UNIVERSITY

MAFINRISK:

Specialized Master in Quantitative Finance & Risk Management

DERIVATIVES

Certificates on Commerzbank Efficiency Growth Index

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Chapter 1

Introduction

The goal of this report is to provide an assessment of an Investment Certificate issued by Commerzbank on the 25th of April 2014 with maturity the 25th of April 2018 and based on the Commerzbank Efficiency Growth Index. The Index itself represent a virtual portfolio consisting on a Basket, and it is based on a mutual fund called Efficiency Growth Fund and aims at achieving a target volatility level by leveraging the investment in the Fund. This report will be divided into a quantitative and qualitative part.
In the quantitative part we will provide a fair value for the Certificate on the issue date, and for each end-of-month date until maturity. We will Compare our evaluation on the issue date with the actual issue price and comment the result. In order to price the certificate we made some assumptions that will be well explained alongside the report and supported by some calculations and graphs.
In the qualitative part we will present a qualitative description of the Certificate, showing its validity from the viewpoint of the issuer and the buyer. In particular from the investor's viewpoint we will show explain if and why this could be a good investment, what are the risks related to the investment, and what are the costs. From the issuer's viewpoint, we will outline the benefits of issuing the Certificate and its sources of revenues.

Chapter 2

Quantitative Section

1 Index Description

The Commerzbank Efficiency Growth Index is a volatility index that represents the daily performance of a virtual portfolio consisting of a Basket and a Reference Interest Rate. The Basket consists of a single mutual fund, called Efficiency Growth Fund, while The Reference Interest Rate is the 1-month EURIBOR rate. The weighting between the Basket and the Reference Interest Rate will be adjusted on a daily basis depending on the realized volatility of the Basket and calculated by using a purely rule-based methodology. The Weighting depends on the daily realized volatility of the Basket and the fixed Risk Control Level, expressed as a percentage. The role of the Risk Control Level is that whenever the Realized Volatility is below the Risk Control Level, the Weighting is at or above 100% and capped at the Max Exposure, while when the realized volatility is above the Risk Control Level, the Weighting falls below 100% according to the formula for the determination of Weighting.

2 Index Calculation

The Index Value has been computed from the 14th April 2014 to the 20th April 2018. On the first date, as reported in the document, the Index Value is 100 and every index point corresponds to 1 EUR. As described before, the Index is composed of a Basket and a Reference Interest Rates. The weighting of this two components depends on the realized volatility of the fund. The realized volatility has been calculated on a 20-day window that starts $n+2$ days before the Index Calculation Date and ends 2 days before the Index Calculation Date.
The realized volatility is calculated by using a purely rule-based methodology.

$$RealVol_{t-lag} = \sqrt{\frac{d}{m}} \cdot \sqrt{\sum_{k=1}^{n} \left(\ln \frac{Basket_{t-n+k-lag}}{Basket_{t-n+k-1-lag}} \right)^2 - \frac{\gamma}{n} \left(\sum_{k=1}^{n} \ln \frac{Basket_{t-n+k-lag}}{Basket_{t-n+k-1-lag}} \right)^2} \tag{2.1}$$

where:

- t means the reference to the relevant Index Calculation Date.
- n means the volatility window which corresponds to the number of days used to calculate the Realized Volatility, in this case $n = 20$.

- d means the annualising factor which represents the expected number of Index Calculation Dates in each calendar year, in our report $d = 252$
- γ means the factor of 1
- m means the scaling factor of 20
- lag means the Lag factor equal to 2

Let us now briefly discuss the weighting factor. It represents the relative proportion of each Fund invested in the Basket versus the Reference Interest Rate as specified in the definition of Basket. The Weighting respect to Index Calculation Date is computed according the following formula:

$$W_t = min\left(MaxW; \frac{TarVol}{RealVol_{t-lag}}\right) \tag{2.2}$$

where:

- $TarVol$ means the Risk Control Level equal to 6%
- $MaxW$ means the Max Exposure equal to 150%
- lag means the Lag factor equal to 2
- $RealVol$ means the Realized Volatility with respect to Index Calculation Date

Finally, we were able to compute the value of the Index for each Index Calculation Date, according to the following relation:

$$Ind_t = Ind_{t-1} \cdot \left(1 + \left(W_{t-1} \cdot \left(\frac{Basket_t}{Basket_{t-1}} - 1\right)\right) + \left((1 - W_{t-1} \cdot Rate_{t-1} \cdot \frac{Act_{t,t-1}}{conv}\right)\right) \tag{2.3}$$

where:

- W_t means the weighting with respect to the immediately preceding Index Calculation Date
- $Basket_{t/t-1}$ means the Basket Value with respect to the current Index Calculation Date and the immediately preceding Index Calculation Date
- $Rate_{t-1}$ means the Reference Interest Rate with respect to the immediately preceding Calculation Date
- $Act_{t,t-1}$ means the number of calendar days from, but excluding, the immediately preceding Index Calculation Date to, and including, the relevant Index Calculation Date
- $conv$ is a number: 360

Let us stress once again that the first index value is set to be 100 points, and it is expressed in the Index Currency, which is EUR.

3 Derivative Payoff

Before we proceed to calculate the price of the derivative with the index as the underlying, let us present the payoff of the derivative:

$$P = CA \cdot max\left(0; \frac{Ind_T}{K}(1 - 1.1\%)^4 - 1\right) \tag{2.4}$$

where:

- K is a constant and it has been calculated taking the average between the Index value on the dates 22nd, 23rd, and 25th April 2014. In our computations $K = 100.876$
- CA is just a number and it is 1000

As one can see from the formula, the Index is the underlying of the Certificate, which can be seen as a plain vanilla European call option.
Indeed, doing the following substitution:

$$S_T = CA\frac{Ind_T}{K}(1 - 1.1\%)^4 \tag{2.5}$$

The payoff results to be equal to the payoff of an European call option:

$$P = max\,(S_T - 1000)^+ \tag{2.6}$$

Said so, the price at each time of the Certificate can be computed using the Black-Scholes formula for call options on non-dividend paying stocks. Black and Scholes formula is based on some fundamental assumptions, like the underlying following a Geometric Brownian Motion, constant volatility over time, and constant interest rates for all maturities. Whether this assumptions are valid or not will be discussed later in this report.

4 Pricing the Derivative

In order to use the Black and Scholes pricing formula for call options, we have to make a fundamental assumption on the process governing the price evolution of the Index: it should follow a Geometric Brownian Motion with constant volatility:

$$Ind_t = r(Ind_t)dt + \sigma(Ind_t)dW_t \tag{2.7}$$

where r is the risk free rate and σ is the volatility.
Actually, the underlying of the Certificate is not directly Ind_t but a rescaling version of it, as stated before:

$$S_t = 1000\frac{Ind_t}{K}(1 - 1.1\%)^4 \tag{2.8}$$

Then, if Ind_t follows a Geometric Brownian Motion, also the new stochastic process $S_t = C \cdot Ind_t$, where $C = \frac{1000}{K}(1 - 1.1\%)^4$, follows a Geometric Brownian Motion:

$$S_t = rS_tdt + \sigma S_tdW_t \tag{2.9}$$

Now we are ready to price the Certificate, from Black and Scholes formula:

$$C_t = S_tN(d_1) - 1000e^{-r(T-t)}N(d_2) \tag{2.10}$$

with:

$$d_1 = \frac{\log\frac{S_t}{K} + (r + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}} \tag{2.11}$$

$$d_2 = d_1 - \sigma\sqrt{T - t} \tag{2.12}$$

The Certificate has been priced for any end-of-month date within the life of the Certificate, from the 25th April 2014 to the 25th April 2018. In order to do so, we estimated the risk-free rate and the volatility at each time t.

5 Our Assumptions on the Black and Scholes Model

In our valuation of the option using the Black-Scholes model, we made two key assumptions about the volatility and risk free rate. We proxy volatility using a 90 day rolling window and interpolated risk-free rates were calculated using Euribor and swap rates. We will discuss why we made these assumptions and evaluate their implications.

5.1 Interpolated Risk-Free Rate

The Black and Scholes model requires the use of a risk-free rate r to discount the expected payoff of the option. For this project, interpolated risk-free rates were derived using short-term Euribor data (1-month, 3-month, 6-month, and 12-month) and long-term swap rates (3-year and 5-year maturities).

We used the below formula to calculate the interpolated rates:

$$InterpolatedRate = R_1 + \frac{T_{target} - T_1}{T_2 - T_1} \times (R_2 - R_1) \tag{2.13}$$

where:

- T_{target} is the intermediate maturity whose rate needs to be calculated
- T_i are the available rate maturities just before and after T_{target}
- R_i are the rates corresponding to maturities T_i

Linear interpolation provides a practical method for estimating intermediate rates, it assumes no sharp yield curve movements, which may not hold during market stress or structural shifts.

For longer maturities, interpolation relied on more spaced-out swap maturities (e.g., 3-year and 5-year rates) to calculate monthly risk-free rates. This introduces potential inaccuracies, as the yield curve between these points may not follow a linear trajectory. Market stress, structural shifts, or changing monetary policies during 2014–2018 could cause deviations from the assumed smooth term structure, leading to errors in discounting the option payoff. However, the shorter maturity interpolated risk free rates 0-12 months, should be more accurate.

5.2 Choice of Volatility

The Black and Scholes Model assumes that the underlying asset follows a geometric Brownian motion with a constant volatility of sigma. This assumption means that the level of risk in the underlying asset remains stable over the life of the option. However, in the real world, volatility is not constant as various macroeconomic events affect the market conditions and causes the volatility to fluctuate over time. We must use a proxy for volatility because it cannot be directly observed in the market. It must be estimated using historical volatility or derived from market prices as implied volatility.

We tested the following three ways of proxying volatility:

- Using the standard deviation of the log return of prices over the entire period from 2014-2018
- Using the standard deviation of the log return of prices over a 90 day rolling period
- Using the standard deviation of the log returns of prices, computed cumulatively from the inception of the index to each valuation date

We used log returns for all three methods as it accounts for the compounding nature of price movements:

$$R_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \tag{2.14}$$

while the average log return over a given period is:

$$\bar{R}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} R_i \tag{2.15}$$

Constant average realized volatility over the four year period Identifying σ_t as the annualized volatility for day t in the period 2014-2018, we computed the constant average realized volatility as:

$$\sigma_{const} = \frac{1}{N}\sum_{t=1}^{N}\sigma_t \tag{2.16}$$

We found this to be 5.131%.
The advantage of using this method is that it aligns with the constant volatility assumption of the Black and Scholes model and it is simple to calculate compared to the other methods. However, it ignores the fluctuating nature of real life volatility as it fails to capture the effect of short term macro-economic events. This may understate risk during volatile periods, such as Brexit in 2016, and the Italian debt crisis in 2018.

90 days rolling average volatility This volatility measures the annualized standard deviation of the daily log return of the index over the recent 90-day window.
The advantage of using this method is that it allows our valuation to account for short-term changes in market conditions. Looking at figure 1, we see that the rolling volatility is indeed fluctuating over the four-year period. However, rolling volatility smooths fluctuations over the window period, which can delay the reflection of sudden market shocks or changes in risk conditions. For example, a sharp spike in volatility due to a macroeconomic event may only partially appear in the rolling average until 90 days after the event. This is still a more dynamic reflection of market conditions compared to the four-year realized average.

Cumulative Volatility This method calculates the volatility of the underlying index cumulatively from the inception of the index to each valuation date. As the valuation progresses over time, the dataset of log returns grows, incorporating all past price movements up to the specific valuation date:

$$\sigma_t = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\left(R_i - \bar{R}_t\right)^2} \tag{2.17}$$

where $\bar{R}_t$ is the Mean log return up to time t and N_t is the number of observations up to time t.
The evolving nature of this method allows the smoothing short-term fluctuations and creating a more stable estimate of volatility over time. It is less sensitive to recent market conditions as the dataset grows, potentially underestimating risk during periods of heightened volatility.
After evaluating the three methods, we decided to use the rolling average volatility in the Black

Figure 2.1: Rolling average volatility

and Scholes Model as it was the most accurate reflection of changing market conditions. However, we noted that whichever volatility we used, it did not have a large impact on the final price output.

Alternative proxies for volatility Other proxies include using implied volatility derived from market prices of other options on the same underlying or a volatility index. Since the underlying is a modified weighted index, we were not able to find other options with the same underlying.

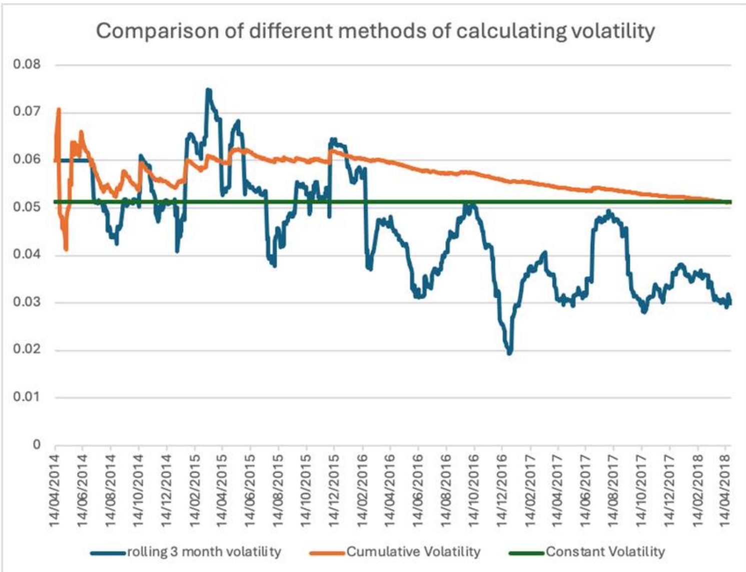

Figure 2.2: Volatility comparison

5.3 Broader assumptions in the Black and Scholes model

In addition to volatility and risk-free rates, the Black and Scholes model relies on several broader assumptions:

- **Lognormal Returns:** The model assumes the underlying index follows a geometric Brownian motion with constant drift and volatility. However, we know from empirical market data that asset prices in the real world do not follow a lognormal distribution, hence the existence of volatility smiles. We need to investigate further methods of derivations valuation in order to incorporate the volatility smile phenomenon into our valuation

- **No Arbitrage:** The model assumes perfectly efficient markets, with no arbitrage opportunities or transaction costs. In practice, transaction costs and bid-ask spreads may slightly affect the certificate's value

6 Price Comparison

We can observe a significant discrepancy between the issuance price, EUR 90, and the theoretical price we calculated in our analysis, EUR 40 (less than one half of the issuance price). While it is expected for the issuance price to be higher than the theoretical value to account for the bank's profit and operational costs, such difference clearly indicates that the certificate is overpriced. The certificate was issued out of the money (OTM). This means that the intrinsic value of the certificate was null at inception and the certificate's value relies almost entirely on its time value, which reflects the remaining time until maturity and the possibility of favorable price movements of the underlying asset.

Figure 2.3: Option price evolution

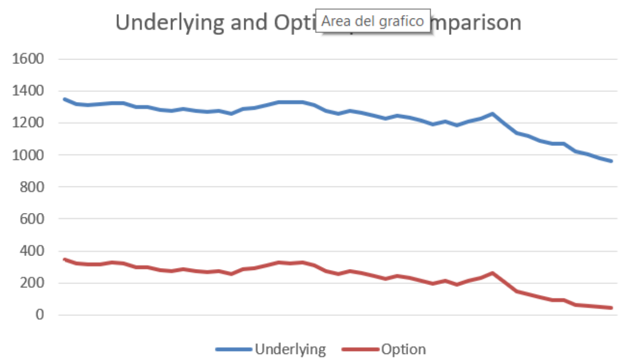

Figure 2.4: Underlying vs Option price

Chapter 3

Qualitative Section

In this part we will present a qualitative description of the Certificate, showing its validity from the viewpoint of the issuer and the buyer. In particular from the investor's viewpoint we will show explain if this could be a good investment or not, what are the risks related to the investment, and what are the costs. From the issuer's viewpoint, we will outline the benefits of issuing the Certificate and its sources of revenues.

1 Investor's Perspective

1.1 Is this a Good Investment?

The certificate offers a structured investment opportunity tailored to risk-averse investors by employing a volatility control mechanism. This mechanism targets a 6% annualized volatility, ensuring stability while maintaining the potential for reasonable returns. Additionally, the leverage is managed dynamically, with a cap to limit excessive exposure during periods of low realized volatility. This makes the investment suitable for individuals seeking low-risk exposure to the European investment-grade fixed income market.
One of the certificate's most appealing features is its downside protection. By limiting the maximum loss to the purchase price, investors face a predefined level of risk, offering security in periods of market uncertainty. This capped loss aligns with the product's structured design as a call option on the underlying index, reducing exposure to severe market downturns.
The certificate also holds potential for attractive returns. The underlying Efficiency Growth Fund, managed by Pharus Management Lux S.A., invests primarily in European investment-grade bonds, which are sensitive to interest rate fluctuations.
In addition, the certificate serves as a potential hedge against interest rate movements. Investors anticipating a downward trend in interest rates may find this product particularly advantageous, as the underlying bond-heavy portfolio is poised to benefit from such macroeconomic conditions.
However, the attractiveness of this certificate is contingent on its pricing and underlying assumptions. We found out that the fair issue price should lay around 40 EUR and not 90 EUR which was the actual issue price of the Certificate: it requires alignment with favorable market conditions to justify this valuation. Investors must assess whether their outlook on volatility and interest rates aligns with the issuer's assumptions to determine if the investment is likely to meet their

expectations.

1.2 Risks of the Investment

Market Risk The sole underlying and main source of market risk of the analyzed certificate is the Efficiency Growth Fund – Euro Global Bond managed by Pharus Management Lux S.A. The fund invests in the European Investment grade fixed income market, holding bonds and other debt securities denominated in Euros. Consequently, its performance is affected by interest rate fluctuations and the European economy. Currently, the fund's portfolio consists exclusively of European sovereign bonds, minimizing default risk. Pharus Management Lux S.A. declares a risk category 3 out of 7, based on 5-year period historical volatility. However, fund managers are permitted to use derivatives both for hedging and other purposes, which could significantly alter the fund's risk profile – and consequently, that of the certificate.

Counterparty Risk The analyzed certificate carries a low counterparty risk, as it is issued by Commerzbank, a well-established German bank with a strong presence in international financial markets. Commerzbank's reputation for stability and compliance with stringent European banking regulations further supports the reliability of the certificate.
However, the terms and conditions of the product grant the issuer certain discretionary powers in specific scenarios. For instance, if the underlying fund is discontinued, the issuer may either replace it with a comparable index or terminate and redeem the certificate prematurely. In the replacement scenario, the issuer can adjust the terms and conditions of the product. If the certificate is terminated, the redemption amount is calculated by the issuer, based on prevailing market conditions. Additionally, if the fund remains active but the index value is unavailable, or a market disruption persists between the valuation and maturity dates, the issuer has the authority to estimate the index price using reasonable discretion, reflecting current market conditions. A market disruption is defined as limitation or suspension of trading in any component of the index on any exchange or trading system.
Finally, the issuer can unilaterally transfer the obligations arising from the certificate to a third party, provided it guarantees compliance by the new issuer.

Liquidity Risk Certificate holders do not have the right to request early redemption, meaning their only option for liquidity is to sell the certificate to another investor. The certificate is listed on Borsa Italiana, one of Europe's leading stock exchanges and a well-regulated and transparent trading environment. Borsa Italiana provides a relatively liquid marketplace, which enhances the accessibility of trading. However, the liquidity of the specific instrument is not guaranteed and depends on market demand and supply dynamics.

Regulatory Risk The Certificates and the rights and duties of the holders and the issuer are governed by the laws of the Federal Republic of Germany. As a result, it is advisable for investors to have some familiarity with German financial law.

Currency Risk The instrument poses no direct or indirect currency risk for Eurozone investors, as both the certificate, the index price, and the fund's investments are all denominated in euros.

1.3 Costs of the Investment

The costs the Investor must bear in relation to the investment in the Efficiency Growth Index Certificate can be divided as follows:

- **Initial Costs:** The investor must pay the initial issue price of the certificate, that is EUR 90
- **Ongoing Costs:** A Protection Fee of 0% is charged to the Investor
- **Potential Exit Costs:** The Investor cannot redeem the investment before maturity. However, the latter can decide to sell it on the secondary market, and this could involve additional costs linked to the transaction:
  - Bid-Ask Spread: Investor may face bid-ask spread cost, due to the difference between bid and ask prices quoted by market makers
  - Brokerage Fees: The investor may incur in brokerage fees or commissions charged by their trading platform or intermediary
- **Taxes:** The Investor shall bear all the present and future taxes in connection with the Certificate.

In addition to these costs that are related to the certificate, there are the fees charged by the Euro Global Bond Fund, which is a key component of the underlying index. These fund-related costs include an initial entry fee (up to 5%) and an exit fee (up to 3%), applied when the fund is bought or redeemed, and an annual fund-management fee of 1.43%. These fees have a direct impact on the NAV of the fund and, as a consequence, an indirect one on the certificate, by reducing the value of the index.

2 Issuer's Perspective

2.1 Rationale

The rationale of issuing the certificate from the perspective of the issuer is multi-faceted. First, there is the market demand for tailored investment solutions. The Efficiency Growth Index Certificate is strategically positioned to attract investors who are interested in risk-adjusted returns. By implementing a volatility control mechanism that targets a 6% volatility level, the certificate appeals to risk-averse investors looking for stability in their investments without sacrificing potential returns. A fixed weighting mechanism that reallocates exposure to either the Euro Global Bond Fund or a risk-free rate based on realized volatility ensures that the product aligns with the low-volatility, capital-preservation preferences of a significant investor segment. This demand for customized products aligns with broader market trends favoring innovative investment strategies that mitigate risks while providing reasonable yields. Another reason would be that Commerzbank leverages its expertise in structuring financial products by creating and managing proprietary indices like the Efficiency Growth Index. This not only enhances the bank's reputation as a leader in financial engineering but also serves as a differentiator in a competitive market, allowing it to attract sophisticated investors who value innovative solutions. As the certificate is essentially a call option on the proprietary index, the structured nature of the certificate allows Commerzbank to employ various hedging strategies, such as derivatives and dynamic portfolio adjustments tied to the underlying fund, to manage its exposure effectively. This capability mitigates the issuer's risk from adverse market movements, enhancing its overall financial stability. Lastly, by listing the certificate on Borsa Italiana's SeDeX market, Commerzbank increases its visibility and accessibility among both retail and institutional investors in Italy. This strategic move supports the issuer's goal of expanding its footprint in European derivatives markets while adhering to regulatory requirements.

2.2 Benefits

In terms of benefits to the issuer upon the issuance of the certificate, one would be the diverse revenue streams it would generate. For instance, the certificate is issued at an initial price of €90, which is above its intrinsic value, generating immediate profit margins for Commerzbank. Also, although there is no explicit Protection Fee, ongoing management fees from the underlying Euro Global Bond Fund (1.43% per annum) would provide a steady revenue stream. Additionally, as certificates are traded on regulated exchanges, as a market maker, Commerzbank can earn from bid-ask spreads and transaction fees associated with trading activities. Another benefit would be the market differentiation aspect. The unique structure of the Efficiency Growth Index Certificate distinguishes Commerzbank's offerings from competitors, reinforcing its brand identity as an innovator in structured products. On the other hand, the product is particularly appealing to risk-averse investors and those seeking moderate returns through low-volatility strategies. This broadens Commerzbank's investor base and enhances customer loyalty. Moreover, the enhanced liquidity in listing on a regulated exchange facilitates ease of entry into and exit from positions, which can attract more sophisticated institutional clients. Finally, it would allow for temporary capital generation, whereby the issuance of these certificates allows Commerzbank to temporarily utilize investor capital for investments in lower-risk assets like Euro Global Bonds, optimizing cash flow management.

2.3 Sources of Revenue

Through dynamic hedging practices, Commerzbank can capitalize on discrepancies between implied and realized volatility, generating additional income from effective risk management. As aforementioned, with the Efficiency Growth Index heavily contingent on the Euro Global Bond Fund (LU0622616760), the management fee of 1.43% p.a. underlying the fund also contributes indirectly to Commerzbank's revenue stream, enhancing profitability over time. Next, active trading on SeDeX generates income through market-making bid-ask spreads as well as increased liquidity premiums through higher trading volumes, consequently leading to higher overall earnings. Furthermore, with capture of residual value, at maturity, any differences between payouts to certificate holders and performance of hedged assets can result in additional profits for Commerzbank, especially if market conditions favor lower volatility than anticipated. Last but not least, the certificate's payoff structure includes the option feature with capped exposure. If actual market volatility remains below expected levels, Commerzbank benefits from reduced hedging costs without transferring these savings to investors, thus retaining additional profits.