FLASH CARD MB1

SPAZIO DI N PARTICELLE

PARTICELLE IDENTICHE $\mathcal{H}_N = \mathcal{H}^{\otimes N}$

→ INVARIANZA PERMUTAZ

→ PROIETTORI SIMM/ANTISIMM

→ FERMIONI E BOSONI

→ SP DI FOCK

→ OP CREAZ / DISTRUZ

→ CCR / CAR

$\mathcal{H}_N = \mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_N \qquad \mathcal{H}_i \neq \mathcal{H}_j$

$|u_1 \dots u_N\rangle$

→ <u>PROD INT</u> $\langle v_1 \dots v_N | u_1 \dots u_N\rangle = \prod_j^N \langle v_j | u_j\rangle$

- $|u_1 \dots u_k + u_k' \dots u_N\rangle = |u_1 \dots u_k \dots u_N\rangle + |u_1 \dots u_k' \dots u_N\rangle$
- $A_k \in L(\mathcal{H}_k) \Rightarrow A_k|u_1 \dots u_k \dots u_N\rangle = |u_1 \dots A_k u_k \dots u_N\rangle$ STATI FATTORIZ.

$[A_k, A_j] = 0$ SE $k \neq j$

<u>PARTIC IDENTICHE</u> $\mathcal{H}(N) = \mathcal{H}^{\otimes N}$

$P_\sigma |u_1 \dots u_N\rangle = |u_{\sigma_1} \dots u_{\sigma_N}\rangle, \quad P_\sigma P_{\sigma'} = P_{\sigma\sigma'}$

$\| |u_1 \dots u_N\rangle \|^2 = \prod_j \| |u_j\rangle \|^2 \Rightarrow P_\sigma$ CONSERVA NORMA

$P_\sigma^{-1} = P_\sigma^\dagger = P_{\sigma^{-1}}$ UNITARIO

RAPPR UNITARIA PERMUTAZ.

$\{P_{ij}\} \subset \{P_\sigma\}$ CLASSE DEGLI SCAMBI $\qquad P_{ij}^2 = \mathbb{1} \Rightarrow P_{ij}^\dagger = P_{ij} \Rightarrow$ AUTOVAL ± 1

$P_\sigma = \prod P_{ij}$ NON UNIVOCA

$$\det \sigma = \begin{cases} +1 & \text{\# pari di scambi} \\ -1 & \text{\# dispari di scambi} \end{cases} \longrightarrow P_\sigma \text{ cambia da parità}$$

Op simmetriz $\quad S(N) = \dfrac{1}{N!} \sum_\sigma P_\sigma$

Antisim $\quad A(N) = \dfrac{1}{N!} \sum_\sigma (-1)^\sigma P_\sigma$

$\longrightarrow$ **Prop**

- $P_\sigma S(N) = S(N) P_\sigma = S(N)$
- $P_\sigma A(N) = A(N) P_\sigma = (-1)^\sigma A(N)$
- $S(N), A(N)$ proiettori
- $A(N) |u_1 \dots u_k \dots u_k \dots u_N\rangle = 0$
- $\{|u_j\rangle\} \subset \mathcal{H} \Rightarrow v_k = \sum_l M_{kl} |u_l\rangle \in \mathcal{H}$

$\Rightarrow A(N)|v_1 \dots v_N\rangle = \det(M)\, A(N) |u_1 \dots u_N\rangle$

Bosoni e Fermioni

$\mathcal{H}_B(N) = S(N)\mathcal{H}_N \longrightarrow P_{ij}\psi_B = \psi_B$

$\mathcal{H}_F(N) = A(N)\mathcal{H}_N \longrightarrow P_{ij}\psi_F = -\psi_F$

$\longrightarrow$ sp. ottenuti xké autoval $\pm$

x partic identiche op $\hat{O}$ ha signif solo se invar $\forall P_\sigma$

$$[\hat{O}, P_\sigma] = 0 \quad \forall \sigma$$

$\Rightarrow \hat{O}$ lascia invar $\mathcal{H}_B$ e $\mathcal{H}_F$ $\longrightarrow$ unici sp invar $\forall$ osservab

STATI SIMM/ANTISIMM

$$\langle \xi_1 \dots \xi_N | S(N)_\pm S(N)_\pm | u_1 \dots u_N\rangle \overset{\text{PROIETT}}{=} \langle \xi_1 \dots \xi_N | S(N)_\pm | u_1 \dots u_N\rangle = \frac{1}{N!}\sum_\sigma (\pm)^\sigma \langle \xi_1 \dots \xi_N | P_\sigma | u_1 \dots u_N\rangle$$

$$= \frac{1}{N!}\sum_\sigma (\pm)^\sigma \prod_j \langle \xi_j | u_{\sigma_j}\rangle = \frac{1}{N!} D_\pm \begin{bmatrix} \langle \xi_1 | u_1\rangle & \dots & \langle \xi_1 | u_N\rangle \\ & \dots & \\ \langle \xi_N | u_1\rangle & & \langle \xi_N | u_N\rangle \end{bmatrix}$$

$D_- = \det$

$D_+ =$ PERMANENTE

$\longrightarrow$ SE $\langle \xi_1 \dots \xi_N | = \langle \underline{x}_1 m_1 \dots \underline{x}_N m_N |$ $\Rightarrow$ $\langle \underline{x}\, m | u\rangle = u_m(\underline{x}) \in L^2(\mathbb{R}^3) \otimes \mathbb{C}^{2S+1}$

$\Rightarrow$ det di SLATER $\longrightarrow$ FUNZ D'ONDA ANTISIMM

$\{|r\rangle\}$ CON di $\mathcal{H}$ $\Rightarrow$ $\{|r_1 \dots r_N\rangle\}$ CON di $\mathcal{H}_N$ $\Rightarrow$ $S(N)_\pm |r_1 \dots r_N\rangle$ BASE RIDONDANTE di $\mathcal{H}(N)_\pm$

$$\| S(N)_\pm |r_1 \dots r_N\rangle \|^2 = \frac{1}{N!}\sum_\sigma (\pm)^\sigma \langle r_1 | r_{\sigma_1}\rangle \dots$$

- FERMIONI

  x PAULI $\langle r | r'\rangle = \delta_{rr'}$ $\Rightarrow$ CONTRIB SOLO $P_\sigma = \mathbb{1}$

  $\Rightarrow$ $\| S(N)_- |r_1 \dots r_N\rangle \|^2 = \frac{1}{N!}$

- BOSONI

  $\| S(N)_+ |r_1 \dots r_N\rangle \|^2 = \frac{1}{N!} (n_1! \dots n_\infty!) \Big|_{\sum n_r = N}$

$\to$ SONO ORTOGONALI? $\langle r_1' \dots r_N' | S_\pm(N) S_\pm(N) | r_1 \dots r_N\rangle = \frac{1}{N!}\sum (\pm)^\sigma \langle r_1' | r_{\sigma_1}\rangle \dots = 0$ SE $\exists\, r_i' \notin \{r_k\}$

$\Rightarrow$ BASE NON ORTONORMALE? SP

- **FERMIONICO** $|r_1 \dots r_N\rangle_F = \sqrt{N!}\, S_-(N)\, |r_1 \dots r_N\rangle$ $\quad r_1 \le \dots \le r_N \quad = |n_1 \dots n_\infty\rangle_F \quad n_k \in \{0,1\}$
- **BOSONICO** $|r_1 \dots r_N\rangle_B = \sqrt{\dfrac{N!}{n_1! \dots n_\infty!}}\, S(N)_+\, |r_1 \dots r_N\rangle \quad = |n_1 \dots n_\infty\rangle_B \quad \sum n_k = N \quad n_k \in \mathbb{N}$

SP DI FOCK $\mathcal{F}_\pm = |0\rangle \oplus \mathcal{H} \oplus \mathcal{H}_\pm(2) \oplus \dots$ → GEN DA $|n_1 \dots n_\infty\rangle_\pm$ CON $\sum n_k$ NON FIX

- OGNI $\mathcal{H}_\pm(N)$ ORTOG ALL'ALTRO
- $\mathcal{F}_\pm$ SEPARAB XKÉ SOMM NUMERAB DI SSP SEPARAB

OP CREAZ

$$c^+_{|a\rangle}\, S(N)_\pm\, |a_1 \dots a_N\rangle = \sqrt{N+1}\, S(N+1)_\pm\, |a\, a_1 \dots a_N\rangle \rightarrow c^+_{|a\rangle} : \mathcal{H}(N)_\pm \longrightarrow \mathcal{H}(N+1)_\pm$$

- $c^+_{\alpha|a\rangle+\beta|v\rangle} = \alpha\, c^+_{|a\rangle} + \beta\, c^+_{|v\rangle}$
- **BOSE** $[c^+_{|a\rangle}, c^+_{|v\rangle}] = 0$ $\qquad$ **FERMI** $\{c^+_{|a\rangle}, c^+_{|v\rangle}\} = 0 \Rightarrow (c^+_{|a\rangle})^2 = 0$

OP DISTRUZ $c_{\langle a|} = (c^+_{|a\rangle})^+ \rightarrow c_{\langle a|}|a_1 \dots a_N\rangle = \dfrac{1}{\sqrt{N}} \sum_j \langle a|a_j\rangle\, |a_1 \dots \cancel{a_j} \dots a_N\rangle$ $\qquad c_{\alpha\langle a| + \beta\langle v|} = \alpha\, c_{\langle a|} + \beta\, c_{\langle v|}$

BOSE $[c_{\langle a|}, c_{\langle v|}] = 0$ $\qquad$ **FERMI** $\{c_{\langle a|}, c_{\langle v|}\} = 0 \Rightarrow (c_{\langle a|})^2 = 0$

AZIONE CONGIUNTA

BOSE $[c^+_{|a\rangle}, c_{\langle v|}] = \langle a|v\rangle$

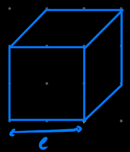
FERMI $\{c^+_{|a\rangle}, c_{\langle v|}\} = \langle a|v\rangle$

Box $\ell^3 \to \hat{H} = \hat{T}$

$\to$ INVAR TRASLAZ NEL BULK

$\to$ BC PERIODICHE

SF DI FERMI



$\vec{p}$ autoagg in scatola ⇒ BC periodiche

$\vec{p}\,|k\rangle = \hbar k\,|k\rangle$ $\quad \langle x|k\rangle = \dfrac{e^{ik\cdot x}}{\sqrt{V}} \xrightarrow{\text{BC periodiche}} k = \dfrac{2\pi}{L}\, n \quad n \in \mathbb{Z}^3$

passo reticolare

Lim TD: $L \to \infty$

En cinetica singola partic $\epsilon = \dfrac{p^2}{2m}$ → fermioni ⇒ x GS riempio liv dal basso

⇒ i k si dispongono su un reticolo

→ mettendone N riempio sfera

$N = 2\sum_k \theta(k - k_F)$ → distrib Fermi

Lim TD $\quad \sum_k f(k) \simeq \int \dfrac{d^3k}{(2\pi)^3} f(k)\, V$

$$= 2V\int \frac{d^3k}{(2\pi)^3}\theta(k_F - k) = \frac{2V}{(2\pi)^3}\underbrace{\frac{4}{3}\pi k_F^3}_{\text{vol sfera}} \Rightarrow k_F = (3\pi^2 n)^{1/3} \quad n \equiv \frac{N}{V} \Rightarrow T = 2V\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\theta(k_F - k)$$

$$= \frac{3}{5} N \epsilon_F$$

CCR / CAR → STATO DI FORK

→ PROASE INFORMA (x m)

⇒ OP DI CAMPO

IN - QT8

$\{|r\rangle\}$ base $\mathcal{H}$ $\quad \langle r|r'\rangle = \delta_{rr'} \quad \sum_r |r\rangle\langle r| = \mathbb{1}$

<u>bosoni</u> $[c_r, c_{r'}] = [c_r^+, c_{r'}^+] = 0 \quad [c_r, c_{r'}^+] = \delta_{rr'}$ $\quad$ CCR

<u>fermioni</u> $\{c_r, c_{r'}\} = \{c_r^+, c_{r'}^+\} = 0 \quad \{c_r, c_{r'}^+\} = \delta_{rr'}$ $\quad$ CAR

<u>numeri di occupaz.</u>

<u>def</u> $\hat{n}_r = c_r^+ c_r \longrightarrow [\hat{n}_{r'}, \hat{n}_r] = 0$

<u>bosoni</u>

sono δ funz. come vettore

$$c_r^+ |n_1 \dots n_\infty\rangle_+ = \sqrt{\frac{N!}{n_1! \dots n_\infty!}}\, c_r^+ S(N)_+ |r_1 \dots r_N\rangle = \sqrt{\frac{(N+1)!}{n_1! \dots n_\infty!}}\, S(N+1)_+ |r r_1 \dots r_N\rangle$$

$$= \sqrt{\frac{(N+1)!}{n_1! \dots n_\infty!}}\, |n_1 \dots (n_r+1) \dots n_\infty\rangle$$

$$c_r |n_1 \dots n_\infty\rangle = \sqrt{n_r}\, |n_1 \dots (n_r - 1) \dots n_\infty\rangle$$

• FERMI $\quad c_r^+|n_1\ldots\rangle = \sqrt{N!}\, c_r^+ A(N)|r_1\ldots r_N\rangle = \sqrt{(N+1)!}\, A(N+1)|r\, r_1\ldots r_N\rangle$

$$= \sqrt{(N+1)!}\begin{cases} 0 & \text{SE } n_r = 1 \\ (-1)^{n_1+\ldots+n_{r-1}} A(N+1)|r_1\ldots r_N\rangle & n_r = 0 \end{cases}$$

⟶ SCAMBI × RIORDINAM

SE USO BASE IMPROPRIA $|\underline{x}\, m\rangle \implies c^+_{|\underline{x} m\rangle} = \psi^+_m(\underline{x})$ $\quad\Big|\quad c_{|\underline{x} m\rangle} = \psi_m(\underline{x})$

• BOSE $\quad [\psi_m(\underline{x}), \psi^+_{m'}(\underline{x}')] = \delta_{mm'}\,\delta_3(\underline{x}-\underline{x}')$

• FERMI $\quad \{\psi_m(\underline{x}), \psi^+_{m'}(\underline{x}')\} = \delta_{mm'}\,\delta_3(\underline{x}-\underline{x}')$

$$|r\rangle = \sum_m \int d_3\underline{x}\, |\underline{x}\, m\rangle\langle \underline{x}\, m|r\rangle \implies c_r^+ = \sum_m \int d_3\underline{x}\, \langle \underline{x}\, m|r\rangle\, \psi_m^+(\underline{x})$$

$$\psi_m^+(\underline{x})|0\rangle\langle 0|\psi_m(\underline{x}) \qquad c_r = \sum_m \int d_3\underline{x}\, \langle \underline{x}\, m|r\rangle\, \psi_m(\underline{x})$$

OP A 1 PARTIC $\quad \hat{O} = \sum O(i) \to \hat{O} S(N)_\pm |u_1 \ldots u_N\rangle = S(N)_\pm \sum |u_1 \ldots O u_i \ldots u_N\rangle$

$O|u_i\rangle = \sum_r |r\rangle\langle r|O|u_i\rangle \longrightarrow = \sum_i \sum_r \langle r|O|u_i\rangle\, S(N)_\pm |u_1 \ldots r \ldots u_N\rangle =$

THOMAS FERMI

SCRIVO FUNZIONALE DELL'ENERGIA IN FUNZ DELLA DENSITÀ

E CONSTRAINT CARICA

$$T = \sum_i \frac{p_i^2}{2m} = \sum_{km} \frac{\hbar k^2}{2m}\hat{n}_{km}$$

$$\to \langle F|T|F\rangle = \sum_{km}\frac{\hbar^2 k^2}{2m}\langle F|\hat{n}_{km}|F\rangle = \sum_{km}\frac{\hbar^2 k^2}{2m}\theta(k_F - k)$$

$$= 2V\int\frac{d_3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m} = \frac{3}{5}N\varepsilon_F = \frac{3}{5}nV\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$= \frac{3}{5}V\frac{\hbar^2}{2m}(3\pi^2)^{2/3}n^{5/3} \to \text{EN GS} \times \text{u VOL}$$

$$T[n] = \frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2)^{2/3}\int d_3x\, n(x)^{5/3}$$

ATOMO CON Ze^-

$$E_{GS}[n] = T_{TF}[n] \underbrace{- Ze^2\int d_3x\frac{n(x)}{|x|}}_{e^- - \text{NUCLEO}} + \underbrace{\frac{e^2}{2}\int d_3x\, d_3y\frac{n(x)n(y)}{|x-y|}}_{e^- - e^-} + \underbrace{\mu}_{\text{MOLTIP LAGR}}\underbrace{\left[Z - \int d_3x\, n(x)\right]}_{\text{CONSTRAINT}}$$

$$\to \frac{\delta E_{GS}[n]}{\delta n(x)} = 0 \to E_{GS}[n+\delta n] - E_{GS}[n] = \int \delta n(x)\frac{\delta E}{\delta n}d_3x + \_\_\_$$

$$\frac{\delta n(x)}{\delta n(y)} = \delta(x-y) \quad (?) \qquad 0 = \frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2)^{2/3}\frac{5}{3}n(x)^{2/3} - \frac{Ze^2}{|x|} + e^2\int d_3y\frac{n(y)}{|x-y|} - \mu$$

$\to$ MOLTO VICINO ATOMO
TROPPO GROSSO
$\delta n \propto x^{-3}$ $\Longrightarrow$ CORREZIONI GRADIENTE

HEG

GAS OMOGENEO DI ELETTRONI

- PRENDO BOX L^3

→ $N\ e^-$ CHE INTERAGISCONO CON $\rho(x)$ pos

$$H = T + U_{ee} + U_{eb} + U_{bb}$$

INTERAZ COULOMB:

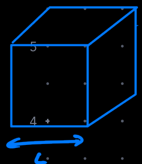
$$\frac{1}{|x-y|} \longrightarrow \frac{e^{-\mu|x-y|}}{|x-y|} \quad \text{YUKAWA}$$

$$\mu \ll \frac{1}{L}$$

STUDIO $\langle \underline{k}\, m | H | \underline{k}'\, m' \rangle$

QUANDO $L \to \infty \Rightarrow \mu \to 0$

CARICA POS $\rho(\underline{x})$ FIX $\rightarrow$ $H = \sum \frac{p_i^2}{2m} + \underbrace{\frac{e^2}{2}\sum_{i\neq j}\frac{1}{|\underline{x}_i-\underline{x}_j|}}_{U_{ee}} \underbrace{- e\sum_{i=1}^{N}\int d_3x\,\frac{\rho(\underline{x})}{|\underline{x}-\underline{x}_i|}}_{U_{eb}} + \underbrace{\frac{1}{2}\int d_3x\,d_3y\,\frac{\rho(\underline{x})\rho(\underline{y})}{|\underline{x}-\underline{y}|}}_{U_{bb}}$



$\rho(\underline{x})$ OMOG $\Rightarrow$ SIST È OMOG
X INVAR TRASL

$\Rightarrow |\underline{k}\, m\rangle$

COULOMB LUNGO RANGE $\Rightarrow$ $\frac{1}{|\underline{x}-\underline{y}|} \longrightarrow \frac{e^{-\mu|\underline{x}-\underline{y}|}}{|\underline{x}-\underline{y}|}$ YUKAWA $\Rightarrow$ INVAR NELLA SCATOLA $\longrightarrow$ VOGLIO SCENDA MOLTO NELLA SCATOLA

$L\to\infty \Rightarrow \mu\to 0$ $\frac{1}{\mu} \ll L$

STO NEL BULK

$$T = \sum_{\underline{k}m}\frac{\hbar^2 k^2}{2m}\, a^\dagger_{\underline{k}m}\, a_{\underline{k}m}$$

$$U_{ee} = \frac{e^2}{2}\sum_{\substack{k_1k_2\\k_3k_4}}\sum_{\substack{m_1m_2\\m_3m_4}} a^\dagger_{k_1m_1} a^\dagger_{k_2m_2} a_{k_4m_4} a_{k_3m_3}\langle \underline{k}_1 m_1, \underline{k}_2 m_2|\frac{e^{-\mu|\underline{x}_1-\underline{x}_2|}}{|\underline{x}_1-\underline{x}_2|}|\underline{k}_3 m_3, \underline{k}_4 m_4\rangle$$

$$= \sum_{\underline{k}m}\delta_{m_1m_3}\delta_{m_2m_4}\int d_3x\,d_3y\,\langle \underline{k}_1\underline{k}_2|\frac{e^{-\mu|\underline{x}_1-\underline{x}_2|}}{|\underline{x}_1-\underline{x}_2|}|\underline{x}\underline{y}\rangle\langle\underline{x}\underline{y}|\underline{k}_3\underline{k}_4\rangle$$

$\int d_3x\,d_3y\,|\underline{x}\underline{y}\rangle\langle\underline{x}\underline{y}|$

$$= \sum_{\underline{k}m}\delta_{m_1m_3}\delta_{m_2m_4}\int d_3x\,d_3y\,\frac{e^{-\mu|\underline{x}-\underline{y}|}}{|\underline{x}-\underline{y}|}\underbrace{\langle \underline{k}_1|\underline{x}\rangle}\underbrace{\langle\underline{k}_2|\underline{y}\rangle}\underbrace{\langle\underline{x}|\underline{k}_3\rangle}\underbrace{\langle\underline{y}|\underline{k}_4\rangle}$$

$\frac{e^{-i\underline{k}_1\underline{x}}}{\sqrt{V}}\;\frac{e^{-i\underline{k}_2\underline{y}}}{\sqrt{V}}\;\frac{e^{+i\underline{k}_3\underline{x}}}{\sqrt{V}}\;\frac{e^{+i\underline{k}_4\underline{y}}}{\sqrt{V}}\;\; e^{\pm i(\underline{k}_3-\underline{k}_1)\underline{y}}$

$$= \sum_{\underline{k}m}\delta_{m_1m_3}\delta_{m_2m_4}\int d_3x\,d_3y\,\frac{e^{-\mu|\;|}}{|\;|}$$

$\frac{e^{-i(\underline{k}_1-\underline{k}_3)(\underline{x}-\underline{y})}}{V^2}\;\underbrace{e^{i(\underline{k}_4-\underline{k}_2+\underline{k}_3-\underline{k}_1)\underline{y}}}$

$\int d_3y\, e^{i\underline{k}\underline{y}} = V\langle\underline{k}|0\rangle = V\delta_{\underline{k}0}$

$\underline{x}-\underline{y} \equiv \underline{x}'$ $\rightarrow$ SMINORE X TERMINI DI BORDO (?)

$$= \sum_{\underline{k}}\int d_3x'\,\frac{e^{-\mu|\underline{x}'|}}{|\underline{x}'|}\,\frac{e^{i(\underline{k}_3-\underline{k}_1)\underline{x}'}}{V^2}\,V\,\delta_{\underline{k}_4+\underline{k}_3,\,\underline{k}_1+\underline{k}_2}$$

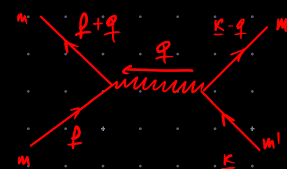
ENTRAMBI k_3, k_4 $\rightarrow$ SCELTI k_1, k_2 $\Longrightarrow$ CONSERVARE MOMENTO

INTEG ANG DA k_1 & k_2 → INDIP DA ROTAZ ⇒ $z \parallel \underline{x}'$ ⊕ COORD SF

$$= 2\pi \int_0^\infty r^2 dr \frac{e^{-\mu r}}{r} \int_{\pm 1} d\xi \, e^{i|\underline{k}_3-\underline{k}_1| r \xi} = 2\pi \int_0^\infty r^2 dr \frac{e^{-\mu r}}{r} \frac{e^{i|\underline{k}_3-\underline{k}_1| r} - e^{-i|\underline{k}_3-\underline{k}_1| r}}{i|\underline{k}_3-\underline{k}_1| r} = \frac{2\pi}{i|\underline{k}_3-\underline{k}_1|}\left[\frac{1}{\mu - i|\underline{k}_3-\underline{k}_1|} - \frac{1}{\mu + i|\underline{k}_3-\underline{k}_1|}\right]$$

$$\Rightarrow U_{ee} = \frac{e^2}{2V} \sum_{\substack{k_1 k_2 \\ k_3 k_4}} \sum_{\substack{m_1 m_2 \\ m_3 m_4}} \delta_{m_1 m_3} \delta_{m_2 m_4} \delta_{k_1+k_2, k_3+k_4} \frac{4\pi}{\mu^2 + |k_3-k_1|^2} a_1^+ a_2^+ a_4 a_3$$

$$= \frac{e^2}{2V} \sum_{m m'} \sum_{\underline{k} \underline{p} \underline{q}} \frac{4\pi}{\mu^2+q^2} a^+_{\underline{p}+\underline{q}\, m} a^+_{\underline{k}-\underline{q}\, m'} a_{\underline{k} m'} a_{\underline{p} m}$$



SE $\mu \to 0 \Rightarrow$ PROBL IN $\underline{q} = \underline{0}$

NB

$$(\nabla^2 - \mu^2)\varphi = \delta(x-y)$$

↓ SdF K

$$\frac{1}{\mu^2 + k^2}$$

$\underline{\mu}$

$$U_{eb} = \sum \int d_3y \, (-e\rho(y)) \frac{e^{-\mu|y-x_i|}}{|y-x_i|}$$

$$\to \sum_{\substack{\underline{k}\, \underline{k}' \\ m\, m'}} a^+_{\underline{k} m} a_{\underline{k}' m'} \langle \underline{k} m | \int d_3y \, (-e\rho(y)) \frac{e^{-\mu|y-\hat{\underline{x}}|}}{|y-\hat{\underline{x}}|} | \underline{k}' m' \rangle = \sum_{\substack{\underline{k}\, \underline{k}' \\ m\, m'}} a^+_{\underline{k} m} a_{\underline{k}' m'} \delta_{m m'} \int d_3y \, (-e) \rho(y) \int d_3x \langle \underline{k} | \underline{x} \rangle \langle \underline{x} | \underline{k}' \rangle \frac{e^{-\mu|y-\underline{x}|}}{|y-\underline{x}|}$$

$$= \sum a^+_{\underline{k} m} a_{\underline{k}' m'} \int d_3y \, (-e) \rho(y) \int \frac{d_3x}{V} e^{i(\underline{k}'-\underline{k})(\underline{x}+\underline{y})} \frac{e^{-\mu|y-\underline{x}|}}{|y-\underline{x}|} \underset{x' \equiv x - y}{=} \sum a^+_{\underline{k} m} a_{\underline{k}' m'} \int d_3y \, (-e) \rho(y) \frac{e^{i(\underline{k}'-\underline{k})y}}{V} \int d_3x' \frac{e^{-\mu|\underline{x}'|}}{|\underline{x}'|} e^{i(\underline{k}'-\underline{k})\underline{x}'}$$

V FINITO

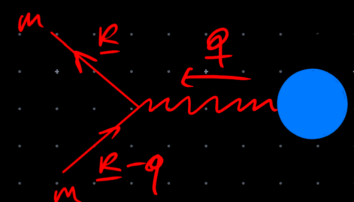
$$\rho(\underline{x}) = \frac{1}{V}\sum_{\underline{k}} \rho_{\underline{k}}\, e^{i\underline{k}\underline{x}} \Rightarrow \rho_{\underline{k}} = \int d_3x\, \rho(\underline{x})\, e^{-i\underline{k}\underline{x}}, \quad \rho_0 = Q$$

IN $\mathbb{R}^3$ $\rho(\underline{x}) = \frac{1}{V}\int \frac{d_3k}{(2\pi)^3}\, \rho(\underline{k})\, e^{i\underline{k}\cdot\underline{x}}$

$$\Rightarrow \sum_{\underline{k}'\underline{k}m} a^+_{\underline{k}m}\, a_{\underline{k}'m} \int d_3y\, (-e)\, \rho(y)\, \frac{e^{i(\underline{k}'-\underline{k})y}}{V}\, \frac{4\pi}{|\underline{k}'-\underline{k}|^2+\mu^2} = \sum_{\underline{k}'\underline{k}m} a^+_{\underline{k}m}\, a_{\underline{k}'m}\, \frac{(-e)}{V}\, \rho(\underline{k}-\underline{k}')\, \frac{4\pi}{|\underline{k}-\underline{k}'|^2+\mu^2}$$

$| \; q \equiv \underline{k}-\underline{k}'$

$$= -\frac{e}{V}\sum_{\underline{k}qm} a^+_{\underline{k}m}\, a_{\underline{k}-q\,m}\, \rho(q)\, \frac{4\pi}{q^2+\mu^2}$$



$$U_{bb} = \frac{1}{2}\int d_3x\, d_3y\, \frac{\rho(x)\rho(y)}{|x-y|}\, e^{-\mu|x-y|} = \frac{1}{2V}\sum_{\underline{k}\underline{k}'} \rho_{\underline{k}}\rho_{\underline{k}'} \int d_3x\, d_3y\, \frac{e^{-\mu|x-y|}}{|x-y|}\, e^{i\underline{k}(\underline{x}+y)+i\underline{k}'y}$$

$$= \frac{1}{2V}\sum_{\underline{k}\underline{k}'} \rho_{\underline{k}}\rho_{\underline{k}'}\, \frac{4\pi}{k^2+\mu^2}\, \underbrace{\int d_3y\, e^{i(\underline{k}+\underline{k}')y}}_{V\,\delta_{\underline{k}+\underline{k}',0}} = \frac{1}{2V}\sum_{\underline{k}} \rho_{\underline{k}}\, \rho_{-\underline{k}}\, \frac{4\pi}{k^2+\mu^2} \quad (k \to q)$$

ANCORA $\mu \not\to 0$

$\boxed{q=0}$ $\quad U_{ee} = \frac{e^2}{2V} \sum_{m,m'} \sum_{k,p} \frac{4\pi}{\mu^2} a^+_{pm} a^+_{km'} a_{km'} a_{pm}$

$a^+_{pm} a_{pm} a^+_{km'} a_{km} = a^+_{pm} a_{pm} (a_{km} a^+_{km} - \delta_{kk'})$

$U_{eb} = \frac{e}{V} \sum_{km} \frac{4\pi}{\mu^2} a^+_{km} a_{km} \rho_0 = -\frac{e}{V} \frac{4\pi}{\mu^2} \hat{N} Q$

$U_{bb} = \frac{1}{2V} \rho_0^2 \frac{4\pi}{\mu^2} = \frac{Q^2}{2V} \frac{4\pi}{\mu^2}$

$H|_{q=0} = \frac{e^2}{2V} \frac{4\pi}{\mu^2} (\hat{N}^2 - \hat{N}) - \frac{e}{V} \hat{N} Q \frac{4\pi}{\mu^2} + \frac{Q^2}{2V} \frac{4\pi}{\mu^2} = \frac{4\pi}{\mu^2} \frac{1}{2V} (eN - Q)^2 - \underbrace{\frac{e^2}{2V} \frac{4\pi}{\mu^2} \hat{N}}$

DA COMMUTAZ

$\hookrightarrow \langle H \rangle \sim N \Rightarrow \frac{\langle H \rangle}{N} \sim \frac{1}{V\mu^2} \xrightarrow[\mu \to 0]{L \to \infty} 0$

$0 \qquad 0$

CASO FINITO $\mu L \gg 1 \Rightarrow \mu^2 L^3 \to \infty$ AL DENOM $\Rightarrow$ TENGO μ

CASO ∞ $\quad \frac{N}{V} \to n \quad$ CHIEDO NEUTRALITÀ $\quad Q = eN \Rightarrow \mu = 0$

$\Rightarrow H = \sum_{km} \frac{\hbar^2 k^2}{2m} a^+_{km} a_{km} + \frac{e^2}{2V} \sum_{mm'} \sum_{kpq} \frac{4\pi}{q^2} a^+_{p+q\, m} a^+_{k-q\, m'} a_{km'} a_{p m} - \frac{e}{V} \sum_{kqm} \frac{4\pi}{q^2} \rho(q) a^+_{km} a_{k-q\, m} + \frac{1}{2V} \sum_{q} \rho_q \rho_{-q} \frac{4\pi}{q^2}$

CONSIDERO UNA HSG IN JELLIUM DI CARICA POS

$\rho(x)$ UNIF $\Rightarrow eN = V\rho$

$\hookrightarrow \rho_k = 0$ SE $k \neq 0$

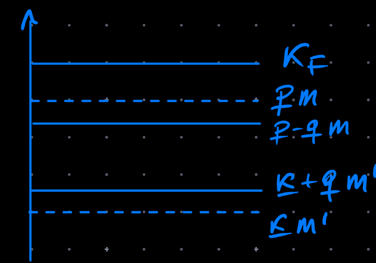
$$H = \sum_{km} \frac{\hbar^2 k^2}{2m} a^+_{km} a_{km} + \frac{1}{2V} \sum_{\substack{kp\\ mm'}} \sum_{q\neq 0} \frac{4\pi e^2}{q^2} a^+_{p+q\,m}\, a^+_{k-q\,m'}\, a_{km'}\, a_{pm}$$

$|F\rangle$ GS DI EN CINETICA $\hat H_0$

⇒ GS H MOLTO PERTURBATIVAM

$$\langle F|\hat H|F\rangle \geq E_{GS}$$

$$|F\rangle = |11\ldots1\,00\ldots0\rangle$$



→ SONO ELETTRONI ⇒ ∃! MODO x SCAMBIARE GLI STATI $pm = k+q\,m'$

$km' = p-q\,m$

→ x CONSERVARE MOM (δ) + $q \neq 0$

$$\langle F|H|F\rangle = \underbrace{\frac{3}{5} N E_F}_{T} + \frac{1}{2V} \sum_{\substack{kp\\ mm'}} \sum_{q\neq 0} \frac{4\pi e^2}{q^2} \langle F| a^+_{pm}\, a^+_{km'}\, a_{k+q\,m'}\, a_{p-q\,m} |F\rangle$$

?

$$= \frac{3}{5} N E_F + \frac{1}{2V} \sum_{kpm} \frac{4\pi e^2}{|p-k|^2} \underbrace{\langle F| a^+_{pm}\, a^+_{km}\, a_{pm}\, a_{km} |F\rangle}_{-\langle F|\hat N_{pm} \hat N_{km}|F\rangle}$$

XKÉ SCOMPARE IL QUADRATO?

$$N_p = \int \frac{d^3k}{(2\pi)^3} \theta(k-k_F) \quad \delta(p-k)$$

$$-\langle F|\hat N_{pm}\, \hat N_{km}|F\rangle \longrightarrow \langle F|N|F\rangle = V \sum_m \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k)$$

$$= \frac{3}{5} N E_F + \frac{1}{2V} \sum_{kpm} \frac{4\pi e^2}{|p-k|^2} \left(-\theta(k_F-k)\,\theta(k_F-p)\right)$$

→ CONTRIBUTO COULOMB NEGATIVO E COSTITUITO SOLO DA e^- CON SPIN ∥

$$\langle F|U|F\rangle = \frac{-1}{2V}\sum_{kqm}\frac{4\pi e^2}{|k-p|^2}\theta(k_F-k)\theta(k_F-p) \xrightarrow[\text{cont}]{} \frac{-1}{2V}V^2\int\frac{d^3p\,d^3k}{(2\pi)^6}\frac{4\pi e^2}{|k-p|^2}\theta(k_F-k)\theta(k_F-p)$$

$\sin^2\theta\, d\theta = d\xi$ → COORD SF

INTEGRO SU SF FORMI

$$= -\frac{V}{2}\frac{8\pi^2}{(2\pi)^6}\int_0^{k_F}p^2dp\int_0^{k_F}k^2dk\int_{-1}^{+1}d\xi\,\frac{4\pi e^2}{k^2+p^2-2kp\xi}$$

$$= -\frac{V}{2}\frac{1}{8\pi^4}\int_0^{k_F}p^2dp\int_0^{k_F}k^2dk\,(-1)\frac{4\pi e^2}{2kp}\ln(k^2+p^2-2kp\xi)\Big|_{\pm 1} = -\frac{V}{4}\frac{4\pi e^2}{8\pi^4}\int_0^{k_F}k\,dk\int_0^{k_F}p\,dp\,\lg\left[\left(\frac{k+p}{k-p}\right)^2\right]$$

SIMM × SCAMBIO $k \leftrightarrow p$

TOT SIMM

$$\int_a^b dx_1\int_a^b dx_2 \ldots \int_a^b dx_n\, f(x_1\ldots x_n) = N!\int_a^b dx_1\int_a^{x_1}dx_2\ldots\int_a^{x_{n-1}}dx_n\, f$$

$$= -V\frac{e^2}{4\pi^3}k_F^4$$

$$\Rightarrow \boxed{\langle F|\hat{H}|F\rangle = \frac{3}{5}N\varepsilon_F - V\frac{e^2}{4\pi^3}k_F^4}$$

$$\frac{\langle \hat H\rangle}{N} = \frac{3}{5}\frac{\hbar^2}{2m}k_F^2 - \frac{1}{n}\frac{e^2}{4\pi^3}k_F^4 = \frac{3}{5}\frac{\hbar^2}{2m}k_F^2 - \frac{3}{4\pi}e^2 k_F = \frac{3}{10}\frac{e^2 a_0^2}{a_0}k_F^2 - \frac{3}{4\pi}\frac{e^2}{a_0}k_F a_0$$

$k_F^3 = 3\pi^2 n$ $\qquad a_0 = \frac{\hbar^2}{me^2} \simeq 0{,}5\ \text{Å}$

$$= \frac{e^2}{2a_0}\left[\frac{3}{5}(a_0 k_F)^2 - \frac{3}{2\pi}(a_0 k_F)\right]$$

$n = \frac{N}{V}$ densità elettronica

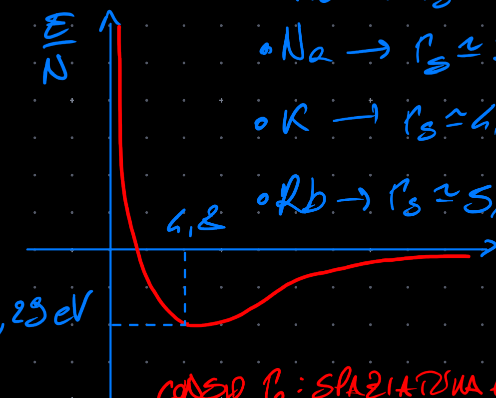
⇒ spaz. media $\left(\frac{V}{N}\right)^{1/3} = \left[\frac{4}{3}\pi(r_s a_0)^3\right]^{1/3}$ ma $\frac{1}{n} = \frac{3\pi^2}{k_F^3} = \frac{V}{N} = \frac{4}{3}\pi(r_s a_0)^3$

param di sist

Metalli
- Al → $r_s \simeq 2{,}07$
- Na → $r_s \simeq 3{,}93$
- K → $r_s \simeq 4{,}86$
- Rb → $r_s \simeq 5{,}20$

$$\Rightarrow (k_F a_0)^2 = 3\pi^2\frac{3}{4\pi}r_s^{-3} \longrightarrow a_0 k_F = \left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_s}$$

$$\Rightarrow \langle F|H|F\rangle = \frac{e^2}{2a_0}\left[\frac{3}{5}\left(\frac{9\pi}{4}\right)^{2/3}\frac{1}{r_s^2} - \frac{3}{2\pi}\left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_s}\right]$$



OSS
- Si può vedere che × $r_s = 6$ ⇒ transiz di fase
- $r_s > 6$ ⇒ cristallo di Wigner (Fetter es 1.6 pg 32)
- Stime esatte da USB $\longrightarrow \frac{e^2}{2a_0}\left(\frac{2{,}10}{r_s^2} - \frac{1{,}8}{r_s}\right)$

Consid. r_0: spaziatura partic.
$V = \frac{4}{3}\pi r_0^3 N$
a_0 raggio Bohr tipica di Coulomb
⇒ $r_s = \frac{r_0}{a_0}$ caratt. densità sist
$r_s \to 0$ ⇒ pot. perturbaz al 1° ordine (gas stra-denso)

TERMINE DI SCAMBIO $\quad E_x = -\frac{e^2}{2a_0} N \frac{3}{2\pi} k_F a_0 = -\frac{e^2}{2a_0} n V \frac{3a_0}{2\pi} (3\pi^2 n)^{1/3}$

$\rightarrow$ FUNZIONALE $\quad E_x[n] = -\frac{e^2}{2} \frac{3}{2\pi} (3\pi^2)^{1/3} \int d_3x \, n^{4/3}(\underline{x})$ DYSON

DATA EN $\Rightarrow$ TD DEL SIST $\quad dE = -p\,dV + \mu\,dN$

ES: SIST OMOGENEO, INVAR × TRASLAZ

EN × PARTIC $\quad \varepsilon = \frac{E}{N} = \varepsilon[n] \longrightarrow \mu = \left.\frac{\partial E}{\partial N}\right|_V = \frac{\partial}{\partial n} n\varepsilon \quad , \quad p = -\left.\frac{\partial E}{\partial V}\right|_N = n(\mu - \varepsilon)$

HARTREE-FOCK

Th di campo medio

$$U_H(\underline{x}) = \int d_3\underline{y}\, v(\underline{x}\,\underline{y})\, n(\underline{y})$$

⊕ CORREZ SELF-INTERACTION

TSTOMAL:

- $H = \sum_i \frac{p^2}{2m} + \sum_{i \neq j} \frac{v(ij)}{2}$
- min su det SLATER

$\Rightarrow$ CON VINCOLO $\langle u_i | u_j \rangle = \delta_{ij}$

PROIETTANDO SU $\langle \underline{x}\, m |$

$\rightarrow$ CASO INVAR × TRASLAZ

$\Rightarrow \langle \underline{k}_i m_i \underline{k}_j m_j | H | \underline{k}_i m_i \underline{k}_j m_j \rangle$

$H = \frac{p^2}{2m} + U_H(\underline{x}) \longrightarrow U_H(\underline{x}) = \int d^3y \, v(\underline{x}-\underline{y})\, n(\underline{y})$ → APPROX MEAN FIELD

$\left(\frac{p^2}{2m} + U_H(\underline{x})\right) u_i = \varepsilon_i u_i \Rightarrow n(\underline{x}) = \sum_i |u_i(\underline{x})|^2$ → DIFETTO: CONTIENE INTERAZ. PARTIC CON SE STESSA

⇒ EQ COMPLETA DI HF $\quad H = \sum_i h(i) + \frac{1}{2}\sum_{i \neq j} v(ij) \Rightarrow$ CERCO MIN TRA DET SLATER

$$|\psi\rangle = \sqrt{N!}\, A(N)\, |u_1 \ldots u_N\rangle$$

$$\langle\psi|H|\psi\rangle = \sum_i \langle u_i|h|u_i\rangle + \frac{1}{2}\sum_{i\neq j} \langle u_i u_j | v | u_i u_j - u_j u_i\rangle \qquad \langle u_i|u_j\rangle = \delta_{ij}$$

E CHE GLI u_i OTTENUTI TRAMITE TRASF UNITARIA PORTANO A STESSO DET DI SLATER

$u_i' = \sum_j U_{ij} u_j \longrightarrow |\psi'\rangle = |\psi\rangle \qquad E' = E$ → INVARIANZA X TRASF UNITARIA

⇒ $E_{HF} = \min_{\{u_n\}} \langle\psi|H|\psi\rangle - \sum_{ij} \varepsilon_{ij}\left[\langle u_i|u_j\rangle - \delta_{ij}\right]$ → VINCOLO STATI ORTONORM

↳ HERMITIANA XCHÉ $E_{HF} \in \mathbb{R}$

→ $E[u_1 \ldots u_N] = \sum \langle u_i|h|u_i\rangle - \frac{1}{2}\sum_{i\neq j} \langle u_i u_j|v|u_i u_j - u_j u_i\rangle$

→ DA IMPORRE $\frac{\delta E}{\delta u_i} = 0 \quad \forall\, i = 1 \ldots N$

MANDO $\forall i$: $a_i \to a_i + \eta_i \Rightarrow E[a_1 \dots a_i+\eta_i \dots] - E[a_1 \dots a_N] = \alpha \eta_i + C(\eta_i^2)$ ⟿ CONSIDERO IL LIN

↳ IMPONGO $= 0$

$\partial \mathcal{V}(ij) = \partial \mathcal{V}(ji) \Rightarrow \langle a_i a_j | \partial \mathcal{V} | a_i a_j \rangle = \langle a_i a_j$??

IMPONGO × MIGLIORO MIN

$$\delta E_i = \langle \eta_i | h | a_i \rangle + \sum_j \langle \eta_i a_j | \partial \mathcal{V} | a_i a_j - a_j a_i \rangle - \varepsilon_{ij} \langle \eta_i | a_j \rangle \overset{\downarrow}{=} 0 \quad \forall \eta_i \Rightarrow h | a_i \rangle + \sum_j \langle a_j | \partial \mathcal{V} | a_i a_j - a_j a_i \rangle = \varepsilon_i | a_i \rangle$$

→ TROVATA 1 SOL ⇒ FAMIGLIA DI SOL CON TRASF UNITARIE
⇒ SI PUÒ RISOLVERE ASSIEME IL PROB DI DIAGONALIZZ $\varepsilon_{ij} \to$ DIVENTA ε_i

$$\langle a_i | h | a_i \rangle + \sum_j \langle a_i a_j | \partial \mathcal{V} | a_i a_j - a_j a_i \rangle = \varepsilon_i \Rightarrow E_{HF} = \sum_i \langle a_i | h | a_i \rangle + \frac{1}{2} \sum_{i \neq j} \langle a_i a_j | \partial \mathcal{V} | a_i a_j - a_j a_i \rangle$$

$$= \frac{1}{2} \sum_i \left[\varepsilon_i - \langle a_i | h | a_i \rangle \right] \qquad \text{CON } n(\underline{x}) = \sum_i |\langle \underline{x} m | a_i \rangle|^2$$

PROIETTO SU $\langle \underline{x} m |$

$$\langle \underline{x} m | h | a_i \rangle + \sum_j \langle \underline{x} m \, a_j | \partial \mathcal{V} | a_i a_j - a_j a_i \rangle = \varepsilon_i a_i(\underline{x})$$

$$\sum_{m' m''} \int d_3 \underline{x}' \, d_3 \underline{x}'' \, | \underline{x}' m' \underline{x}'' m'' \rangle \langle \underline{x}' m' \underline{x}'' m'' |$$

$$a_i(\underline{x}' m') a_j(\underline{x}'' m'') - a_j(\underline{x}' m') a_i(\underline{x}'' m'')$$

$$h a_i(\underline{x}) + \sum_j \sum_{m' m''} \int d_3 \underline{x}' \, d_3 \underline{x}'' \, \partial \mathcal{V}(\underline{x}' \underline{x}'') \underbrace{\langle \underline{x} m \, a_j | \underline{x}' m' \underline{x}'' m'' \rangle}_{\delta_{m m'} \delta_3(\underline{x} - \underline{x}') a_j^*(\underline{x}'' m'')} \langle \underline{x}' m' \underline{x}'' m'' | a_i a_j - a_j a_i \rangle$$

$$= \varepsilon_i a_i(\underline{x} m)$$

$$(h u_i)(\underline{x}\, m) + \sum_j \sum_{m'm''} \int d_3\underline{x}'\, d_3\underline{x}''\, \delta_{mm'}\, \delta_3(\underline{x}-\underline{x}')\, u_j^*(\underline{x}''m'')\left[u_i(\underline{x}'m')\,u_j(\underline{x}''m'') - u_j(\underline{x}'m')\,u_i(\underline{x}''m'')\right] = \varepsilon_i\, u_i(\underline{x}\, m)$$

$$\left[-\frac{\hbar^2}{2m}\nabla_{\underline{x}}^2 + U(\underline{x})\right] u_i(\underline{x}\, m) + \int d_3\underline{x}''\, \mathcal{V}(\underline{x}\, \underline{x}'') \underbrace{\sum_{j m''} \left|u_j(\underline{x}''m'')\right|^2}_{n(\underline{x}'')} u_i(\underline{x}\, m)$$

$$= \left[-\frac{\hbar^2}{2m}\nabla_{\underline{x}}^2 + U(\underline{x})\right] u_i(\underline{x}\, m) + U_H(\underline{x})\, u_i(\underline{x}\, m)$$

SE CONSERVO INVAR × TRASLAZ $[H, \vec{P}] = 0 \Rightarrow V = V(\underline{x} - \underline{y})$

$\Rightarrow$ OGNI NOPLA $|\underline{k}_i\, m_i\rangle$ $i = 1 \ldots N$ SODDISFA HF

$$\frac{p^2}{2m}|\underline{k}_i\, m_i\rangle + \sum_j \langle \underline{k}_j m_j| \mathcal{V} \left(|\underline{k}_i m_i\, \underline{k}_j m_j\rangle - |\underline{k}_j m_j\, \underline{k}_i m_i\rangle\right) = \varepsilon_i |\underline{k}_i\, m_i\rangle$$

$\underline{k} = \underline{k}_i$

$[H, \vec{P}] = 0 \Rightarrow V$ CONSERVA MOM $\Rightarrow \underline{k} + \underline{k}_j = \underline{k}_i + \underline{k}_j \Rightarrow m = m_i$

$$\frac{\hbar^2}{2m} k_i^2 |\underline{k}_i m_i\rangle + \sum_j \langle \underline{k}_i m_i \, \underline{k}_j m_j | \hat{V} | \underline{k}_i m_i \, \underline{k}_j m_j\rangle \, |\underline{k}_i m_i\rangle$$

$$- \sum_j \langle \underline{k}_i m_i \, \underline{k}_j m_j | \hat{V} | \underline{k}_j m_j \, \underline{k}_j m_i\rangle |\underline{k}_i m_i\rangle = \varepsilon_i |\underline{k}_i m_i\rangle$$

Abbiamo

$$\langle \underline{k}_i m_i \, \underline{k}_j m_j | \hat{V} | \underline{k}_i m_i \, \underline{k}_j m_j\rangle = \langle \underline{k}_i \underline{k}_j | \hat{V} | \underline{k}_i \underline{k}_j \rangle$$

$$= \int d_3x \, d_3y \; V(\underline{x}-\underline{y}) \underbrace{\langle \underline{k}_i \underline{k}_j | \underline{x}\underline{y}\rangle}_{\frac{e^{-i\underline{k}_i\underline{x}-i\underline{k}_j\underline{y}}}{V}} \underbrace{\langle \underline{x}\underline{y} | \underline{k}_i \underline{k}_j\rangle}_{\frac{e^{i\underline{k}_i\underline{x}+i\underline{k}_j\underline{y}}}{V}} \qquad = \frac{1}{V^2}$$

$$= \frac{1}{V^2} \int d_3x \, d_3y \; V(\underline{x}-\underline{y}) \underset{\underline{x}'=\underline{x}-\underline{y}}{=} \frac{1}{V} \int d_3x' \, V(\underline{x}') = \frac{\tilde{V}_0}{V}$$

$$\Rightarrow \frac{\hbar^2}{2m} k_i^2 |\underline{k}_i m_i\rangle + \sum_j \frac{\tilde{v}_0}{V} |\underline{k}_i m_i\rangle - E_{xc} = \varepsilon_i |\underline{k}_i m_i\rangle$$

$$E_{xc} = \langle \underline{k}_i m_i \, \underline{k}_j m_j | v | \underline{k}_j m_j \, \underline{k}_i m_i \rangle = \delta_{m_i m_j} \langle k_i k_j | v | k_j k_i \rangle$$

$$= \delta_{m_i m_j} \int d_3\underline{x}\, d_3\underline{y}\, V(\underline{x}-\underline{y}) \langle \underline{k}_i \underline{k}_j | \underline{x}\underline{y} \rangle \langle \underline{x}\underline{y} | \underline{k}_i \underline{k}_j \rangle = \delta_{m_i m_j} \int d_3\underline{x}\, d_3\underline{y}\, V(\underline{x}-\underline{y}) \frac{e^{i(\underline{k}_j-\underline{k}_i)(\underline{x}-\underline{y})}}{V^2}$$

$$= \frac{\delta_{m_i m_j}}{V} \int d_3\underline{x}'\, V(\underline{x}') e^{-i(\underline{k}_i-\underline{k}_j)\underline{x}'} = \frac{\delta_{m_i m_j}}{V} \tilde{v}(\underline{k}_i-\underline{k}_j)$$

$$\frac{\hbar^2 k_i^2}{2m} |\underline{k}_i m_i\rangle + \sum_j \frac{\tilde{v}_0}{V} |\underline{k}_i m_i\rangle - \sum_j \delta_{m_i m_j} \tilde{v}_{\underline{k}_i-\underline{k}_j} |\underline{k}_i m_i\rangle = \varepsilon_i |\underline{k}_i m_i\rangle$$

$\Rightarrow |\underline{k}_i m_i\rangle$ AUTOSTATO H CON AUTOVAL ε_i

$$\varepsilon_i = \frac{\hbar^2 k^2}{2m} + n\tilde{v}_0 - \frac{1}{V}\sum_j \delta_{m_i m_j} \tilde{v}_{\underline{k}_i-\underline{k}_j}$$

NULLO IN HEG
→ RITROVO ESEMPI PRECEDENTI

SOLO SPIN ∥

HEG DENSO $|\psi\rangle = |F\rangle$

$$\longrightarrow \varepsilon(\underline{k}) = \frac{\hbar^2 k^2}{2m} - \frac{1}{V}\sum_{|k_i|<k_F} \frac{4\pi e^2}{|\underline{k}-\underline{k}_i|^2} = \frac{\hbar^2 k^2}{2m} - \int \frac{d_3 k'}{(2\pi)^3}\,\theta(k_F - k')\,\frac{4\pi e^2}{|\underline{k}-\underline{k}'|^2} = \frac{\hbar^2 k^2}{2m} - \frac{2e^2 k_F}{\pi} f\left(\frac{k}{k_F}\right)$$

$$f(x) = \frac{1}{2} + \frac{1-x^2}{4x}\ln\frac{1+x}{|1-x|}$$ $\longrightarrow$ HF DÀ UNA PESSIMA NAPPA DEI VALORI DI SINGOLA PARTICELLA

CONSIDERANDO DENSITÀ DI STATI x UNITÀ DI SPIN E VOLUME $\rho(\varepsilon) = \frac{1}{V}\sum_{\underline{k}} \delta(\varepsilon(\underline{k}) - \varepsilon)$

$$\simeq \int \frac{d_3 \underline{k}}{(2\pi)^3}\,\underbrace{\delta(\varepsilon(\underline{k}) - \varepsilon)}_{=\,\frac{\delta(\underline{k}-\bar{k})}{\varepsilon'(\underline{k})|_{\bar{k}}}}$$

$$\rho(\varepsilon) = \frac{4\pi}{(2\pi)^3}\,\frac{\bar{k}^2(\varepsilon)}{\varepsilon'(k)\big|_{\bar{k}(\varepsilon)}}$$ $\longrightarrow$ PROBL k_F $f\left(\frac{k}{k_F}\right)\Big|_{k_F} = f(1)$ DERIVATA DIVERGENTE

$\Rightarrow \rho(\varepsilon_F) = 0$ PROBLEMATICO $\longrightarrow$ COME MAI?

NON COINCIDE CON DATI EXP

EVOLUZIONE TEMPORALE

DEF PROPAGATORE $\Longrightarrow$ EQ SCHRÖDINGER

DESCRIZIONE DI INTERAZIONE x

$$H = H_0 + V_t$$

EQ SCHRÖDINGER x INTERAZIONE

$\longrightarrow$ Texp

$$H^0 = \sum_a \hbar\omega_a C_a^+ C_a \longrightarrow C_a(t) = e^{iH^0 t/\hbar} C_a e^{-iH^0 t/\hbar} = e^{-i\omega_a t} C_a$$

$$C_a^+(t) = e^{i\omega_a t} C_a^+$$

$\longrightarrow$ EVOLUZ TEMP CONSERVA AZIONE OPERATORE A MENO DI UNA FASE

SE HO H COMPLICATO $\Rightarrow$ DESCRIZ D'INTERAZIONE $H = H_0 + V_t$

$\rightarrow$ EVOLUZ GOVERNATA DA FAMIGLIA OP A 2 PARAM

$$\begin{cases} U(t,t')U(t't'') = U(t,t'') \\ U(t,t) = \mathbb{1} \quad \forall t \\ U^+(tt') = U(t't) \end{cases} \longrightarrow \text{PROPAGATORI}$$

XKÉ NON SAREBBE UN GRUPPO?

INDIP DA t' XKÉ EQUIVALE A QUALCOSA IN t''

I $\rightarrow \partial_t U(t,t')U(t't'') = \partial_t U(t,t'') \xrightarrow{\times U^+(tt'')} \partial_t U(tt')U(t't) = \partial_t U(tt'')U(t''t)$

$\Rightarrow$ DIP SOLO DA t

$$\partial_t U(tt')U(t't) = \frac{H(t)}{i\hbar}$$

$\Rightarrow \partial_t U(tt') i\hbar = H(t)U(t,t') \longrightarrow$ IMPLICA EVOLUZ ALLA SCHROEDINGER

$\rightarrow$ NON COMMUTANO $\Rightarrow$ IN GEN NON SO RISOLVERE

$$H_t = H_0 + V_t \longrightarrow U(0,t) = e^{-\frac{iH_0 t}{\hbar}} U_I(t,0) \Rightarrow U_I(t,t') = U_I(t,0)\,U_I^\dagger(t',0)$$

→ EQ SCHRÖDINGER PER PROPAGATORE DI INTERAZIONE?

$$= e^{iH_0 t/\hbar}\, U(t,0)\, U^\dagger(t',0)\, e^{-iH_0 t'/\hbar}$$

$$i\hbar\,\partial_t U_I(t,t') = i\hbar\,\partial_t\left[e^{iH_0 t/\hbar}\, U(t,t')\, e^{-iH_0 t'/\hbar}\right]$$

$$= -H_0 U_I + e^{iH_0 t/\hbar}\, \underbrace{\partial_t U}_{\frac{H_t U}{i\hbar} = \frac{H_0+V_t}{i\hbar} U}\, e^{-iH_0 t'/\hbar}\, i\hbar = e^{\frac{iH_0 t}{\hbar}}\, V_t\, \underset{e^{-iH_0 t/\hbar}\, U_I\, \cancel{e^{+iH_0 t'/\hbar}}}{U}\, \cancel{e^{-\frac{iH_0 t'}{\hbar}}} = e^{\frac{iH_0 t}{\hbar}}\, V_t\, e^{-iH_0 t/\hbar}\, U_I$$

$$\begin{cases} i\hbar\,\partial U_I(t,t') = V_{H_0}(t)\, U_I(t,t') \Rightarrow \text{EQ MOTO DI INTERAZ} \\ U_I(t',t') = \mathbb{1} \end{cases}$$

→ PROBL DI CAUCHY ⇒ EQ VOLTERRA $\quad U_I(t,t') = \mathbb{1} + \frac{1}{i\hbar}\int_{t'}^{t} dt''\, V_{H_0}(t'')\, U_I(t'',t') \overset{\text{V ACCOLO}}{=}$

$$= \mathbb{1} + \sum_{\ell=1}^{\infty} \frac{1}{(i\hbar)^\ell} \int_{t'}^{t} dt_1 \int_{t'}^{t_1} dt_2 \ldots \int_{t'}^{t_{\ell-1}} dt_\ell\, V_{H_0}(t_1) \ldots V_{H_0}(t_\ell)$$

$$\Rightarrow U_I(t,t') = T\exp \frac{1}{i\hbar}\int_{t'}^{t} dt''\, V_{H_0}(t'')$$

FUNZIONE DI GREEN

A 2 PTI

DEFINIZIONE DA OP CREAZ/DISTRUZ OP

→ SIGNIFICATO DI TRANSIZIONE

DEF T-PRODOTTO

PROP $\langle \psi_S | O | \psi_S \rangle$

ESEMPIO DENSITÀ

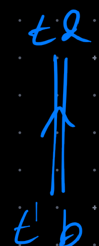
$i G_{ab}(t,t') = \langle GS| T\, c_a(t)\, c_b^\dagger(t') |GS\rangle$

evolve da H indip da t tc $H|gs\rangle = E_{GS}|gs\rangle$

$\mathbb{1}$ se $t>t'$

$\boxed{t>t'}$ $\langle gs|T c_a(t) c_b^\dagger(t')|gs\rangle = \langle gs| c_a(t) c_b^\dagger(t')|gs\rangle = \langle gs| e^{iHt/\hbar} c_a e^{-iHt/\hbar} e^{+iHt'/\hbar} c_b^\dagger e^{-iHt'/\hbar}|gs\rangle$

$= e^{-\frac{i}{\hbar}E_{GS}(t'-t)} \underbrace{\langle gs| c_a}_{+1 \text{ in } \langle a|}\, e^{-iH(t-t')}\, \underbrace{c_b^\dagger|gs\rangle}_{+1 \text{ in } |b\rangle}$ → ampiezza transiz tra stato con partic in a e stato con partic in b in tempo $t-t'$



→ propagaz particella

def fermion → op campo

$i G_{mm'}(\underline{x}t, \underline{x}'t') = \langle gs|T\psi_m(\underline{x}t)\,\psi^\dagger_{m'}(\underline{x}'t')|$

$\underline{\text{DEF}}$ [T PROD] $T A_1(t_1)\ldots A_N(t_N) = (\pm)^\sigma A_{\sigma_1}(t_{\sigma_1})\ldots A_{\sigma_N}(t_{\sigma_N})$ → + boson, − fermion

$t_{\sigma_1} > t_{\sigma_2} > \ldots > t_{\sigma_N}$

$\underline{\text{PROP}}$ sotto T PROD gli op commutano esattam a meno di segn

$T A_1(t_1)\ldots A_n(t_n) = (\pm)^\pi T A_{\pi_1}(t_{\pi_1})\ldots A_{\pi_n}(t_{\pi_n})$

PROP FISICHE GREEN

$$\hat{O} = \sum c_r^\dagger \langle r|O|r'\rangle c_{r'} \longrightarrow \text{DATA } H \quad \langle gs|\hat{O}|gs\rangle = ?$$

$$\langle gs|c_r^\dagger c_{r'}|gs\rangle = \langle gs|c_r^\dagger(t)\, c_{r'}(t)|gs\rangle = \lim_{t'\to t^+} \langle gs|c_r^\dagger(t')\, c_{r'}(t)|gs\rangle$$

$$= \lim_{t'\to t^+} \langle gs|T\, c_r^\dagger(t')\, c_{r'}(t)|gs\rangle = (\pm) \lim_{t'\to t^+} \langle gs|T\, c_{r'}(t)\, c_r^\dagger(t')|gs\rangle = \pm i\, G^0_{r'r}(t, t^+)$$

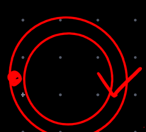
$$\Rightarrow \langle gs|\hat{O}|gs\rangle = \pm i \sum_{rr'} \langle r|O|r'\rangle\, G_{r'r}(t\, t^+)$$

ES [DENSITÀ A SPIN FIX]

$$t^+ = t + \eta$$

$$\langle gs|\hat{n}_m(\underline{x})|gs\rangle = \langle gs|\psi_m^\dagger(\underline{x})\, \underbrace{\psi_m(\underline{x})}_{t}|gs\rangle \overset{\downarrow}{=} \lim (\pm) \langle gs|T \psi_m(\underline{x}t)\, \psi_m^\dagger(\underline{x}t^+)|gs\rangle = \pm i\, G^0_{mm}(\underline{x}t, \underline{x}t^+)$$

$\underline{x}t$



HAMILTONIANA OSCILLATORE FERMIONICO

→ AUTOVETT SINGLE PARTICLE $\{|k\rangle\}$

$\Rightarrow |GS\rangle =$ SLATER

CALCOLO ESPLICITO DI FG

$H_0 = \sum_\alpha \hbar\omega_\alpha c^\dagger_\alpha c_\alpha$ N FERMIONI

RIEMPIO DAL BASSO

$\omega_1 < \omega_2 \le \_\_\_$

$\uparrow$ xké di solito
$\neq$ non degen

$\Rightarrow E_F = E_{GS}$ (?)

$$iG^0_{mm'}(\underline{x}t, \underline{x}'t') = \langle gs|T\psi_m(\underline{x}t)\psi^\dagger_{m'}(\underline{x}'t')|gs\rangle \qquad |gs\rangle =$$

$$\hat\psi_m(\underline{x}) = \sum_\alpha \hat c_\alpha \langle \underline{x} m|\alpha\rangle$$

$$\downarrow$$

$$\hat\psi_m(\underline{x}t) = \sum_\alpha e^{-i\omega_\alpha t}\langle \underline{x} m|\alpha\rangle \hat c_\alpha$$

ANCHE SE TEMPO GIÀ FUORI

$$\Rightarrow iG^0_{mm'}(\underline{x}t, \underline{x}'t') = \sum_{\alpha\alpha'} e^{-i\omega_\alpha t + i\omega_{\alpha'} t'}\langle \underline{x} m|\alpha\rangle\langle\alpha'|\underline{x}'m'\rangle \langle gs|T c_\alpha c^\dagger_{\alpha'}|gs\rangle$$

$$= \sum_{\alpha\alpha'}\left[\theta(t-t')\langle gs|c_\alpha c^\dagger_{\alpha'}|gs\rangle - \theta(t'-t)\langle gs|c^\dagger_{\alpha'} c_\alpha|gs\rangle\right]$$

$$\langle T A_t B_{t'}\rangle = \theta(t-t')\langle A_t B_{t'}\rangle \pm \theta(t'-t)\langle B_{t'} A_t\rangle$$

$\Rightarrow \neq 0$ SE STATO VUOTO

$$= \sum_{\alpha\alpha'}\left[\theta(t-t')\langle gs|\delta_{\alpha\alpha'}\underbrace{c_\alpha c^\dagger_\alpha|gs\rangle}_{1-n_\alpha} - \theta(t'-t)\delta_{\alpha\alpha'}\underbrace{\langle gs|c^\dagger_\alpha c_{\alpha'}|gs\rangle}_{n_\alpha}\right]$$

$$= \sum_{\alpha} e^{-i\omega_\alpha(t-t')} \langle \underline{x} m | \alpha \rangle \langle \alpha | \underline{x}' m' \rangle \left[\theta(t-t')\,\theta(\omega_\alpha - \omega_F) - \theta(t'-t)\,\theta(\omega_F-\omega_\alpha)\right]$$

$$= i\, G_{mm'}(\underline{x}t, \underline{x}'t')$$

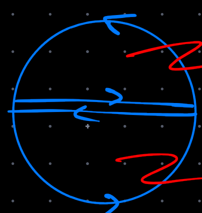
SE INVAR X EVOLUZ TEMP $H \neq H(t)$ $\longrightarrow$ SP ω

$$= i \int_{\pm\infty} \frac{d\omega}{2\pi}\, e^{-i\omega(t-t')}\, G^{0}_{mm'}(\underline{x}\,\underline{x}'\,\omega)$$ $\Rightarrow$ COME SEMPLIFICA IN SP ω?

$$\theta(t-t') = i \int_{\pm\infty} \frac{d\omega}{2\pi}\, \frac{e^{-i\omega(t-t')}}{\omega + i\eta}$$ $\longrightarrow$ CHIUSURA CAMMINO?

$\Rightarrow$ η NSSIMO > 0

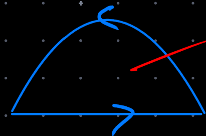
$\boxed{t > t'}$



$\Rightarrow$ CONTRIB 0

$\Rightarrow$ POLO $-i\eta \Rightarrow \dfrac{i(-2\pi i)}{2\pi}\, e^{-\eta(t-t')} = 1$

$\boxed{t<t'}$



$\Rightarrow$ NO POLI $\Rightarrow$ CONTRIB 0

$$G^0_{mm'}(\underline{x}t,\underline{x}'t') = \sum_a \underbrace{}_{\exp(-i\omega_a(t-t'))} \left[i\int_{\pm\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega+i\eta}\,\theta(\omega_a-\omega_F) - i\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{+i\omega(t-t')}}{\omega+i\eta}\,\theta(\omega_F-\omega_a)\right]$$

$$= i\sum_a \langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle \left[\theta(\omega_a-\omega_F)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i(\omega+\omega_a)(t-t')}}{\omega+i\eta} - \theta(\omega_F-\omega_a)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{+i(\omega+\omega_a)(t-t')}}{\omega+i\eta}\right.$$

$\omega \to \omega - \omega_a$

$\omega \to -\omega$

$$= i\sum_a \langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle \left[\theta(\omega_a-\omega_F)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega-\omega_a+i\eta} - \theta(\omega_F-\omega_a)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i(\omega+\omega_a)(t-t')}}{-\omega+i\eta}\right]$$

CAMBIA SEGN DENOM E DIVENTA +

=)

$$G^0_{mm'}(\underline{x}\,\underline{x}'\omega) = \sum_\alpha \langle \underline{x} m|\alpha\rangle\langle\alpha|\underline{x}'m'\rangle \left[\frac{\theta(\omega_\alpha-\omega_F)}{\omega-\omega_\alpha+i\eta} + \frac{\theta(\omega_F-\omega_\alpha)}{\omega-\omega_\alpha-i\eta}\right]$$

FUNZ DI GREEN X PARTIC NON INTERAGENTI

SCELGO SPW

$$h u_\alpha = \hbar\omega_\alpha u_\alpha$$

$$H_0 = \sum \hbar\omega_\alpha c^\dagger_\alpha c_\alpha$$

$$G^0_{mm'}(\underline{x}\,\underline{x}'\omega) = \sum_\alpha u_\alpha(\underline{x} m)\, u^*_\alpha(\underline{x}'m') \left[\frac{\theta(\omega_\alpha-\omega_F)}{\omega-\omega_\alpha+i\eta} + \frac{\theta(\omega_F-\omega_\alpha)}{\omega-\omega_\alpha-i\eta}\right]$$

APPLICHIAMOLO

$$\langle gs|\hat{n}_m(\underline{x})|gs\rangle = -i\,G^0_{mm}(\underline{x}t\,\underline{x}t^+) = -i\int_{\pm\infty}\frac{d\omega}{2\pi}\,G^0_{mm}(\underline{x}\,\underline{x}\,\omega)\,e^{i\omega\eta}$$

NON OMETTIBILE

$$= -i\sum_\alpha |u_\alpha(\underline{x}m)|^2\int_{\pm\infty}\frac{d\omega}{2\pi}\,e^{i\omega\eta}\left[\frac{\theta(\omega_\alpha-\omega_F)}{\omega-\omega_\alpha+i\eta} + \frac{\theta(\omega_F-\omega_\alpha)}{\omega-\omega_\alpha-i\eta}\right] \longrightarrow$$

INTEG CON RESIDUI

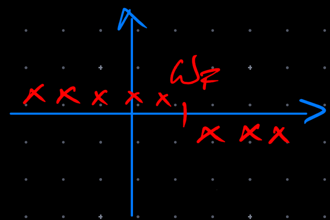
polo $-i\eta$

polo $+i\eta$

$$= \sum |u_e(\underline{x}\, m)|^2 (-i) \frac{2\pi i}{2\pi} \theta(\omega_F - \omega_e) \underbrace{e^{i\omega_e \eta}}_{\substack{\downarrow \eta \to 0 \\ 1}} = \sum |u_e(\underline{x}\, m)|^2 \theta(\omega_F - \omega_e)$$

DENSITÀ SINGOLO STATO

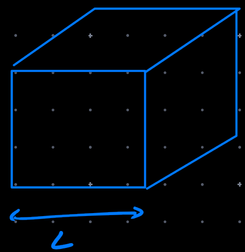
DENSITÀ GS = $\sum$ DENSITÀ SINGOLI STATI $< E_F$



VEDIAMO CHE ANCHE X ESSI INTERAG IL VAL DISCRIMINANTE È ω_F

GAS IDEALE DI FERMI

$h = \frac{p^2}{2m}$ $\quad |\alpha\rangle = |\underline{k}\, m\rangle \longrightarrow$ NUOVA SCATOLA $\quad \underline{k} = \frac{2\pi}{L}\underline{n} \quad \underline{n} \in \mathbb{Z}^3$ $\qquad \omega_k = \varepsilon_k/\hbar$



$$G^0_{mm'}(\underline{x}\,\underline{x}'\omega) = \sum_{\underline{k}m''} \langle \underline{x} m | \underline{k} m''\rangle\langle \underline{k} m'' | \underline{x}' m'\rangle \left[\frac{\theta(\varepsilon_k - \varepsilon_F)}{\omega - \omega_k + i\eta} + \frac{\theta(\varepsilon_F - \varepsilon_k)}{\omega - \omega_k - i\eta}\right]$$

$$= \sum_{\underline{k} m''} \delta_{mm''}\delta_{m''m'} \frac{e^{i\underline{k}(\underline{x}-\underline{x}')}}{V}[-] = \frac{\delta_{mm'}}{V}\sum_{\underline{k}} e^{i\underline{k}(\underline{x}-\underline{x}')}[-]$$

↳ TRASF FOURIER INVERSA

$$= \delta_{mm'}\int \frac{d^3k}{(2\pi)^3} e^{i\underline{k}(\underline{x}-\underline{x}')} G^0(\underline{k}\omega)$$

$$G^0(\underline{k}\omega) = \frac{\theta(\varepsilon_k - \varepsilon_F)}{\omega - \frac{\varepsilon_k}{\hbar} + i\eta} + \frac{\theta(\varepsilon_F - \varepsilon_k)}{\omega - \frac{\varepsilon_k}{\hbar} - i\eta} = \frac{1}{\omega - \frac{\varepsilon_k}{\hbar} + i\eta\, \mathrm{sgn}(\omega_k - \omega_F)}$$

SONO PASSATO A SP $\underline{k}$ XKÈ HO INVAR X TRASLAZ (VALE $\forall H$ TC $[H, \vec{p}] = 0$)

IN II QTZ $\quad \vec{p} = \sum_{\underline{k}m} \hbar \underline{k}\, c^\dagger_{km} c_{km} \longrightarrow \underline{a}\cdot\underline{p} = \sum_{\underline{k}m} \hbar\, \underline{k}\cdot\underline{a}\, c^\dagger_{\underline{k}m} c_{\underline{k}m}$

$$e^{i\underline{a}\cdot\underline{p}/\hbar}\, c_{\underline{k}m}\, e^{-i\underline{a}\cdot\underline{p}/\hbar} = e^{-i\underline{a}\cdot\underline{k}}\, c_{\underline{k}m}$$

INVECE $\quad e^{\frac{i}{\hbar}\underline{Q}\cdot\underline{P}}\,\hat{\psi}_m(\underline{x})\,e^{-\frac{i}{\hbar}\underline{Q}\cdot\underline{P}} = \sum_{\underline{k}} \langle \underline{x}|\underline{k}\rangle\, c_{\underline{k}m}\, e^{-\underline{Q}\underline{k}} = \sum_{\underline{k}} \langle \underline{x}-\underline{Q}|\underline{k}\rangle\, c_{\underline{k}m}$

$= \hat{\psi}_m(\underline{x}-\underline{Q})$ AGISCE COME δ CAMPO SCALARE

<u>xCIÒ</u> SE $[H,\vec{P}]=0 \Rightarrow G_{mm'}(\underline{x}t,\underline{x}'t')$ DIP DA $\underline{x}-\underline{x}'$

$$\Rightarrow G_{mm'}(\underline{x}t,\underline{x}'t') = \int \frac{d_3\underline{k}}{(2\pi)^3}\int \frac{d\omega}{2\pi}\, G_{mm'}(\underline{k}\omega)\, e^{i\underline{k}(\underline{x}-\underline{x}')-i\omega(t-t')}$$

EQUAZIONI DEL MOTO IN FUNZIONI DI GREEN

$H = T + V \longrightarrow$ II QTZ

$\longrightarrow E_{GS} = \langle gs | H | gs \rangle$

$$= \frac{1}{2}\langle H_1 \rangle + \frac{1}{2}\langle \sum_r c_r^+ [c_r, H] \rangle$$

$$= \frac{1}{2}\langle H_1 \rangle \pm \frac{1}{2} i\hbar \partial_t G_{rr}(t,t')|$$

$$H=\sum_{ab} h_{ab}\, c_a^\dagger c_b + \frac{1}{2}\sum_{\substack{ab\\cd}} v_{abcd}\, c_a^\dagger c_b^\dagger c_d c_c \qquad = \langle ab|v|cd\rangle = \langle ba|v|dc\rangle$$

$$[c_r, c_a^\dagger c_b] = \delta_{ar} c_b \qquad \oplus \qquad [c_r, c_a^\dagger c_b^\dagger c_d c_c] = \left(\delta_{ra} c_b^\dagger \pm \delta_{rb} c_a^\dagger\right) c_d c_c$$

$$\Rightarrow [c_r, H] = \sum_{ab} h_{ab} [c_r, c_a^\dagger c_b] + \frac{1}{2}\sum v_{abcd} [c_r, c_a^\dagger c_b^\dagger c_d c_c]$$

$$= \sum_b h_{rb} c_b + \frac{1}{2}\left(\sum_{bcd} v_{rbcd}\, c_b^\dagger c_d c_c \pm \sum_{acd} v_{arcd}\, c_a^\dagger c_d c_c\right) = \sum_b h_{rb} c_b + \sum_{bcd} v_{rbcd}\, c_b^\dagger c_d c_c$$

$a \leftrightarrow b \Rightarrow v_{brcd} = v_{rbdc}$ (cd mon)

CON −

OSSERVAZIONI:

- $H = H_1 + H_2 \Rightarrow \sum_r c_r^\dagger [c_r, H] = H_1 + 2H_2$

  1 PARTIC   2 PARTIC

- $i\hbar\partial_t c_r(t) = i\hbar\partial_t\, e^{\frac{i}{\hbar}Ht} c_r e^{-\frac{i}{\hbar}Ht} = -[H, c_r(t)] = [c_r(t), H] = \sum_b h_{rb} c_b(t) + \sum_{bcd} v_{rbcd} (c_b^\dagger c_d c_c)(t)$

PRIMA ABBIAMO OTTENUTO $\langle GS|O|GS\rangle$ DA G^0 $\rightarrow$ TROVIAMO E_{GS}

$$E_{GS} = \langle GS|H|GS\rangle = \langle GS|H_1|GS\rangle + \langle GS|H_2|GS\rangle$$

$$\sum_r \langle gs|\, c_r^\dagger(t)\, i\hbar\, \partial_t\, c_r(t)\,|gs\rangle = \langle gs|\, H_1\, |gs\rangle + 2\langle gs|\, H_2\, |gs\rangle$$

$$= \lim_{t'\to t} \langle gs|\sum_r c_r^\dagger(t')\, i\hbar\, \partial_t\, c_r(t)\,|gs\rangle = \lim_{t'\to t} i\hbar\, \partial_t \sum_r \langle gs|\, c_r^\dagger(t')\, c_r(t)\,|gs\rangle$$

so $t'\to t^+ \Rightarrow T = \mathbb{1}$

$$= \lim_{t'\to t} i\hbar\, \partial_t \sum_r \langle GS|\, T\, c_r^\dagger(t')\, c_r(t)\,|GS\rangle = \lim_{t'\to t} \pm\, i\hbar\, \partial_t\, i\, G_{rr}(t,t')$$

$$\left(= \langle gs|\, H_1\, |gs\rangle + 2\langle gs|\, H_2\, |gs\rangle \quad i\hbar\, \partial_t\, c_r(t) = [c_r(t), H] \right)$$ SBAGLIATO

$$\Rightarrow E_{GS} = \frac{1}{2}\langle gs|\, H_1\, |gs\rangle \pm \frac{1}{2} \sum_r \lim_{t'\to t^+} i\hbar\, \partial_t\, i\, G_{rr}(t,t')$$

$$= \sum_{rr'} (\pm)\, \frac{i}{2} \langle r|\, h\, |r'\rangle\, G_{r'r}(t,t') \pm \sum_r i \lim_{t'\to t^+} (i\hbar\, \partial_t)\, G_{rr}(t,t')$$

EQ MOTO

$$iG_{ab}(t,t') = \langle gs|\, T c_a(t)\, c_b^\dagger(t')\, |gs\rangle$$

$$= \theta(t-t')\langle gs|\, c_a(t)\, c_b^\dagger(t')\, |gs\rangle \pm \theta(t'-t)\langle gs|\, c_b^\dagger(t')\, c_a(t)\, |gs\rangle$$

DERIVO

$$i\partial_t G_{ab}(t,t') = \delta(t-t')\langle c_a(t)\, c_b^\dagger(t') \mp c_b^\dagger(t')\, c_a(t)\rangle +$$

$$+\theta(t'-t)\langle \partial_t c_a(t)\, c_b^\dagger(t')\rangle \pm \theta(t'-t)\langle c_b^\dagger(t')\, \partial_t c_a(t)\rangle$$

$t'=t \Rightarrow$ evolve su $gs \Rightarrow$ scompare

$$= \delta(t-t')\langle [c_a, c_b^\dagger]_\pm \rangle + \langle T\, \partial_t c_a(t)\, c_b^\dagger(t')\rangle$$

$$= \delta_{ab}\,\delta(t-t') + \langle T\, \partial_t c_a(t)\, c_b^\dagger(t')\rangle$$

$$-\hbar\partial_t G_{ab}(t,t') = i\hbar\,\delta_{ab}\,\delta(t-t') + \sum_c h_{ac}\langle T c_c(t)\, c_b^\dagger(t')\rangle$$

$$+\sum_{b'c'd'} 2S_{ab'c'd'}\langle T\left(c_{b'}^\dagger c_{d'} c_{c'}\right)(t)\, c_b^\dagger(t')\rangle$$

$$\Rightarrow \left[-\hbar\partial_t G_{ab}(tt') - i\sum_c h_{ac} G_{cb}(tt')\right] - \sum_{b'c'd'} 2S_{ab'c'd'}\langle T\left(c_{b'} c_{d'} c_{c'}\right)(t)\, c_b^\dagger(t')\rangle = i\hbar\,\delta_{ab}\,\delta(t-t') \qquad (*)$$

→ EQ MOTO FG → IN PRESENZA DI OP A 2 CORPI CONTIENE EQ MOTO DI FG DI ORDINE SUP

FORMA NON CHIUSA: GERARCHIA DI MARTIN & SCHWINGER

⇒ TRONCO $\langle 4P\rangle = \langle 2P \times 2P\rangle \to$ <u>HF</u>

→ FACCIAMO IL TRONCAMENTO

$$i^2 G_{abcd}(t_1 t_2 t_3 t_4) = \langle GS| T c_a(t_1) c_b(t_2) c_d^+(t_4) c_c^+(t_3) |GS\rangle$$ → [illegible] RISCRIVENDO IN θ

$$= \pm \langle GS| T c_b(t_2) c_a(t_1) c_d^+(t_4) c_c^+(t_3) |GS\rangle = \pm\, i^2 G_{bacd}(t_2 t_1 t_3 t_4)$$

AAAAH

- $(*) \to \times(-i)\, \delta_{ac}$
- × RIORDINARE T PROD $\quad T(c_{b'}^+ c_{d'} c_{c'})(t)\, c_b^+(t') \equiv T c_{b'}^+(t^{++}) c_{d'}(t^+) c_{c'}(t) c_b^+(t')$ È PARI A 2 SCAMBI

$$\Rightarrow \sum_c \left(+i\hbar\, \delta_{ac}\, \partial_t - h_{ac} \right) G_{cb}(t,t') + i \sum_{b'c'd'} \nu_{ab'c'd'}\, i^2 G_{d'c'bb'}(t^+ t\, t' t^{++}) = \hbar\, \delta_{ab}\, \delta(t-t')$$

CASO NON INTERAGENTE $\quad \sum_c \left(i\hbar\, \delta_{ac}\, \partial_t - h_{ac} \right) G^0_{cb}(t t') = \hbar\, \delta_{ab}\, \delta(t-t') \longrightarrow$ SP ω $\quad G\, e^{-i\omega(t-t')}$ È DERIVO ∂_t

→ POI GUARDO INTEGRANDA

$$\sum_c \left(\hbar\omega\, \delta_{ac} - h_{ac} \right) G^0_{cb}(\omega) = \hbar\, \delta_{ab} \longrightarrow$$ SCOM MATR $\quad (\hbar\omega\, \mathbb{1} - \hat{h})\, G^0(\omega) = \hbar\, \mathbb{1}$

$$\Rightarrow G^0(\omega) = \left(\omega - \frac{\hat{h}}{\hbar} \right)^{-1}$$ RISOLVENTE $\longrightarrow \{|j\rangle\}$ SISTEMA $h \Rightarrow G^0(\omega) = \sum_j \frac{|j\rangle\langle j|}{\omega - \varepsilon_j/\hbar}$ → SINGOLARITÀ SU Re ⇒ SI SPOSTANO I POLI NEL PIANO $\mathbb{C}$

$\mathbb{I} \neq$ mom ⇒ ESP ≠ FG, IN BASE $\{|\underline{x}\, m\rangle\}$ L'OP h È LOCALE $\quad \hat{h} = -\frac{\hbar^2}{2m} \nabla^2 + U(\underline{x})$

$$\Rightarrow \left[\hbar\omega + \frac{\hbar^2}{2m} \nabla_x^2 - U(\underline{x}) \right] G^0_{mm'}(\underline{x}\, \underline{x}'\, \omega) = \hbar\, \delta_3(\underline{x} - \underline{x}')\, \delta_{mm'}$$ → PONGO $G^0_{mm'} = G^0 \delta_{mm'}$

$$\to \left[\hbar\omega + \frac{\hbar^2}{2m} \nabla^2 - U(\underline{x}) \right] f(\underline{x}) = g(\underline{x}) \longrightarrow$$

g SORGENTE ASSEGNATA

f SCONOSCIUTO

EQ DIFF LIN NON OMOG ⇒ SOL OMOG + SOL PARTIC

$$f(\underline{x}) = f_0(\underline{x}) + \frac{1}{\hbar} \int d_3x'\, G^0(\underline{x}\, \underline{x}'\, \omega)\, g(\underline{x}') \Rightarrow$$

RISCRIVO EQ DIFF INCHIAVABILE COME EQ INTEG IN G^0

SE NON È U CO ACCOMB ALLA SORG, RISOLVO PARTIC PASSO È POI TRATTO PERTURBATIVO

TEOR GELLMANN & LOW

$$H = H_0 + gV \Rightarrow H_\varepsilon = H_0 + g e^{-\varepsilon|t|} V$$ INTERPOLA

$$H_0 |E_0\rangle = E_0 |E_0\rangle$$

$$\lim_{\varepsilon \to \infty} \left(|\psi\rangle_{\varepsilon\pm} = \frac{U_{\varepsilon I}(0,\pm\infty)}{\langle E_0| U_{\varepsilon I}(0 \pm \infty) |E_0\rangle} |E_0\rangle \right)$$ EIGEN H COINCIDENTI SE E_0 NON DEG

COLLEGA GS H CON GS H_0 → ACCENSIONE ADIABATICA INTERAZ

INTERPOLA H E H_0

$H = H_0 + gV$ ⇒ SE V A 2 CORPI PUÒ QUASI INATTACCAB SENZA TEO PERTURBATIVA

$$H_\varepsilon = H_0 + g\, e^{-\varepsilon|t|} V = \begin{cases} H_0 & |t| \to \infty \\ H & |t| = 0 \end{cases} \qquad \forall \varepsilon > 0$$

$[H_t, H_{t'}] \neq 0 \to U_\varepsilon(t,t') \xrightarrow{\text{DESCRIZ INTERAZ}} U_\varepsilon(t,0) = e^{-\frac{i}{\hbar}H_0 t}\, U_{I\varepsilon}(t,0)$

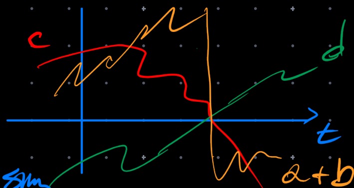
TEOR G.L. : $|E_0\rangle$ EIGEN H_0, $\exists \lim_{\varepsilon \to 0^+} |\psi_{\varepsilon\pm}\rangle = \dfrac{U_{\varepsilon I}(0,\pm\infty)|E_0\rangle}{\langle E_0|U_{\varepsilon I}(0,\pm\infty)|E_0\rangle}$ ⇒ e $|\psi_\pm\rangle$ AUTOST DI H (COINCIDONO A MENO DI UNA FASE) SE E_0 NON DEG

NB TEOR NON GARANTISCE $GS^0 \to GS$ POSSIBILE LEVEL CROSSING

EVOLUZ TEMP AUTOVALORI H

$$H = \begin{pmatrix} a & c+id \\ c-id & b \end{pmatrix} \quad a,b,c,d \in \mathbb{R} \text{ di } t \to \begin{cases} \lambda_+ + \lambda_- = a+b \\ \lambda_+\lambda_- = ab - |c+id|^2 \end{cases} \Rightarrow \lambda_\pm = \frac{1}{2}\left(a+b \pm \sqrt{(a+b)^2 - 4ab - 4|c+id|^2}\right)$$

SEPARAZ $|\lambda_+ - \lambda_-| = \sqrt{(a-b)^2 + 4c^2 + 4d^2} \longrightarrow 0 \iff \begin{cases} a=b \\ c=d \end{cases}$ → MOLTO IMPROBABILE CHE AVVENGA NELLO STESSO PTO, + DIFFICILE NEL CASO HERMIT CHE IN QUESTO SIM



SEPARAZ LIVELLI DISTRIBUZ × MATR RANDOM

$P(s) = s^\beta$ × PICCOLI s

$\beta = 1$ × SIMM

$\beta = 2$ × HERMITIANE

$\beta = 4$ × QUATERNIONICHE

⇒ PROB CHE LA SPAZIATURA SIA 0 È 0

LEVEL CROSSING NON AVVIENE SE CONSERVO LE SIMMETRIE

Dim [MORINAGA]

USO SCHRÖDINGER $\quad H_t = H_0 + g e^{+\varepsilon t} \Rightarrow U_g(t,s) \quad s<t<0$ $\quad$ PONGO $g = e^{\varepsilon\theta} \Rightarrow H_t = H_0 + e^{\varepsilon(t+\theta)} V \rightarrow U_{g=1}$ con $t+\theta$

$H_0 + e^{t\varepsilon} g V$ ha prop $U_1(t+\theta, s+\theta) = U_g(t,s) \Rightarrow \frac{\partial}{\partial g} \rightarrow \frac{\partial}{\partial \theta}$

$$\frac{\partial}{\partial g} U_g(t,s) = \frac{\partial\theta}{\partial g}\frac{\partial U_1}{\partial\theta}(t+\theta, s+\theta) = \frac{\partial\theta}{\partial g}\left[\frac{\partial U_1}{\partial t}(t+\theta, s+\theta) + \frac{\partial U_1}{\partial s}(t+\theta, s+\theta)\right]$$

$$= \frac{\partial\theta}{\partial g}\left[\frac{\partial U_g}{\partial t}(t,s) + \frac{\partial U_g}{\partial s}(t,s)\right] \qquad g = e^{\varepsilon\theta} \rightarrow 1 = \frac{\partial\theta}{\partial g} g\varepsilon$$

$$\varepsilon g \frac{\partial}{\partial g} U_g(t,s) = \frac{H_t}{i\hbar} U_g(t,s) - U_g(t,s)\frac{H_s}{i\hbar} \rightarrow t = 0 \longrightarrow H_{t=0} = H$$

$$i\hbar\, \varepsilon g \frac{\partial}{\partial g} U(0,s) = H\, U(0,s) - U(0,s)\left[H_0 + g e^{\varepsilon s} V\right]$$

PASSO IN RAPPRESENTAZIONE DI INTERAZIONE

$$\Big|\; U(\tau,0) = e^{-\frac{i}{\hbar}\tau H_0} U_I(\tau,0) = U^\dagger(0,\tau)$$

$$i\hbar\, \varepsilon g \frac{\partial}{\partial g} U_I(0s)\, e^{\frac{i}{\hbar}H_0 s} = H U_I(0s)\, e^{\frac{i}{\hbar}H_0 s} - U_I(0s)\, e^{\frac{i}{\hbar}H_0 s}[-] \times e^{-iH_0 s/\hbar}$$

$$i\hbar\varepsilon g\frac{\partial}{\partial g}U_I(0,s) = H U_I(0,s) - U_I(0,s)H_0 - g\,U_I(0,s)\,V_{H_0}(s)\,e^{\varepsilon s} \quad \times|\mathcal{E}_0\rangle$$

$$i\hbar\varepsilon g\frac{\partial}{\partial g}U_I(0,s)\,|\mathcal{E}_0\rangle = (H-\mathcal{E}_0)\,U_I(0,s)\,|\mathcal{E}_0\rangle - g\,e^{\varepsilon s}\,U_I(0,s)\,V_{H_0}(s)\,|\mathcal{E}_0\rangle$$

$\varepsilon < \infty \Rightarrow$ CONSID $s \to -\infty$

$$i\hbar g\varepsilon\frac{\partial}{\partial g}U_I(0,-\infty)\,|\mathcal{E}_0\rangle = (H-\mathcal{E}_0)\,U_I(0,-\infty)\,|\mathcal{E}_0\rangle$$

NON POSSO ELIMINARE ε

$\varepsilon \to 0 \Rightarrow H$ HA AUTOVAL. $\mathcal{E}_0$

$$\varepsilon i g\hbar\frac{\partial}{\partial g}\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle = \langle\mathcal{E}_0|(H-\mathcal{E}_0)\,U_I(0,-\infty)|\mathcal{E}_0\rangle$$

$$\varepsilon i\hbar g\frac{\partial}{\partial g}\frac{U_I(0,-\infty)}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}|\mathcal{E}_0\rangle = \frac{(H-\mathcal{E}_0)\,U_I(0,-\infty)}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}|\mathcal{E}_0\rangle - \frac{U_I(0,-\infty)|\mathcal{E}_0\rangle}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}\frac{\langle\mathcal{E}_0|(H-\mathcal{E}_0)\,U_I(0,-\infty)|\mathcal{E}_0\rangle}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}$$

$\times$ Hp $\exists$ lim FINITO $\Rightarrow \varepsilon\lambda \to 0$

$$0 = (H-\mathcal{E}_0)\frac{U_I(0,-\infty)|\mathcal{E}_0\rangle}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle} - \frac{U_I(0,-\infty)|\mathcal{E}_0\rangle}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}\frac{\langle\mathcal{E}_0|(H-\mathcal{E}_0)\,U_I(0,-\infty)|\mathcal{E}_0\rangle}{\langle\mathcal{E}_0|U_I(0,-\infty)|\mathcal{E}_0\rangle}$$

$$\Rightarrow 0 = (H-\mathcal{E}_0)|\varphi_-\rangle - |\varphi_-\rangle\langle\mathcal{E}_0|H-\mathcal{E}_0|\varphi_-\rangle$$

$\longrightarrow$ $\times$ MISURARE U_I USO T exp DYSON

$|\varphi_\pm\rangle$ NON NORMALIZ., ANCHE $\times +\infty$ ∎

$$\langle gs | T O_{1H}(t_1) \ldots O_{NH}(t_N) | gs \rangle = \frac{\langle E_0 | T S O_{1,H_0}(t_1) \ldots O_{N,H_0}(t_N) | E_0 \rangle}{\langle E_0 | S | E_0 \rangle}$$

$$S = U_I(-\infty, +\infty)$$

FORMULA DI MADZIONS

(CONSEGUENZA GELLMANN & LOW)

$H = H_0 + V$, $H|gs\rangle = E_{gs}|gs\rangle$ $\qquad H_0|E_0\rangle = E_0|E_0\rangle$

$$\Rightarrow \forall\, \hat{O} \text{ VALE } \langle gs|T O_{1H}(t_1)\_\_ O_{NH}(t_N)|gs\rangle = \frac{\langle E_0|T S O_{1H_0}(t_1)\_\_ O_{NH_0}(t_N)|E_0\rangle}{\langle E_0|S|E_0\rangle}$$

OP DI SCATTERING

$$S = U_I(+\infty,-\infty) = T\exp \frac{1}{i\hbar}\int_{-\infty}^{+\infty} dt\, V_{H_0}(t)$$ $\Rightarrow$ CALCOLABILE ALMENO PERTURBATIVAMENTE

DIM (FORMULA VALE SOLO x T PROD $\Rightarrow$ POI FACCIO PG)

$$\boxed{\begin{aligned} O_H(t) &= e^{iHt/\hbar} O e^{-iHt/\hbar} = U_I^\dagger(t0) e^{iH_0t/\hbar} O e^{-iH_0t/\hbar} U_I(t0) \\ &= U_I^\dagger(t0) O_{H_0}(t) U_I(t0) \end{aligned}}$$

$$t_1 > t_2 > \_\_ > t_N \Rightarrow \langle gs|O_{1H}(t_1)\_\_ O_{NH}(t_N)|gs\rangle =$$

$$= \frac{\langle\psi_+|U_I^\dagger(t_1 0) O_{1H_0}(t_1) U_I(t_1 0) U_I^\dagger(t_2 0) O_{2H_0}(t_2)\_\_\_\,|\psi_-\rangle}{\langle\psi_+|\psi_-\rangle}$$

SOTTOINT H_0

$$= \frac{\overbrace{\frac{\langle E_0|U_I^\dagger(0,+\infty)}{\cancel{\langle E_0|U_I(0+\infty)|E_0\rangle^*}}}^{\langle\psi_+|}\; U_I(0t_1) O_1(t_1) U_I(t_1 t_2) O_2(t_2)\_\_ \;\overbrace{\frac{U_I(0-\infty)|E_0\rangle}{\cancel{\langle E_0|U_I(0-\infty)|E_0\rangle}}}^{|\psi_-\rangle}}{\underbrace{\langle E_0|U_I^\dagger(0,+\infty)\, U_I(0-\infty)|E_0\rangle}_{\cancel{\langle H\rangle^*}\quad \cancel{\langle H\rangle}}}$$

$$= \frac{\langle \Omega_0| U_I(\infty, t_1)\, O_1(t_1)\, U_I(t_1,t_2) \_\_\_\, O_N(t_N)\, U_I(t_N,-\infty)|\Omega_0\rangle}{\langle \Omega_0|U_I(\infty,-\infty)|\Omega_0\rangle}$$ → CRONOLOGICAMENTE ORDINATO $\Rightarrow T = \mathbb{1}$

$$U_I(t_1,t_2) = T\exp \frac{1}{i\hbar}\int_{t_2}^{t_1} V_{H_0}(t')\,dt'$$ → SE V A 2 CORPI $\Rightarrow$ Texp HA # PARI DI c e $c^\dagger$

$\Rightarrow$ SALTO NO SEGNO

$$\Rightarrow\ = \frac{\langle \Omega_0| T S\, O_1(t_1) \_\_ O_N(t_N)|\Omega_0\rangle}{\langle \Omega_0|S|\Omega_0\rangle}$$

× COMPLETARE $\quad iG_{mm'}(\underline{x}t\, \underline{x}'t') = \langle GS|T\psi_m(\underline{x}t)\,\psi^\dagger_{m'}(\underline{x}'t')|GS\rangle = \frac{\langle\Omega_0|TS\,\psi_{mH_0}(\underline{x}t)\,\psi^\dagger_{m'H_0}(\underline{x}'t')|\Omega_0\rangle}{\langle\Omega_0|S|\Omega_0\rangle}$

SOSTITUENDO A S LA VAGONATA DI V $\Rightarrow$ UN BOTTO DI CORPI $\Rightarrow$ TEOR WICK × TRATTARLE ?

~~PROPAGATORE CUBONO~~

$\hat{H}$ SINGLE PARTICLE

→ U VA IN SORGENTE

⇒ EQ DIFF NON OMOG

⇒ SOL OMOG G^0 FREE PART

$\oplus \int G^0 U G \, dy$

→ FUNZ RICORSIVA

⇒ SOSTITUISCO N VOLTE

→ SE V PICCOLO TRONCO

⇒ PERTURBAZ

$$\left[\hbar\omega + \frac{\hbar^2}{2m}\nabla^2 - U(\underline{x})\right] G(\underline{x}\,\underline{x}'\omega) = \hbar\,\delta_3(\underline{x}-\underline{x}')$$ → SE USABILE x SINGOLE PARTICELLE ⇒ G

PARTIC NON INTERAGENTI

⇒ $G_{mm'} = \delta_{mm'} G$

ALTRIMENTI METODO PERTURB

→ $$\left[\hbar\omega + \frac{\hbar^2}{2m}\nabla^2\right] G(\underline{x}\,\underline{x}'\omega) = \hbar\,\delta_3(\underline{x}-\underline{x}') + U(\underline{x})\,G(\underline{x}\,\underline{x}'\omega)$$ → EQ DIFF NON OMOG

→ SOL OMOG + PARTIC

↳ G^0

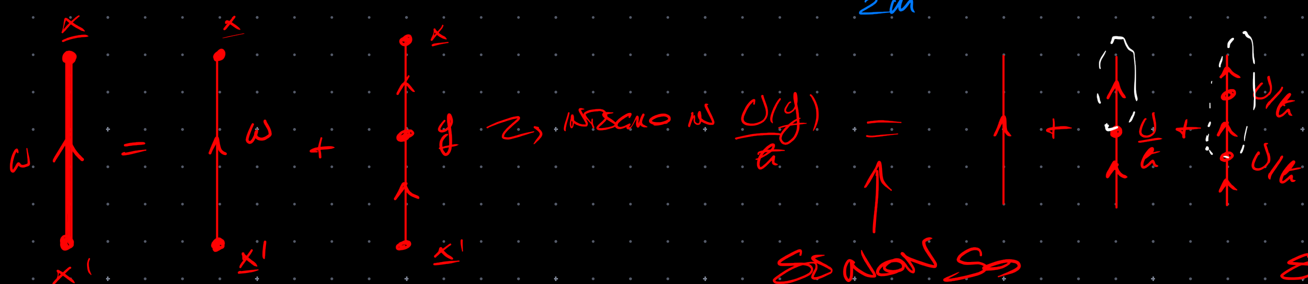
SE SO FG x GAS IDEALE ⇒

↳ RICAVO EQ DIFF COME δ IN FORMA INTEGRALE

G^0 PARTIC LIBERA ⇒ $$G(\underline{x}\,\underline{x}'\omega) = G^0(\underline{x}\,\underline{x}'\omega) + \frac{1}{\hbar}\int d_3y\, G^0(\underline{x}\,y\,\omega)\,U(y)\,G(y\,\underline{x}'\omega)$$

↳ SU DI CUI PASSO ITERATIVO

→ FACENDO AGIRE $H_0 = \hbar\omega + \frac{\hbar^2\nabla^2}{2m}$ TORNO ALL'ORIGINALE



G^0

SE NON SO RISOLVERE ⇒ x ITERAZ

BORN SERIES

SVILUPPO "PERTURB" DI G USANDO INTERAZ U

TEOR DI WICK

STATICO

OP ASSOCIATI A STATO

OP ORDINATI NORM

CONTRAZIONI $\to$ $\langle\ \rangle$ SU STATO RISP

$\to$ RISCRITTURA PROD = N + CONTRAZ

$\times$ N OP $\Rightarrow$ TEOR WICK

$$A_1 A_2 = N[\ ] + \overbrace{A_1 A_2}$$

δ QM $\times$ INDICE

$$\to A_1 \dots A_n = \sum_{k=0}^{[\frac{n}{2}]} N_{nk}$$

N_{nk} = OP ORDINATI NORM CON K CONTRAZ

$|GS\rangle$, BASE DI OP ASSOCIATI ALLO STATO CANONICO $\alpha_k^\pm \longrightarrow$ NON AUTOAGG

$$[\alpha_k^+, \alpha_j^+]_\mp = [\alpha_k^-, \alpha_j^-]_\mp = 0 \qquad [\alpha_k^-, \alpha_j^+]_\mp = \delta_{kj}$$

TC $\quad \alpha_k^- |GS\rangle = 0$

$\langle GS| \alpha_k^+ = 0$

($\mp$) SIANO UNO L'AGGIUNTO DELL'ALTRO)

SU SF FERMI $\quad \alpha_k^- |F\rangle = 0 \longrightarrow \begin{cases} \text{OP CREAZ} \quad Q^+_{\underline{k}m} |F\rangle = 0 & |\underline{k}| \leq k_F \\ \text{OP DISTRUZ} \quad Q_{\underline{k}m} |F\rangle = 0 & |\underline{k}| > k_F \end{cases}$

$\langle F| \alpha_k^+ = 0 \longrightarrow \begin{cases} \langle F| Q_{\underline{k}m} = 0 & |\underline{k}| \leq k_F \\ \langle F| Q^+_{\underline{k}m} = 0 & |\underline{k}| > k_F \end{cases}$

SICCOME SONO UNA BASE $\Rightarrow$ SVILUPPO OGNI OP

$$\psi_m(\underline{x}) = \sum_{\underline{k}} Q_{\underline{k}m} \langle \underline{x}|\underline{k}\rangle = \sum_{k \leq k_F} Q_{\underline{k}m} \langle \underline{x}|\underline{k}\rangle + \sum_{k > k_F} Q_{\underline{k}m} \langle \underline{x}|\underline{k}\rangle = \psi_m^{(+)}(\underline{x}) + \psi_m^{(-)}(\underline{x})$$

CON $\langle F | \varphi_m^{(+)} = 0$ $\varphi_m^{(-)} | F\rangle = 0$

PRESO A CON $a/a^+ \Rightarrow A = \underset{\substack{\langle GS| \downarrow \\ 0}}{A^{(+)}} + \underset{\substack{\downarrow |GS\rangle \\ 0}}{A^{(-)}} \longrightarrow [A_i^{(+)}, A_j^{(+)}]_{\mp} = 0 = [A_i^{(-)}, A_j^{(-)}]$ $[A_\ell^{(-)}, A_k^{(+)}] = c\mathbb{1} \neq 0$
C-NUMERO

ES [BCS]

$$|BCS\rangle = \prod_k \left(u_k + v_k\, a^+_{k\uparrow} a^+_{-k\downarrow}\right)|0\rangle$$

$\longrightarrow$ MINIMIZ HAM $\Rightarrow$ STATO BCS $\min\limits_{v_k u_k} \langle BCS|H|BCS\rangle$ TC $|u_k|^2 + |v_k|^2 = 1$

CASO PARTIC $u_k = 0 \quad v_k = 1$ SE $k \leq k_F$
$\Rightarrow |BCS\rangle = |F\rangle$
$u_k = 1 \quad v_k = 0$ SE $k > k_F$

$\exists$ $a^+_{k\downarrow} a_{k\uparrow}$ TC ANNICHILANO BCS $\rightarrow \alpha^\pm$ TRASF BOGOLIUBOV

DEF [PRODOTTO NORMALM ORDINATO]
$A^{(+)}$ A SX DI $A^{(-)}$

DEF [OP DI ORDINAMENTO NORMALE]

$$N[A_1^{(\pm)} \_\_ A_n^{(\pm)}] = (\pm)^{\sigma} A_{i_1}^{+} \_\_\_ A_{i_N}^{(-)}$$ NORMALM ORD

→ NON UNIVOCO

$$N[1^+, 2^+, 3^-, 4^+] = \begin{cases} \pm 1^+ 2^+ 4^+ 3^- \\ \pm 4^+ 1^+ 2^+ 3^- \\ \_\_\_\_ \end{cases}$$ → CAMBIA SOLO SGN PERMUTAZ

$$A_1^- A_2^+ = [A_1^-, A_2^+]_{\mp} \pm A_2^+ A_1^-$$

→ TEOR WICK = MASSIMO ROBACCIA IN SOMMA NORM ORD

$$A_1 A_2 = (A_1^+ + A_2^-)(A_2^+ + A_2^-) = A_1^+ A_2^+ + A_1^+ A_2^- + A_1^- A_2^+ + A_1^- A_2^- = N[A_1, A_2] + [A_1^-, A_2^+] \Rightarrow \text{C-NUM}$$

DEF [CONTRAZ] $A_1 A_2 = N[A_1 A_2] + \overbrace{A_1 A_2}$ $\longrightarrow \overbrace{A_1 A_2} = [A_1^-, A_2^+]$

$$\to \langle GS | A_1 A_2 | GS \rangle = 0 + \overbrace{A_1 A_2}$$
$\hookrightarrow \langle N \rangle = 0$

⇒ VAL MEDIO SU STATO A MIN EN ⟶ SU $A_1 \_ A_n$ ≡ TEOR WICK

PROP [ANALOGA T-PROD] $N[A_1 \_ A_N] = (\pm)^{\sigma} N[A_{\sigma_1} \_ A_{\sigma_N}]$ → DA DIM

DEF [CONTRAZ CON N CAGACAZZI] $\overbrace{A (A_1 \_ A_n) A'} \equiv (A_1 \_ A_n) \overbrace{A A'} (\pm)^n$

TEOR DI WICK HA DIM × STEPS

① $A_0^- A_1^+ \dots A_n^+ = [A_0^-, A_1^+]_\pm A_2^+ \dots A_n^+ + A_1^+ A_0^- A_2^+ \dots A_n^+$

$$= \overbrace{A_0 A_1} A_2^+ \dots A_n^+ \pm A_1^+ \underbrace{[A_0^- A_2^+]_\pm}_{\overbrace{A_0 A_2}} A_3^+ \dots A_n^+ + A_1^+ A_2^+ A_0^- A_3^+ \dots A_n^+$$

$$= \overbrace{A_0 A_1} A_2^+ \dots A_n^+ + \overbrace{A_0 A_1^+ A_2} \dots A_n^+ + \dots + \overbrace{A_0 A_1^+ \dots A_{n-1}^+ A_n} + \underbrace{(\pm)^n A_1^+ \dots A_n^+ A_0^-}$$

② DIM X INDUZ SU SLIDES POCO DIABOLICA $N[A_0^- A_1^+ \dots A_n^+]$

$$A_0^- N[A_1 \dots A_n] = N[A_0^- A_1 \dots A_n] + \sum_{k=1}^{n} N[\overbrace{A_0 \dots A_n} \dots A_n]$$

③ BANALE

$$A_0 N[A_1 \dots A_n] = N[A_0 \dots A_n] + \sum_{k=1}^{n} N[\overbrace{A_0 \dots A_k} \dots A_n]$$

↪ PUNTO 2 QED

ITERANDO $\Rightarrow$ TEOR DI WICK

TEOR DI WICK [STATICO] (NO TEMPO)

$$A_1 \ldots A_n = N[A_1 \ldots A_n] + \sum N[A_1 \ldots A_n] + \ldots$$

contraz $\forall$ coppia

N_{n1}

N_{n0}

$$+ \sum N[A_1 \ldots A_n] + \ldots + \sum_{\text{contraz tot}} N[A_1 \ldots A_n]$$

N_{n2}

$\sum$, se n pari $\Rightarrow$ numero

$$\Rightarrow \langle GS | A_1 \ldots A_n | GS \rangle = \begin{cases} 0 & n \text{ disp} \\ \sum_{\text{contraz tot}} N[A_1 \ldots A_n] & n \text{ pari} \end{cases}$$

n disp $\Rightarrow$ A spaiato

Dim [x induzione]

$$A_1A_2 = N[A_1A_2] + \overline{A_1A_2} \to H_p \equiv A_1 \ldots A_n = \sum_{k=0}^{\lfloor n/2 \rfloor} N_{nk}$$

Somma norm. con k contraz. dentro $N[\ldots]$

→ Voglio dim che vale x $n+1$

$$A_0A_1 \ldots A_n = A_0(N_{n0} + N_{n1} + \ldots)$$

$$A_0 N[A_1 \ldots A_n] = N[A_0 \ldots A_n] + \sum_k N[\overline{A_0 \ldots A_k} \ldots A_n]$$

$$= N_{n+1\,0} + N[\overline{A_0 N_{n0}}]$$

$$= N_{n+1\,0} + N[\overline{A_0 N_{n0}}] + N[A_0 N_{n1}] + N[\overline{A_0 N_{n1}}] + \ldots$$

$$= N_{n+1\,0} + N_{n+1\,1} + \ldots$$

$$N[\overline{A_0 N_{nk}}] + N[A_0 N_{nk}] = N_{n+1\,k+1} \qquad \blacksquare$$

ESEMPIO

$$\langle F|\hat{n}_\uparrow(\underline{x})\,\hat{n}_\uparrow(\underline{x}')|F\rangle = \langle F|\psi^\dagger_\uparrow(\underline{x})\psi_\uparrow(\underline{x})\,\psi^\dagger_\uparrow(\underline{x}')\psi_\uparrow(\underline{x}')|F\rangle$$

$$\langle F|c^\dagger c^\dagger|F\rangle = 0 = \overline{c^\dagger c^\dagger} \leftarrow N[n\,n]$$

$$= \langle F|\psi^\dagger_\uparrow(\underline{x})\psi_\uparrow(\underline{x})|F\rangle\langle F|\psi^\dagger_\uparrow(\underline{x}')\psi_\uparrow(\underline{x}')|F\rangle$$

$$+\langle F|\psi^\dagger_\uparrow(\underline{x})\psi_\uparrow(\underline{x}')|F\rangle\langle F|\psi_\uparrow(\underline{x})\psi^\dagger_\uparrow(\underline{x}')|F\rangle$$

$$\Rightarrow \langle F|\hat{n}_\uparrow(\underline{x})\,\hat{n}_\uparrow(\underline{x}')|F\rangle - \langle F|\hat{n}_\uparrow(\underline{x})|F\rangle\langle F|\hat{n}_\uparrow(\underline{x}')|F\rangle$$

$$=$$

$$\langle F|\psi^\dagger_\uparrow(\underline{x})\psi_\uparrow(\underline{x}')|F\rangle\langle F|\psi_\uparrow(\underline{x})\psi^\dagger_\uparrow(\underline{x}')|F\rangle$$

$\rightarrow$ NON BANALE a $\underline{x}' \rightarrow \underline{x}$ $\Rightarrow$ ESCLUSIONE PAULI $\Rightarrow$?

$\longrightarrow$ DA PAULI

$$\lim_{x'\to x} \langle F | \hat{n}_\uparrow(\underline{x})\, \hat{n}_\uparrow(\underline{x}') | F\rangle$$

$$= \lim_{x'\to x} \left(\langle F | n(x) | F\rangle\langle F | n(x') | F\rangle + \langle F | \psi^\dagger(x)\psi(x') | F\rangle\langle F | \psi(x)\psi^\dagger(x') | F\rangle \right.$$

DA SVOLGERE CON INTEGRALI

x EVOLUZ TEMP NON PERT MUTANO CAMPI DI $\alpha^\pm$

$$\alpha_k^\pm(t) = f_k(t)\,\alpha_k^\pm \Rightarrow [\alpha_j^-(t), \alpha_k^+(t')] = \delta_{kj}\, g(t,t')\,\mathbb{1}$$

$$\Rightarrow T A_1(t_1)\dots A_n(t_n) = N[A_1\dots A_n] + \sum N[\widetilde{A_1\dots A_n}] + \sum N[\widetilde{\widetilde{A_1\dots A_n}}] + \dots$$

$$\widetilde{A_1 A_2} = \theta(t_1 - t_2)\, \overline{A_1 A_2} + \theta(t_2 - t_1)\, \overline{A_2 A_1}$$

$$\Rightarrow \langle GS | T A_1 \dots A_n | GS \rangle = \sum_{\text{contraz tot}} N[\widetilde{A_1 \dots A_n}]$$

SE n PARI, 0 SE DISP

TEOR DI WICK

T-ORDINATO

$$\Rightarrow G(1234) = G(13)G(24) \pm G(14)G(23) + G^{con}(1234)$$

DATO $H = H_0 + V$ TC VALE WICK TEOR $\Rightarrow$ POSSO CALCOLARMI G

$\rightarrow$ FORMULA [illegible] $G = \dfrac{\langle E_0 | T S\, \varphi_x \varphi^+_{x'} | E_0 \rangle}{\langle E_0 | S | E_0 \rangle}$ ✶

$\rightarrow$ SVILUPPO TEXP POI WICK

$\Rightarrow$ SCOMPAIONO I DIAG DI VUOTO

OSS $\langle GS | A_1 \ldots A_n | GS \rangle = 0$ se n disp $\forall |GS\rangle$

$= \sum$ contraz tot se n pari $\longrightarrow$ contraz = funz a 2 pti

$\Rightarrow$ funz a n particelle = $\prod$ funz a 2 pti $\longrightarrow$ caratterizzaz funz libero

Interaz a 2 corpi: $V = \frac{e^2}{2V} \sum_{k p q \neq 0} \sum_{m m'} a^+_{q m} a^+_{k m'} a_{k+q\, m'} a_{p-q\, m}$

$$\langle F|V|F\rangle = \langle F| a^+_{qm} a_{p-q\, m} |F\rangle \langle F| a^+_{km'} a_{k+q\, m'} |F\rangle - \langle F| a^+_{qm} a_{k+q\, m'} |F\rangle \langle F| a^+_{km'} a_{p-q\, m} |F\rangle$$

$\neq 0 \iff q = 0 \iff \neq 0$

2) se ho evoluz + bastardo $\longrightarrow$ evoluz temp non deve mutare il carattere di $\alpha^\pm$

$H_0 = \sum \hbar \omega_k c^+_k c_k \longrightarrow \begin{array}{l} c_k(t) = e^{-i\omega_k t} c_k \\ c^+_k(t) = e^{+i\omega_k t} c^+_k \end{array} \Rightarrow$ ancora ord normale

TEOR DI WICK T-ORDINATO

$|GS\rangle,\ \alpha_k^\pm,\ H \longrightarrow$ x TEOR WICK È NEC CHE EVOLUZ TEMP DI BASE $\alpha_k^\pm$ SIA TIPO

$$\alpha_k^\pm(t) = f_k(t)\,\alpha_k^\pm$$

$$\Rightarrow \alpha_k^-(t)|GS\rangle = 0\,, \quad \langle GS|\alpha_k^+(t) = 0 \quad \forall t$$

$$[\alpha_j^-(t), \alpha_k^-(t')]_\pm = [\alpha_j^+(t), \alpha_k^+(t')]_\pm = 0$$

$$[\alpha_j^-(t), \alpha_k^+(t')]_\pm = \delta_{kj}\, g(t,t')\, \mathbb{1}$$

CONTRAZ DIP DA T

$\Rightarrow$ STATEMENT TEOR DI WICK

$$T A_1(t_1) \ldots A_n(t_n) = \underline{N}[A_1(t_1) \ldots A_n(t_n)] + \sum N[A_1 \ldots A_n] + \sum N[A_1 \ldots A_n] + \ldots$$

STEP 1 $\quad T A_1(t_1) A_2(t_2) = \theta(t_1-t_2)\, A_1(t_1) A_2(t_2) \pm \theta(t_2-t_1)\, A_2(t_2) A_1(t_1)$

$$= \theta(t_1-t_2)\left(N[A_1(t_1)A_2(t_2)] + \overbracket{A_1(t_1)\, A_2}(t_2) \right) \pm$$

$$\pm\, \theta(t_2-t_1)\left(N[A_2(t_2)A_1(t_1)] + \overbracket{A_2(t_2)\, A_1}(t_1) \right)$$

$$= \left[\theta(t_1-t_2) + \theta(t_2-t_1)\right] N[A_1(t_1)A_2(t_2)] + \theta(t_1-t_2)\, \overbracket{A_1 A_2}$$

$$+\, \theta(t_2-t_1)\, \overbracket{A_2 A_1}$$

$$= N[A_1(t_1)A_2(t_2)] + \widetilde{A_1(t_1)}\, A_2(t_2)$$ $\Rightarrow$ COMB CNOM = CNOM

$$\Rightarrow \langle GS|\, T A_1(t_1) A_2(t_2)\, |GS\rangle = \widetilde{A_1(t_1)}\, A_2(t_2)$$

$\longrightarrow$ OBS PROP VALIDA SOLO x T ORD

SCAMBIANDO $\quad \widetilde{A_1(t_1)} A_2(t_2) = \pm\, \widetilde{A_2(t_2)} A_1(t_1)$

$$\Rightarrow \langle GS | T \psi_{m_1}(1)\, \psi^+_{m_2}(2) | GS \rangle = i\, G^0_{m_1 m_2}(12)$$

$$\overset{m}{\psi\psi} = 0 \quad \text{su } |GS\rangle \text{ autost. } \hat{N}$$

$$\neq 0 \quad \text{su delocalizzato su tutto sp di Fock}$$

$$= \langle GS | T \psi_{m_1} \psi^+_{m_2} | GS \rangle = i\, F(12)$$ correlazioni anomale

Step 2

$\times$ Hp induz su N $\quad T A_1(t_1) \ldots A_N(t_N) = \sum_{k=0}^{[\frac{N}{2}]} N_{Nk} \Rightarrow N_{Nk} = \sum N[\overline{\quad\ldots\quad}]$ k contraz tot

$\times$ N+1 op $\quad T A_0(t_0) A_1(t_1) \ldots A_N(t_N) = (\pm)^\ell A_\ell\, T A_0 \ldots \cancel{A_\ell} \ldots A_N$

ℓ tolto perché A_ℓ corrisp a t_{max}

$$= (\pm)^\ell A_\ell \Big(N[A_1 \ldots \cancel{A_\ell} \ldots A_N] + \sum_m N[\overline{A_0 \ldots} \cancel{A_\ell} \ldots A_N] + \ldots \Big)$$

ORA USO CONTRAZ INDIP DA t $\quad A_0 N[A_1 \dots A_n] = N[A_0 \dots A_n] + \sum_k N[\overbrace{A_0 \dots A_k} \dots]$

$$\Rightarrow (\pm)^\ell N[A_\ell A_0 \dots \cancel{A_\ell} \dots A_N] + (\pm)^\ell \sum_k N[\overbrace{A_\ell A_0 \dots A_k} \dots \cancel{A_\ell} \dots A_N]$$ INDIP DA t

$$+ (\pm)^\ell \sum_m N[A_\ell A_0 \overset{m}{\dots} \cancel{A_\ell} \dots A_N] + (\pm)^\ell \sum_{k \neq \ell}^{N} N[\overbrace{A_\ell A_0 \dots A_k} \dots \cancel{A_\ell} \dots A_N]$$

→ METTO A_ℓ A POSTO NEL I° TERM → $N[A_0 \dots A_N]$

II TERM: $\overbrace{A_\ell A_n} = \langle T A_\ell A_k \rangle$ MA A_ℓ HA t MAX $\Rightarrow T = 1$

→ SPOSTO $\Rightarrow \pm$ DAVANTI $\Rightarrow$ NO sgn $\Rightarrow N[A_0 \overset{m}{\dots} A_N]$ ALTRE TOND

INOLTRE $\overbrace{A(A_1 \dots A_n) A'} = \overset{m}{AA'}(A_1 \dots A_n)(\pm)^n$

$$\Rightarrow N[A_0 \dots A_n] + N[A_0 \overset{m}{\dots} A_n] + \dots$$

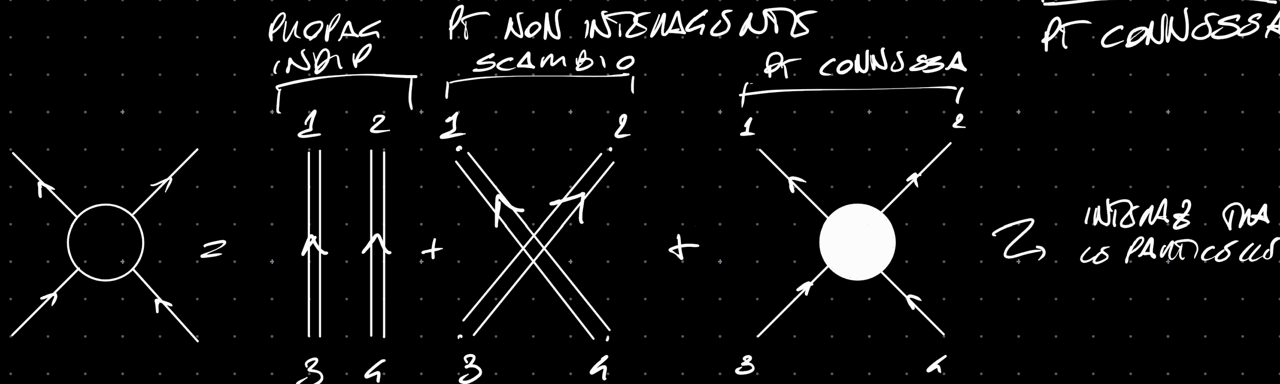
APPLICAZIONE

$$i^2 G^0(1234) = \langle GS | T\, \psi(1)\, \psi(2)\, \psi^\dagger(4)\, \psi^\dagger(3) | GS \rangle$$

$$= iG^0(13)\, iG^0(24) \pm iG^0(14)\, iG^0(13) = i^2 \ \mathrm{det}/\mathrm{perm} \begin{bmatrix} G^0(13) & G^0(14) \\ G^0(23) & G^0(24) \end{bmatrix}$$

IN TH A PARTIC NON INTERAGENTI SE POSSO SCRIVERE BENE $\hat{H}$ => FG CHE FATTORIZZANO

+ IN GEN $\quad G(1234) = \underbrace{G(13)\, G(24) \pm G(14)\, G(23)}_{\text{PT NON INTERAGENTE}} + \underbrace{G(1234)}_{\text{PT CONNESSA}}$



SE $H = H_0 + V$ TC VALE WICK TOTO $\Rightarrow$ CI CALCOLO IL PROPAG

$$G_{mm'}(x\,x') = \underset{\uparrow\ \text{GS } H}{\frac{1}{i}}\langle E|T\varphi_m(x)\varphi^\dagger_{m'}(x')|E\rangle \underset{\uparrow\ \text{FORMULA MADRE}}{=} \frac{1}{i}\,\frac{\langle E_0|TS\,\varphi_m(x)\varphi^\dagger_{m'}(x')|E_0\rangle}{\langle E_0|S|E_0\rangle}$$

$$S = T\exp\frac{1}{i\hbar}\int_{\pm\infty} dt_1\, V_{H_0}(t_1) \underset{\uparrow\ V_{\text{II QTZ}}}{=} T\exp\frac{1}{2i\hbar}\int_{\pm\infty} dt_1\int d_3\underline{x}_1\, d_3\underline{x}_1' \sum_{\substack{\mu\mu'\\ \nu\nu'}} v_{\mu\mu'\nu\nu'}(\underline{x}_1,\underline{x}_1')\,\varphi^\dagger_\mu(\underline{x}_1 t_1^{+++})$$

2 OP PARZIALM

$\neq$ MA EVOLUZ UGUALE XRÉ

STESSO t_1

$\rightarrow$ TORD $\Rightarrow t_1^{+++},\ t_1^{++},\ t_1^{+}$

$$\varphi^\dagger_\nu(\underline{x}_1', t_1^{++})\,\varphi_{\nu'}(\underline{x}_1' t_1^{+})\,\varphi_{\mu'}(\underline{x}_1 t_1)$$

TUTTI A STESSO TEMPO XRÉ INTERAZ STATICA $\Rightarrow$ ISTANTANEA

$$= T\exp\frac{1}{2\hbar}\int d_4x_1\, d_4x_1' \sum_{\substack{\mu\mu'\\ \nu\nu'}} \mho^0_{\mu\mu'\nu\nu'}(x_1\, x_1')\,\varphi^\dagger_\mu(x_1^{+++})\,\varphi^\dagger_\nu(x_1'^{++})\,\varphi_{\nu'}(x_1'^{+})\,\varphi_{\mu'}(x_1')$$

$$\mho^0_{\mu\mu'\nu\nu'} = v_{\mu\mu'\nu\nu'}(x\,x')\,\delta(t-t') \qquad \text{INTERAZ ISTANTANEA}$$

$$= T\exp \frac{1}{2i\hbar}\int d1\, d2 \sum_{\substack{\mu\mu' \\ \nu\nu'}} U_{\mu\mu'\nu\nu'}(12)\, \psi^{+}_{\mu}(1)\, \psi^{+}_{\nu}(2)\, \psi_{\nu'}(2)\, \psi_{\mu'}(1)$$

POSSO DENTRO

→ NUM A ORDINE 0 É $S = \mathbb{1} \Rightarrow G^{0}_{mm'}$

\+ CORREZ PERTURB S

→ NUM ORD 1

$$\frac{1}{2i\hbar}\int d1\, d2 \sum_{\text{SPIN}} U^{0}_{\mu\mu'\nu\nu'}(12) \langle E_0 | T \psi^{+}_{\mu}(1)\psi^{+}_{\nu}(2)\psi_{\nu'}(2)\psi_{\mu'}(1)$$

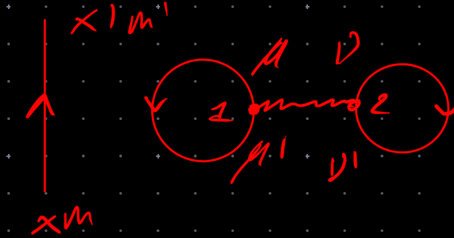
$$\psi_{m}(x)\, \psi^{+}_{m'}(x') | E_0 \rangle$$

→ TEOR WICK TORD

$\langle\ \ \rangle$ = CONTRAZ TOT XRŚ 6 OP

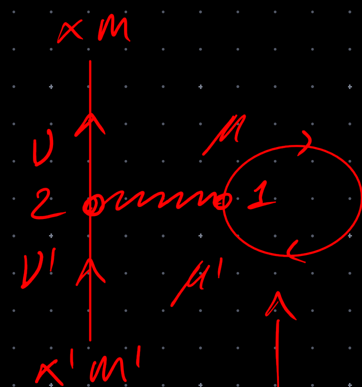
x = PTO ENTRATA $\quad x'$ = PTO USCITA

$$\varphi^\dagger_\mu(1)\,\varphi^\dagger_\nu(2)\,\varphi_{\nu'}(2)\,\varphi_{\mu'}(1)\,\varphi_m(x)\,\varphi^\dagger_{m'}(x')$$



$$\varphi^\dagger_\mu(1)\,\varphi^\dagger_\nu(2)\,\varphi_{\nu'}(2)\,\varphi_{\mu'}(1)\,\varphi_m(x)\,\varphi^\dagger_{m'}(x')$$

$$\varphi^\dagger_\mu(1)\,\varphi^\dagger_\nu(2)\,\varphi_{\nu'}(2)\,\varphi_{\mu'}(1)\,\varphi_m(x)\,\varphi^\dagger_{m'}(x')$$



SE DA $x' \longrightarrow 1 \longrightarrow x \Rightarrow$ STESSO DIAGRAMMA

DIFFERISCONO x SCAMBIO MA VALGONO

UGUALE XKÉ VAR MUTE

PARTIC PROPAGANDO INTERAGISCE CON CAMP. MEDIO

$\langle T \varphi \varphi^\dagger \rangle$ = DENSITÀ

→ POT HARTREE

→ FG VEDO SOLO SPIN ENTRANTE / USCENTE

$$\psi^+_\mu(1)\,\psi^+_\nu(2)\,\psi_{\nu'}(2)\,\psi_{\mu'}(1)\,\psi_m(x)\,\psi^+_{m'}(x')$$

IDEM SCAMBIO $1 \leftrightarrow 2$

CON $U^0_{\mu\mu'\nu\nu'}(12) = U^0_{\nu\nu'\mu\mu'}(21)$

IN TOT HO 6 DIAG $\longrightarrow$ $3! = \#$ CONTRAZ TOT

→ SVILUPPO A ORD N $\Longrightarrow$ 2N+1 CREATORI

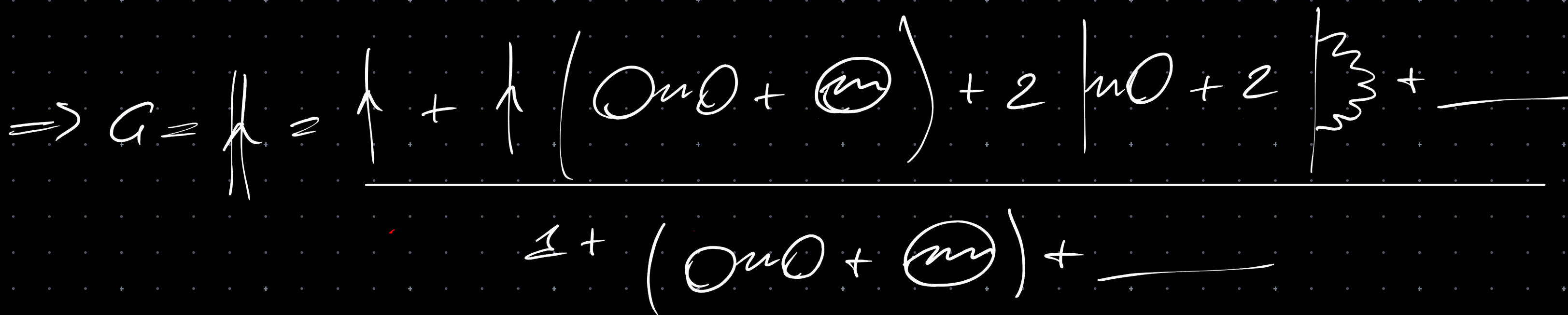
2N+1 DISTRUTTORI $\Longrightarrow$ $(2N+1)!$ CONTRAZ

$\Longrightarrow$ $(2N+1)!$ DIAG FEYNMAN

RISULTATO IMPORTANTE (RICAVATO IN MODO RIGOROSO)

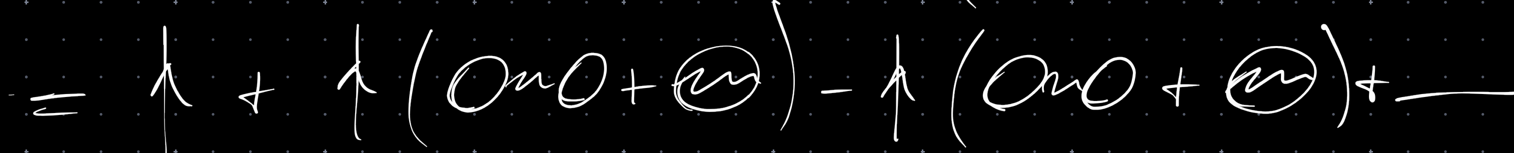
$$\langle E_0|S|E_0\rangle = 1 + \langle E_0|T\,\psi^+_\mu(1)\,\psi^+_\nu(2)\,\psi_{\nu'}(2)\,\psi_{\mu'}(1)\,|E_0\rangle$$

AL I° ORDINE



Th PERTURBATIVA → SVILUPPO DENOM COME SERIE GEOM

$\Rightarrow$ AL NUM 



⇒ INUTILE CALCOLARE FATTORI DI VUOTO

FATTORE 2 DEGLI ALTRI SI CANCELLA CON 1/2 AL NUM ⇒ FACCIO SOLO 2 DIAGR

DIAG VUOTO ⟷ CONTRAZ INTERNE = CONTRAZ TRA DI LORO

$$G_{mm'}(x x') = \frac{1}{i} \frac{\langle E_0 | T S \psi_m(x) \psi^\dagger_{m'}(x') | E_0 \rangle}{\langle E_0 | S | E_0 \rangle}$$

$$= \frac{1}{i \langle E_0 | S | E_0 \rangle} \sum_{k=0}^{\infty} \frac{1}{(i\hbar)^k} \frac{1}{k!} \int d1 \dots dk \, \langle E_0 | T V_{H_0}(t_1) \dots V_{H_0}(t_k) \psi_m(x) \psi^\dagger_{m'}(x') | E_0 \rangle$$

$$= \sum_{k=0}^{\infty} G^{(k)}_{mm'}(x x')$$ ⇒ VOGLIAMO CAPIRE COME CALCOLARE $G^{(k)}$ DI SOLITO PUNTUALE

→ SOMMA DI DIAG DI ORDINE K CON

- K LINEE in V°
- 2K+1 a G° CONTRAZ TOT DI 4K+2 OP
- 2K PIÙ SP-TEMPO

IL DENOM CANCELLA TUTTI I DIAG CON PATTERN DI VUOTO

IN GEN
$$\frac{\langle E_0 | T S O_1(t_1) \ldots O_N(t_N) | E_0 \rangle}{\langle E_0 | S | E_0 \rangle} =$$

$$= \langle E_0 | T S O_1(t_1) \ldots O_N(t_N) | E_0 \rangle^* \rightarrow \text{OMESSO DIAG DI VUOTO}$$

PATTERN DI VUOTO APPAIONO QUANDO LINKO LE V TRA LORO

NUMERATORE:

$$\sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\frac{1}{k!}\int dt_1\ldots dt_k\langle E_0|T\,V_1\ldots V_k\,O_1(t_1)\ldots O_N(t_N)|E_0\rangle$$

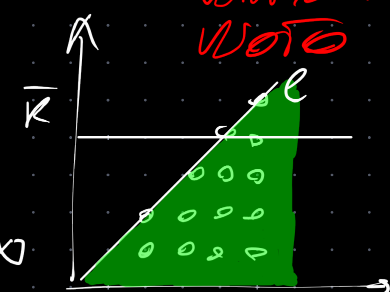
→ O mescolato in op creaz/distruz A

→ classifico contraz tot in * e non (1 o + V contratti tra loro connessi)

$$=\sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\frac{1}{k!}\int dt_1'\ldots dt_k'\sum_{l=0}^{k}\binom{k}{l}\langle E_0|T\,\underbrace{V_1\ldots V_l}_{\text{contratti tot}}\,\underbrace{\ldots V_k\ldots A_1\ldots A_N}_{\text{contratti tot senza vertici di vuoto}}|E_0\rangle$$

modi in cui l op si contraggono con se stessi

$$=\langle E_0|T\,V_1\ldots V_l|E_0\rangle\langle E_0|T\,V_{l+1}\ldots V_k\,A_1\ldots A_N|E_0\rangle$$



Ora scambio le sommatorie: se fix $l \Rightarrow k: 0 \to \infty$

$$= \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{l!\,(k-l)!} \frac{1}{(i\hbar)^{l+(k-l)}} \int dt_1 \dots dt_l \langle \Omega_0 | T V_1 \dots V_l | \Omega_0 \rangle \cdot$$

$$\cdot \int dt_{l+1} \dots dt_k \langle \Omega_0 | T V_{l+1} \dots V_k (A_1 \dots A_N) | \Omega_0 \rangle_*$$

$k' = k - l$

$$= \sum_{l=0}^{\infty} \frac{1}{l!} \frac{1}{(i\hbar)^l} \int dt_1 \dots dt_l \langle \Omega_0 | V_1 \dots V_l | \Omega_0 \rangle \sum_{k'=0}^{\infty} \frac{1}{k'!} \frac{1}{(i\hbar)^{k'}}$$

$$\int dt_1 \dots dt_{k'} \langle \Omega_0 | T V_1 \dots V_{k'} A \dots A | \Omega_0 \rangle_*$$

NON FATTORIZZATO $\longrightarrow$ I° FATTORE $= \langle \Omega_0 | S | \Omega_0 \rangle$

II° FATT. SVILUPPO SENZA DIAG. VUOTO

REGOLE DI FEYNMAN

- ∀ DIAG $G^{(k)}$ PREFATTORE $\frac{1}{(i\hbar)^k}\,\frac{1}{i}\, i^{2k+1} = \left(\frac{i}{\hbar}\right)^k$

- PREFATTORE $(\pm 1)^{\# LOOP}$ $\Rightarrow$ TIENE CONTO DEL FATTO CHE CONTRAIAMO ANCHE A DIST.

$$V\,\overbrace{V_1 \_\_ V_N}\,V'$$

- ∀ VERTICE  HO INTEG $d\underline{x}_1\, dt_1$ E $\sum_{\sigma\sigma'}$

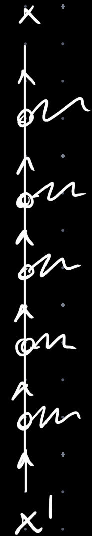
- $G^0(\underline{x}t\,\underline{x}'t) \equiv G^0(\underline{x}t, \underline{x}'t^+)$

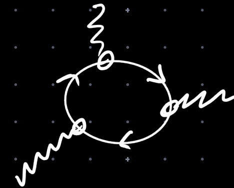
LINEA CONTINUA
$x \to x'$

IN MEZZO INDICI
→ CATENA DI CONTRAZ

+ ALTRE CONTRAZ COLLEGATE A INDICI

→ LOOP





SCRIVO V EXPLICIT

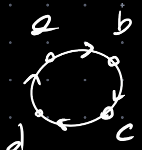
$$T\,\underbrace{\varphi^+\varphi^+\varphi\varphi}_{V_1} \text{---} \underbrace{\varphi^+\varphi^+\varphi\varphi}_{V_k}\,\varphi_x\varphi^+_{x'}$$

LA CATENA È FATTA DA

$$\varphi_x(\varphi^+_k\varphi_k) \text{---} (\varphi^+_1\varphi_1)\varphi^+_{x'}$$ AL BORDO

No cambio sgn xké scavalco coppie

$\varphi^+\varphi \to$ BOSONE È BOSE



$$\varphi^+_a\varphi_a\varphi^+_b\varphi_b\varphi^+_c\varphi_c\varphi^+_d\varphi_d$$

G^0_{ab} G^0_{bc} G^0_{cd}

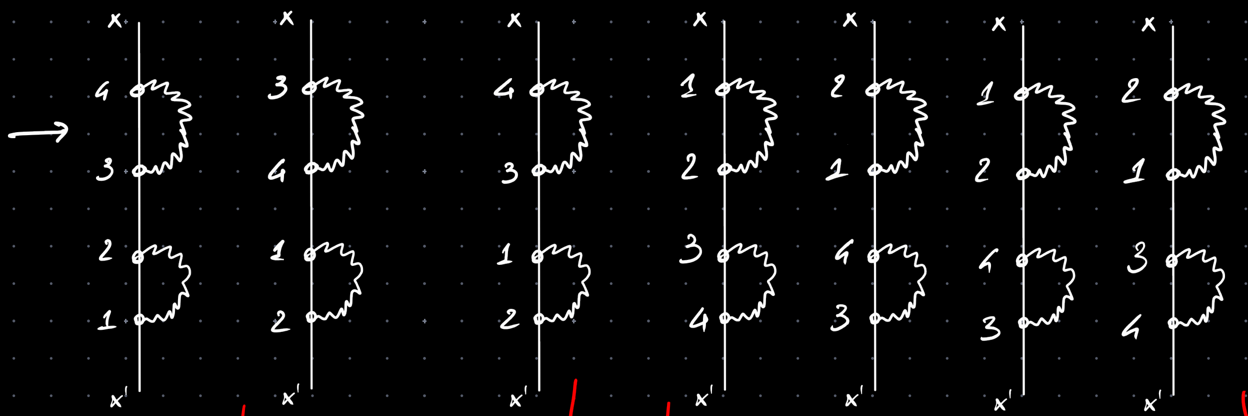
ULTIMA $\langle T\varphi^+_a\varphi_d\rangle = \pm\langle T\varphi_d\varphi^+_a\rangle = \pm G^0_{da}$

1 ∀ loop

$G^0(qtq't) = G^0(qtq't^+)$ → × TEMPI UGUALI IL II È SEMPRE t' XKÉ SI RIFÀ AL CREATORE

ESEMPIO

$$T\Big[\underbrace{\varphi^\dagger_3\varphi^\dagger_4\varphi_4\varphi_3}_{V_{34}}\,\underbrace{\varphi^\dagger_1\varphi^\dagger_2\varphi_2\varphi_1}_{V_{12}}\,\varphi_x\varphi^\dagger_{x'}\Big]$$

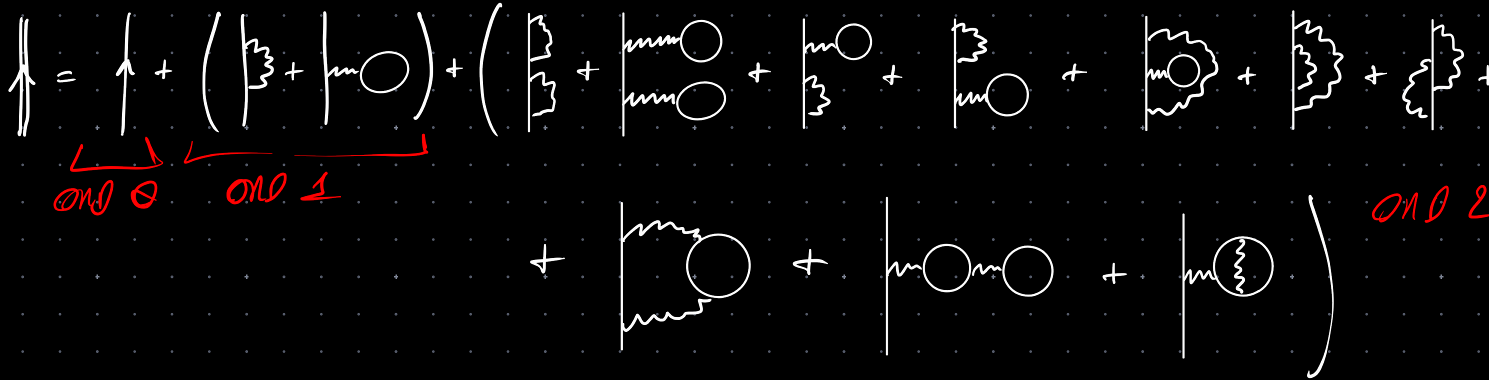


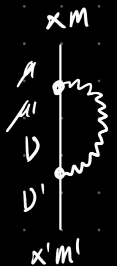
SCAMBIO INDICI NELLA COPPIA

$$\rightarrow \frac{1}{2^K}$$

SCAMBIO COPPIE → INVERTO I V

$$\rightarrow K!$$



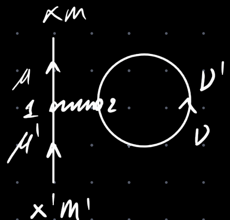


$$= \frac{i}{\hbar} \sum_{\substack{\mu\mu' \\ \nu\nu'}} \int d1\, d2\; G^0_{m\mu}(x1)\, G^0_{\mu'\nu}(12^+)\, G^0_{\nu' m'}(2x')\, U^0_{\mu\mu'\nu\nu'}(12)$$

X GAS ELETTRONICO CON INTERAZ COULOMB

$$G^0_{\mu\nu} = \delta_{\mu\nu}\, G^0 \qquad U_{\mu\mu'\nu\nu'}(12)$$

$$= \frac{4\pi e^2}{|x_1 - x_2|}\, \delta(t_1 - t_2)\, \delta_{\mu\mu'}\, \delta_{\nu\nu'}$$



$$= \frac{i}{\hbar}(-1)\sum_{SPIN}\int d1\,d2\,\delta_{m\mu}\,G^0(x1)\,\delta_{\mu'm'}\,G^0(1x')\,\delta_{\mu\mu'}\,\delta_{\nu\nu'}\,\frac{4\pi e^2}{|x_1-x_2|}\,\delta(t_1-t_2)\,\delta_{\nu\nu'}\,G^0(22^+)$$

$$= -\frac{i}{\hbar}\sum_{\mu\nu}\int d1\,d2\,\delta_{m\mu}\,\delta_{\mu m'}\,G^0(\underline{x}t,\underline{x}_1t_1)\,G^0(\underline{x}_1t_1,\underline{x}'t')\,\frac{4\pi e^2}{|x_1-x_2|}\,\frac{1}{i}\langle T\psi_\nu(\underline{x}_2t_1)\psi^\dagger_\nu(\underline{x}_2t_1^+)\rangle$$

$$= \frac{1}{\hbar}\,\delta_{mm'}\int d_3\underline{x}_1\,dt_1\,G^0(\underline{x}t,\underline{x}_1t_1)\,G^0(\underline{x}_1t_1,\underline{x}'t')\,U_H(\underline{x}_1)$$

$$U_H(\underline{x}) = \int d_3y\,\hat{n}(y)\,2v(\underline{x}\,y)$$

INTEG SPAZIALE INTERAZ COULOMB × $\hat{n}$ DENSITÀ SU $|E_0\rangle$ (IMPORTANTE

→ POT HARTREE

→ ESPR ANALITICA ARBITRAR INFAME → MODO SEMPLICE × INVAR TRASLAZ

SU $\underline{R}$ ⇒ DIMOSTRO GLI INTEG

SELF ENERGIA

SEPARO G IN G^0 E Σ

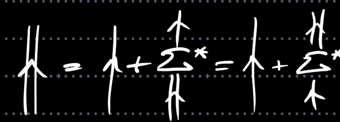
$\Sigma \to \Sigma_{LOC}$ 1 PTO SPTEMPO

$\searrow \Sigma_{BILOC}$ 2 PTI SPTEMPO

$\Sigma_{LOC} =$ 

DEF $\Sigma^* =$ INSIEME CHE NON POSSO SEPARARE ELIMINANDO G^0

$\equiv$ SELF EN PROPRIA

$\Rightarrow$ EQ DYSON 

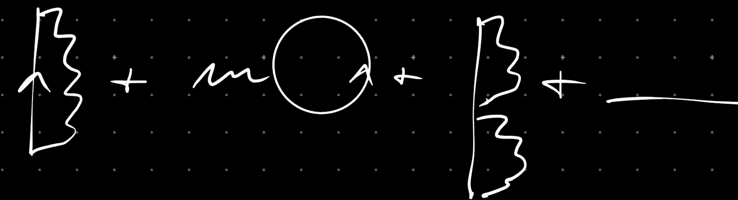
$\Sigma^* = \Sigma_{LOC} + \Sigma^*_{XC}$ $\rightarrow$ TERM SCAMBIO

$\longrightarrow$ SE HO DENSITÀ $\Rightarrow$ APPROX HF

$\Rightarrow$ SELF ENERGIA = SOMMA INSERZIONI PERTURBATIVE AL PROPAGATORE

$\longrightarrow$ FUNZ A 2 PTI

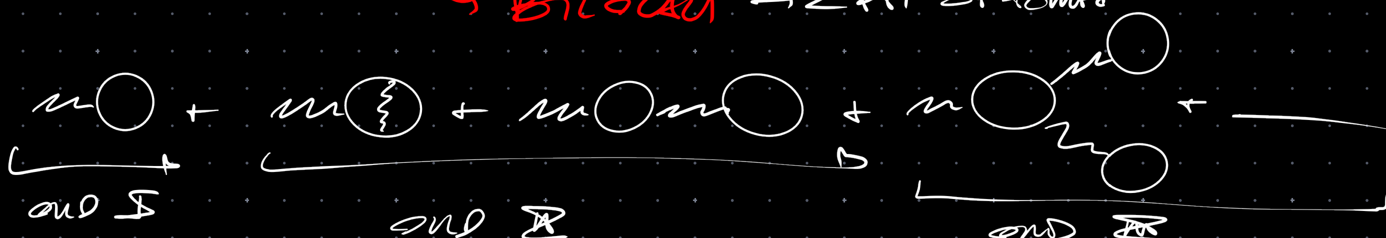
$\Sigma_{\mu\mu'}(x, y) =$



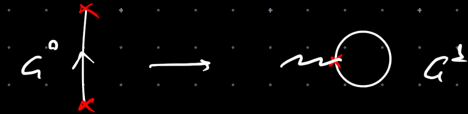
2 TIPI DI CONTRIB $\longrightarrow$ LOCALI $\rightarrow$ 1 SOLO PTO SP-TEMPO

$\searrow$ BILOCALI $\rightarrow$ 2 PTI SP-TEMPO

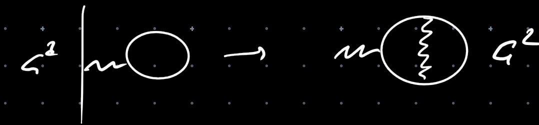
$\Sigma_{LOC} =$



× OTTENERE ORD I IN Σ_{loc} HO AMPUTATO G^0



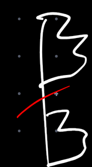
ORD II AMPUTO G^1



⇒ IN GEN Σ_{loc} =  G ESATTA CHIUSA SU SE STESSA

INVECE PRENDENDO Σ LOC

POSSO SEPARARLO IN 2 DIAG
2 SELF EN



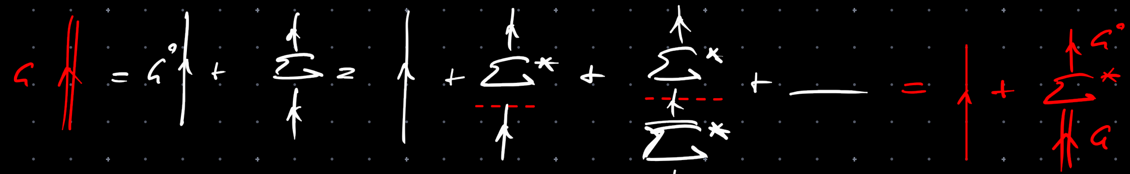
→ NON VALE X

⇒ SELF EN PROPRIA = SUBSET DIAG DI Σ NON SEPARABILI RIMUOVENDO G^0 → DA LORO RICOSTRUISCO TUTTI GLI ALTRI

$$\Sigma = \Sigma^* + \Sigma^* G^0 \Sigma^* + \Sigma^* \Sigma^* \Sigma^* + \ldots = \Sigma^* + \Sigma^* \Sigma$$

INDUTTIVA

SU G

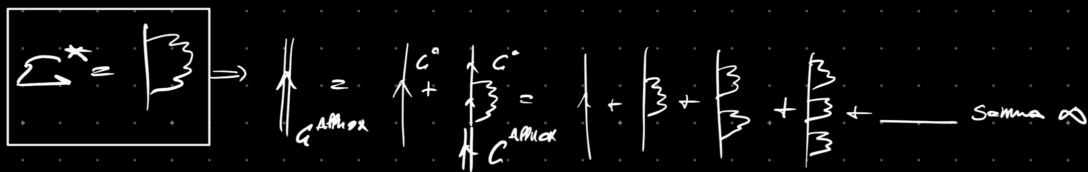


$$\Rightarrow G(12) = G^0(12) + \int d3\, d4\; G^0(13)\,\Sigma^*(34)\,G(42)$$

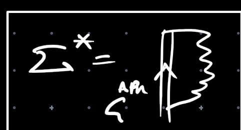
$\Rightarrow$ EQ DYSON

$\Rightarrow$ CALCOLANDO Σ^* TROVO G

APPROX

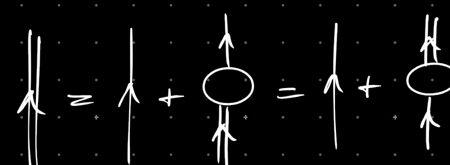


SE ORA CONSID



$\rightarrow$ OS TROISIO

IL RAGIONAM x EQ DYSON VALE ANCHE DAL BASSO

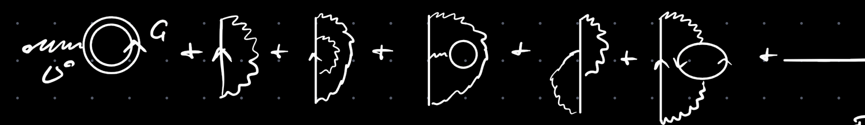


G = EIGEN MAIN PROPAGATORE , Σ^* = INSERZIONI CHE NON POSSO SCINDERE

EXPLICIT

$$G_{\mu\nu}(xx') = G^0_{\mu\nu}(xx') + \sum_{\rho\sigma}\int d_4x_1\, d_4x_2\, G^0_{\mu\rho}(xx_1)\,\Sigma^*_{\mu\rho}(x_1x_2)\,G^0_{\sigma\nu}(x_2x')$$

$$\Sigma^* = \Sigma_{loc} + \Sigma^*_{xc} =$$



+ ——— # diag di Σ^* cresce fattorialmente con l'ord

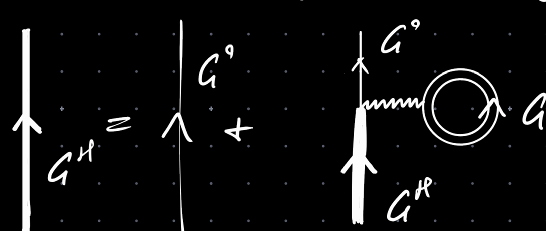
correlaz e scambio

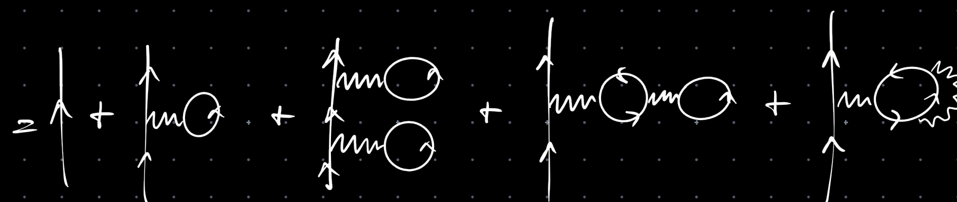
⇒ risoluz parziale x es sumiamo i contrib locali

→ def G^{HF} quando so la densità (x adesso via x es DFT)

⇒ ho Σ_{loc} che abbiamo visto essere il pot di Hartree

$$\int d_3y\; v(\underline{x}-y)\; n(y)$$





+ ———

→ ricaviamolo

$G \to G^{HF}$

$$\left(i\hbar\partial_t + h\right) G^0_{\mu\nu}(xx') = \hbar\,\delta_{\mu\nu}\,\delta(t-t')\,\delta_3(\underline{x}-\underline{x}')$$

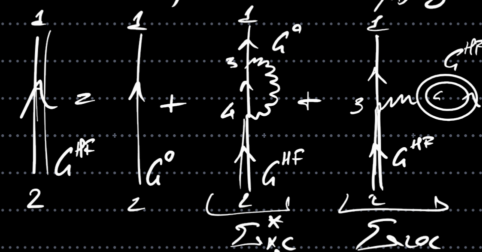
→ AGGIUNGO U_H PASSANDO A G^{HF}

$$\left(i\hbar\partial_t + h\right) G^{HF}_{\mu\nu} = \hbar\,\delta_{\mu\nu}\,\delta(t-t')\,\delta_3(\underline{x}-\underline{x}') + \hbar\,\frac{U_H(\underline{x})}{\hbar}\,G^{HF}(xx')$$

→ SPW $\left(i\hbar\omega + h(x) - U_H(x)\right) G^{HF}(\underline{x}\,\underline{x}'\omega) = \hbar\,\delta_{\mu\nu}\,\delta_3(\underline{x}-\underline{x}')$

SELF ENERGIA IN APPROX HF

$$\left(U^0_{\mu\nu} = \delta_{\mu\nu}\,U^0\right)$$



$$G^{HF}(12) = G^0(12) + \frac{i}{\hbar}\int d3\,d4\; G^0(13)\left(G^0(34^+)\,U^0(34)\right) G^{HF}(42) + (-1)\frac{i}{\hbar}\int d3\; G^0(13)\; G^{HF}(32)\int d4\; U^0(34)\,G^{HF}(44^+)$$

$\to \left[i\hbar\partial_t - h\right] \quad \Rightarrow \left[i\hbar\partial_t - h\right] G^{HF}(12) = \hbar\,\delta_{\mu\nu}\,\delta(12) +$

SE SO $G^H \Rightarrow$ DYSON $\Rightarrow$ EQ DIFF

G^0 RISOLVE $\left(i\hbar\partial_t - h(\underline{x})\right) G^0_{\mu\nu}(\underline{x}t\;\underline{x}'t') = \hbar\,\underbrace{\delta_{\mu\nu}\,\delta_3(\underline{x}-\underline{x}')}_{[\varphi_\mu(\underline{x}),\,\varphi^+_\nu(\underline{x}')]_\pm}\,\delta(t-t')$

G^0 FG IN SENSO STRETTO $\Rightarrow$ EQ DIFF $\Rightarrow$ PICCO IN UN INTEGRALE

$[i\hbar\partial_t - h(\underline{x})]\,G^H_{\mu\nu}(xx') = \hbar\,\delta_{\mu\nu}\,\delta_3(\underline{x}-\underline{x}')\,\delta(t-t') + \frac{\hbar}{\hbar}U_H(\underline{x})\,G^H_{\mu\nu}(xx')$ $\longrightarrow$ EQ DIFF NON OMOGENEA

METTENDOSI NEL CASO SEMPLICE


$= \frac{1}{\hbar}U_H(\underline{x})$

$\Rightarrow [i\hbar\partial_t - U_H(\underline{x})]\,G^H_{\mu\nu}(xx') = \hbar\,\delta_{\mu\nu}\,\delta(t-t')\,\delta_3(\underline{x}-\underline{x}')$ $\longrightarrow$ RISOLVIBILE

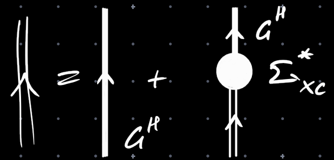
$\longrightarrow$ x RES SP Δ $\longrightarrow \delta(t-t')=1$ $\Rightarrow [\hbar\omega - h(\underline{x}) - U_H(\underline{x})]\,G(\underline{x}\,\underline{x}'\omega) = \hbar\,\delta_3(\underline{x}-\underline{x}')$

$\longrightarrow [h(\underline{x}) + U_H(\underline{x})]\,u_j(\underline{x}) = \varepsilon_j\,u_j(\underline{x})$ EQ AUTOVALORI $\Rightarrow$ HO EN COSTANTE x CAMPO MEDIO

MA EQ HARTREE NON TIENE CONTO DELLO SCAMBIO

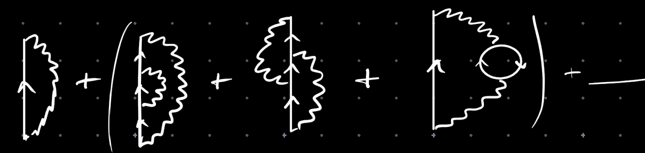
$$G^H(\underline{x}\,\underline{x}'\omega) = \sum_j \frac{u_j(\underline{x})\,\overline{u_j(\underline{x}')}}{\omega - \frac{\varepsilon_j}{\hbar} + i\eta\,\theta(\varepsilon_j - \varepsilon_F)}$$

$\Rightarrow$ POSSO RISOLVERE DYSON



NE HO ELIMINATO DEI TADPOLE
→ DIAG DOVE È SOSTITUITO
$G^0 \leftrightarrow G^H$

→ CONSID Σ^* SOLO NONLOC
⇒ ANCHE GLI IBRIDI CON TADPOLE
SCOMPAIONO

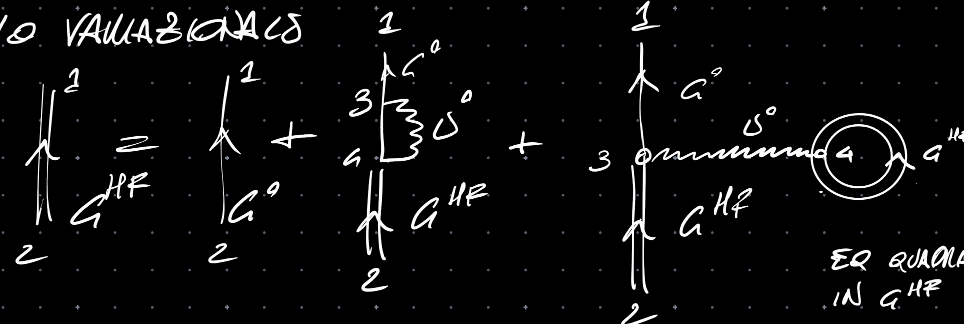
$\Sigma^*_{xc} =$ 

ORD	# DIAG
0	1
1	3
3	20
4	189
5	2232
6	31130

→ SBATTI ⇒ TA PERTURBATIVA

I° APPROCCIO × G DA HF = APPROCCIO VARIAZIONALE

EQ DYSON IN APPROX HF



EQ QUADRATICA IN G^{HF}

MISTO $U^0_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\,\delta_{\nu\nu'}\,U^0$

$$G^{HF}_{\mu\nu}(12) = G^{0}_{\mu\nu}(12) + \frac{i}{\hbar}\sum_{\rho\sigma}\int d3\, d4\, G^{0}_{\mu\rho}(13)\left[G^{HF}(34^+)\,U^0(34)\right] G^{HF}_{\sigma\nu}(42)$$

$$+\frac{i}{\hbar}(-1)\int d3\, G^{0}_{\mu\rho}(13)\, G^{HF}_{\rho\nu}(32)\int d4\, U^0(34)\, G^{HF}(44^+)$$

$\longrightarrow$ INTEG TOMIBILE $\Rightarrow$ VOGLIO EQ DIFF $\Rightarrow$ $(i\hbar\partial_t - h)$

$$\left[i\hbar\partial_t - h(\underline{x}_1)\right] G^{HF}_{\mu\nu}(12) = \hbar\,\delta(12)\,\delta_{\mu\nu}$$

$$\overbrace{\phantom{v(\underline{x}_1\underline{x}_4)\delta(t_1-t_4)}}^{U^0}$$

$$+\,i\int d4\, G^{HF}_{\mu\sigma}(\underline{x}_1 t_1, \underline{x}_4 t_4)\, v(\underline{x}_1 \underline{x}_4)\,\delta(t_1-t_4)\, G^{HF}(\underline{x}_4 t_4\, \underline{x}_2 t_2)$$

$$-\,i\int d4\, \underbrace{G^{HF}_{\sigma\sigma}(44^+)}_{\hat{n}^{HF}(\underline{x}_4)}\, v(\underline{x}\, \underline{x}_4)\,\delta(t_1-t_4)\, G^{HF}_{\mu\nu}(\underline{x}_1 t_1\, \underline{x}_2 t_2)$$

$\hat{n}^{HF}(\underline{x}_4) \longrightarrow$ FG A PT COINCIDENTI = DENSITÀ

$$\left[i\hbar\partial_t - h(\underline{x}_1) - U_H(\underline{x}_1)\right] G^{HF}_{\mu\nu}(\underline{x}_1 t_1\, \underline{x}_2 t_2) = \hbar\,\delta_{\mu\nu}\,\delta(\underline{x}_1-\underline{x}_2)\,\delta(t_1-t_2)$$

$$+\,i\int d_3\underline{x}_4\, G^{HF}_{\mu\sigma}(\underline{x}_1 t_1\, \underline{x}_4 t_1^+)\, G^{HF}_{\sigma\nu}(\underline{x}_4 t_1\, \underline{x}_2 t_2)\, v(\underline{x}_1-\underline{x}_2)$$

→ COME RISOLVO STA ABERRAZIONE? SP ω

$$\left[\hbar\omega - h(\underline{x}_1) - U_H(\underline{x}_1)\right] G^{HF}_{\mu\nu}(\underline{x}\,\underline{y}\,\omega) = \hbar\,\delta_{\mu\nu}\,\delta_3(\underline{x}-\underline{y})$$
$$+ i\int d_3x' \int \frac{d\omega'}{2\pi}\, G^{HF}_{\mu\sigma}(\underline{x}\,\underline{x}'\omega')\, e^{i\omega'\eta}\, G^{HF}_{\sigma\nu}(\underline{x}'\underline{y}\,\omega)\, v(\underline{x}-\underline{x}')$$

→ SOL × ES

$$G^{HF}_{\mu\nu}(\underline{x}\,\underline{y}\,\omega) = \sum_j \frac{\langle \underline{x}\mu | a_j \rangle\langle a_j | \underline{y}\nu \rangle}{\omega - \frac{\varepsilon_j}{\hbar} + i\eta\,\theta(\varepsilon_j - \varepsilon_F)}$$

SUPP $\exists$ SPETTRO IN COI ε_j, a_j SCONOSCIUTI MA IMPONGO ORTONORM $\langle a_j | a_k \rangle = \delta_{jk}$

⇒ PRENDO ANSATZ G^{HF} E TROVO COND SU a_j ⇒ $\{a_j\}$ SOL HF

→ N SOL DI HF × N PARTIC

EQ MOTO PROPAG

$$\left[i\hbar\partial_t - h(\underline{x})\right] G_{mm'}(\underline{x}t,\underline{x}'t') = \hbar\delta_{mm'}\,\delta_3(\underline{x}-\underline{x}')\,\delta(t-t') + i\sum_{m''}\int d_3y\, v(\underline{x}y)\, G_{mm''m''m'}(\underline{x}t, yt^+, yt^{++}, \underline{x}'t')$$

DA $\partial_t \xrightarrow{\theta} \delta$

$\hookrightarrow$ FUNZ A 4 PT (4)

$$i^2 G(1234) = \langle E_0|T\{\varphi(1)\varphi(2)\varphi^+(4)\varphi^+(3)\}|E_0\rangle_*$$

FORM MODE

* SOLO CONNESSO

$$= \sum_{K=0}^{\infty}\frac{1}{(i\hbar)^K}\frac{1}{K!}\int dt_1\ldots dt_K\,\langle E_0|T\,V(t_1)\ldots V(t_K)\,\varphi(1)\varphi(2)\varphi^+(4)\varphi^+(3)\,|E_0\rangle$$

ORA CONSID CONTRAZ SENZA PT A VUOTO $\rightarrow$ 3 TIPI

1. CONTRAZ TRA FATTORI E

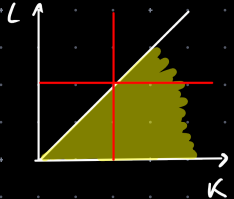
$\varphi_1\varphi_3^+$ CON L V $\qquad L = 0\ldots K$

$\varphi_2\varphi_4^+$ CON $(K-L)V$

\#SCELTE PARI

$$\rightarrow \sum_{K=0}^{\infty}\frac{1}{(i\hbar)^K}\frac{1}{K!}\sum_{L=0}^{K}\binom{K}{L}\int dt_1\ldots dt_L\,\langle E_0|V(t_1)\ldots V(t_L)\,\varphi_1\varphi_3^+|E_0\rangle\int ds_1\ldots ds_{K-L}\,\langle E_0|T\,V(s_1)\ldots V(s_{K-L})\,\varphi_2\varphi_4^+|E_0\rangle$$

$\hookrightarrow$ DOPO I PRIMI L HO COMB RIPETUTE



SCAMBIO SOMME E SEMPLIF. K!

$$\sum_{l=0}^{\infty}\frac{1}{(i\hbar)^l}\frac{1}{l!}\sum_{k=l}^{\infty}\frac{1}{(i\hbar)^{k-l}}\frac{1}{(k-l)!}\left[\;\_\_\;\right] \leftarrow k'=k-l$$

$$=\sum_{l=0}^{\infty}\frac{1}{(i\hbar)^l}\frac{1}{l!}\int dt_1\ldots dt_l\,\langle E_0|T\,V(t_1)\ldots V(t_l)\,\psi_1\psi_3^{\dagger}|E_0\rangle\sum_{k'=0}^{\infty}\frac{1}{(i\hbar)^{k'}}\frac{1}{k'!}\int ds_1\ldots ds_{k'}\,\langle E_0|T\,V(s_1)\ldots V(s_{k'})\,\psi_2\psi_4^{\dagger}|E_0\rangle$$

RIASSORBO T exp $=\langle E_0|T S\,\psi_1\psi_3^{\dagger}|E_0\rangle\langle E_0|T S\,\psi_2\psi_4^{\dagger}|E_0\rangle = i\,G(13)\,i\,G(24)$ → I TERMINE

(2) PRENDO 14 E 23 E FACCIO IL SALTO ⇒ $\pm i\,G(14)G(23)$

(3) CONTRAGGO TUTTO CON TUTTO ⇒ G CONNESSA → PARTIC ENTRANO ESCONO E INTERAGISCONO

$$\Rightarrow i^2 G(1234) = i^2 G(13)G(24) \pm i\,G(14)G(23) + i\,G^{con}(1234)$$

PRIMI 2 PROPAGATORI IN ASSENZA DI TUTTI GLI ALTRI SENZA CORRELAZ TRA LE PROPAGAZ
⇒ DIAG FEYNMAN ESATTI

HF → $G^{conn} = 0$ NEL METODO VARIAZ HO MIN SU STATO SLATER

$$\Rightarrow G_4 = G_2 G_2 \pm G_2 G_2$$

HO TRONCATO GERARCHIA ⇒ QUADRATICA SOLO IN G

METTO IN EQ MOTO USANDO G LIBERA

$$\left[i\hbar\partial_t - h(\underline{x})\right] G_{mm'}(\underline{x}t,\underline{x}'t') = \hbar\,\delta_{mm'}\,\delta_3(\underline{x}-\underline{x}')\,\delta(t-t') + i\sum_{m''}\int dy\, v(\underline{x}y)\, G_{mm''m''m'}(\underline{x}t\,yt^{+}\,yt^{++}\,\underline{x}'t')$$

INTEGRO $\Rightarrow$ $$G_{mm'}(\underline{x}t\,\underline{x}'t') = G^0_{mm'}(\underline{x}t\,\underline{x}'t') + \frac{1}{\hbar}\sum_{m''}\int d_3x''\,dt''\,G^0_{mm''}(\underline{x}t\,\underline{x}''t'')\; i\sum_{m''}\int d_3y\, v(\underline{x}''y)\left[GG-GG\right]$$

SE APPLICO L'HF MI APPAIONO δ E TORNO ALL'EQ DI PRIMA $\longrightarrow$ L'ULTIMA È DYSON

IL DISCORSO $G_4 = G_2G_2 \pm G_2G_2 + G_4^{conn}$ È DETTO

CLUSTER EXPANSION, SI PUÒ FA' $\forall\, G_n$

$\longrightarrow$ IDEM × ORDINI SUP

SE HO INVAR × TRASLAZ $[H,\vec{p}]=0 \Rightarrow G_{AD}(xx') = G_{AD}(x-x') \Rightarrow$ POSSO DIM

CHE Σ DIP

DA $x-x'$

× FACILE USO $G = G^0 + G\Sigma G^0 \Rightarrow$ ANCHE Σ^* INVAR × TRASLAZ

$\Rightarrow$ FACCIO FOURIER DI TUTTO

$$G_{\mu\nu}(\underline{x}t\,\underline{x}'t') = \int \frac{d_3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, e^{-i\omega(t-t')+i\underline{k}(\underline{x}-\underline{x}')}\, G_{\mu\nu}(\underline{k}\omega)$$

SE PIAZZO SVILUPPO DI DYSON $\Sigma^*_{\rho\sigma}$ E $G^0_{\mu\nu}$ $\Rightarrow$ TEOR CONVOLUZ $\rightarrow$ DERIVATA DI EXP $\Rightarrow$ CONSERV DI δ

$$G_{\mu\nu}(\underline{k}\omega) = G^0_{\mu\nu}(\underline{k}\omega) + \sum_{\rho\sigma} G^0_{\mu\rho}(\underline{k}\omega)\,\Sigma^*_{\rho\sigma}(\underline{k}\omega)\,G_{\sigma\nu}(\underline{k}\omega)$$ $\rightarrow$ EQ ALGEBRICA MATRICIALE

SE SUPP MATRICI DIAGONALI $G_{\mu\nu} = \delta_{\mu\nu} G$

$\Rightarrow G(\underline{k}) = G^0(\underline{k}) + G^0(\underline{k})\,\Sigma^*(\underline{k})\,G(\underline{k})$ EQ I gr IN QUADRIMOMENTO

$$\Rightarrow G(k) = \frac{G^0(k)}{1 - G^0(k)\,\Sigma^*(k)} = \frac{1}{G^0(k)^{-1} - \Sigma^*(k)}$$

SE G^0 È GAS LIBERO

$$G(\underline{k}\omega) = \frac{1}{\omega - \underbrace{\frac{\varepsilon^0_k}{\hbar}}_{\text{AUTOVAL PARTIC SINGOLA}} - \Sigma^*(\underline{k}\omega)}$$

CON INVAR TRASLAZ NO INDIC SBATTI

$G^0 \to$ POLO SEMPLICE IN $\hbar\omega = \varepsilon_k^0 \pm$ PT ASSOLUTA $\to$ LEGGE DISPERSIONE (LEGAME FASE-MOMENTO)

COSA FA LA G? DI SOLITO SI TROVA POLO DI I ORDINE $\Rightarrow$ SVILUPPO DI LAURENT

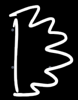
$$G = \frac{Z_k}{\omega - \omega_{polo}(k)} + \text{PT ANALITICA}$$

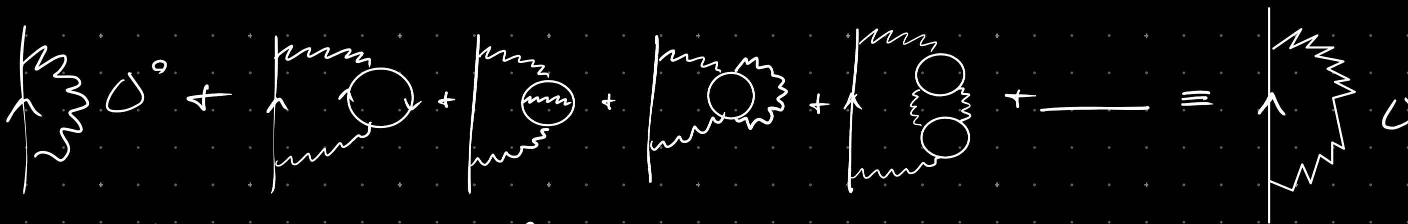
↳ CONTRIB DI QUASI PARTIC

SE POLO CON RESIDUO $\simeq 1 \Rightarrow$ PT $\underline{\mathrm{Im}} \simeq 0 \Rightarrow$ DESCRIVE QUALCOSA DI MOLTO SIMILE A UNA PARTICELLA MA NON LO È (IN PT LO È + PASTICCI VARI)

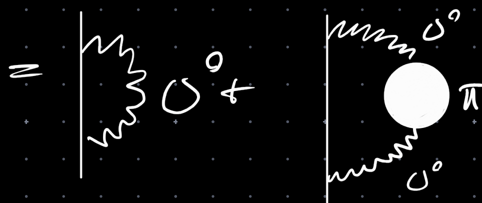
INDUZIONE ELETTROSTATICA

E POLARIZZAZIONE

CONSID  DEL I° ORD + SUCCESSIVI



INTERAZIONE EFFICACE



Π = POLARIZZAZIONE

$$U_{\mu\mu'\nu\nu'}(12) = U^0_{\mu\mu'\nu\nu'}(12) + \sum_{\substack{\sigma\sigma' \\ \rho\rho'}} \int d3\,d4\, U^0_{\mu\mu'\rho\rho'}(13)\,\Pi_{\rho\rho'\sigma\sigma'}(34)\,U^0_{\sigma\sigma'\nu\nu'}(42)$$

Π È MOLTO COMPLICATO
MA MOLTE INFO GIÀ DA I° ORDINE

SE $U^0_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\delta_{\nu\nu'}U^0 \Rightarrow U_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\delta_{\nu\nu'}U$ ANCHE SE Π NE DIPENDE

$$O(12) = O^0(12) + \sum_{\rho\sigma} \int d3\, d4\, O^0(13)\, \Pi_{\rho\rho\sigma\sigma}(34)\, O^0(42) = O^0(12) + \int d3\, d4\, O^0(13) \underbrace{\sum_{\rho\sigma} \Pi_{\rho\rho\sigma\sigma}(34)}_{\Pi(34)} O^0(42)$$

SAPPIAMO CHE O^0 INVAR x SCAMBIO $\rightarrow$ USIAMO $\Pi_{\rho\rho'\sigma\sigma'}(12) = \Pi_{\sigma\sigma'\rho\rho'}(21)$

$\Rightarrow O$ INVAR x SCAMBIO

USIAMO Π CONTO LE CONTRAZ DA 3 G^0

$$\frac{1}{i} \sum_{k\geq 2}^{\infty} \frac{1}{(i\hbar)^k} \int_{\pm\infty} \langle E_0 | T V_1 \dots V_k \psi_x \psi_y^+ | E_0 \rangle_* \, dt_1 \dots dt_k$$

$2 O^0$

NO $k!$ XKÉ STO SCEGLIENDO V PARTIC E NON CONTA $\forall k$! VIENE DALLO SCAMBIO

$$= \int_{V_1} dx_1 dy_1\, O^0(x_1 y_1) \langle E_0 | T \psi_{x_1}^+ \psi_{y_1}^+ \psi_{y_1} \psi_{x_1} V_2 \dots V_k \psi_x \psi_y^+ | E_0 \rangle_* = \int dx_1 dy_1\, O^0(x_1 y_1)\, iG(x x_1) \langle E_0 | T \psi_{y_1}^+ \psi_{y_1} \psi_x V_2 \dots V_k \psi_y^+ | E_0 \rangle_*$$

NO SGN XKÉ SALTO QUADRATO

$$= \int_{V_2} dx_1 dy_1 dx_2 dy_2\, O^0(x_1 y_1) O^0(x_2 y_2)\, iG^0(x x_1) \langle E_0 | T \psi_{y_1}^+ \psi_{y_1} \psi_{x_1} \psi_{x_2}^+ \psi_{y_2}^+ \psi_{y_2} \psi_{x_2} \underbrace{V_3 \dots V_k}_{k-2} \psi_y^+ | E_0 \rangle_*$$

$$= \int dx_1 dy_1 dx_2 dy_2\, O^0(x_1 y_1) O^0(x_2 y_2)\, iG^0(x x_1)\, iG^0(x_2 y)\, iG^0(x_1 x_2) \langle E_0 | T V \dots V \psi_{y_1}^+ \psi_{y_1} \psi_{y_2}^+ \psi_{y_2} | E_0 \rangle_*$$

$$= \frac{1}{i} \frac{i^3}{(i\hbar)^2} \int dx_1 dy_1 dx_2 dy_2\, G^0(x x_1) G^0(x_1 x_2) G^0(x_2 y)\, O^0(x_1 y_1)\, O^0(x_2 y_2) \sum_{k=0}^{\infty} \frac{1}{(i\hbar)^k} \int dt_1 \dots dt_k \langle E_0 | T V(t_1) \dots V(t_k)\, \psi_{y_1}^+ \psi_{y_1} \psi_{y_2}^+ \psi_{y_2} | E_0 \rangle_*$$

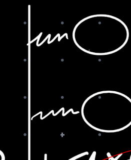
TRASLATO

RICORDANDO $\Sigma = \frac{i}{\hbar} \int dx_1 dx_2\, G^0(x x_1) G^0(x_1 x_2) G^0(x_2 y)\, O^0(x_1 x_2)$

$$\Pi_{\rho\rho'\sigma\sigma'}(12) = \frac{1}{i\hbar}\langle E|T\delta\left(\psi^+_\rho(y_1)\psi_{\rho'}(y_1)\right)\delta\left(\psi^+_\sigma(y_2)\psi_{\sigma'}(y_2)\right)|E\rangle$$

$$\delta\hat O = \hat O - \langle E|\hat O|E\rangle \mathbb{1}$$

→ ESCLUDE CONTRAZ DEL TIPO



$$\Rightarrow \langle E|T\delta A\,\delta B|E\rangle = \langle E|T(A-\langle A\rangle)(B-\langle B\rangle)|E\rangle = \langle E|TAB|E\rangle - \langle A\rangle\cancel{\langle E|TB|E\rangle} - \langle B\rangle\langle E|TA|E\rangle \cancel{+\langle A\rangle\langle B\rangle}$$

$$= \langle E|TAB|E\rangle - \langle A\rangle\langle B\rangle \rightarrow \text{CORRELAZIONE}$$

QUESTA SCRITTURA EVIDENZIA INVAR x SCAMBIO Π GRAZIE A $T\psi^+\psi\psi^+\psi$ CHE NON CAMBIA sgn x SCAMBIO COPPIA

$$\Pi(y_1y_2) = \sum_{\rho\sigma}\Pi_{\rho\rho\sigma\sigma}(y_1y_2) = \frac{1}{i\hbar}\langle E|T\delta(\hat n(y_1))\,\delta(\hat n(y_2))|E\rangle = \frac{1}{i\hbar}\left[\langle E|T\hat n(y_1)\hat n(y_2)|E\rangle - \langle\hat n(y_1)\rangle\langle\hat n(y_2)\rangle\right]$$

x GAS FERMI $|E\rangle = |F\rangle$

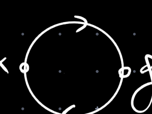
$$\Pi(xy) = \frac{1}{i\hbar}\left[\langle F|T\hat n(\underline{x})\hat n(y)|F\rangle - \langle F|\hat n(\underline{x})|F\rangle\langle F|\hat n(y)|F\rangle\right]$$

WICK $\sum_{\mu\nu}\psi^+_\mu(x)\psi_\mu(x^+)\psi^+_\nu(y^+)\psi_\nu(y)$ → SI CANCELLA IL 2° TERM

EVOLUZ H_0

$$\psi^+_\mu\psi_\mu = \langle F|T\psi^+_\mu(x)\psi_\mu(x)|F\rangle = \langle F|T\psi^+_\mu(\underline{x}t)\psi_\mu(\underline{x}t)|F\rangle = \langle F|\psi^+_\mu(x)\psi_\mu(\underline{x})|F\rangle \rightarrow \sum_\mu = \hat n(x)$$

$$= \frac{1}{i\hbar}\sum_{\mu\nu}\langle F|T\psi^+_\mu(x)\psi_\nu(y)|F\rangle\langle F|T\psi_\mu(x)\psi^+_\nu(y)|F\rangle = -\frac{1}{i\hbar}i^2\sum_{\mu\nu}G^0_{\nu\mu}(yx)\,G^0_{\mu\nu}(xy)$$ ×



$$= -\frac{i}{\hbar}\sum_{\mu\nu}G^0_{\nu\mu}(yx)\,G^0_{\mu\nu}(xy) = -\frac{i}{\hbar}\,2\,G^0(yx)\,G^0(xy)$$

$G_{\mu\nu}$ DIAG SPIN $\sum_{\mu\nu}\delta_{\mu\nu}\delta_{\nu\mu}$

Π DETTA FUNZ DI LINDHARD

$\Pi(11) =$  $\notin \Pi^*$

DEFINISCO Π PEZZO DEF POLARIZ PROPRIA Π^* $\longrightarrow$ NON RIDUCIBILI CUTANDO U^0

$$\Rightarrow \Pi = \Pi^* + \Pi^* U^0 \left(\Pi^* + \Pi^* U^0 \left(\Pi^* + \_\_\_\right)\right) = \Pi^* + \Pi^* U^0 \Pi = \boxed{\Pi^* + \Pi^* U^0 \Pi}$$

EQ DYSON

<u>EXPLICIT</u> $\Pi_{\rho\rho'\sigma\sigma'}(12) = \Pi^*_{\rho\rho'\sigma\sigma'}(12) + \int d3d4 \sum_{\mu\mu'\nu\nu'} \Pi^*_{\rho\rho'\mu\mu'}(13)\, U^0_{\mu\mu'\nu\nu'}(34)\, \Pi_{\nu\nu'\sigma\sigma'}(42)$

SE $U^0_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\delta_{\nu\nu'} U^0 \Rightarrow \Pi(12) = \Pi^*(12) + \int d3d4\, \Pi^*(13)\, U^0(34)\, \Pi(42)$

$\rightarrow$ ABBIAMO VISTO $U = U^0 + U^0 \Pi U^0 = U^0 + U^0 \Pi^* \left(U^0 + U^0 \Pi^* \left(U^0 + U^0 \Pi^* \left(\_\_\_\right)\right)\right)$

$$\Rightarrow \boxed{U = U^0 + U^0 \Pi^* U}$$

EQ DYSON

$\rightarrow$ TC HO U SOLO CON Π^*

SE H INVAR × TRASLAZ $\Rightarrow$ FUNZ Π e Π^* (Π × DSS)

$\Rightarrow$ ANCHE D

SPOSTO IN SP K

$$D(xy) = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi} D(\underline{k}\omega)\, e^{i\underline{k}(\underline{x}-\underline{y}) - i\omega(t_x - t_y)}$$

IDEM × Π^* e D^0

$$D(\underline{k}\omega) = D^0(k) + D^0(k)\Pi^*(k)D(k) \Rightarrow D(k) = \frac{D^0(\underline{k}\omega)}{1 - D^0(\underline{k}\omega)\Pi^*(\underline{k}\omega)}$$

$\Rightarrow$ × POT STATICO $D^0(\underline{k}\omega) = D^0(\underline{k})$

ES [COULOMB]

$$D^0(\underline{k}\omega) = \frac{4\pi e^2}{k^2} \Rightarrow D(\underline{k}\omega) = \frac{\frac{4\pi e^2}{k^2}}{1 - \frac{4\pi e^2}{k^2}\Pi^*(\underline{k}\omega)}$$

$\longrightarrow$ FUNZ DIELETTRICA $\varepsilon(\underline{k}\omega)$

DEF $\boxed{\varepsilon(\underline{k}\omega) = 1 - D^0(\underline{k}\omega)\Pi^*(\underline{k}\omega)}$ FUNZ DIELETTRICA GENERALIZ

$$D(\underline{k}\omega) = \frac{4\pi e^2}{k^2 - 4\pi e^2\Pi^*(\underline{k}\omega)}$$

NSG 025 Psgnwman W SP K

TEOREMA WICK → $G = \sum G^{(g)}$ NON SCRIVO SPIN XKÈ INTRODUCO DA TRASF.

$$G_g(x x') = \left(\frac{i}{\hbar}\right)^k (-1)^{\ell} \sum_{\text{spin}} \int d_4 z_1 \dots d_4 z_{2k} \underbrace{U^0(z_i z_j) \dots U^0(z_i z_j)}_{k} \underbrace{G^0(x z) \dots G^0(z z) \dots G^0(z x')}_{2k+1}$$

g_n, grafo ordine k

START END

$$V = \frac{1}{2}\int dx\, dy\, U^0 \psi^+ \psi^+ \psi \psi$$

LE G^0 FORMANO 2 TIPI DI STRUTTURE

1 → START → END (LINEA CONTINUA)

2 → LOOP

NEL CASO INVAR × TRASLAZ. SIA ∝ G^0 CHE ∝ INTERAZ.

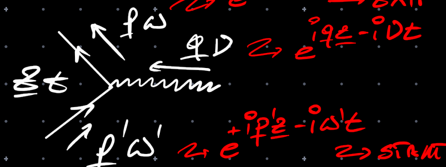
$$G^0(z_1 z_2) = \int \frac{d_3 k}{(2\pi)^3} \frac{d\omega}{2\pi} e^{i\underline{k}(\underline{z}_1 - \underline{z}_2) - i\omega(t_1 - t_2)} G^0(\underline{k}\omega)$$

IN SP. POSIZ. HO CAMMINO PARTICELLA E ARRIVO

$$U^0(z_1 z_2) = \int \frac{d_3 q}{(2\pi)^3} \frac{d\omega}{2\pi} e^{i\underline{q}(\underline{z}_1 - \underline{z}_2) - i\omega(t_1 - t_2)} U^0(q\omega)$$

IN SP. R U^0 NON HA CAMMINO NATURALE E SCELGO IO IL VERSO

INTEGR. DI EXP → δ



IN QUESTO VERTICE INTEGRO SUI FATTORI $\int d_3 z\, dt$

$$\int d_3 z\, e^{i z (p'+q-p)} \int dt\, e^{it(\omega-\nu-\omega')} = (2\pi)^3 \delta_3(p'+q-p)\, 2\pi\, \delta(\omega-\nu-\omega')$$ $\Rightarrow \sum$ MOMENTI ENTRANTI $= \sum$ MOMENTI USCENTI

$\Rightarrow$ CONSERVAZ MOMENTO NEL VERTICE $\Rightarrow$ INVAR X TRASLAZ ANCHE NEL VERTICE

IDEM X TEMPO $\Rightarrow$ INVAR TEMPORALE

$\rightarrow$ $2K$ δ x $2K$ INTEG $\rightarrow$ RIMANGONO K INTEG SULLE U E 1 TERM G

$\forall$ INTEG HO FATTORE $(2\pi)^4$ + δ CONSERVAZ MOM + TEMPO, A DX RIMANGONO K INTEG SULLE U E $2K+1$ INTEG FOURIER x G^0

$2K$ δ ANNULLANO $2K$ INTEG LASCIANDO I MOM CONSERVATI $\forall$ VERTICE

$\longrightarrow$ REGOLE FEYNMAN È QUELLO DI PRIMA + FATTORE

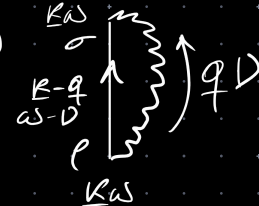
$$(2\pi)^4 \left[\frac{1}{(2\pi)^4}\right]^{\underset{\#U^0}{K} + \underset{\#G^0}{2K+1}} \left[(2\pi)^4\right]^{2K} = \left[\frac{1}{(2\pi)^4}\right]^{K} \longrightarrow$$ K QUADRIMOM DA INTEG $\int \frac{d_4 p}{(2\pi)^4}$

QUANDO ABBIAMO TERMINI UGUALI $G(x t, y t^+) \longrightarrow G(k\omega) \Rightarrow$ FATTORE $e^{-i\omega(t-t^+)}$

DIVENTA $G(k\omega)\, e^{i\omega\eta}$

ESEMPIO

$\Sigma^*_{\beta\,\rho\sigma}(k\omega)$



FIX $q\nu \Rightarrow$ IL POSTO DA CONSERVAZ

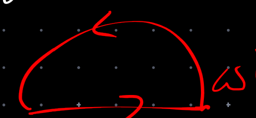
$$=\left(\frac{i}{\hbar}\right)\sum_{\rho\sigma}\int\frac{d_3q\,d\nu}{(2\pi)^4}\,G^0_{\sigma\rho}(k-q,\omega-\nu)\,\delta_{\nu\sigma}\,\delta_{\rho\mu}\,U^0(q\nu)\,e^{i(\omega-\nu)\eta}$$

↳ INTERAZ ISTANTANEA

K INTERAZ COULOMB $\quad U^0(q\nu)=\frac{4\pi e^2}{q^2}$

$$=\delta_{\mu\nu}\frac{i}{\hbar}\int\frac{d_3q}{(2\pi)^3}\frac{4\pi e^2}{q^2}\int\frac{d\omega'}{2\pi}\left[\frac{\theta(|k-q|-k_F)}{\omega'-\frac{\epsilon_{k-q}}{\hbar}+i\eta}+\frac{\theta(k_F-|k-q|)}{\omega'-\frac{\epsilon_{k-q}}{\hbar}-i\eta}\right]e^{i\omega'\eta}$$

DA QUI CAPISCO COME CHIUDERE IL CAMMINO



$\theta \longleftarrow$ I PEZZO SOTTO

ASS $\longleftarrow$ II SOPRA

$$=\delta_{\mu\nu}\frac{i}{\hbar}\int\frac{d_3q}{(2\pi)^3}\frac{4\pi e^2}{q^2}\frac{\cancel{2\pi}\,i}{\cancel{2\pi}}\,\theta\Big(k_F-\underbrace{|k-q|}_{q'}\Big)=\delta_{\mu\nu}\left(\frac{-1}{\hbar}\right)\int_{q'<k_F}\frac{d_3q'}{(2\pi)^3}\frac{4\pi e^2}{|k-q'|}$$

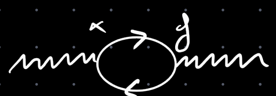
TRATTIAMO INTERAZ EFFETTIVA

$$U = U^0 + U^0\,\Pi\,U^0$$

$$\Pi(xy) = \frac{1}{i\hbar}\langle E|T\,\delta n(x)\,\delta n(y)|E\rangle \quad \text{(SCALARE)}$$

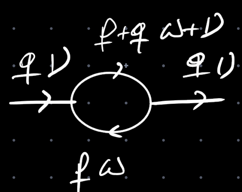
→ COME ASSEGNO DATO DIAGRAMMA L'ESPR ANALITICA?

INFATTI $\frac{i}{\hbar}$ $\forall$ DIAG INTERNO



$$= \Pi^0(xy) = \frac{i}{\hbar}(-1)\,2\,G^0(xy)\,G^0(yx)$$

→ SPK



$$= \frac{i}{\hbar}(-1)\,2\int\frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\left[\frac{\theta(p-k_F)}{\omega-\frac{\varepsilon_p}{\hbar}+i\eta}+\frac{\theta(k_F-p)}{\omega-\frac{\varepsilon_p}{\hbar}-i\eta}\right]\left[\frac{\theta(|p+q|-k_F)}{\omega+\nu-\frac{\varepsilon_{p+q}}{\hbar}+i\eta}+\frac{\theta(k_F-|p+q|)}{\omega+\nu-\frac{\varepsilon_{p+q}}{\hbar}-i\eta}\right]$$

$$= \Pi^0(q\nu)$$

INTEGRANDO IN ω $\longrightarrow$ $\frac{1}{\omega^2}$ CONV POLI SOTTO E SOPRA

GLI INTEG CONV ⟹ CHIUDO DOVE MI VA $\longrightarrow$ SE HO 2 POLI DALLO STESSO LATO

INTEGRO DAL LATO OPPOSTO ⟹ 0

⟹ CONTRIB SOLO INTEG CON POLI OPPOSTI $\longrightarrow$ 2/4 (E INTEGRO IN ω)

$$\Pi^0(q\nu) = \frac{-2i}{\hbar}\int \frac{d_3 p}{(2\pi)^3}\,\frac{2\pi i}{2\pi}\left[\frac{\theta(k_F-p)\,\theta(|p+q|-k_F)}{\nu - \frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}+i\eta}+\frac{\theta(p-k_F)\,\theta(k_F-|p+q|)}{-\nu+\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}+i\eta}\right]$$

OGNI TERMINE HA UNA PARTIC DENTRO E 1 FUORI DA $|F\rangle$

⟹ SEPARIAMO A Re E Im

$$\frac{1}{x-y\pm i\eta} = \frac{P}{x-y} \mp i\pi\,\delta(x-y)$$ PLEMELJ (VALE SE INTEGR SU UNA DISTRIB)

$$\rho = \mathrm{Re}\,\Pi^0(q\nu) = \frac{2}{\hbar}\int\frac{d_3p}{(2\pi)^3}\,\frac{\theta(k_F-p)\,\theta(|p+q|-k_F) - \theta(p-k_F)\,\theta(k_F-|p+q|)}{\nu - \frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}$$

A Im DA PLEMELJ

$$\mathrm{Im}\,\Pi^0(q\nu) = -\frac{2\pi}{\hbar}\int\frac{d_3p}{(2\pi)^3}\,\theta(k_F-p)\,\theta(|p+q|-k_F)\,\delta\!\left(\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}\right)+\theta(p-k_F)\,\theta(k_F-|p+q|)\,\delta\!\left(\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}\right)$$

$\nu>0$ $\qquad$ $\nu<0$

Im Π^0 → DICE CHE HO UNA PARTIC DENTRO E 1 FUORI K_F E CHE
LA DIFF EN È $\hbar\nu$

INTEGRO Re

$$\theta(k_F-p)\left[1-\theta(k_F-|p+q|)\right]\left[1-\theta(k_F-p)\right]\theta(k_F-|p+q|)$$

$$=\theta(k_F-p)-\theta(k_F-|p+q|)$$

$$\mathrm{Re}\,\Pi^0(q\nu)=\frac{2}{\hbar}\int\frac{dp}{(2\pi)^3}\,\frac{\theta(k_F-p)-\theta(k_F-|p+q|)}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}$$

SOLO NEL II TERMINE → $p+q=-p'$ ⇒ CAMBIO SEGNO

$$=\frac{2}{\hbar}\int\frac{d_3p}{(2\pi)^3}\,\theta(k_F-p)\left[\frac{1}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}-\frac{1}{\nu+\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}\right]$$

[CONTO DA FINIRE x $\nu = 0$ (CASO STATICO)]

$$\frac{1}{\hbar}\left[\frac{\hbar^2}{2m}\right]\left(q^2 + 2\mathbf{q}\cdot\mathbf{p}\right)$$

EN QUADRAT

$$\varepsilon_{p'} = \varepsilon_{-p'}$$

A Im $\rightarrow$ I° A $\quad p+q > K_F,\ p < K_F \implies \varepsilon_{p+q} - \varepsilon_p \geq 0 \implies \nu > 0$

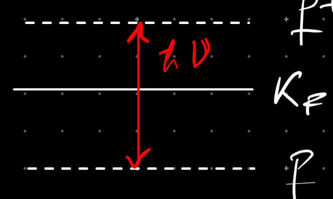
II A $\quad p < K_F,\ p+q > K_F \implies \varepsilon_{p+q} - \varepsilon_p < 0 \implies \nu < 0$

DOVE Im $\Pi^0 = 0$?

$\downarrow$

DOVE NON SONO RISPETTATI I VINCOLI

$\boxed{\nu > 0}$

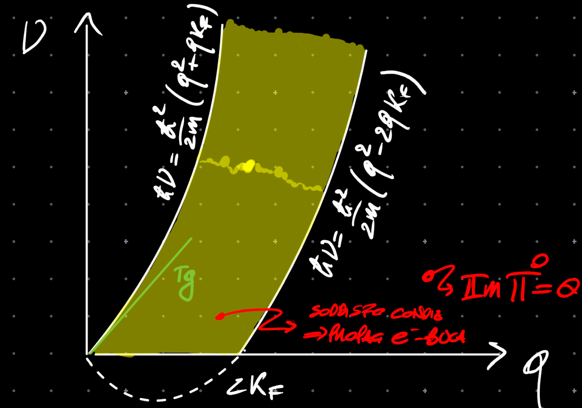


$$\delta \implies \hbar\nu = \frac{\hbar^2}{2m}\left(q^2 + 2\mathbf{q}\cdot\mathbf{p}\right)$$

$$\implies \hbar\nu \leq \frac{\hbar^2}{2m}\left(q^2 + 2|\mathbf{q}||\mathbf{p}|\right) \underset{\theta(K_F - p)}{\leq} \frac{\hbar^2}{2m}\left(q^2 + 2qK_F\right)$$

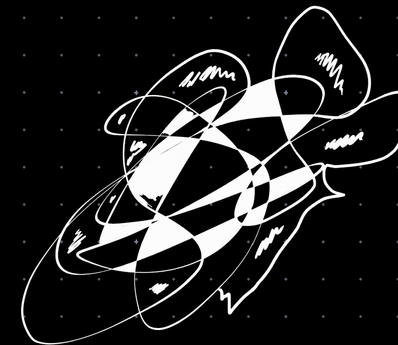
$$\hbar\nu \geq \frac{\hbar^2}{2m}\left(q^2 - 2qp\right) \geq \frac{\hbar^2}{2m}\left(q^2 - 2qK_F\right)$$

XKE LA δ SIA SODDISF ⇒ ν COMPRESSA TRA LE 2 PARAB

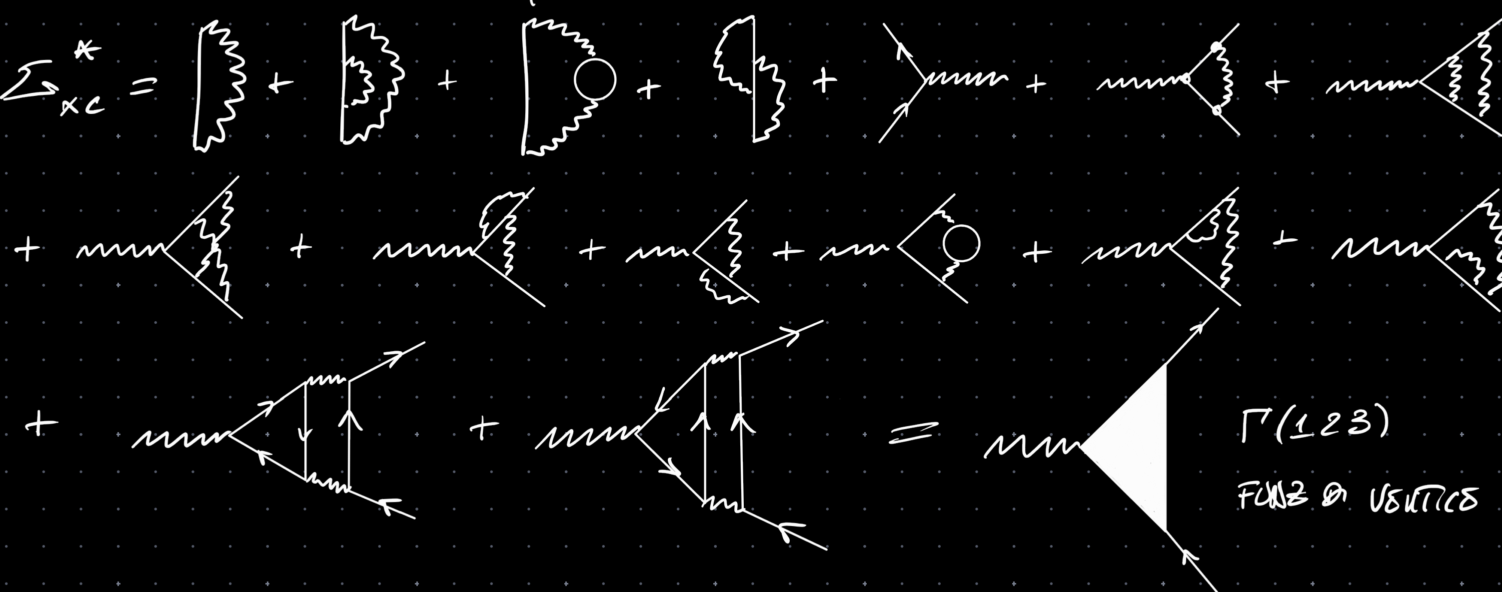


INTERESSANTE Tg VICINO $q \to 0$

$$\to \hbar\nu = \frac{\hbar^2}{m}k_F q = 2\varepsilon_F q$$



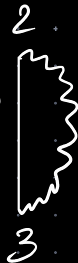
$\Sigma^*_{xc} =$

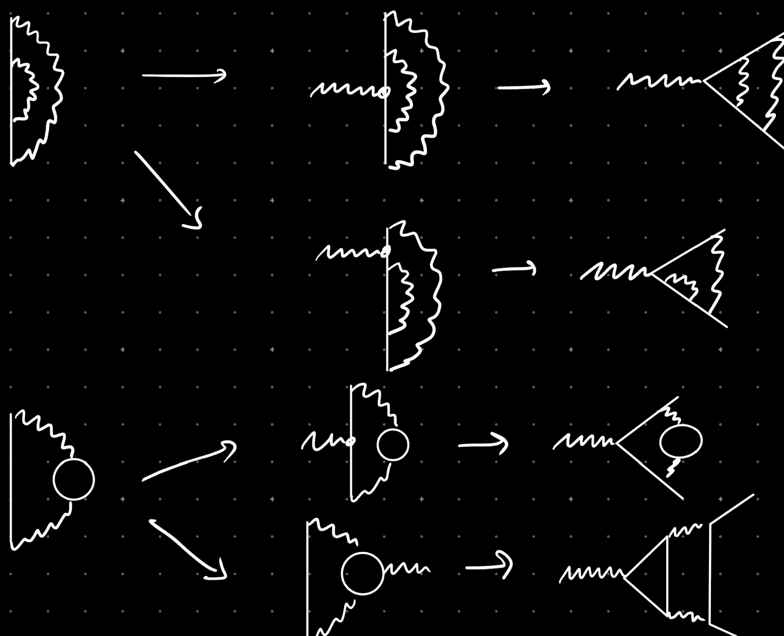


$\Gamma(1\,2\,3)$

FUNZ DI VERTICE

$\Gamma^0 =$  POSSO RICAVARE TUTTI SUCC

I) PRENDO G^0  E SOSTITUISCO G^0 CON  $\Longrightarrow$ 

II) 

SCC $\rightarrow$ DAI DIAG DI Σ

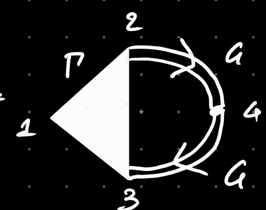
RICAVO DIAG DI VERTICE

EQ A HSAN

- $\boxed{G = G^0 + G^0 \Sigma^* G}$ DYSON I
- $\boxed{U = U^0 + U^0 \Pi^* U}$ DYSON II
- $\Pi^* =$

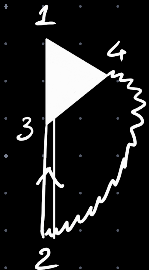
CONTINUO A VESTIRE LE G O I VERTICI O ENTRAMBI

$\Rightarrow$ $\Pi^* =$



IGNORANDO SPIN ?

$$\Pi^*(14) = \frac{i}{\hbar}(-1)\, 2 \int d2\, d3\, \Gamma(123)\, G(42)\, G(34)$$

- $\Sigma^* = \frac{i}{\hbar}$ 

$\Sigma^*(12) = \frac{i}{\hbar} \int d3\, d4\, \Gamma(413)\, G(32)\, U(42)$

- $$\Gamma(123) = \Gamma^0(123) + \int d4\, d5\, G^0(41)\, G^0(15)\, \frac{\delta \Sigma^*(23)}{\delta G^0(45)}$$

KERNEL DI BETHE SALPETER

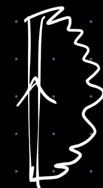
KERNEL CREATO LEVANDO G^0 E SOSTITUENDO CON Σ

NESSUNO SA
QUALCOSA SUA
ROBA X VERTICE MANCO
SE Σ ⟹ NO DESCRIZIONE ACCURATA

→ APPR GW (DRASTICA) $\Gamma = \Gamma^0$

⇒ EQ HEDIN = 4 EQ INTEGRABILI

$\Pi^* =$ 

$\Sigma^* =$ 

$$\Sigma^* = F\left(G^0(2\,\cdot)\,G^0(\cdot\,3)\,G^0 \_\_ G^0\right)$$

$$G^0 \to G^0 + \delta G^0 \oplus \Sigma \to \Sigma + \delta\Sigma \oplus \frac{\delta G(ab)}{\delta G(cd)} = \delta_{ac}\,\delta_{bd}$$ (SEMPLICE)

POI PRENDO IL FUNZIONALE E VADO 1×1 IN G^0

→ $\delta\Sigma$ LIN IN δG

$$\delta\Sigma(23) = \int d4\, d5\, \frac{\delta\Sigma(23)}{\delta G(45)}\, \delta G(45)$$

IN APPROX GW MENTRE IL # DI DIAGRAMMI DIVERGEVA FATTORIALM ORA LA SERIE HA RAGGIO DI CONV FINITO

RAPP. DI LEHMANN

PROPAGATORE

$H(t) \longrightarrow$ SP ω $\Rightarrow$ RAPPR LEHMANN

$$iG_{\mu\mu'}(xx') = \langle E_0^N | T\, \psi_\mu(x)\, \psi^\dagger_{\mu'}(x') | E_0^N \rangle$$

$H|E_n^N\rangle = E_n^N |E_n^N\rangle$ CONSERVA N PARTIC

⊕ EXPLICIT T ORD

$$= \theta(t-t')\langle E_0^N | \psi_\mu(x)\, \psi^\dagger_{\mu'}(x') | E_0^N \rangle - \theta(t'-t)\langle E_0^N | \psi^\dagger_{\mu'}(x')\, \psi_\mu(x) | E_0^N \rangle$$

$\psi^\dagger$ CREA PARTIC $\Rightarrow$ INSERISCO COMPLETEZZA N+1 PARTIC

$$= \sum_n \Big[\theta(t-t')\, e^{-\frac{i}{\hbar}(E_n^{N+1}-E_0^N)(t-t')} \langle E_0^N | \psi_\mu(\underline{x}) | E_n^{N+1} \rangle\langle E_n^{N+1} | \psi^\dagger_{\mu'}(\underline{x}') | E_0^N \rangle$$

$$- \theta(t'-t)\, e^{\frac{i}{\hbar}(E_n^{N+1}-E_0^N)(t'-t)} \langle E_0^N | \psi^\dagger_{\mu'}(\underline{x}') | E_n^{N-1} \rangle\langle E_n^{N-1} | \psi_\mu(x) | E_0^N \rangle \Big]$$

ORA IL TEMPO È EXPLICIT $\Rightarrow$ SP ω

$$= i\sum_n \left[\int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')-\frac{i}{\hbar}(\mathcal{E}_n^{N+1}-\mathcal{E}_0^N)(t-t')}}{\omega+i\eta} \langle \psi_\mu \rangle\langle \psi^+_{\mu'} \rangle - \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')+\frac{i}{\hbar}(\mathcal{E}_n^{N-1}-\mathcal{E}_0^N)(t-t')} \langle \psi^+_{\mu'} \rangle\langle \psi_\mu \rangle \right.$$

MANGIO IN ω IL CONTRIB $\Delta\mathcal{E}$ $\oplus$ MANGIO I Δ DX δ $\mathcal{E}$

$$\Rightarrow G_{\mu\mu'}(xx') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \sum_n \left[\frac{\langle \mathcal{E}_0^N | \psi_\mu(x) | \mathcal{E}_n^{N+1} \rangle\langle \mathcal{E}_n^{N+1} | \psi^+_{\mu'}(x') | \mathcal{E}_0^N \rangle}{\omega - \frac{\mathcal{E}_n^{N+1}-\mathcal{E}_0^N}{\hbar} + i\eta} \right.$$

$$\left. + \frac{\langle \mathcal{E}_0^N | \psi^+_{\mu'}(x') | \mathcal{E}_n^{N-1} \rangle\langle \mathcal{E}_n^{N-1} | \psi_\mu(x) | \mathcal{E}_0^N \rangle}{\omega + \frac{\mathcal{E}_n^{N-1}-\mathcal{E}_0^N}{\hbar} - i\eta} \right]$$

$$\Rightarrow G_{\mu\mu'}(xx') = \sum_n \left[\_\_\_ \right]$$

POSSO ASSORBIRE $\mathcal{E}$N $\mathcal{E}$O $\mathcal{E}$P $\neq$
XKÉ LE SOMME SONO DIVERSE SHIFT.
TUTTO A UN NUMERO

ora $E_n^{N+1} - E_0^N = E_n^{N+1} - E_0^{N+1} + E_0^{N+1} - E_0^N$ ⊕ $\varepsilon_n^{N+1} \equiv E_n^{N+1} - E_0^{N+1}$ en. di eccitaz.

≥ 0 x def

$\mu \equiv E_0^{N+1} - E_0^N$ pot chimico

per $N \to \infty$ indip da N ma da ρ densità

idem x $N-1$ $\quad E_n^{N-1} - E_0^N = \underbrace{E_n^{N-1} - E_0^{N-1}}_{\varepsilon_n^{N-1}} + \underbrace{E_0^{N-1} - E_0^N}_{-\mu}$

$$\Rightarrow G_{\mu\mu'}(\underline{x}\,\underline{x}'\omega) = \sum_n \frac{\langle E_0^N | \psi_\mu(\underline{x}) | E_n^{N+1}\rangle\langle E_n^{N+1} | \psi^\dagger_{\mu'}(\underline{x}') | E_0^N\rangle}{\omega - \frac{\varepsilon_n^{N+1}}{\hbar} - \frac{\mu}{\hbar} + i\eta}$$

$$+ \sum_n \frac{\langle E_0^N | \psi^\dagger_{\mu'}(\underline{x}') | E_n^{N-1}\rangle\langle E_n^{N-1} | \psi_\mu(\underline{x}) | E_0^N\rangle}{\omega + \frac{\varepsilon_n^{N-1}}{\hbar} - \frac{\mu}{\hbar} - i\eta}$$

SOMMO TRA STATI DISCRETI $\Rightarrow$ POLI POSSIBILI IN PT Im

II PT $\to$ SOPRA

I° PT $\to$ SOTTO

FACENDO IL TD I POLI SI CONDENSANO ACQUISENDO PT Im FINITA MA MANTENGONO IL SEGNO

INVARIANZA × TRASLAZ

$$[H, \underline{P}] = 0 \Rightarrow |E_n(\underline{x})\, \underline{K}\rangle \Rightarrow \sum_n \longleftrightarrow \sum_{n\,\underline{K}}$$

AZIONE TRASLAZ $\quad e^{\frac{i}{\hbar}\underline{a}\cdot\underline{P}}\, \psi_A(\underline{x})\, e^{-\frac{i}{\hbar}\underline{a}\cdot\underline{P}} = \psi_A(\underline{x}-\underline{a})$

$$\Rightarrow \psi_A(\underline{x}) = e^{-\frac{i}{\hbar}\underline{x}\cdot\underline{P}}\, \psi_A(\underline{0})\, e^{\frac{i}{\hbar}\underline{x}\cdot\underline{P}} \Rightarrow$$ SCARICO SU AUTOSTATI

POSSO SUPP CHE GS ABBIA $\underline{K} = \underline{0}$ (FERMO NEL BARICENTRO)

$$G_{\mu\mu'}(\underline{x}\,\underline{x}'\omega)=\sum_{n\underline{k}}\frac{\langle E_0^N\underline{0}|e^{-\frac{i}{\hbar}\underline{x}\cdot\underline{p}}\varphi_\mu(0)e^{\frac{i}{\hbar}\underline{x}\cdot\underline{p}}|E_n^{N+1}\underline{k}\rangle\langle E_n^{N+1}\underline{k}|e^{-\frac{i}{\hbar}\underline{x}\cdot\underline{p}}\varphi^+_{\mu'}(0)\,e^{\frac{i}{\hbar}\underline{x}\cdot\underline{p}}|E_0^N\underline{0}\rangle}{\omega-\frac{E_n^{N+1}}{\hbar}-\frac{\mu}{\hbar}+i\eta}+\sum_{n\underline{k}}\frac{\langle E_0^N\underline{0}|e^{-\frac{i}{\hbar}\underline{x}\cdot\underline{p}}\varphi^+_{\mu'}(0)e^{\frac{i}{\hbar}\underline{x}\cdot\underline{p}}|E_n^{N-1}(-k),-\underline{k}\rangle\langle E_n^{N-1}(-k)\,-\underline{k}|e^{-\frac{i}{\hbar}\underline{x}\cdot\underline{p}}\varphi_\mu(0)e^{\frac{i}{\hbar}\underline{x}\cdot\underline{p}}|E_0^N\underline{0}\rangle}{\omega+\frac{E_n^{N-1}}{\hbar}-\frac{\mu}{\hbar}-i\eta}$$

$E_{\underline{k}}=E_{-\underline{k}}$

$$=\sum_{n\underline{k}}e^{i\underline{k}(\underline{x}-\underline{x}')}\left[\frac{\langle E_0^N\underline{0}|\varphi_\mu(0)|E_n^{N+1}(k)\underline{k}\rangle\langle E_n(k)\underline{k}|\varphi^+_{\mu'}(0)|E_0^N\underline{0}\rangle}{\omega-\frac{E_n^{N+1}(\underline{k})}{\hbar}-\frac{\mu}{\hbar}+i\eta}+\frac{\langle E_0^N 0|\varphi^+_{\mu'}(0)|E_n^{N-1}(-k)-\underline{k}\rangle\langle E_n^{N-1}(-k)-\underline{k}|\varphi_\mu(0)|E_0^N 0\rangle}{\omega+\frac{E_n^{N-1}(\underline{k})}{\hbar}-\frac{\mu}{\hbar}-i\eta}\right]$$

PONENDO $\mu=\mu'$

$$G_{\mu\mu}(\underline{k}\omega)=V\sum_n\left[\frac{|\langle E_0^N 0|\varphi_\mu(\underline{0})|E_n^{N+1}(\underline{k})\underline{k}\rangle|^2}{\omega+\frac{E_n^{N+1}(k)}{\hbar}-\frac{\mu}{\hbar}+i\eta}+\frac{|\langle E_0^N\underline{0}|\varphi^+_\mu(0)|E_n^{N-1}(-k)-\underline{k}\rangle|^2}{\omega-\frac{E_n^{N-1}(-k)}{\hbar}-\frac{\mu}{\hbar}-i\eta}\right]$$

(*)

$$\Rightarrow\sum_\mu G_{\mu\mu}(\underline{k}\omega)=\int d\omega'\,\frac{A(\underline{k}\omega')}{\omega-\omega'+i\eta\,\mathrm{sgn}\left(\omega'-\frac{\mu}{\hbar}\right)}$$

$A \to$ FUNZ SPETTRALE ≥ 0 XKÉ CL DI FUNZ POS

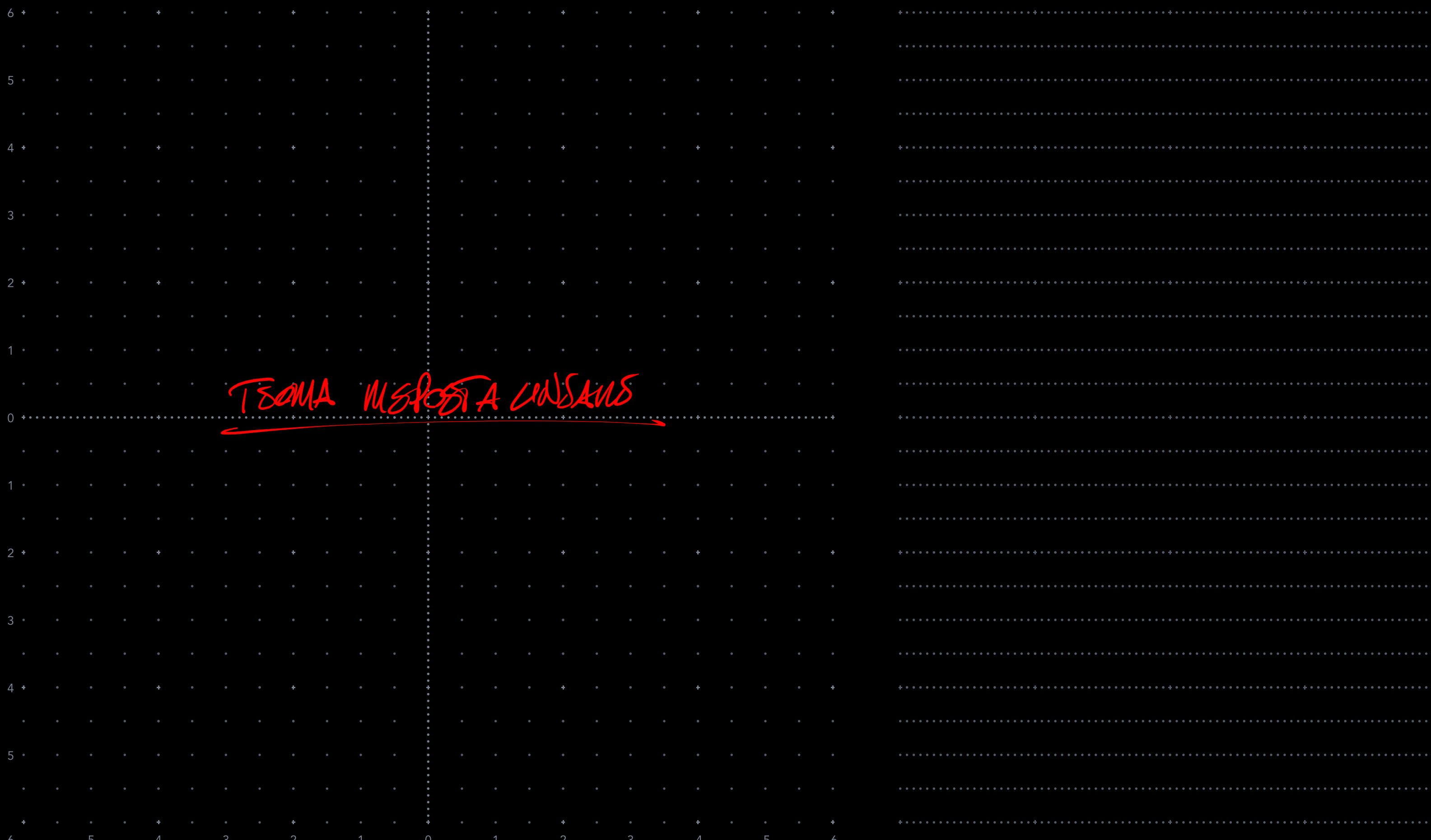
$$A(\underline{k}\omega)=V\sum_{n\mu}|\langle\ |\ |\ \rangle|^2\,\delta\left(\omega'-\frac{E_n^{N+1}(\underline{k})}{\hbar}-\frac{\mu}{\hbar}\right)+|\langle\ |\ |\ \rangle|^2\,\delta\left(\omega'+\frac{E_n^{N-1}(-\underline{k})}{\hbar}-\frac{\mu}{\hbar}\right)$$

$\downarrow$ $\omega'>\mu/\hbar$ $\qquad$ $\downarrow$ $\omega'<\mu/\hbar$ $\Rightarrow$ CONTA IL TERMINE IN BASE AL SGN

$\left(\int\delta=0\right.$ ALTRIM$\left.\right)$

VEDIAMO CHE $G(\underline{k}\omega)\xrightarrow[|\omega|\to\infty]{}\frac{1}{\omega}\Rightarrow\int d\omega'\,A(\underline{k}\omega')=\underline{1}$

TEORIA RISPOSTA LINEARE



H INFATTI APPLICATO → PERTURBO CON $V(t) = 0 \times t<0$

→ SUPP DI ESSERE $|\psi(0)\rangle = |GS\rangle$ DI H

$$t \geq 0 \to |\psi(t)\rangle = U(t,0)|\psi(0)\rangle = e^{-\frac{i}{\hbar}Ht}\, U_I(t,0)\,|GS\rangle$$

V PICCOLO ⇒ DESCRIZ DI INTERAZ

ORA CONSID OBS O_t CHE HA SUA ESPLICITA EVOLUZ TEMP INDIP

$$\langle\psi(t)|O_t|\psi(t)\rangle = \langle GS|U_I^{+}(t,0)\, e^{i/\hbar Ht}\, O_t\, e^{-i/\hbar Ht}\, U_I(t,0)|GS\rangle$$

$$= \langle GS|U_I^{+}(t,0)\, O_{Ht}\, U_I(t,0)|GS\rangle$$

EVOLUZ HEISENBERG DI O_t SECONDO H

SUPP DI POTER SVILUPP IL Texp AL I° ORD

$$U_I(t,0) = T\exp \frac{1}{i\hbar}\int_0^t dt'\, V_H(t') = \langle GS|\left(1 - \frac{1}{i\hbar}\int_0^t dt'\, V_H(t') + \dots\right) O_H(t) \left(1 + \frac{1}{i\hbar}\int dt'\, V_H(t') + \dots\right)|GS\rangle$$

$$= \langle \psi(t)|O_t|\psi(t)\rangle - \langle GS|O_H(t)|GS\rangle = \frac{1}{i\hbar}\int_0^t dt' \langle GS|\left[O_H(t), V_H(t)\right]|GS\rangle$$

LE EVOLUZIONI DI HEISENBERG SI ELIMINANO SU GS $\Rightarrow 1$

$$\langle \psi(t)|O_t|\psi(t)\rangle - \langle GS|O_t|GS\rangle = \frac{1}{i\hbar}\int_{\pm\infty} dt'\, \theta(t-t')\langle GS|\left[O_H(t), V_H(t')\right]|GS\rangle$$

$\rightarrow$ LIN IN V

PERTURBAZ EXT

$$H_t = H + \int d_3 x \; \hat{n}(x) \; \varphi(\underline{x} t)$$

↳ DIP TEMP SOLO DA CAMPO EXT

$$\varphi(\underline{x} t) = 0 \quad \text{SE } t < 0$$

VADO A MIS MOMENTO $\hat{n}$

$$\langle \psi(t) | n(x) | \psi(t) \rangle - \langle GS | n(\underline{x}) | GS \rangle$$

$$= \frac{1}{i\hbar} \int_{\pm\infty} dt' \int d_3 \underline{x}' \, \theta(t-t') \langle GS | \left[\hat{n}_H(\underline{x} t), \hat{n}_H(\underline{x}' t') \right] | GS \rangle \, \varphi(\underline{x}' t')$$

$$\delta n(\underline{x} t) = \frac{1}{\hbar} \int_{\pm\infty} dt' d_3 \underline{x}' \, D^{RET}(\underline{x} t \, \underline{x}' t') \, \varphi(\underline{x}' t')$$

VARIAZ DENSITÀ IN $\underline{x}\, t$

OMOGENEO IN H

$$i D^{RET}(xx') = \theta(t-t') \langle GS | [n_H(x), n_H(x')] | GS \rangle$$

→ VARIAZIONE $\delta \hat{O}$ INDOTTA DA FUNZ EXT È LIN IN QUESTA DA UNA RISPOSTA IN GEN NON LOC

θ CONSERVA LA CAUSALITÀ $(t' < t)$ ⟹ FUNZ DI RISPOSTA RITARDATA $D^R(t,t')$ DIP DA $t-t'$

H INVAR X TRASLAZ SP-TEMPO

$D^{RET}(\underline{x}t\, \underline{x}'t')$ DIP DA $\underline{x}-\underline{x}'$ E $t-t'$ ⟶ DIM CON TRASLAZ SU OP CAMPO

$[\underline{p}, H] = 0$ ⟹ POSSO RIMUOVERE EVOLUZ TEMP E AGIRE SU $|GS\rangle$ ⟹ DIP DA $\underline{x}-\underline{x}'$

⟹ INTEG ⟶ CONVOLUZ SU SP-TEMPO ⟹ SP FOURIER ⟹ INTEG SPARISCE

$$\delta n(\underline{x}t) = \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{2\pi} e^{+i\underline{k}\underline{x} - i\omega t} \delta n(\underline{k}\omega)$$

$$\boxed{\delta n(\underline{k}\omega) = \frac{1}{\hbar} D^{RET}(\underline{k}\omega) \varphi(\underline{k}\omega)}$$

IN SP $\underline{k}\omega$ I VARI MODI RISP IN MANIERA INDIP ATTRAVERSO I COEFF $\dfrac{D^{RET}(\underline{k}\omega)}{\hbar}$ → SUSCETTIBILITÀ GENERALIZZATA x ES $\vec{J} = \sigma \vec{E}$ x CORRENTE / CAMPO EM

$\vec{D} = \varepsilon \vec{E}$ IDENTITÀ VALIDA FREQ x FREQ

→ COME CALCOLO D^{RET}? D^{RET} E D^{T} COLLEGATI DA FUNZ LEHMANN

$$i D^T(\underline{x}t\, \underline{x}'t') \equiv \langle GS| T\, \delta n_H(\underline{x}t)\, \delta n_H(\underline{x}'t') |GS\rangle$$

GIÀ VISTO x POLARIZ

x INVAR TRASLAZ SP-TEMPO $D^T(\underline{k}\omega)$

$$= i\hbar\, \Pi(\underline{x}t\, \underline{x}'t')$$

RAPPR LEHMANN x δA δB

(I) $$i\, C^R_{AB}(t,t') = \langle GS| \left[A_H(t), B_H(t') \right] |GS\rangle\, \theta(t-t')$$

(II) $$i\, C^T_{AB}(t,t') = \langle GS| T\, \delta A_H(t)\, \delta B_H(t') |GS\rangle = \theta(t-t') \langle GS| \delta A_H(t)\, \delta B_H(t') |GS\rangle + \theta(t'-t) \langle GS| \delta B_H(t')\, \delta A_H(t) |GS\rangle$$

SVILUPPO LA (I)

$|GS\rangle = |E_0\rangle$, $|E_n\rangle$ SONO $\mathcal{H}$ IN SSP N PARTIC → SUPP A, B OPERINO SU $\mathcal{H}_N$

PARI DI $c, c^\dagger$

$$i C^R_{AB}(t,t') = \sum_n \left(e^{-\frac{i}{\hbar}(E_n-E_0)(t-t')} A_{0n}B_{n0} - e^{\frac{i}{\hbar}(E_n-E_0)(t-t')} B_{0n}A_{n0} \right)\theta(t-t')$$

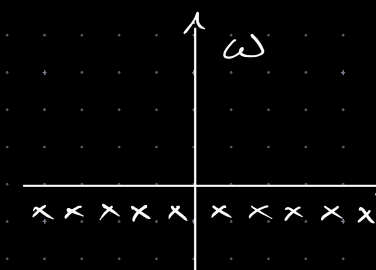
$n=0$ si cancella θ — sposto δ shift $(E_n-E_0)/\hbar$

$$= i\int \frac{d\omega}{2\pi}\, e^{-i\omega(t-t')}\left[\sum_n \frac{A_{0n}B_{n0}}{\omega-\frac{E_n-E_0}{\hbar}+i\eta} - \frac{B_{0n}A_{n0}}{\omega+\frac{E_n-E_0}{\hbar}+i\eta}\right]$$

$$\Rightarrow C^R_{AB}(\omega) = \sum_n \frac{A_{0n}B_{n0}}{\omega-\frac{E_n-E_0}{\hbar}+i\eta} - \frac{B_{0n}A_{n0}}{\omega+\frac{E_n-E_0}{\hbar}+i\eta} = \int_{\pm\infty} d\omega' \, \frac{C_{AB}(\omega')}{\omega-\omega'+i\eta}$$

FUNZ SPETTRALE

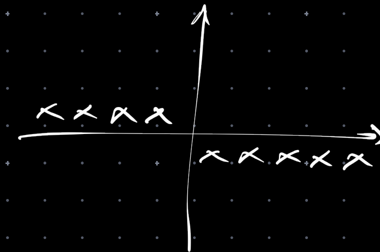
$$C_{AB}(\omega) = \sum_n A_{0n}B_{n0}\,\delta\Big(\omega - \underbrace{\frac{E_n^N-E_0^N}{\hbar}}_{>0}\Big) - B_{0n}A_{n0}\,\delta\Big(\omega + \underbrace{\frac{E_n^N-E_0^N}{\hbar}}_{>0}\Big)$$



TORD x SS (UGUALE MA 2 θ)

FUNZ RET HA POLI PIANO INFERIORE $\Rightarrow$ ANALITICA IN PIANO SUPERIORE

$$C^T_{AB}(\omega) = \int_{\pm\infty} d\omega' \, \frac{C_{AB}(\omega')}{\omega-\omega'+i\eta\,\mathrm{sgn}(\omega)}$$



SE $A = \hat{n}(x)$ $\quad B = \hat{n}(x')$

$\Rightarrow$ ELEM MATR AUTOSTATO H

$$e^{-\frac{i}{\hbar}\underline{x}\cdot \underline{P}}\,\varphi(\underline{0})\,e^{+\frac{i}{\hbar}\underline{x}\cdot \underline{P}} = \varphi(\underline{x}) \quad\Rightarrow\quad \hat{n}(\underline{x}) = e^{-\frac{i}{\hbar}\underline{x}\cdot \underline{\hat{P}}}\,\hat{n}(\underline{0})\,e^{\frac{i}{\hbar}\underline{x}\cdot \underline{P}}$$

IN BASE $|E_n(\underline{k})\,\underline{k}\rangle \longrightarrow \langle E_0\,\underline{0}\,|\,n(\underline{x})\,|\,E_n(\underline{k})\,\underline{k}\rangle$ = SVIL FOURIER

$$i D^R(\underline{k}\omega) = \int_{\pm\infty} d\omega' \, \frac{D(\underline{k}\omega')}{\omega-\omega'+i\eta} \qquad D^T(\underline{k}\omega) = \int_{\pm\infty} d\omega' \, \frac{D(\underline{k}\omega')}{\omega-\omega'+i\eta\,\mathrm{sgn}(\omega)}$$

FACENDO CONTO $D(\underline{k}\omega) \in \mathbb{R}$

$\Rightarrow$ uso $\dfrac{1}{\omega-\omega'\pm i\eta} = \dfrac{P}{\omega-\omega'} \mp i\pi\,\delta(\omega-\omega')$

$$\Rightarrow D^R(\underline{k}\omega) = \int d\omega' \frac{\mathcal{D}(\underline{k}\omega')}{\omega-\omega'} - i\pi\,\mathcal{D}(\underline{k}\omega)$$

$$\bar{D}^T(\underline{k}\omega) = \int d\omega' \frac{\mathcal{D}(\underline{k}\omega')}{\omega-\omega'} - i\pi\,\mathrm{sgn}(\omega)\,\mathcal{D}(\underline{k}\omega)$$

$\rightarrow \mathcal{D}(\underline{k}\omega) \in \mathbb{R} \Rightarrow \quad \mathrm{Re}\, D^R(\underline{k}\omega) = \mathrm{Re}\, \bar{D}^T(\underline{k}\omega)$

$\qquad\qquad\qquad\qquad \mathrm{Im}\, D^R(\underline{k}\omega) = \mathrm{Im}\, \bar{D}^T(\underline{k}\omega)\,\mathrm{sgn}(\omega)$

SCREENING

IMPUREZZA IN $\underline{0}$ DI CHARGE Ze IN HEG $\longrightarrow H_{HEG} - Ze^2 \int d_3x \, \frac{n(\underline{x})}{|\underline{x}|}$

PERTURB INDIP DA t ACCESA A $t=-\infty$

$$\delta n(\underline{x}t) = \frac{1}{\hbar} \int dt' d_3x' D^R(\underline{x}t\, \underline{x}'t') \varphi(\underline{x}') \quad \rightsquigarrow \quad \varphi(\underline{x}) = -\frac{Ze^2}{|\underline{x}|}$$

D^R INVAR X TRASL XKÉ CO É HEG

$$\delta n(\underline{k}\omega) = \frac{1}{\hbar} D(\underline{k}\omega) \varphi(\underline{k}\omega) \quad \rightsquigarrow \quad \varphi(\underline{k}\omega) = -Ze^2 \frac{4\pi}{k^2} 2\pi \delta(\omega)$$

$$\delta n(\underline{x}t) = \int \frac{d_3k}{(2\pi)^3} e^{i\underline{k}\underline{x}} \frac{1}{\hbar} D^R(\underline{k}0) (-Ze^2) \frac{4\pi}{k^2} = -Ze^2 \frac{4\pi}{(2\pi)^3} \int_0^\infty k^2 dk \frac{1}{\hbar} D^R(\underline{k}0) \frac{2\pi}{k^2} \int d\theta \sin\theta \, e^{ikx\cos\theta}$$

DIP DA $|k|$ XKÉ HEG INVAR X ROTAZ

DA AZIMUT

$$\frac{1}{ikr}\left(e^{ikr} - e^{-ikr}\right) = \int_{\pm 1} d\xi \, e^{ikr\xi}$$

$$\Rightarrow \delta n(r) = -\frac{Ze^2}{i4\pi^2 r} \int_0^\infty k\, dk \frac{4\pi}{k^2} \frac{1}{\hbar} D^R(\underline{k}0) \left(e^{ikr} - e^{-ikr}\right)$$

$k \leftrightarrow -k$

$$= \frac{-Ze^2}{i4\pi^2 r} \int_{\pm\infty} k\, dk \frac{4\pi}{k^2} \frac{1}{\hbar} D^R(\underline{k}0) e^{ikr}$$

SAPPIAMO CHE $D^T(xx') = \hbar\,\Pi(xx')$

$\Rightarrow$ CON CALCOLATORE $D^R(xx') \Rightarrow D^R(xx') \equiv \hbar\,\Pi^R(xx') \overset{R}{\Rightarrow} D^R(\underline{k}\omega) = \hbar\,\Pi^R(\underline{k}\omega)$

E SO CHE $\Pi(\underline{k}\omega) = \dfrac{\Pi^*(\underline{k}\omega)}{1 - U_0(\underline{k})\,\Pi^*(\underline{k}\omega)}$ EQ DYSON

$\Rightarrow$ FACILE DIM CHE SE $\Pi^{*R} = \mathrm{Re}\,\Pi^* + i\,\mathrm{Im}\,\Pi^*\,\mathrm{sgn}(\omega) \Rightarrow \Pi^R = \dfrac{\Pi^{*R}}{1 - U_0\,\Pi^{*R}}$

$$\Rightarrow \frac{D^R(\underline{k}0)}{\hbar} = \Pi^R(\underline{k}0) = \frac{\Pi^{*R}(\underline{k}0)}{1 - \underbrace{\frac{4\pi e^2}{k^2}}_{U_0(k)}\Pi^{*R}(\underline{k}0)}$$

$$\Rightarrow \delta n(r) = \frac{-Z}{i4\pi^2 r}\int_{\pm\infty} k\,dk\, \frac{\frac{4\pi e^2}{k^2}\Pi^{*R}(\underline{k}0)}{1 - \frac{4\pi e^2}{k^2}\Pi^{*R}(\underline{k}0)}\, e^{ikr}$$

$\longrightarrow$ NON SO Π^{*R} MA SOLO CHE

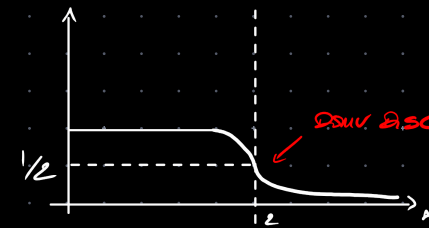
$\Pi^{*R} = \mathrm{Re}\,\Pi^*(\underline{k}0)$ (E Im NULLA)

$\Rightarrow$ APPROX $\Pi^* \simeq \Pi^0$

$\nexists$ CALCOLO ESATTO $\longrightarrow$ VADO A $r \to \infty \Rightarrow e^{ikr}$ OSCILLA UN BOTTO $\Rightarrow$ INTEGRALE VA A 0 VELOCEMENTE

⇒ DOMINA ZONA $k \to 0$ NELL' INTEG

⇒ IN $k \to 0$ $\quad \Pi^{*R}(k,0) \simeq \Pi^{0}(k,0) = -\frac{m k_F}{4\pi^2 \hbar^2}\, g\left(\frac{k}{k_F}\right) \rightsquigarrow g(x) = \frac{1}{2} - \frac{1}{2x}\left(1-\frac{x^2}{4}\right) \lg\left|\frac{2-x}{2+x}\right|$



g MISURATA DA INTEG $\longrightarrow g(0) = 1$

⇒ APPROX THOMAS-FERMI $\quad g(x) \simeq 1$

$$\delta n(r) = -\frac{Z}{4\pi^2 i r}\int_{\pm\infty} k\,dk \,\frac{4\pi e^2 \left(-\frac{m k_F}{4\pi^2 \hbar^2}\right)}{k^2 + \frac{m k_F 4\pi e^2}{4\pi^2 \hbar^2}} \rightsquigarrow k_{TF}^2 = \frac{m k_F e^2}{\pi \hbar^2} = \frac{k_F}{a_0}, \qquad a_0 = \frac{\hbar^2}{m e^2}$$

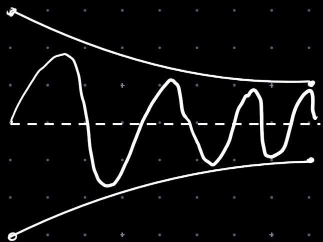
$\frac{k_{TF}^2}{k_F^2} \simeq 1 \longrightarrow$ NON COSÌ ≠ DA DENSITÀ MATERIA COND

$$\delta n(r) = -\frac{Z}{4\pi^2 i r}\int k\,dk\,\frac{k_{TF}^2}{k^2+k_{TF}^2}\, e^{ikr} \longrightarrow 2 \text{ POLI } \pm i k_{TF}$$

$$= \frac{e^{-r}}{r} k_{TF}^2 \frac{Z}{4} \longrightarrow \text{ANDAM. THOMAS FERMI}$$

RESIDUI

IN USCITA SE USO MEGLIO g $\Rightarrow$ TRASF FOURIER $\Rightarrow$ $\delta n(r) \sim \Sigma \frac{1}{r^3} \cos(2k_F r)$



ZONE A DENSITÀ POS E NEG.

$\rightarrow$ CARICA ESALTATA O MENO DALLA SCHERMATURA

OSCILLAZ DI FRIEDEL $\rightarrow$ A $T > 0$ SCOMPAIONO

OSCILLAZ DI PLASMA

e^- SPARATI SU SUP $\Rightarrow$ e^- RIFLESSI $\rightarrow$ ALCUNI PERDONO UN MULTIPLO DI UNA CERTA QUANTITÀ

IDEM SU LASTRE DI METALLO $\Rightarrow$ PERDO UN × ATTIVAZ DI OSCILLAZ COLLETTIVE NEL METALLO

SE LA ILLUMINO CON UN $\begin{cases} < \hbar\omega_{PL} \Rightarrow \text{RIFLESSI} \\ > \hbar\omega_{PL} \Rightarrow \text{ASSORBITI} \end{cases}$

$$\omega_{PL}^2 = \frac{4\pi e^2}{m} n \longrightarrow \hbar\omega_{PL} \sim 10\,eV > E_F$$

INDIP DA $\hbar$ $\Rightarrow$ PIUTTOSTO UNIV × SIS EL

$$\delta n(\underline{x}\,t) = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, e^{i(\underline{k}\underline{x}-\omega t)}\, \Pi^R(\underline{k}\omega)\, \varphi(\underline{k}\omega)$$

DOVE LA RISPOSTA È MAGGIORE? SE Π^R HA POLI VICINI A Re

POLO $\in \mathrm{Re}$ $\Rightarrow$ RISP ESPLODE

OGNI FUNZ RITARDATA HA POLI IN ASSE INFERIORE

$$\Pi(\underline{k}\omega) = \frac{\Pi^*}{1 - v\Pi^*} \longrightarrow \text{POLI} = \text{ZERI FUNZ DIELETTRICA} \longrightarrow \varepsilon(\underline{k}\omega) = 1 - v(\underline{k})\,\Pi^*(\underline{k}\omega)$$

$$\varepsilon^R(\underline{k}\omega) = 1 - v(\underline{k})\,\Pi^{*R}(\underline{k}\omega)$$

POLI : $0 = 1 - v(\underline{k})\,\Pi^{*R}(\underline{k}\omega) \longrightarrow$ FIX $\underline{k}$ $\quad \omega(\underline{k}) = \overbrace{\omega_1(\underline{k})}^{\mathrm{Re}} + \overbrace{i\,\omega_2(\underline{k})}^{\mathrm{Im}}$

SICCOME USI $\boxed{\omega_2 < 0}$ $\qquad 0 = 1 - 2S(\underline{k})\,\Pi^{*R}(\underline{k}, \omega_1 + i\omega_2)$

IDEM X Im MA TRA POCO SPARISCE

→ IMPONGO ω_2 PICCOLO ⇒ SEPARO Re E Im E SVILUPPO

$$1 - 2S(\underline{k})\left[\mathrm{Re}\,\Pi^{*R}(\underline{k}\omega_1) + i\,\mathrm{Im}\,\Pi^{*R}(\underline{k}\omega_1) + i\omega_2 \frac{\partial}{\partial\omega}\mathrm{Re}\,\Pi^{*R}(\underline{k}\omega)\Big|_{\omega=\omega_1} + \_\_\_\right]$$

PER Re $\quad 0 = 1 - 2S(\underline{k})\,\mathrm{Re}\,\Pi^{*R}(\underline{k}\,\omega_1(\underline{k})) \quad \left(+ \text{PER } \mathrm{Im} \cdot \omega_2 \to 0 \right)$

PER Im $\quad 0 = -2S(\underline{k})\,\mathrm{Im}\,\Pi^{*R}(\underline{k}\omega_1) + \omega_2 \frac{\partial}{\partial\omega}\mathrm{Re}\,\Pi(\underline{k}\omega)\Big|_{\omega_1} 2S(\underline{k}) \longrightarrow$ DA QUI ω_2

$$\begin{cases} 0 = 1 - 2S(\underline{k})\,\mathrm{Re}\,\Pi^{*R}(\underline{k}\,\omega_1(\underline{k})) \\ \omega_2(\underline{k}) = \dfrac{\mathrm{Im}\,\Pi^{*R}(\underline{k}\,\omega_2(\underline{k}))}{\frac{\partial}{\partial\omega}\mathrm{Re}\,\Pi^{*R}(\underline{k}\omega)\Big|_{\omega_1(\underline{k})}} \end{cases} \Rightarrow \Pi^{R}(\underline{k}\omega) = \frac{\overset{\text{RESIDUO}}{Z(\underline{k})}}{\omega - (\omega_1(\underline{k}) + i\omega_2(\underline{k}))} + \Pi^{RES}(\underline{k}\omega)$$

TROVATO IL POLO MISURIVO Π^{*R}

CONTRIB POLO: $$\delta n^{polo}(\underline{x}t) = \int \frac{d^3k}{(2\pi)^3} Z(\underline{k})\, e^{-i\underline{k}\underline{x}} \int_{\pm\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - \omega_1 - i\omega_2} \phi(\underline{k}\omega)$$

$t>0 \rightarrow$ CHIUDO SOTTO

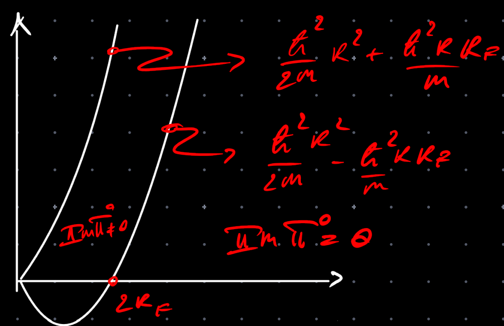
$$-\frac{2\pi i}{2\pi}\, e^{-i(\omega_1 + i\omega_2)t}\, \varphi(\underline{k}\omega_1) \longrightarrow \omega_2 < 0 \Rightarrow$$ TERMINE DECRESCENTE INVERSO TEMPO DI DECADIMENTO

VEDREMO CHE ω_1 INDIP DA K (ALMENO I ORD) $\Rightarrow$ ESCE DA INTEG E LA FUNZ OSCILLA

APPLICHIAMO A CASO COULOMB

1° APPROX $\Pi^{*R} = \Pi^{0R}$

E VEDIAMO A LUNGO IL MIO MOTO CONTINUO HO 2 REGIONI IN CUI $\mathrm{Im}\,\Pi^0 = 0$



PATOLOGICO XKÉ SE POLO $\in \mathrm{Re} \Rightarrow$ INTEG ESPLODE

$\rightarrow \exists$ COMUNQUE $\Rightarrow$ LO USO X BUONA APPROX ω_1

IN ZONA K GRANDE $\Rightarrow$ LUNGH PICCOLO MA VOGLIO GUARDARE SOPRA A SCALA RETICOLARE

$\Rightarrow$ ZONA K PICCOLO $\Rightarrow$ LUNGH D'ONDA GRANDI

$$1 = \frac{4\pi e^2}{k^2}\,\Pi^0(\underline{k}\omega) \longrightarrow \text{OTTIAM } \omega = \omega(\underline{k})$$

$$1 = \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\int \frac{d_3q}{(2\pi)^3}\,\theta(k_F-q)\left[\frac{1}{\omega-\frac{\varepsilon^0_q}{\hbar}-\frac{\varepsilon^0_{q+k}}{\hbar}}-\frac{1}{\omega-\frac{\varepsilon^0_q}{\hbar}+\frac{\varepsilon^0_{q+k}}{\hbar}}\right]$$

$$= \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\int \frac{d_3q}{(2\pi)^3}\,\theta(k_F-q)\,\frac{2\left(\frac{\varepsilon^0_{q+k}-\varepsilon^0_q}{\hbar}\right)}{\omega^2-\left(\frac{\varepsilon^0_q-\varepsilon^0_{q+k}}{\hbar}\right)^2}$$

$$\geq \left(\frac{\hbar k^2}{2m}+\frac{\hbar k q}{m}\right)^2 \quad q<k_F \text{ x la } \theta$$

svilup geom

$$= \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\int \frac{d_3q}{(2\pi)^3}\,\theta(k_F-q)\,2\left(\frac{\hbar k^2}{2m}+\frac{\hbar k q}{m}\right)\left[1+\left(\frac{\hbar^2 k^2}{2m}+\frac{\hbar k q}{m}\right)^2+\_\_\_\right]$$

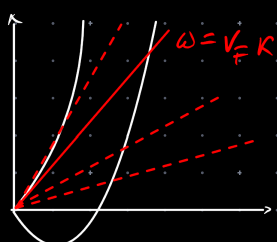
ordine 0 ⇒ ω_{pl} ⊕ come è

θ x vel disp in q

$$1 = \frac{4\pi e^2}{k^2\omega^2}\frac{4}{\hbar}\int \frac{d_3q}{(2\pi)^3}\,\theta(k_F-q)\left[\left(\frac{\hbar k^2}{2m}+\frac{\hbar k q}{m}\right)+\frac{1}{\omega^2}\left(\frac{\hbar k^2}{2m}+\frac{\hbar k q}{m}\right)^3+\_\_\_\right]$$

miracolo Coulomb → k^2 in I° termine si cancella con fattore

→ accum x es pot così piccolo $\quad \varepsilon(k)=\varepsilon(0)+\_ \Rightarrow \omega^2 = \text{cost}\, k^2$ → lungo lun ⇒ onde



intensità $\begin{cases} > v_F \Rightarrow \text{onde} \\ < v_F \Rightarrow \text{non le vedo} \end{cases}$

(He_3 su fattore)

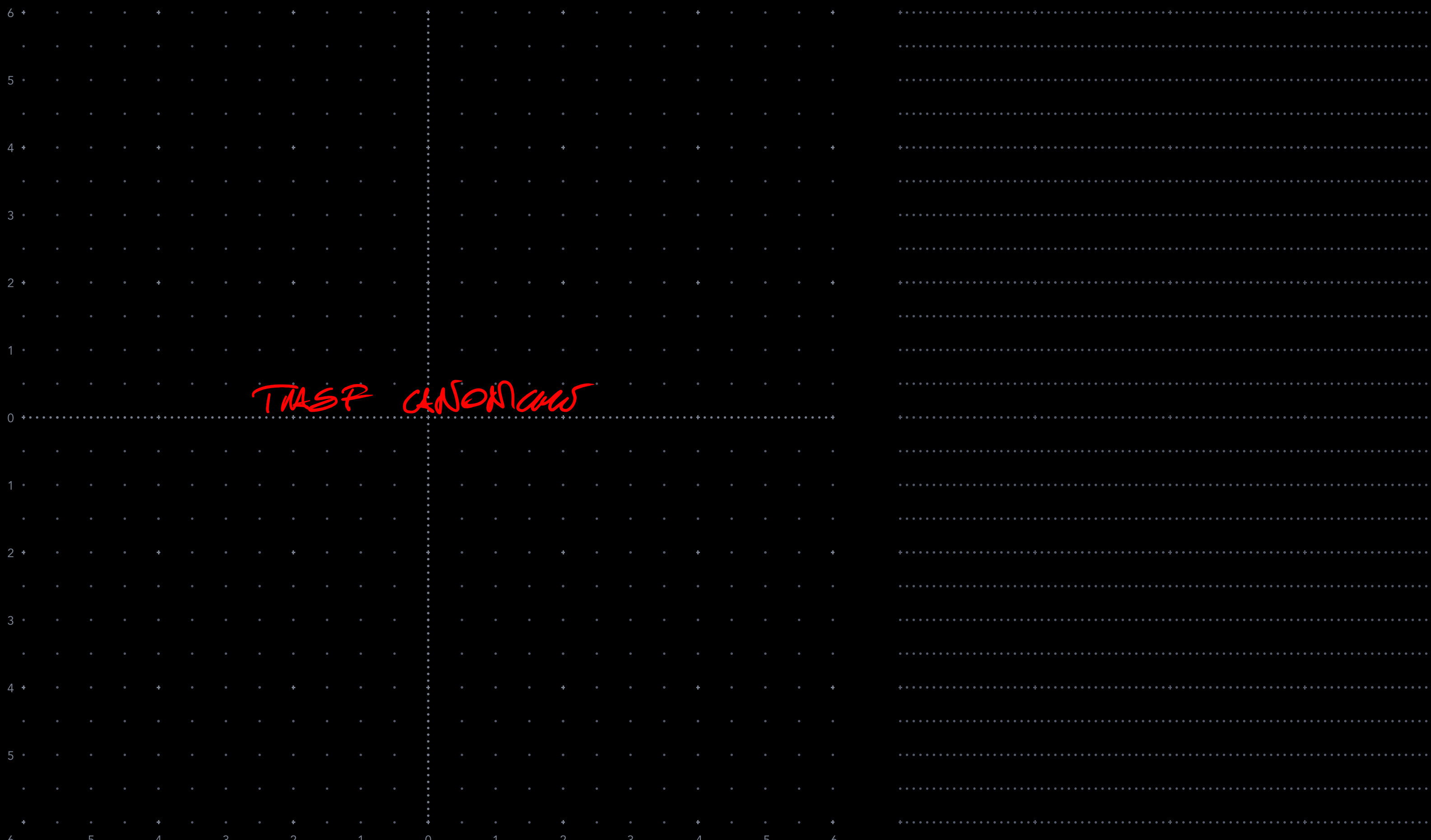
$$1 = \frac{4\pi e^2}{\omega^2}\frac{4}{\hbar}\frac{\hbar}{2m}\underbrace{\int \frac{d_3 q}{(2\pi)^3}\theta(k_F - q)} + \frac{4\pi e^2}{k^2\omega^2}\frac{\hbar}{\hbar\omega^2}\int d_3 q\,\theta(k_F - q)\Big[\underbrace{\left(\frac{\hbar k^2}{2m}\right)^3} + \cancel{3\left(\frac{\hbar k^2}{2m}\right)^2\frac{\hbar k}{m}q}^{\,0} + 3\left(\frac{\hbar k^2}{2m}\right)\left(\frac{\hbar k q}{m}\right)^2 +$$

$$+ \cancel{\left(\frac{\hbar k q}{m}\right)^3}^{\,0}\Big] + \_\_\_$$

$$k_F = (3\pi^2 n)^{1/3} \longrightarrow \frac{1}{(2\pi)^3}\frac{4}{3}\pi\, 3\pi^2 n$$

K ficado $\oplus$
POT GRANDES
$\Rightarrow$ TRASE

$$1 = \frac{4\pi e^2 n}{m\omega^2} + \frac{4\pi e^2}{k^2\omega^4}\frac{4}{\hbar}\int \frac{d_3 q}{(2\pi)^3}\theta(k_F - q)\,3\left(\frac{\hbar k^2}{2m}\right)\left(\frac{\hbar}{m}\right)^2 k_i k_j q_i q_j$$


TMASP CANONICAL

CONSERVANO COMMUTATORI FOND

$$x' = U^+ x U \qquad p' = U^+ p U \qquad S' = U^+ S U$$

$$\longrightarrow [x_i, p_j] = [x_i', p_j'] = i\hbar \delta_{ij} \qquad [S_i, S_j] = [S_i', S_j'] = -i\hbar \varepsilon_{ijk} S_k$$

ES [x, S INVAR]

$$x' = x, \quad S' = S, \quad p_i' \psi = \left[p_i - i\hbar \left(U^+ \partial_i U \right) \right] \psi$$

X AVERE INVAR LOC INTRO $\pi_i = p_i - A_i$ DERIV COVAR

$$\longrightarrow \pi_i' = U^+ \pi_i U = p_i - i\hbar U^+ \partial_i U - A_i' = \pi_i + A_i - i\hbar U^+ \partial_i U - A_i'$$

IMPONGO $\pi_i = \pi_i'$ $\Rightarrow$ $A_i' = A_i - i\hbar \left(U^+ \partial_i U \right)$

X AVERE INVAR LOC

$$[\pi_i, \pi_j] = [p_i - A_i, p_j - A_j] = -i\hbar \partial_i A_j + i\hbar \partial_j A_i + [A_i, A_j]$$

INVAR x PASSO GLOB → SIMM $U(1)$ $\qquad \varphi' = e^{i\alpha}\varphi$

SE PASSO A SIMM $U(1)_{LOC}$ $\longrightarrow$ $\varphi'(\underline{x}) = e^{i\alpha(\underline{x})/\hbar}\varphi(\underline{x})$ IN GEN LA FISICA NON È INVAR

x RENDERLA INVAR DEVO PASSARE DA p_i A π_i

U IN $U(1)_{LOC}$ È $\quad U = e^{-i\alpha(x)/\hbar} \Rightarrow A'_i(x) = A_i - \mathrm{grad}_i\,\alpha(x)$

SUPPONIAMO CHE LA TH CHE MI INTERESSA SIA INVAR x UNITARIE

$$\varphi'(x) = U\varphi(x) \qquad U \in SU(N)$$

IMPONENDO INVAR LOC $\quad A'_i = A_i - i\hbar\, U_x^\dagger \partial_i U_x$ $\longrightarrow$ MATR $\in SU(N)$ CHE DIP DA x

HERMIT $\oplus$ $Tr = 0$ $\Rightarrow$ ELEM ALGEBRA

NOTE II QFT → ESTENSIONE SIMM UNITARIA

SP → NP

LIVELLI DI LANDAU

SP : $h = \frac{p^2}{2m}$ $\longrightarrow$ e^- in campo em $\quad \frac{1}{2m}\left(p + \frac{e}{c}A'\right)^2 = \frac{1}{2} m v^2$

$$\underline{v} = \underline{\dot{x}} = \frac{p + \frac{e}{c}\vec{A}}{m}$$

PRENDO $\vec{B} \parallel z$ $\longrightarrow$ HO LIBERTÀ DI SCELTA $\vec{A}$ $\rightarrow$ POSSO PRENDERE $A = \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix}$

$$B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & 0 \end{vmatrix} \rightarrow B_z = \partial_x A_y - \partial_y A_x$$

2 $\rightarrow$ ENTRAMBI INDIP DA z

$$[v_x, v_y] = \frac{1}{m^2}\left[p_x + \frac{e}{c}A_x, p_y + \frac{e}{c}A_y\right] = \frac{1}{m^2}\left(-i\hbar\,\partial_x A_y + i\hbar\,\partial_y A_x\right)\frac{e}{c} = -\frac{eB_z}{m^2 c}\, i\hbar$$

$\longrightarrow$ COMMUTATORE GAUGE INVARIANTE x È B INDIP DAL GAUGE

Tensore conducibilità

$$H_A = \sum \frac{1}{2m}\left(p_i + \frac{e}{c}\vec{A}(x_i, t)\right)^2 + U(x_1 - x_N) = \sum \frac{p_i^2}{2m} + \frac{e}{2mc}\left(\vec{p}_i \cdot \vec{A}(x_i t) + \vec{A}(\ )\,\vec{p}_i\right)$$

$$+ \frac{e^2}{2mc^2}\sum A^2(x_i t) + U(x_1 - x_N)$$

$$= H_{A=0} + \int d_3\underline{x}\ \frac{e}{2mc}\sum\left[\vec{p}_i\,\delta_3(\underline{x}-\underline{x}_i) + \delta_3(\underline{x}-\underline{x}_i)\,\vec{p}_i\right]\vec{A}(\underline{x}t) + \frac{e^2}{2mc^2}\int d_3\underline{x}\sum\delta_3(\underline{x}-\underline{x}_i)A^2(\underline{x}t)$$

NON È UN OP XKÈ INDIP DA x_i

$$= H_{A=0} + \frac{e}{c}\int d_3\underline{x}\ \vec{j}(\underline{x})\,\vec{A}(\underline{x}t) + \frac{e^2}{2mc^2}\int d_3\underline{x}\ A^2(\underline{x}t)\,\hat{n}(\underline{x})$$

IN EQ CONTINUITÀ x PROB POSIZ

EMERGE STA CORRENTE DI PROB IN ASSENZA DI $\vec{A}$

ES $\neq p/m$

$\Rightarrow \vec{j} \neq$ CORRENTE CON OSS x CAMPO MAGN

$$n_{H_A}(\underline{x}t) = e^{\frac{i}{\hbar}H_A t}\, n(\underline{x})\, e^{-\frac{i}{\hbar}H_A t}$$

IN CON DIP DA t XKÈ $\vec{A}$ DIP DA t

$$\to -\frac{i}{\hbar}\frac{\partial}{\partial t} n_{H_A}(\underline{x}t) = e^{\frac{i}{\hbar}H_A t}\left[H_A, n(\underline{x})\right] e^{-\frac{i}{\hbar}H_A t}$$

$[\hat{n}, O] = 0 \Rightarrow$ A CINETICA NON COMMUTA

DENSITÀ DI CARICA $\quad \rho(\underline{x}\, t) = -e\, \hat{n}(\underline{x} t)$ $\quad \neq \vec{j}$

$\rightarrow$ CONSERVAZ. CARICA $\Rightarrow$ EQ CONTINUITÀ $\quad \frac{1}{c}\frac{\partial}{\partial t}\rho(\underline{x} t) = -\operatorname{div}\vec{J}(\underline{x} t)$

$$\vec{J} = -e\vec{j} - \frac{e^2}{mc}\vec{A}\,\hat{n}(\underline{x})$$ $\longrightarrow$ CORRENTE CONSERVATA CHE MIS IN PRESENZA DI CAMPO

$$= \rho\,\vec{v} \longrightarrow \vec{v} = \frac{\vec{p} + \frac{e}{c}\vec{A}}{m}$$

RISPOSTA LINEARE

ACCENDO $\vec{A}$ AD UN CERTO t E PRIMA $|GS\rangle$ DI $H_{A=0}$

$$\langle \varphi(t) | J_\ell(\underline{x} t) | \varphi(t) \rangle = \langle GS | J_\ell(\underline{x} t) | GS \rangle + \frac{1}{i\hbar}\int dt'\,\theta(t-t')\,\langle GS | \left[J_\ell(\underline{x} t), V_{H_{A=0}}(t') \right] | GS \rangle$$

DIPENDE $\Rightarrow \vec{A}$ DIPENDE

$$\vec{J} = e\vec{j} - \frac{e^2}{mc^2}\hat{n}\vec{A}(\underline{x}t) \longrightarrow \text{NEL GS } \langle\vec{j}\rangle = 0$$

$$= -\frac{e^2}{mc^2}\langle n(\underline{x})\rangle_{GS} A_\ell(\underline{x}t) + \sum_m \frac{1}{i\hbar}\frac{e}{c}\int dt' d_3\underline{x}'\, \theta(t-t')\langle GS|\left[J_\ell(\underline{x}t), j_m(\underline{x}'t')\right]|GS\rangle A_m(\underline{x}'t')$$

OMETTO L'ALTRO TERM PERCHÉ QUADRATICO NELLA PERTURBAZ (ANCHE SE CONTRIB A $\vec{J}$)
$\longrightarrow$ METTO SOLO $e\vec{j}$

$$J_\ell(\underline{x}t) = -\frac{e^2}{mc^2} n_{GS}(\underline{x}) A_\ell(\underline{x}t) - \sum_m \frac{e^2}{\hbar}\int dt' d_3\underline{x}'\, D^R_{\ell m}(\underline{x}\, t\, \underline{x}'t') A_m(\underline{x}'t')$$

$$i D^R_{\ell m}(x x') = \theta(t-t')\langle GS|\left[j_\ell(x), j_m(x')\right]|GS\rangle$$

VALE $\sum_\ell \partial_\ell D^R_{\ell m} = -\frac{\partial}{\partial t} D^{RET}_{0m}(x x')$ DA DIM

$\partial_\ell = \frac{\partial}{\partial x_\ell}$

$\hookrightarrow$ COMP 0 DEL QUADRIDIM È LA DENSITÀ

SE FACCIO TUTTO

$$i\mathcal{D}^{T}_{\ell m}(x x') = \langle GS | T\, \delta j_\ell(x)\, \delta j_m(x') | GS\rangle$$

$$j_\ell(\underline{x} t) = \sum_m \int dt' d_3\underline{x}' \left[-\frac{e^2}{mc} n(x')\, \delta_{\ell m}\, \delta_3(x-x')\, \delta(t-t') - \frac{e^2}{\hbar} \mathcal{D}^R_{\ell m}(\underline{x} t \underline{x}' t') \right] A_m(\underline{x}' t')$$

VOGLIAMO CAMPO $\vec{E}$

$$-\frac{1}{c}\frac{\partial A_\ell}{\partial t} = E_\ell \xrightarrow[\text{SPW}]{} \frac{i\omega}{c} A_\ell(\underline{x}\omega) = E_\ell(\underline{x}\omega) \rightarrow A_\ell = \frac{E_\ell c}{i\omega}$$

POSSO USCIRE j LIN IN E $\Rightarrow$ TENSORE CONDUTTIVITÀ

$$j_\ell(\underline{x}\omega) = \int d_3\underline{x}' dt'\, \sigma_{\ell m}(\underline{x}\underline{x}'\omega)\, E_m(\underline{x}'\omega)$$ COME DI OHM

X ORA NON HA INVARIANZE

$$\sigma_{\ell m}(\underline{x}\underline{x}'\omega) = -\frac{e^2}{im\omega} n(\underline{x})\, \delta_{\ell m}\, \delta_3(\underline{x}-\underline{x}') - \frac{e^2}{\hbar i\omega} \mathcal{D}^R_{\ell m}(\underline{x}\underline{x}'\omega)$$

IMPONGO ORA INVAR X TRASLAZ

$$J_e(\underline{k}\,\omega) = \sigma_{em}(\underline{k}\,\omega)\,E_m(\underline{k}\,\omega) \quad \rightarrow \text{CALCOLARSI } D^R_{em} \text{ SBATTI}$$

$\Rightarrow$ USO MODELLO IN CUI e^- SI MUOVONO TRA CENTRI DI SCATTERING (BOSONI)

$\Rightarrow$ DISSIPAZIONE

SE $\omega \rightarrow 0$ (CORRENTE COST) $\Rightarrow$ DEVO CANCELLARE SINGOLARITÀ IN σ_{em} $\rightarrow$ IL MODELLO DI SCATTERING FUNZIONA

LOGAMS $D=\varepsilon E$

$$H_t = H - e\int d_3\underline{x}\ \hat{n}(\underline{x})\int d_3\underline{x}'\ \frac{\rho_{EXT}(\underline{x}'t')}{|\underline{x}-\underline{x}'|} \Rightarrow$$ INTERAZ DENSITÀ e^- INTERNA - EXT

TH RISP LIN

COMMUTATORE RITARDATO n-n

$$\boxed{\rho_{ind} = -e\,\delta n}$$

$$\delta n(\underline{x}t) = \frac{1}{\hbar}\int d_3\underline{x}'dt'\,\overset{R}{D}(\underline{x}t\,\underline{x}'t')\underbrace{\int d_3\underline{x}''\,dt''\frac{(-e)}{|\underline{x}'-\underline{x}''|}\rho_{EXT}(\underline{x}''t'')}_{\varphi(\underline{x}'t')}$$

VARIAZ DENSITÀ CARICA INDOTTA

$$\rho_{ind}(x,t) = \frac{1}{\hbar}\int d_3\underline{x}'dt'\,\overset{R}{D}(\underline{x}t\,\underline{x}'t')\int d_3x''dt''\underbrace{\frac{e^2\,\delta(t'-t'')}{|x'-x''|}}_{U^0}\rho_{EXT}(x''t'') = \int d_4x'\,\overset{R}{\Pi}(xx')\int d_4x''\,U^0(x'x'')\,\rho_{EXT}(x'')$$

$$\hbar\,\overset{R}{\Pi}(xx') = \overset{R}{D}(xx')$$

→ EQ MAXWELL $\operatorname{div}\vec{E} = 4\pi(\rho_{EXT}+\rho_{ind})$

SU SP $\underline{k}$ (INVAR SPAZIALE) $\Rightarrow$ ESPR + SEMPLICE → $i\underline{k}\,\underline{E}(\underline{k}\omega) = 4\pi\left[1+\overset{R}{\Pi}(\underline{k}\omega)\,2\mathcal{S}(\underline{k})\right]\rho_{EXT}(\underline{k})$

$$\overset{R}{\Pi} = \frac{\overset{*R}{\Pi}}{1-2\mathcal{S}(k)\,\overset{*R}{\Pi}} \Rightarrow i\underline{k}\,\underline{E} = \frac{4\pi\,\rho_{EXT}(\underline{k}\omega)}{\varepsilon^R(\underline{k}\omega)}$$

$\varepsilon^R(\underline{k}\omega) \Rightarrow$ FUNZ DIELETTRICA

→ SE MOSTRO A SIST CON $i\underline{k}\underline{D} = 4\pi\rho_{EXT}$ $\Rightarrow$ USARE SU COMPONENTE $/\!/\,\underline{k}$

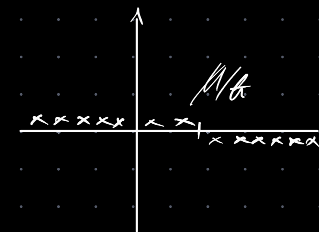
→ SUPP ISOTROPIA $\vec{D}\,/\!/\,\underline{k} \Rightarrow \underline{D} = \varepsilon^R\underline{E}$ ⟶ ε CALCOLAB SOLO CON $\overset{\circ}{\Pi}$

PERCHÉ È MOLTO ROBA ANALITICA

QUASI-PARTICLES

SVIL LEHMANN PER PROPAGATORE

$$G_{\mu\mu'}(\underline{x}\,\underline{x}'\omega)=\sum_n \frac{\langle E_0^N|\psi_\mu(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\psi^\dagger_{\mu'}(\underline{x}')|E_0^N\rangle}{\omega-\frac{1}{\hbar}(\mu+E_n^{N+1})+i\eta}+\frac{\langle E_0^N|\psi^\dagger_{\mu'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\psi_\mu(\underline{x})|E_0^N\rangle}{\omega-\frac{1}{\hbar}(\mu-E_n^{N-1})-i\eta}$$



ACCENNO $\quad G^R_{\mu\mu'}(x\,x')=\langle E_0^N|\{\psi_\mu(x),\psi^\dagger_{\mu'}(x')\}|E_0^N\rangle\,\theta(t-t')$

HA POLI TUTTI SOTTO

SE $\omega\to\pm\infty \Rightarrow G=\frac{1}{\omega}\sum_n \underbrace{\langle|\ \rangle\langle|\ \rangle}_{1}+\underbrace{\langle|\ \rangle\langle|\ \rangle}_{1}$ + CORRECTION $=\frac{1}{\omega}\langle E_0^N|\{\psi_\mu(\underline{x}),\psi^\dagger_{\mu'}(\underline{x}')\}|E_0^N\rangle=\frac{1}{\omega}\delta_{\mu\mu'}\delta_3(\underline{x}-\underline{x}')$

→ NEWS DISP DIM SENZA LEHMANN

ORA <u>INVAR × TRASLAZ E SPIN</u>

$$G_{\mu\mu'}(\underline{k}\,\omega)=\delta_{\mu\mu'}G(\underline{k}\,\omega)\Rightarrow G(\underline{k}\,\omega)=\int d\omega'\frac{A(\underline{k}\,\omega')}{\omega-\omega'+i\eta\,\mathrm{sgn}(\omega-\frac{\mu}{\hbar})}$$

⇒ FUNZ SPETTRALE

SOMMA

⇒ PROPAG DI PARTIC LIBERA

$G^0(\underline{k}\,\omega)$

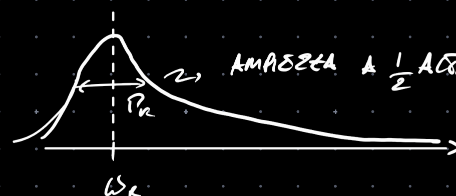
$$\begin{cases} A(\underline{k}\omega) \geq 0 \\ \int_{\pm\infty} d\omega\, A(\underline{k}\omega) = 1 \end{cases}$$ → SEMBRA SOVRAPP DI UNA MIS DI PROB DI $\frac{1}{\text{PROPAG FREE PARTIC}}$

→ CASO ELEMENTARE PARTIC LIBERA (INVAR x TRASLAZ)

$$A^0(\underline{k}\omega) = \delta\left(\omega - \frac{\varepsilon_0(\underline{k})}{\hbar}\right)$$ ⟶ CENTRATA SU LEGGE DISPERSIONE FREE PARTICLE
⇒ PROPAG LIBERO

SE HO INTERAZ ⇒ δ SI ALLARGA ⟶ APPROX LORENTZIANA

$$A(\underline{k}\omega) = \frac{\Gamma_k}{\pi}\,\frac{1}{(\omega-\omega_k)^2+\Gamma_k^2}$$



≈ AMPIEZZA A $\frac{1}{2}$ ALTEZZA ⟶ FAT TAILS

$$A \xrightarrow{\Gamma_k \to 0} A^0$$

$$G(\underline{k}\omega) = \frac{1}{\omega-\omega_k+i\Gamma_k\,\mathrm{sgn}\left(\omega-\frac{\mu}{\hbar}\right)} = G^0$$

$n(\underline{k})$ = # OCCUPAZ STATO k → CALCOLABILE CON ORCEG

APPROX DIFETTOSA = GAUS È HA MOMENTI FINITI MENTRE QUI DECADIM TROPPO LENTO x AUMENTARE

↳ VARIANZA

$$G(\underline{k}\omega) = \underbrace{\int_{\pm\Delta} d\omega' \frac{A(\underline{k}\omega')}{\omega-\omega'}}_{Re} \underbrace{- i\pi A(\underline{k}\omega)\,\mathrm{sgn}\left(\omega - \frac{\mu}{\hbar}\right)}_{Im} \rightarrow \mathrm{Im}\,G \begin{cases} <0 & \omega > \mu/\hbar \\ >0 & \omega < \mu/\hbar \end{cases}$$

$\Rightarrow$ IN $\omega = \frac{\mu}{\hbar}$ HO $\mathrm{Im}\,G = 0$

HO SEMPRE LA CONSISTENZA & LA FUNZ DI GREEN $\quad G^{con}(\underline{k}\omega) = \dfrac{1}{\omega - \omega_k + i\Gamma_k\,\mathrm{sgn}\left(\omega - \frac{\mu}{\hbar}\right)}$ 1 POLO

COME $\times$ TRASF FOURIER: Σ ONDE PIANE $\Rightarrow$ 1 FUNZ COME USO

$\longrightarrow$ TRASF DI HILBERT

$\exists$ ALTRA FORMA DELLA GF $\qquad G(\underline{k}\omega) = \dfrac{1}{\omega - \frac{\varepsilon_0^k}{\hbar} - \Sigma^*(\underline{k}\omega)}$ $\qquad$ SOL EQ DYSON IN SP $\underline{k}$ (VEDI HF 2AG)

SEPARO PER ω IN PT Re e Im

$$= \frac{1}{\left(\omega - \frac{\varepsilon_0^k}{\hbar} - \mathrm{Re}\,\Sigma^*\right) - i\,\mathrm{Im}\,\Sigma^*}$$

$$\operatorname{Im} G(\underline{k}\,\omega) = \frac{\operatorname{Im}\Sigma^*(\underline{k}\,\omega)}{\left[\omega - \frac{\varepsilon_0^k}{\hbar} - \operatorname{Re}\Sigma^*\right]^2 + \left(\operatorname{Im}\Sigma^*\right)^2}$$

$\longrightarrow$ SI ANNULLA IN $\omega = \frac{\mu}{\hbar} \Rightarrow$ DENOM PARAM NULLO

IN SP $\underline{k}$ DOVE SUP DI FERMI $\underline{k}$ T.C. $\frac{\mu}{\hbar} - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\,\frac{\mu}{\hbar}) = 0$

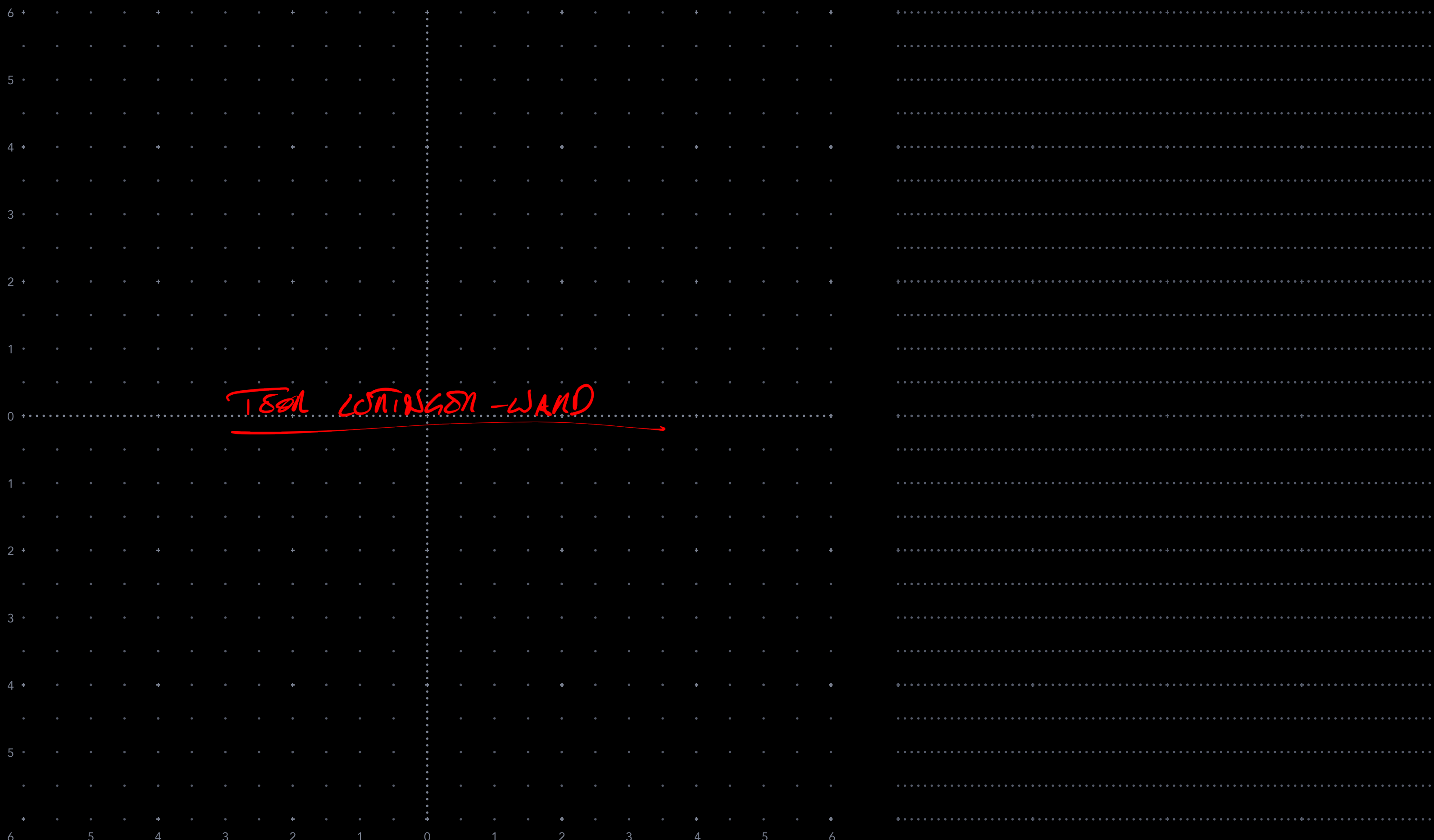
$\hookrightarrow$ Re xké $\operatorname{Im}\Sigma^*(\underline{k}\,\frac{\mu}{\hbar}) = 0$

$$\frac{\varepsilon_0^k}{\hbar} + \Sigma^*(\underline{k}, \frac{\mu}{\hbar}) = \frac{\mu}{\hbar} = \frac{\varepsilon_F}{\hbar}$$

$$G(\underline{k}\,\mu/\hbar) = \int d\omega' \frac{A(\underline{k}\,\omega')}{\mu/\hbar - \omega'}$$

$$\longrightarrow \int \frac{d\underline{k}}{(2\pi)^3}$$

TEAM GOTTINGEN -WAND



IL VOLUME RACCHIUSO NELLA SUP DI FERMI È UGUALE ALLA DENSITÀ (x DUE E SPIN $\left[\frac{1}{2}\right]$)

$$\int \frac{dk}{(2\pi)^3}\, \theta(G(\underline{k}, \mu/\hbar)) = \hat{n}$$

COSA CURIOSA

SE PRENDO HEG E ACCENDO COULOMB

SUP HEG ⟶ SF A FERMI (INTERAZ NULLA)

IL VOL $\frac{4}{3}\pi k_F^3$ RIMANE INVAR ACCENDENDO INTERAZ

CON COULOMB SF È ANCORA ISOTROPO MA I INTERAZ CHE DEFORMANO LA SUPERFICIE

$$G(\underline{k}\omega) = \frac{1}{\omega - \frac{\varepsilon^0(k)}{\hbar} - \Sigma^*(\underline{k}\omega)}$$

UNA QUASI PARTIC DEVE ESSERE UN POLO SEMPLICE DEL PROPAGATORE

$$\rightarrow \omega - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega) = 0 \qquad \text{CON } \omega(\underline{k}) = \underbrace{\omega_1(\underline{k})}_{Re} + i\underbrace{\omega_2(\underline{k})}_{Im}$$

$\Rightarrow$ G FUNZ L'ANALITICA

$$G(\underline{k}\omega) = \underbrace{\frac{z(\underline{k})}{\omega - \omega_{qp}(\underline{k})}} + G^{reg}(\underline{k}\omega)$$

$\hookrightarrow$ PT ANALITICA

$\hookrightarrow$ QUESTA FG ASSOMIGLIA A QUELLA DI UNA PARTIC

MA CON PT IMMAGINARIA FINITA ω_2

$\Rightarrow$ ROBA CHE DECADE

È COME SE DA FUNZ SPETTRALE AVESSIMO ESTRATTO E CON DAVANTI $z(\underline{k}) \leq 1$

$\rightarrow$ SE $z(\underline{k})$ È DI ORDINE 1 $\Rightarrow$ CONTRIB IMPORTANTE

$$\omega_1 + i\omega_2 - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\,\omega_1 + i\omega_2) = 0 \qquad \omega_2 = \text{VITA PARTIC}^{-1} \Rightarrow \text{VOGLIO SIA PICCOLO}$$

$$0 = \omega_1 + i\omega_2 - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega_1) - i\omega_2 \frac{\partial \Sigma^*}{\partial \omega}\Big|_{\omega_1} + \text{———}$$

ESPANDO IN δ Re E δ Im

$$0 = \omega_1(\underline{k}) - \frac{\varepsilon_0(\underline{k})}{\hbar} - \mathrm{Re}\,\Sigma^*(\underline{k}\,\omega_1(\underline{k})) \qquad \delta\,\mathrm{Re}$$

$$0 = \omega_2(\underline{k}) - \mathrm{Im}\,\Sigma^*(\underline{k}\,\omega_1(\underline{k})) - \omega_2 \left.\frac{\partial \mathrm{Re}\,\Sigma^*}{\partial \omega}\right|_{\omega=\omega_1} \qquad \delta\,\mathrm{Im}$$

$$\Rightarrow \omega_2 = \frac{\mathrm{Im}\,\Sigma^*(\underline{k},\omega_1(\underline{k}))}{1 - \left.\frac{\partial}{\partial\omega}\mathrm{Re}\,\Sigma^*(\underline{k}\omega)\right|_{\omega=\omega_1(\underline{k})}} \longrightarrow \text{NULLO DOVE } \mathrm{Im}\,\Sigma^* = 0 \Rightarrow \text{SU SUP DI FERMI}$$

$$Z(\underline{k}) = \lim_{\omega \to \omega_{polo}} G(\underline{k}\omega)(\omega - \omega_{po}) = \lim_{\omega \to \omega_{po}} \frac{\omega - \omega_p}{\omega - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega)}$$

$$\simeq \lim_{\omega \to \omega_1} \frac{\omega - \omega_1}{(\omega - \omega_1)\left[1 - \left.\frac{\partial \mathrm{Re}\,\Sigma^*}{\partial \omega}\right|_{\omega_1}\right]} = \frac{1}{1 - \left.\frac{\partial \mathrm{Re}\,\Sigma^*}{\partial \omega}\right|_{\omega_1}}$$

ω_2 TRASC

$$\Rightarrow \omega_2 = Z(\underline{k})\,\mathrm{Im}\,\Sigma^*(\underline{k}\omega)$$

QUASI PARTICELLE = POLI SEMPLICI DI G CON PESO $Z(\underline{k})$ DI ORDINE 1 E VICINO ALLA REGIONE DI k VICINO ALLA SUP DI FERMI (ω_2 PICCOLO)

→ IN PRESENZA DI QUASI PARTIC VEDIAMO CHE LA SUP DI FERMI È SUP DI DISCONTINUITÀ DEL n DI OCCUPAZ (COME SE DI FERMI È x n DI OCCUPAZ 1 IN, 0 OUT)

→ IL SALTO VALE ESATTAM Z

Team Dr MIGDAL

SSP DI FERMI È SSP DI DISCONTINUITÀ × FF OCCUPAZ $n_{\underline{k}} = \langle GS| a^\dagger_{\underline{k}\mu} a_{\underline{k}\mu} |GS\rangle$

USIAMO FORMULA VERA A PRESCINDERE DA QP

$$n(\underline{k}) = \sum_\mu \langle GS| a^\dagger_{\underline{k}\mu} a_{\underline{k}\mu} |GS\rangle = \int_{\pm\infty} \frac{d\omega}{2\pi i}\, G(\underline{k}\omega)\, e^{i\omega\eta} = \int_{\pm\infty} \frac{d\omega}{2\pi i}\, \frac{z(\underline{k})\, e^{i\omega\eta}}{\omega-\omega_1-i\omega_2} + n^{NSG}(\underline{k})$$

$$= n^{NSG}(\underline{k}) + z(\underline{k})\, \theta\left(\frac{\mu}{\hbar} - \omega_1(\underline{k})\right)$$

IL POLO QP DA QUANTO VALE LA PT RE CHE STA ± LA PT Im ALLORA CONTRIB 0

→ SALTO X $k = k_F$ (SSP DI FERMI)

$$\frac{\mu}{\hbar} - \varepsilon_0(k) - \Sigma^*\left(\underline{k}, \frac{\mu}{\hbar}\right) = 0 \quad \text{SSP FERMI}$$

CASO ISOTROPICO $k_F = (3\pi^2 n)^{1/3}$ RAGGIO SF

ABBIAMO CAPITO CHE LE QP VIVONO A LUNGO VICINO A SF DI FERMI ⇒ CHIAMIAMO IL SEGNO DI ω_1 E ω_2

RIPASSANDO DA GS A L

$$G(\underline{k}t) = \int_{\pm\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\left(\frac{Z(k)}{\omega-\omega_p} + G^{RSF}(\underline{k}\omega)\right) \qquad t>0 \Rightarrow$$

$$= \underbrace{-\frac{2\pi i}{2\pi}\, e^{-it(\omega_1(\underline{k})+i\omega_2(k))}\, Z(\underline{k})}_{\text{PT DI QP}} + \_\_\_\_$$

INTERPRETO ω_1 EFFETTIVAM COME LEGGE DI DISPERSIONE

$E(\underline{k}) = \hbar\omega_1(\underline{k})$ x ANALOG CASO TEMP LIBERA

CHIUDENDO SOTTO HO POLO SOLO IN $\omega_2 < 0 \Rightarrow \omega_1 > \frac{\mu}{\hbar} \Rightarrow \omega_2 < 0$

$\Rightarrow \omega_1 > \frac{\mu}{\hbar} \Rightarrow \omega_2 < 0$

HO 0 SE $\omega_1(\underline{k}) < \mu/\hbar \Rightarrow$ SMORZAMENTO MOLTO LENTO DOVUTO A $e^{\omega_2 t}$ SE $\omega_1 > \mu/\hbar$

LA QUASI PARTICELLA È UNA MANIFESTAZ COLLETTIVA DOVUTA AL MINSETIONE

→ STESSA CARICA DI e^- MA ≠ MASSA E RISENTE DELLA SELF EN

LE PARTIC SONO INDISTINGUIBILI ⇒ STAT COLLETTIVO → LA PROPAG DI TUTTO L'INSIEME PUÒ MANIF CON IN SOSTANZA UN CONTRIB DI PART E CREA LA QUASI PARTICELLA COME MANIF DI COMPORTAM GLOBALE

VICINO A SUP DI FERMI $\Rightarrow$ SVILUPPO ATTORNO A K_F

× QUASI PARTIC $\quad \omega_1(k) = \omega_1(k_F) + \left.\frac{\partial \omega_1}{\partial k}\right|_{k_F} (k - k_F) + \_\_\_$

× FREE PARTIC $\quad \omega^0(k) = \omega_0(k_F) + \frac{\hbar k_F}{2m}(k - k_F) + \_\_\_$

$\uparrow$

$\frac{\hbar k^2}{2m}$

× ANALOGIA $\quad \frac{\partial \omega_1}{\partial k} = \frac{\hbar k_F}{m^*} \rightarrow$ MASSA EFF

SE NON FOSSIMO IN UNA SFERA AVREI $m^* \neq \forall$ DIR

$$\omega_1(\underline{k}) - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\,\omega_1(\underline{k})) = 0 \longrightarrow \text{SVILUPPO}$$

$$\omega_1(k_F) + \frac{\hbar k_F}{m^*}(k - k_F) - \frac{\varepsilon_0(k_F)}{\hbar} - \frac{\hbar}{m} k_F (k - k_F) - \Sigma^*(k_F \omega_1(k_F)) - \left.\frac{\partial \Sigma^*}{\partial k}\right|_{k_F}(k - k_F)$$

$$- \frac{\partial \Sigma^*}{\partial \omega} \left.\frac{\partial \omega_1}{\partial k}\right|_{k_F} (k - k_F) + \_\_\_ = 0$$

TERM QUADRATICI TRASCURABILI

× SUP SUP FERMI IL TERM DI ORD 0 È 0 $\quad \frac{\mu}{\hbar} - \frac{\mu_0}{\hbar} - \Sigma^*\left(k_F, \frac{\mu}{\hbar}\right) = 0$

$$\Rightarrow \frac{\hbar k_F}{m^*} - \frac{\hbar k_F}{2m} - \frac{\partial \Sigma^*}{\partial k}(k_F,\omega_1(k_F)) - \frac{\partial \Sigma^*}{\partial \omega}\frac{\hbar k_F}{m^*} = 0 \longrightarrow$$ MI DICE m^* IN TERMINI DI Σ^*

→ L'INTERAZ COULOMBIANA NON MODIFICA MOLTO m^* RISP A m_e → MOLTO MODIF X INTERAZ e^--FONONE

↓

ANCHE 50-100%

X FONONI X SE RILEVANTE BASTA

FONONE SCAMBIATO

SU DISP CASO NON OMOGENEO

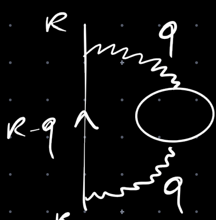
SOTTOLINEARE

X COULOMB IN HEG

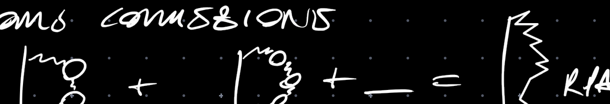
$\Sigma^*(k) =$

+ ___

↳ NON C'È IN HEG XKÉ MOM SCAMBIATO 0 → q=0 SI CANCELLA IN HEG



$\Pi^0(q)$ → BRUTTA DIVERGENZA $\frac{1}{q^2}$ → SE PASSO A POT SCH E FACCIO PERTURBAZIONE PRENDO DIAG SEMPRE PEGGIORI DI ORDINE SUCC COME CORREZIONE



$$= v_{RPA} = v^0 + v^0 \pi^0 v^0 + v^0 \pi^0 v^0 \pi^0 v^0 + \ldots$$

SOMMO SU TUTTE LE BOLLE E OTTENGO

$v_{RPA} =$ 

v^0 + $v^0 \, \pi^0 \, v_{RPA}$

$$v_{RPA} = \frac{4\pi e^2}{q^2 - 4\pi e^2 \pi^0(q)} \longrightarrow \frac{1}{q^2}$$ È ACCOMPAGNATO DA π^0 → SCHERMAGGIO CHE FA SOMMA NON DIVERGENTE COULOMBIANA

⇒ POSSO FARE TEORIA PERTURBATIVA AL I ORDINE

BASI TH DELL'ELASTICITÀ

LA CATENA UNI-DIM. PUÒ TRATTATA COME UN MEZZO ELASTICO

APPROCCIO STRUTTURA: STRUTTURA CRISTALLINA → BORN-OPPENHEIMER → ESPANSIONE EN = EN POT IONI

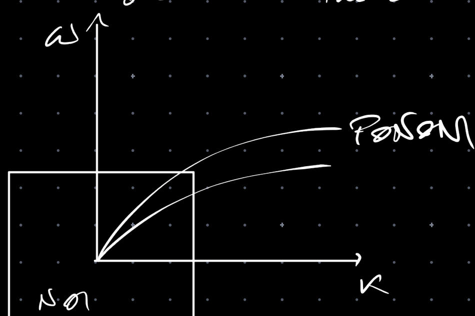
→ POSIZ EQ IONI → SVILUPPO II ORD ATTORNO AL MIN → OSCILLATORI ARMONICI DISTR ACCOPPIATI 1 VICINO

→ DISACCOPPIANDOLI IN MODI NORMALI = FONONI

TUTTO MOLTO DIP DA CHE TIPO DI CRISTALLO (BCC, FCC)

x NOI APPROCCIO + UNIV → TRATTO TUTTO COME MEZZO ELASTICO OMOG. ISOTROPO

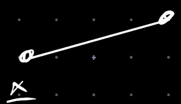
METTENDOSI SU K PICCOLI (+ DEL PASSO RETICOLARE)



PRIMA CERCATI I MEZZI CONTINUI E POI GENERALIZZ LA LEGGE DI HOOKE

→ VOGLIO TENSORE DI STRESS PROP AI GRADIENTI DELLA DEFORMAZIONE

MEZZO CONTINUO A CUI APPLICO DEFORMAZ



$\underline{x}' = \underline{x} + u(\underline{x})$ → $u(\underline{x})$ PICCOLO E CON GRADIENTI PICCOLI (NON ANDIAMO OLTRE AL LIN)

→ DESCRIVO COME CAMBIANO RAPPORTI TRA PUNTI VICINI CON TENSORE METRICO

CHE A RIPOSO $\tilde{\delta}$ è δ

$$ds'^2 = dx'^i dx'_i = \frac{\partial x'^i}{\partial x_j} dx^j \frac{\partial x'_i}{\partial x_k} dx^k \equiv g_{jk}(\underline{x})\, dx^j dx^k \longrightarrow g_{jk}(\underline{x}) = \frac{\partial x'^i}{\partial x^j} \frac{\partial x'_i}{\partial x_k}$$

NEL NOSTRO CASO

$$g_{jk}(\underline{x}) = \left(\delta_{ij} + \frac{\partial u_i}{\partial x^j}\right)\left(\delta_{ki} + \frac{\partial u_i}{\partial x^k}\right)$$

$$= \delta_{jk} + \frac{\partial u_k}{\partial x^j} + \frac{\partial u_j}{\partial x^k} \equiv \delta_{jk} + 2D_{jk}$$

↳ STRAIN TENSOR

MATR NON NEG XKÈ PRODOT È MATR UNA TRASPOSTA DELL'ALTRA

($M \geq 0 \Rightarrow \exists R$ TC $M = R^T R$

$\Rightarrow \underline{v}^T M \underline{v} \geq 0 \ \forall \underline{v}$)

COME CAMBIANO I VOLUMI?

SE D SUFF. PICCOLO RISP $\mathbb{1}$

$$\Rightarrow \det(\mathbb{1}+\varepsilon M) = 1 + \varepsilon \operatorname{Tr} M + O(\varepsilon^2)$$

$$d_3\underline{x}' = \left|\det \frac{\partial x'}{\partial x}\right| d_3 x = \sqrt{g(\underline{x})}\, d_3\underline{x}$$

$$g \equiv \det g_{ij} \rightarrow \sqrt{g} = \sqrt{\det(\mathbb{1}+2D)} = \sqrt{1+2\operatorname{Tr}D+\dots}$$

$$\simeq 1 + \operatorname{Tr} D = 1 + \operatorname{div}\vec{\underline{a}}$$

$$\Rightarrow \delta V = d_3\underline{x}' - d_3\underline{x} = \operatorname{div}\underline{a}\, V \Rightarrow \operatorname{div}\underline{a} = \frac{\delta V}{V}$$ → SI CALCOLA IN UN PTO ATTORNO A QUALE SI CONSIDERA V

→ CAMBIA ANCHE LA DENSITÀ E NON VIENE RISPETTATA L'UNIFORMITÀ

$$\rho_0\, d_3\underline{x} = (\rho_0 + \delta\rho(\underline{x}))\, d_3\underline{x}' = [\rho_0 + \delta\rho(\underline{x})](1+\operatorname{div}\underline{a})\, d_3\underline{x}$$

$$\Rightarrow \delta\rho(\underline{x}) = -\rho_0 \operatorname{div}\underline{a}$$

→ STO APPLICANDO FORZE E IL MEZZO REAGISCE CON FORZE INTERNE

→ COME DESCRIVERLE IN UN MEZZO CONTINUO?

→ TENSORE DEGLI SFORZI DI CAUCHY $\sigma_{ij}(\underline{x})$

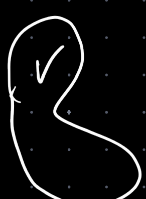
$$df_i = \sigma_{ij}(\underline{x})\, n^j\, da$$

↳ comp i-esima forza su da una volta che si è annullata la deformaz ed è nuovamente in equilibrio

x fluido ideale ∅ forza di taglio

$$\sigma_{ij} = -p\,\delta_{ij} \longrightarrow \underbrace{\sigma^k{}_k}_{\text{traccia}} = -3p \longrightarrow \text{non def in gen} \quad p = -\frac{1}{3}\sigma^k{}_k$$

Preso V delimitato da sup S del mezzo continuo → tipicam. vi agiscono forze a breve raggio



$$F_i = \int_S da\, \sigma_{ij}\, n^j \underset{\text{è un flusso}}{=} \int_V d^3x\, \partial^j \sigma_{ij}$$

divergenza σ = forza x unità di vol (ovv)

⇒ LEGGE DI NEWTON x OGNI

→ CONSID ρ IMPORTANTE XKÉ GIÀ ACCELERAZ È PICCOLA ⇒ AL I° ORDINE

$$\rho_0 \ddot{u}_j = \partial^i \sigma_{ij}$$

x CONOSCERE σ_{ij} DEVO SPECIALIZZARMI A MEZZI ELASTICI

∃ RELAZ LINEARE $\sigma_{ij}(\underline{x}) = C_{ij}^{\;\;kl}(\underline{x}) D_{kl}(\underline{x})$

↳ PROP ELASTICHE CHE VARIANO $\forall \underline{x}$

SIMM IN kl E ij XKÉ σ_{ij} SIMM

SE HA PROP OMOGENEE ⇒ NO DIP DA $\underline{x}$

SE HO SIMM TC $ij \leftrightarrow kl$ (SIMM x SCAMBIO) ⇒ CONSERVAZ ENERGIA

σ_{ij} SIMM x CONSERVAZ MOM ANGOLARE

λ, μ = COST DI LAMÉ (DIMENSIONI DI PRESSIONE)

x MEZZI ELASTICI

$$\sigma_{ij}(\underline{x}) = \lambda \delta_{ij} \underbrace{D^k_{\;k}}_{\mathrm{Tr}D}(\underline{x}) + 2\mu D_{ij}(\underline{x})$$

→ IN GIOCO 2 QUANTITÀ CHE SI TRASF IN MANIERA INDIP SOTTO ROTAZ $D^k_{\;k}$ E D_{ij}

→ SOSTITUISCO IN EQ MOTO

$$\rho_0 \ddot{u}_j = \lambda \partial_j \partial^k u_k + 2\mu\, \partial^i \partial_j = \lambda \partial_j \partial_k u_k + \mu\, \partial^i \left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} \right) = (\lambda+\mu) \partial_j \partial_k u_k + \mu \nabla^2 u_j$$

LAPLACIANO

$$\Rightarrow \rho_0 \ddot{\underline{u}} = (\lambda+\mu)\, \mathrm{grad}\,(\mathrm{div}\,\underline{u}) + \mu \nabla_2 \underline{u}$$

→ DA QUESTA EQ → DIVERG → $\rho_0 \frac{\partial^2}{\partial t^2} \mathrm{div}\,\underline{u} = (\lambda + 2\mu) \nabla_2\, \mathrm{div}\,\underline{u}$ (1)

→ ROTORE → $\rho_0 \frac{\partial^2}{\partial t^2} \mathrm{rot}\,\underline{u} = \mu \nabla_2\, \mathrm{rot}\,\underline{u}$ (2)

→ EQ ONDE CARATT DA VELOCITÀ ≠

(1) VELOCITÀ LONGITUDINALE $v_L = \sqrt{\frac{\lambda + 2\mu}{\rho_0}} \Rightarrow \left(\frac{1}{v_L^2} \frac{\partial^2}{\partial t^2} - \nabla_2 \right) \mathrm{div}\,\underline{u} = 0$

CAMPO SCALARE CHE DESCRIVE LA VARIAZ DI VOLUME

⇒ ONDE DI PRESSIONE LONGITUD

v_L = VELOCITÀ DEL SUONO NEL MEZZO

(2) $v_t = \sqrt{\frac{\mu}{\rho_0}}$ ONDE TRASVERSALI

$(\mathrm{rot}\,\underline{u})_i = \varepsilon_{ijk} \partial_j u_k \Rightarrow \perp \underline{u}$ → ONDE + LENTE $v_t \le \sqrt{2}^{-1} v_L$

I VARI MODI SONO TUTTI LEGATI AI VALORI λ E μ