THOMAS FERMI

$$T=\sum_{km}\frac{\hbar k^2}{2m}\hat n_{km} \longrightarrow \langle F|T|F\rangle=\sum_{km}\frac{\hbar^2 k^2}{2m}\theta(k_F-k)$$

$$=2V\int_0^{k_F}\frac{d_3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}=\frac{3}{5}NE_F=\frac{3}{5}nV\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$=\frac{3}{5}V\frac{\hbar^2}{2m}(3\pi^2)^{2/3}n^{5/3}$$ EN GS x n UNIF

$$\Rightarrow T[n]=\frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2)^{2/3}\int d_3x\, n^{5/3}(x)$$

x ATOMO CON Z e^-

$$E_{GS}[n]=T_{HF}[n]\underbrace{-Ze^2\int d_3x\frac{n(x)}{x}}_{U_{en}}+\underbrace{\frac{e^2}{2}\int d_3x\,d_3y\frac{n(x)n(y)}{|x-y|}}_{U_{ee}}+\underbrace{\mu\left[Z-\int d_3x\, n(x)\right]}_{\text{CONSTRAINT}}$$

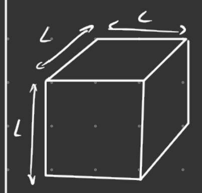
$$\frac{\delta E_{GS}[n]}{\delta n}=0\longrightarrow E_{GS}[n+\delta n]-E_{GS}[n]=\int\delta n(x)\frac{\delta E}{\delta n}d_3x+\_\_\qquad \frac{\delta n(x)}{\delta n(y)}=\delta(x-y)$$

$$\Rightarrow 0=\frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2)^{2/3}\frac{5}{3}n^{2/3}(x)-\frac{Ze^2}{|x|}+e^2\int d_3y\frac{n(y)}{|x-y|}-\mu$$

UNA MISURA x'RÉ ATOMO TROPPO GROSSO
E $n\propto x^{-3}\Rightarrow$ CORREZ GRADIENTE

JEG

BCKGROUND
CARICA POS $\rho(x)\geq 0$ FIX $\Rightarrow H=\sum\frac{p_i^2}{2m}+\frac{e^2}{2}\sum_{i\neq j}\frac{1}{|x_i-x_j|}-e\sum_{i=1}^{N}\int d_3x\frac{\rho(x)}{|x-x_i|}+\frac{1}{2}\int d_3x\,d_3y\frac{\rho(x)\rho(y)}{|x-y|}$



$\rho(x)$ OMOG $\Rightarrow$ SYS É OMOG x INVAR TRASLAZ $\Rightarrow$ USO BASE $|km\rangle$

COULOMB $\to$ LUNGO RANGE $\Rightarrow$ NO BOX $\quad\frac{1}{|x-y|}\longrightarrow\frac{e^{-\mu|x-y|}}{|x-y|}$ YUKAWA $\quad\frac{1}{\mu}\ll L\Rightarrow$ MI METTO NEL BULK

$L\to\infty\ \mu\to 0$

$$\boxed{T=\sum_{km}\frac{\hbar^2}{2m}k^2 a^+_{km}a_{km}}$$

$$1=\int d_3x\,d_3y\,|xy\rangle\langle xy|$$

$$U_{ee}=\frac{e^2}{2}\sum_{\substack{k_1k_2\\k_3k_4}}\sum_{\substack{m_1m_2\\m_3m_4}}a^+_{k_1m_1}a^+_{k_2m_2}a_{k_4m_4}a_{k_3m_3}\langle k_1m_1k_2m_2|\frac{e^{-\mu|\hat x_1-\hat x_2|}}{|\hat x_1-\hat x_2|}|k_3m_3k_4m_4\rangle=\frac{e^2}{2}\sum_{km}\delta_{m_1m_3}\delta_{m_2m_4}a_1^+a_2^+a_4a_3\int d_3x\,d_3y\langle k_1k_2|\frac{e^{-\mu|\hat x_1-\hat x_2|}}{|\hat x_1-\hat x_2|}|xy\rangle\langle xy|k_3k_4\rangle$$

$$=\frac{e^2}{2}\sum_{km}\delta_{m_1m_3}\delta_{m_2m_4}a_1^+a_2^+a_4a_3\int d_3x\,d_3y\frac{e^{-\mu|x-y|}}{|x-y|}\langle k_1|x\rangle\langle k_2|y\rangle\langle x|k_3\rangle\langle y|k_4\rangle=\frac{e^2}{2}\sum_{km}\delta_{m_1m_3}\delta_{m_2m_4}a_1^+a_2^+a_4a_3\int d_3x\,d_3y\frac{e^{-\mu|x-y|}}{|x-y|}\frac{e^{-i(k_1-k_3)x-i(k_2-k_4)y}}{V^2}\quad e^{\pm i(k-k)y}?$$

$$=\frac{e^2}{2}\sum_{km}\delta_{m_1m_3}\delta_{m_2m_4}a_1^+a_2^+a_4a_3\int d_3x\,d_3y\frac{e^{-\mu|x-y|}}{|x-y|}\frac{e^{-i(k_1-k_3)(x-y)}}{V^2}e^{i(k_4-k_2+k_3-k_1)y}$$

$x'=x-y$ SHIFT x TERM IN BORDO

$\hookrightarrow\int d_3y\, e^{ik y}=V\langle k|0\rangle=V\delta_{k0}$

$$=\frac{e^2}{2}\sum_{km}\delta_{m_1m_3}\delta_{m_2m_4}a_1^+a_2^+a_4a_3\int d_3x\frac{e^{-\mu|x'|}}{|x'|}\frac{e^{i(k_3-k_1)x'}}{V^2}V\delta_{k_1+k_2,k_3+k_4}$$

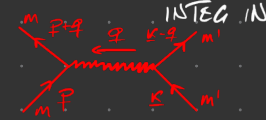
ENTRA k_1+k_2 ESCE k_3+k_4 $\Rightarrow$ CONSERVAZ MOMENTO

INTEG INDIP DA $k_1,k_2\Rightarrow$ INDIP DA ROTAZ $\Rightarrow\hat z\parallel x'\oplus$ COORD SF $2\pi\int_0^\infty r^2dr\frac{e^{-\mu r}}{r}\int_{\pm1}dy\, e^{i|k_3-k_1|r y}=2\pi\int_0^\infty r^2dr\frac{e^{-\mu r}}{r}\frac{e^{i|k_3-k_1|r}-e^{-i|k_3-k_1|r}}{i|k_3-k_1|r}=\frac{2\pi}{i|k_3-k_1|}\left[\frac{1}{\mu-i|k_3-k_1|}-\frac{1}{\mu+i|k_3-k_1|}\right]$

NB $(\nabla^2-\mu^2)\varphi=\delta(x-y)$ $\quad\frac{4\pi}{\mu^2+k^2}$

$$\Rightarrow\boxed{U_{ee}}=\frac{e^2}{2V}\sum_{\substack{k_1k_2\\k_3k_4}}\sum_{\substack{m_1m_2\\m_3m_4}}\delta_{m_1m_3}\delta_{m_2m_4}\delta_{k_1+k_2,k_3+k_4}\frac{4\pi}{\mu^2+|k_3-k_1|^2}a_1^+a_2^+a_4a_3=\boxed{\frac{e^2}{2V}\sum_{mm'}\sum_{kpq}\frac{4\pi}{\mu^2 q^2}a^+_{p+q\,m}a^+_{k-q\,m'}a_{km'}a_{pm}}$$

$\hookrightarrow$ SE $\mu\to 0\Rightarrow$ PROBL $q=0$

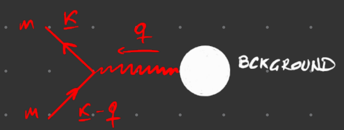


$$\boxed{U_{eb}}=\sum_{\substack{kk'\\mm'}}a^+_{km}a_{k'm'}\int d_3y(-e)\rho(y)\int\frac{d_3x}{V}e^{i(k-k')(x+y)}\frac{e^{-\mu|y-x|}}{|y-x|}\overset{x'=x-y}{=}\sum a^+_{km}a_{k'm'}\int d_3y(-e)\rho(y)\frac{e^{i(k-k')y}}{V}\underbrace{\int d_3x'\frac{e^{-\mu|x'|}}{|x'|}e^{-i(k'-k)x'}}_{\frac{4\pi}{|k-k'|^2+\mu^2}}\leftarrow\rho(x)=\frac{1}{V}\int\frac{d_3k}{(2\pi)^3}\rho(k)e^{ikx}\Rightarrow$$

$$\rho(k)=\int d_3x\,\rho(x)e^{-ikx}$$

$$\rho_0=Q$$

$$=\sum_{kk'm}a^+_{km}a_{k'm}\frac{(-e)}{V}\rho(k-k')\frac{4\pi}{|k-k'|^2+\mu^2}\underset{q=k-k'}{=}\boxed{-\frac{e}{V}\sum_{kqm}a^+_{km}a_{k-qm}\,\rho(q)\frac{4\pi}{q^2+\mu^2}}$$



$$\boxed{U_{bb}}=\frac{1}{2}\int d_3x\,d_3y\frac{\rho(x)\rho(y)}{|x-y|}e^{-\mu|x-y|}=\frac{1}{2V}\sum_{kk'}\rho_k\rho_{k'}\int d_3x\,d_3y\frac{e^{-\mu|x-y|}}{|x-y|}e^{ik(x+y)+ik'y}=\frac{1}{2V}\sum_{kk'}\rho_k\rho_{k'}\frac{4\pi}{k^2+\mu^2}\overbrace{\int d_3y\, e^{i(k+k')y}}^{V\delta_{k,-k'}}=\frac{1}{2V}\sum_k\rho_k\rho_{-k}\frac{4\pi}{k^2+\mu^2}\overset{k=q}{=}\boxed{\frac{1}{2V}\sum_q\rho_q\rho_{-q}\frac{4\pi}{q^2+\mu^2}}$$

SEPARO I TERM $\boxed{q=0}$

$$U_{ee}|_{q=0}=\frac{e^2}{2V}\sum_{mm'}\sum_{kp}\frac{4\pi}{\mu^2}\overbrace{a^+_{pm}a^+_{km'}a_{km'}a_{pm}}^{\hat N^2-\hat N}\qquad U_{eb}|_{q=0}=\frac{e}{V}\sum_{km}\frac{4\pi}{\mu^2}a^+_{km}a_{km}\rho_0=-\frac{e}{V}\frac{4\pi}{\mu^2}\hat NQ\qquad U_{bb}|_{q=0}=\frac{1}{2V}\rho_0^2\frac{4\pi}{\mu^2}=\frac{Q^2}{2V}\frac{4\pi}{\mu^2}$$

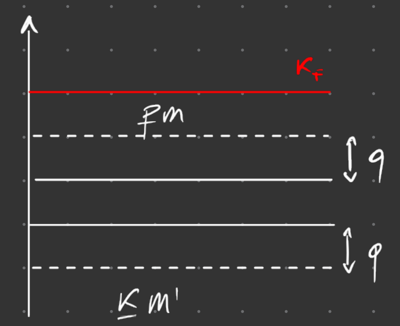
$$\Rightarrow H|_{q=0}=\frac{e^2}{2V}\frac{4\pi}{\mu^2}(\hat N^2-\hat N)-\frac{e}{V}\hat NQ\frac{4\pi}{\mu^2}+\frac{Q^2}{2V}\frac{4\pi}{\mu^2}=\frac{4\pi}{\mu^2}\frac{1}{2V}(eN-Q)^2-\frac{e^2}{2V}\frac{4\pi}{\mu^2}\hat N$$

$\uparrow$ 0 x NEUTRALITÀ $\qquad\sim\frac{1}{V\mu^2}\xrightarrow[\mu\to 0]{L\to\infty}0$

$$\Rightarrow H|_{q=0}\xrightarrow[\mu\to 0]{L\to\infty}0$$

NEL CASO FINITO $\mu L\gg 1\Rightarrow\mu L^3\to\infty\Rightarrow$ TENGO μ

∞ $\Rightarrow\frac{N}{V}\to n\longrightarrow$ RICHIEDO NEUTRALITÀ $Q=eN$



$e^-\Rightarrow\exists!$ MODO x SCAMBIARE GLI STATI $\quad p\,m=k+q\ m'$ $\quad k\,m'=p-q\ m$ $\quad\Rightarrow q=p-k$

$$H=\sum_{km}\frac{\hbar^2k^2}{2m}a^+_{km}a_{km}+\frac{e^2}{2V}\sum_{mm'}\sum_{kpq\neq 0}\frac{4\pi}{q^2}a^+_{p+qm}a^+_{k-qm'}a_{km'}a_{pm}-\frac{e}{V}\sum_{\substack{kqm\\q\neq 0}}\frac{4\pi}{q^2}\rho(q)a^+_{km}a_{k-qm}+\frac{1}{2V}\sum_{q\neq 0}\rho_q\rho_{-q}\frac{4\pi}{q^2}$$

$$\to\boxed{\rho(x)\text{ UNIF}}\Rightarrow eN=V\rho\Rightarrow\rho_k=0\text{ SE }k\neq 0\Rightarrow H=\sum_{km}\frac{\hbar^2k^2}{2m}a^+_{km}a_{km}+\frac{1}{2V}\sum_{\substack{kp\\mm'}}\sum_{q\neq 0}\frac{4\pi e^2}{q^2}a^+_{p+qm}a^+_{k-qm'}a_{km'}a_{pm}$$

PRENDO $|F\rangle$ GS EN CINETICA $\hat H_0\longrightarrow\langle F|\hat H|F\rangle\geq E_{GS}$

$$\langle F|H|F\rangle=\frac{3}{5}NE_F+\frac{1}{2V}\sum_{\substack{kp\\mm'}}\sum_{q\neq 0}\frac{4\pi e^2}{q^2}\langle F|a^+_{pm}a^+_{km'}a_{k+qm'}a_{p-qm}|F\rangle\overset{q=p-k}{=}\frac{3}{5}NE_F+\frac{1}{2V}\sum_{kpm}\frac{4\pi e^2}{|p-k|^2}\langle F|\overbrace{a^+_{pm}a^+_{km}a_{pm}a_{km}}^{-\hat N_{pm}\hat N_{km}}|F\rangle=\frac{3}{5}NE_F+\frac{1}{2V}\sum_{kpm}\frac{4\pi e^2}{|p-k|^2}\left(-\theta(k_F-k)\theta(k_F-p)\right)$$

CONTRIB COULOMB NULLO E SOLO DA SPIN $\parallel$

$$\langle F|U|F\rangle=-\frac{1}{2V}\sum_{kqm}\frac{4\pi e^2}{|k-p|^2}\theta(k_F-k)\theta(k_F-p)\xrightarrow{\text{CONT}}-\frac{1}{2V}V^2\int\frac{d_3p\,d_3k}{(2\pi)^6}\frac{4\pi e^2}{|k-p|^2}\theta(k_F-k)\theta(k_F-p)\overset{\substack{\sin\theta d\theta=-dy\\ \text{INTEG SU IF}}}{=}-\frac{V}{2}\frac{8\pi^2}{(2\pi)^6}\int_0^{k_F}p^2dp\int_0^{k_F}k^2dk\int_{\pm1}dy\frac{4\pi e^2}{k^2+p^2-2kpy}=-\frac{V}{4}\frac{4\pi e^2}{8\pi^4}\int_0^{k_F}k\,dk\int_0^{k_F}p\,dp\,\lg\left[\left(\frac{k+p}{k-p}\right)^2\right]=-\frac{Ve^2}{4\pi^3}k_F^4$$

$\hookrightarrow$ SIMM x SCAMBIO $k\leftrightarrow p$ $\quad\oplus\int_a^b dx_1\ldots\int_a^b dx_n\, f(x_1\ldots x_n)=N!\int_a^b dx_1\int_a^{x_1}dx_2\ldots\int_a^{x_{n-1}}dx_n f(x_1\ldots x_n)$ TOT SIMM

$$\Rightarrow\boxed{\langle F|H|F\rangle=\frac{3}{5}NE_F-\frac{Ve^2}{4\pi^3}k_F^4}$$

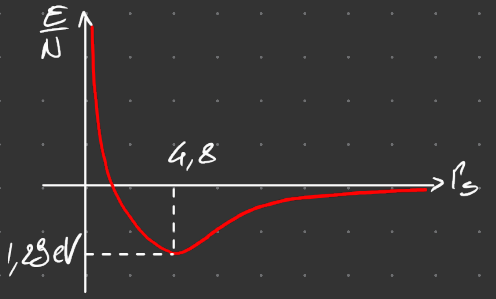
$$\frac{\langle \hat H\rangle}{N} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} - \frac{1}{n}\frac{e^2}{4\pi^3}k_F^4 = \frac{3}{5}\frac{\hbar^2}{2m}k_F^2 - \frac{3}{4\pi}e^2 k_F = \frac{3}{10}\frac{e^2 a_0^2}{a_0}k_F - \frac{3}{4\pi}\frac{e^2}{a_0}k_F a_0 = \frac{e^2}{2a_0}\left[\frac{3}{5}(a_0k_F)^2 - \frac{3}{2\pi}(a_0 k_F)\right]$$

$k_F^3 = 3\pi^2 n$ $\qquad$ $a_0 = \frac{\hbar^2}{me^2} \simeq 0{,}5\,\text{Å}$

$\longrightarrow$ DEF SEPARAZ MEDIA $\left(\frac{V}{N}\right)^{1/3} = \left[\frac{4}{3}\pi (r_s a_0)^3\right]^{1/3}$ r_s PARAM SEITZ $\Rightarrow \frac{1}{n} = \frac{3\pi^2}{k_F^3} = \frac{V}{N} = \frac{4}{3}\pi(r_s a_0)^3$

$\Rightarrow (k_F a_0)^2 = 3\pi^2 \frac{3}{4\pi} r_s^{-3} \longrightarrow a_0 k_F = \left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_s}$

$$\Rightarrow \langle F|H|F\rangle = \frac{e^2}{2a_0}\left[\frac{3}{5}\left(\frac{9\pi}{4}\right)^{2/3}\frac{1}{r_s^2} - \frac{3}{2\pi}\left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_s}\right]$$



METALLI

Al $r_s \simeq 2{,}07$
Na $r_s \simeq 3{,}93$
K $r_s \simeq 4{,}86$
Rb $r_s \simeq 5{,}20$

OSS

- $r_s = 6 \Rightarrow$ TRANSIZ DI FASE
- $r_s > 6 \Rightarrow$ CRISTALLO DI WIGNER
- STIME ESATTE DA MC: $\frac{e^2}{2a_0}\left(\frac{2{,}10}{r_s} - \frac{1{,}8}{r_s}\right)$

CONSIDERO r_0 SPAZIATURA PARTIC $V = \frac{4}{3}\pi r_0^3 N$

a_0 = RAGGIO TIPICO COULOMB

$\Rightarrow r_s = \frac{r_0}{a_0}$ CARATT DENSITÀ SYS

$r_s \to 0 \Rightarrow$ POT = PERTURBAZ PICCOLA (GAS ESTREMO DENSO)

TERMINE DI SCAMBIO

$$E_x[n] = -\frac{e^2}{2}\frac{3}{2\pi}(3\pi^2)^{1/3}\int d_3x\, n(x)^{4/3}$$

TERMODINAMICA $dE = -p\,dV + \mu\,dN$ $\longrightarrow$ x SYS OMOGENEO (INVAR TRASLAZ)

$$\begin{cases} \varepsilon = \frac{E}{N} = \varepsilon[n] \\ \mu = \frac{\partial E}{\partial N}\Big|_V = \frac{\partial}{\partial n} n\varepsilon \\ p = -\frac{\partial E}{\partial V}\Big|_N = n(\mu - \varepsilon) \end{cases}$$

EVOLUZIONE TEMPORALE $H^0 = \sum_a \hbar\omega_a c_a^+ c_a \Rightarrow c_a(t) = e^{iHt/\hbar} c_a e^{-iHt/\hbar} = e^{-i\omega_a t} c_a$ ANALOG $c_a^+(t) = e^{i\omega_a t} c_a^+$

SE H COMPLICATO $\Rightarrow$ DESCRIZ DI INTERAZ $H = H_0 + V_t$ $\longrightarrow$ EVOLUZ GOVERNATA DA FAMIGLIA A OP A 2 PARAM

$$\begin{cases} U(t,t')U(t't'') = U(tt'') \\ U(tt) = \mathbb{1} \\ U^+(tt') = U(t't) \end{cases}$$

$\longrightarrow$ PROPAGATORI (NO SVILUPPO KRS ESPON NOSTRO NON UNICO)

DA (I) DERIVANDO

$\to \partial_t U(tt')U(t't'') = \partial_t U(tt'')$

$\times U^+(tt'')$

$\Rightarrow [\partial_t U(tt')]U(t't) = [\partial_t U(tt'')]U(t''t)$

$\Rightarrow$ DIP SOLO DA t $\partial_t U(tt')U(t't) = \frac{H(t)}{\hbar}$ $\Rightarrow \boxed{i\hbar\partial_t U(tt') = H(t)U(tt')}$ IMPLICA EVOLUZ ALLA SCHRÖDINGER

$\to$ NON COMMUTANO $\Rightarrow$ IN GEN NON SO RISOLVERE

$H_t = H_0 + V_t \to U(0t) = e^{-iH_0t/\hbar} U_I(t,0) \Rightarrow U_I(tt') = U_I(t0)U_I^+(t'0) = e^{iH_0t/\hbar}U(t0)U^+(t'0)e^{-iH_0t'/\hbar}$

$e^{-iH_0t/\hbar} U_I e^{+iH_0t'/\hbar}$

$\Rightarrow$ EQ EVOL x PROPAG DI INTERAZ $i\hbar\partial_t U_I(tt') = i\hbar\partial_t\left[e^{iH_0t/\hbar}U(tt')e^{-iH_0t'/\hbar}\right] = -H_0U_I + e^{iH_0t/\hbar}\underbrace{\partial_t U}_{\frac{H_t U}{i\hbar}} e^{-iH_0t'/\hbar}i\hbar = e^{iH_0t/\hbar}V_t U \cancel{e^{-iH_0t'/\hbar}}$

$= e^{iH_0t/\hbar}V_t e^{-iH_0t/\hbar}U_I = V_{H_0}(t)U_I$

$$\longrightarrow \begin{cases} i\hbar\partial_t U_I(tt') = V_{H_0}(t)U_I(tt') \\ U_I(t't') = \mathbb{1} \end{cases} \longrightarrow \text{EQ MOTO DI INTERAZ}$$

$\longrightarrow$ PROBL CAUCHY $\Rightarrow$ EQ VOLTERRA $U_I(tt') = \mathbb{1} + \frac{1}{i\hbar}\int_{t'}^{t} dt''\, V_{H_0}(t'')U_I(t''t') = \mathbb{1} + \sum_{l=1}^{\infty}\frac{1}{(i\hbar)^l}\int_{t'}^{t}dt_1\int_{t'}^{t_1}dt_2\ldots\int_{t'}^{t}dt_l\, V_{H_0}(t_1)\ldots V_{H_0}(t_l) = T\exp\frac{1}{i\hbar}\int_{t'}^{t}dt''\,V_{H_0}(t'')$

FUNZ TOT SIMM + CAMBIO ESTR

HF $H = \sum_i h(i) + \frac{1}{2}\sum_{i\neq j} v(ij) \longrightarrow$ CERCO MIN SU DET SLATER $|\varphi\rangle = \sqrt{N!}A(N)|a_1 \ldots a_N\rangle$ TC $\langle a_i|a_j\rangle = \delta_{ij}$

$\langle\varphi|H|\varphi\rangle = \sum\langle a_i|h|a_i\rangle + \frac{1}{2}\sum_{i\neq j}\langle a_i a_j|v|a_i a_j - a_j a_i\rangle$ $\longrightarrow$ GLI a_i OTTENUTI TRAMITE UNITARIA PORTANO A STESSO DET DI SLATER

$a_i = \sum_j O_{ij}a_j \longrightarrow |\varphi'\rangle \equiv |\varphi\rangle$ $\qquad E' = E \longrightarrow$ INVAR x TRASF UNITARIA

$E_{HF} = \min_{\{a_i\}}\langle\varphi|H|\varphi\rangle - \sum_{ij}\varepsilon_{ij}[\langle a_i|a_j\rangle - \delta_{ij}]$ $\longrightarrow$ IMPONGO $\frac{\delta E}{\delta a_i} = 0\ \forall i \Rightarrow a_i \to a_i + \eta_i \Rightarrow E[a_1\ldots a_i+\eta_i\ldots] - E[a_1\ldots a_N] = \alpha\eta_i + O(\eta_i^2)$ $\to$ CONSID PT LIN MANDANDO A 0

LAGRANGIANA x CUI E_{HF} ER

$\delta E_i = \langle\eta_i|h|a_i\rangle + \sum_j\langle\eta_i a_j|v|a_i a_j - a_j a_i\rangle - \varepsilon_{ij}\langle\eta_i|a_j\rangle = 0\ \forall\eta_i \Rightarrow h|a_i\rangle + \sum_j\langle a_j|v|a_i a_j - a_j a_i\rangle = \varepsilon_i|a_i\rangle$ $\longrightarrow$ TROVATA 1 SOL $\Rightarrow$ FAMIGLIA DI SOL CON TRASF UNIT $\Rightarrow \varepsilon_{ij}$ SI PUÒ DIAGONALIZ IN ε_i

$$\Rightarrow E_{HF} = \sum\langle a_i|h|a_i\rangle + \frac{1}{2}\sum_{i\neq j}\langle a_i a_j|v|a_i a_j - a_j a_i\rangle = \frac{1}{2}\sum_i\left[\varepsilon_i - \langle a_i|h|a_i\rangle\right]$$

PROIETTO SU $\langle \underline{x}m|$ $\Rightarrow \langle\underline{x}m|h|a_i\rangle + \sum_j\langle\underline{x}m\, a_j|v|a_i a_j - a_j a_i\rangle = \varepsilon_i a_i(\underline{x}) = h a_i(\underline{x}) + \sum_j\sum_{m'm''}\int d_3x'd_3x''\,v(\underline{x}'\underline{x}'')\langle\underline{x}m\,a_j|\underline{x}'m'\underline{x}''m''\rangle\langle\underline{x}'m'\underline{x}''m''|a_i a_j - a_j a_i\rangle = \varepsilon_i a_i(\underline{x}m)$

$(h a_i)(\underline{x}m) + \sum_j\sum_{mm''}\int d_3x'd_3x''\,\delta_{mm'}\delta_3(x-x')\,a_j^*(\underline{x}''m'')\left[a_i(\underline{x}'m')a_j(\underline{x}''m'') - a_j(\underline{x}'m')a_i(\underline{x}''m'')\right] = \varepsilon_i a_i(\underline{x}m)$

$\hookrightarrow a_j^* a_i = 0$

$$\left[-\frac{\hbar^2}{2m}\nabla_x^2 + U(\underline{x})\right]a_i(\underline{x}m) + \int d_3x''\,v(\underline{x}\,\underline{x}'')\overbrace{\sum_{jm''}|a_j(\underline{x}''m'')|^2}^{\hat n(\underline{x}'')}a_i(\underline{x}m) = \left[-\frac{\hbar^2}{2m}\nabla_x^2 - U(\underline{x})\right]a_i(\underline{x}m) + U_H(\underline{x})a_i(\underline{x}m)$$

SE CONSID INVAR x TRASLAZ $[H,p] = 0 \Rightarrow V = V(\underline{x}-\underline{y}) \Rightarrow$ OGNI N-UPLA $|\underline{k}_i m_i\rangle$ SODDISFA HF

$\sum\frac{p^2}{2m}|\underline{k}_i m_i\rangle + \sum_j\langle\underline{k}_j m_j|v(|\underline{k}_i m_i\,\underline{k}_j m_j\rangle - |\underline{k}_j m_j\,\underline{k}_i m_i\rangle) = \varepsilon_i|\underline{k}_i m_i\rangle$ $\qquad$ x INVAR V CONSERVA MOMENTO $\Rightarrow \underline{k}+\underline{k}_j = \underline{k}_i + \underline{k}_j \Rightarrow \underline{k} = \underline{k}_i\ m = m_i$

$\frac{\hbar^2}{2m}k_i^2|\underline{k}_i m\rangle + \sum_j\langle\underline{k}_i m_i\,\underline{k}_j m_j|v|\underline{k}_i m_i\,\underline{k}_j m_j\rangle|\underline{k}_i m_i\rangle - \sum_j\langle\underline{k}_i m_i\,\underline{k}_j m_j|v|\underline{k}_j m_j\,\underline{k}_i m_i\rangle|\underline{k}_i m_i\rangle = \varepsilon_i|\underline{k}_i m_i\rangle$

$\Rightarrow \langle\underline{k}_i m_i\,\underline{k}_j m_j|v|\underline{k}_i m_i\,\underline{k}_j m_j\rangle = \langle\underline{k}_i\underline{k}_j|v|\underline{k}_i\underline{k}_j\rangle = \int d_3x\,d_3y\,V(\underline{x}-\underline{y})\underbrace{\langle\underline{k}_i\underline{k}_j|\underline{x}\underline{y}\rangle}_{\frac{1}{V}e^{-i\underline{k}_i\underline{x}-i\underline{k}_j\underline{y}}}\underbrace{\langle\underline{x}\underline{y}|\underline{k}_i\underline{k}_j\rangle}_{\frac{1}{V}e^{+i\underline{k}_i\underline{x}+i\underline{k}_j\underline{y}}} = \frac{1}{V^2}\int d_3x\,d_3y\,V(\underline{x}-\underline{y}) = \frac{1}{V}\int d_3x'\,V(\underline{x}') = \frac{\tilde V_0}{V}$

$\Rightarrow \frac{\hbar^2}{2m}k_i^2|\underline{k}_i m_i\rangle + \sum_j\frac{\tilde v_0}{V}|\underline{k}_i m_i\rangle - E_{xc} = \varepsilon_i|\underline{k}_i m_i\rangle$ $\longrightarrow E_{xc} = \langle\underline{k}_i m_i\,\underline{k}_j m_j|v|\underline{k}_j m_j\,\underline{k}_i m_i\rangle = \delta_{m_im_j}\langle\underline{k}_i\underline{k}_j|v|\underline{k}_j\underline{k}_i\rangle$

$= \delta_{m_im_j}\int d_3x\,d_3y\,V(\underline{x}-\underline{y})\langle\underline{k}_i\underline{k}_j|\underline{x}\underline{y}\rangle\langle\underline{x}\underline{y}|\underline{k}_j\underline{k}_i\rangle = \frac{\delta_{m_im_j}}{V}\int d_3x'\,V(\underline{x}')e^{-i(\underline{k}_i-\underline{k}_j)\underline{x}'} = \frac{\delta_{m_im_j}}{V}\tilde v(\underline{k}_i-\underline{k}_j)$

$\Rightarrow \frac{\hbar^2}{2m}k_i^2|\underline{k}_i m_i\rangle + \sum_j\frac{\tilde v_0}{V}|\underline{k}_i m_i\rangle - \sum_j\frac{\delta_{m_im_j}}{V}\tilde v_{\underline{k}_i-\underline{k}_j}|\underline{k}_i m_i\rangle = \varepsilon_i|\underline{k}_i m_i\rangle \Rightarrow |\underline{k}_i m_i\rangle$ AUTOSTATO H CON AUTOVAL $\varepsilon_i = \frac{\hbar^2}{2m}k_i^2 + n\tilde v_0 - \frac{1}{V}\sum_j\delta_{m_im_j}\tilde v_{\underline{k}_i-\underline{k}_j}$

0 x HEG = NEUTRALITÀ $\qquad$ SOLO SPIN //

x HEG DENSO $|\varphi\rangle = |F\rangle \Rightarrow \varepsilon(\underline{k}) = \frac{\hbar^2k^2}{2m} - \frac{1}{V}\sum_{|\underline{k}_i|<k_F}\frac{4\pi e^2}{|\underline{k}-\underline{k}_i|^2} = \frac{\hbar^2k^2}{2m} - \int\frac{d_3k'}{(2\pi)^3}\theta(k_F-k')\frac{4\pi e^2}{|\underline{k}-\underline{k}'|^2} = \frac{\hbar^2k^2}{2m} - \frac{2e^2k_F}{\pi}f\left(\frac{k}{k_F}\right)$

$f(x) = \frac{1}{2} + \frac{1-x^2}{4x}\ln\frac{1+x}{|1-x|}$ $\longrightarrow$ HF DÀ PESSIMA MAPPA DEI VAL DI SINGLE PARTIC

CONSIDERANDO DENSITÀ STATI x UNITÀ DI SPIN E VOL $\rho(\varepsilon) = \frac{1}{V}\sum_k\delta(\varepsilon(k)-\varepsilon) \simeq \int\frac{d_3k}{(2\pi)^3}\overbrace{\delta(\varepsilon(\underline{k})-\varepsilon)}^{\frac{\delta(k-\bar k)}{\varepsilon'(k)|_{\bar k}}} = \frac{4\pi}{(2\pi)^3}\frac{\bar k^2(\varepsilon)}{\varepsilon'(k)|_{\bar k(\varepsilon)}}$

$\longrightarrow$ PROBL IN k_F $f\left(\frac{k}{k_F}\right)\Big|_{k_F} = f(1) \longrightarrow$ DERIV DIVERGE $\Rightarrow \rho(\varepsilon_F)$ PROBLEMATICO $\longrightarrow$ NON COINCIDE CON DATI SPERIMENTALI

FUNZIONE DI GREEN A 2 PT $\boxed{iG_{ab}(tt') = \langle GS|T c_a(t)c_b^+(t')|GS\rangle}$ DOVE GLI OP DI CREAZ/DISTR HANNO EVOLUZ DA H INDIPEND TC $H|GS\rangle = E_{GS}|GS\rangle$

SE $\boxed{t > t'}$ $\langle GS|c_a(t)c_b^+(t')|GS\rangle = \langle GS|e^{iHt/\hbar}c_a e^{-iHt/\hbar}e^{iHt'/\hbar}c_b^+ e^{-iHt'/\hbar}|GS\rangle = e^{-\frac{i}{\hbar}E_{GS}(t'-t)}\underbrace{\langle GS|c_a}_{+1\text{ IN }\langle a|} e^{-iH(t-t')/\hbar}\underbrace{c_b^+|GS\rangle}_{+1\text{ IN }|b\rangle}$ $\longrightarrow$ AMPIEZZA TRANSIZ TRA STATO CON PARTIC IN a E IN b IN TEMPO $t-t'$

$\underline{x}t$

$\underline{x}'t'$

DEF PROPAG $iG_{mm'}(\underline{x}t\,\underline{x}'t') = \langle GS|T\psi_m(\underline{x}t)\psi_{m'}^+(\underline{x}'t')|GS\rangle$

DEF [T-PROD] $TA_1(t_1)\ldots A_n(t_n) = (\pm)^\sigma A_{\sigma_1}(t_{\sigma_1})\ldots A_{\sigma_N}(t_{\sigma_N})$ TC $t_{\sigma_1} > t_{\sigma_2} > \ldots > t_{\sigma_N}$ $\qquad$ + x BOSE $\quad$ − x FERMI

PROP $TA_1(t_1)\ldots A_n(t_n) = (\pm)^\pi T A_{\pi_1}(t_{\pi_1})\ldots A_{\pi_n}(t_{\pi_n})$

PROP FG DATO $\hat O = \sum c_r^+\langle r|O|r'\rangle c_{r'}$ E H CON $|GS\rangle \Rightarrow \langle GS|\hat O|GS\rangle = ?$

$\to \langle GS|c_r^+c_{r'}|GS\rangle = \langle GS|c_r^+(t)c_{r'}(t)|GS\rangle = \lim_{t'\to t^+}\langle GS|c_r^+(t')c_{r'}(t)|GS\rangle = \lim_{t'\to t^+}\langle GS|Tc_r^+(t')c_{r'}(t)|GS\rangle = \pm\lim_{t'\to t^+}\langle GS|Tc_{r'}(t)c_r^+(t')|GS\rangle$

$= \pm iG_{r'r}(tt^+)$

$\Rightarrow \langle GS|\hat O|GS\rangle = \pm i\sum_{rr'}\langle r|O|r'\rangle G_{r'r}(tt^+)$

ESEMPIO DENSITÀ A SPIN FIX

$\langle GS|\hat n_m(\underline{x})|GS\rangle = \langle GS|\psi_m^+(\underline{x})\psi_m(\underline{x})|GS\rangle = \langle GS|\psi_m^+(\underline{x}t)\psi_m(\underline{x}t)$

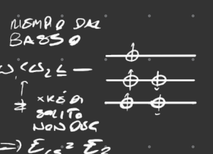
$t' = t + \eta \quad = \lim_{\eta\to 0}(\pm)\langle GS|T\psi_m(\underline{x}t)\psi_m^+(\underline{x}t^+)|GS\rangle = \pm iG_{mm}(\underline{x}t\,\underline{x}t^+)$

$\underline{x}t$

FG DI PARTICELLA NON INTERAGENTE $H=\sum_a \hbar\omega_a c_a^\dagger c_a$ × N FERMIONI

$$i G^0_{mm'}(\underline{x}t\,\underline{x}'t') = \langle GS|T\psi_m(\underline{x}t)\psi^\dagger_{m'}(\underline{x}'t')|GS\rangle \qquad \psi_m(\underline{x})=\sum_a \hat c_a\langle \underline{x}m|a\rangle \qquad \psi_m(\underline{x}t)=\sum_a e^{-i\omega_a t}\langle \underline{x}m|a\rangle \hat c_a$$

$$=\sum_{aa'} e^{-i\omega_a t+i\omega_{a'}t'}\langle \underline{x}m|a\rangle\langle a'|\underline{x}'m'\rangle\langle GS|T c_a c^\dagger_{a'}|GS\rangle$$



SVILUPPO T-ORD

$$\sum_{aa'}\left[\theta(t-t')\langle GS|c_a c^\dagger_{a'}|GS\rangle-\theta(t'-t)\langle GS|c^\dagger_{a'}c_a|GS\rangle\right]$$

$$\langle T A_t B_{t'}\rangle=\theta(t-t')\langle A_t B_{t'}\rangle\pm\theta(t'-t)\langle B_{t'}A_t\rangle$$

$$=\sum_{aa'}\left[\theta(t-t')\langle GS|\delta_{aa'}c_a c_a^\dagger|GS\rangle-\theta(t'-t)\langle GS|\delta_{aa'}c_a^\dagger c_a|GS\rangle\right]$$

$$=\sum_a e^{-i\omega_a(t-t')}\langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle\left[\theta(t-t')\theta(\omega_a-\omega_F)-\theta(t'-t)\theta(\omega_F-\omega_a)\right]$$

SE INVAR × TRASLAZ TEMP $\boxed{H\neq H(t)}$ ⇒ SP ω

$$=i\int_{\pm\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}G^0_{mm'}(\underline{x}\underline{x}'\omega)$$

HO CHE $\theta(t-t')=i\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega+i\eta}$

$$\Rightarrow G^0_{mm'}(\underline{x}t\,\underline{x}'t')=\sum_a \ldots\left[i\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega+i\eta}\theta(\omega_a-\omega_F)-i\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{+i\omega(t-t')}}{\omega+i\eta}\theta(\omega_F-\omega_a)\right]$$

$$=i\sum_a\langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle\left[\theta(\omega_a-\omega_F)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i(\omega+\omega_a)(t-t')}}{\omega+i\eta}-\theta(\omega_F-\omega_a)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{+i(\omega+\omega_a)(t-t')}}{-\omega+i\eta}\right]$$

$$=i\sum_a\langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle\left[\theta(\omega_a-\omega_F)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega-\omega_a+i\eta}-\theta(\omega_F-\omega_a)\int_{\pm\infty}\frac{d\omega}{2\pi}\frac{e^{-i(\omega+\omega_a)(t-t')}}{-\omega+i\eta}\right]$$

$\omega\to\omega-\omega_a$

$$\boxed{\Rightarrow G^0_{mm'}(\underline{x}\underline{x}'\omega)=\sum_a\langle \underline{x}m|a\rangle\langle a|\underline{x}'m'\rangle\left[\frac{\theta(\omega_a-\omega_F)}{\omega-\omega_a+i\eta}+\frac{\theta(\omega_F-\omega_a)}{\omega-\omega_a-i\eta}\right]}$$

FG SP ω × PARTIC NON INTERAG

$\hbar u_a=\hbar\omega_a u_a \qquad H_0=\sum\hbar\omega_a\hat n_a$

$$G^0_{mm'}(\underline{x}\underline{x}'\omega)=\sum_a u_a(\underline{x}m)u_a^*(\underline{x}'m')\left[\frac{\theta(\omega_a-\omega_F)}{\omega-\omega_a+i\eta}+\frac{\theta(\omega_F-\omega_a)}{\omega-\omega_a-i\eta}\right]$$

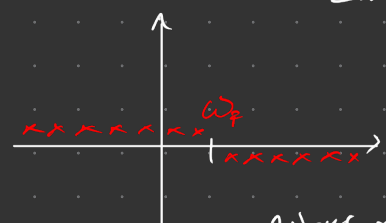
APPLICHIAMOLO $\langle GS|\hat n_m(\underline{x})|GS\rangle=-iG^0_{mm}(\underline{x}t\underline{x}t^+)=-i\int_{\pm\infty}\frac{d\omega}{2\pi}G^0_{mm}(\underline{x}\underline{x}\omega)e^{i\omega\eta}$

$$=-i\sum_a|u_a(\underline{x}m)|^2\int_{\pm\infty}\frac{d\omega}{2\pi}e^{i\omega\eta}\left[\frac{\theta(\omega_a-\omega_F)}{\omega-\omega_a+i\eta}+\frac{\theta(\omega_F-\omega_a)}{\omega-\omega_a-i\eta}\right]$$

POLO $-i\eta$ — POLO $+i\eta$

$$=\sum_a|u_a(\underline{x}m)|^2(-i)\frac{2\pi i}{2\pi}\theta(\omega_F-\omega_a)\,e^{i\omega_a\eta}=\sum_a|u_a(\underline{x}m)|^2\theta(\omega_F-\omega_a)$$

$\eta\to 0$: $\to 1$ — DENSITÀ SINGOLO STATO



DENSITÀ GS = Σ DENSITÀ SINGOLI STATI $<E_F$

ANCHE × SIS INTERAG VAL DISCRIMINANTE È ω_F

CASO IDEALE DI FERMI $\hat h=\frac{p^2}{2m}$ $|a\rangle=|\underline{k}m\rangle$ → IN BOX L $\underline{k}=\frac{2\pi}{L}\underline{n}$ $\underline{n}\in\mathbb{Z}^3$

$$G^0_{mm'}(\underline{x}\underline{x}'\omega)=\sum_{\underline{k}m''}\langle \underline{x}m|\underline{k}m''\rangle\langle \underline{k}m''|\underline{x}'m'\rangle\left[\frac{\theta(\varepsilon_k-\varepsilon_F)}{\omega-\omega_k+i\eta}+\frac{\theta(\varepsilon_F-\varepsilon_k)}{\omega-\omega_k-i\eta}\right]=\sum_{\underline{k}m''}\delta_{mm''}\delta_{m''m'}\frac{e^{i\underline{k}(\underline{x}-\underline{x}')}}{V}[\ldots]=\frac{\delta_{mm'}}{V}\sum_{\underline{k}}e^{i\underline{k}(\underline{x}-\underline{x}')}[\ldots]$$

TRASF FOURIER DISCRETA

$$=\delta_{mm'}\int\frac{d^3k}{(2\pi)^3}e^{i\underline{k}(\underline{x}-\underline{x}')}G^0(\underline{k}\omega)$$

PASSATO A SP $\underline{k}$ XKÉ HO INVAR × TRASLAZ

$$G^0(\underline{k}\omega)=\frac{\theta(\varepsilon_k-\varepsilon_F)}{\omega-\frac{\varepsilon_k}{\hbar}+i\eta}+\frac{\theta(\varepsilon_F-\varepsilon_k)}{\omega-\frac{\varepsilon_k}{\hbar}-i\eta}=\frac{1}{\omega-\frac{\varepsilon_k}{\hbar}+i\eta\,\mathrm{sgn}(\omega_k-\omega_F)}$$

AGISCE COME CAMPO SCALARE

INOLTRE $\underline{P}=\sum_{\underline{k}m}\hbar\underline{k}\,c^\dagger_{\underline{k}m}c_{\underline{k}m}\to \underline{a}\underline{P}=\sum_{\underline{k}m}\hbar\underline{k}\underline{a}\,c^\dagger_{\underline{k}m}c_{\underline{k}m}\Rightarrow e^{i\underline{a}\underline{P}/\hbar}c_{\underline{k}m}e^{-i\underline{a}\underline{P}/\hbar}=e^{-i\underline{a}\underline{k}}c_{\underline{k}m}\to e^{\frac{i}{\hbar}\underline{a}\underline{P}}\psi_m(\underline{x})e^{-\frac{i}{\hbar}\underline{a}\underline{P}}=\sum_{\underline{k}m}\langle \underline{x}|\underline{k}\rangle c_{\underline{k}m}e^{-i\underline{a}\underline{k}}=\psi_m(\underline{x}-\underline{a})$

$$\Rightarrow \text{SE } [H,\underline{P}]=0\Rightarrow G_{mm'}(\underline{x}t\underline{x}'t')\text{ DIP DA }\underline{x}-\underline{x}'\Rightarrow G_{mm'}(\underline{x}t\underline{x}'t')=\int\frac{d^3k}{(2\pi)^3}\int\frac{d\omega}{2\pi}G_{mm'}(\underline{k}\omega)e^{i\underline{k}(\underline{x}-\underline{x}')-i\omega(t-t')}$$

EQ DEL MOTO IN FG $H=\sum_{ab}h_{ab}c_a^\dagger c_b+\frac12\sum_{abcd}\mathcal{V}_{abcd}c_a^\dagger c_b^\dagger c_d c_c$ CON $\langle ab|\mathcal{V}|cd\rangle=\langle ba|\mathcal{V}|dc\rangle$ $c\leftrightarrow d\Rightarrow(-1)$

$$[c_r,c_a^\dagger c_b]=\delta_{ar}c_b \quad\oplus\quad [c_r,c_a^\dagger c_b^\dagger c_d c_c]=(\delta_{ra}c_b^\dagger\pm\delta_{rb}c_a^\dagger)c_d c_c\Rightarrow[c_r,H]=\sum_{ab}h_{ab}[c_r,c_a^\dagger c_b]+\frac12\sum\mathcal{V}_{abcd}[c_r,c_a^\dagger c_b^\dagger c_d c_c]=\sum_b h_{rb}c_b+\frac12\left(\sum_{bcd}\mathcal{V}_{rbcd}c_b^\dagger c_d c_c\pm\sum_{acd}\mathcal{V}_{ard}c_a^\dagger c_d c_c\right)$$

$$=\sum_b h_{rb}c_b+\sum_{bcd}\mathcal{V}_{rbcd}c_b^\dagger c_d c_c$$

UTILIZZI
- $H=H_1+H_2\Rightarrow\sum c_r^\dagger[c_r,H]=H_1+2H_2$
- $i\hbar\partial_t c_r(t)=i\hbar\partial_t e^{\frac{i}{\hbar}Ht}c_r e^{-\frac{i}{\hbar}Ht}=[c_r(t),H]=\sum_b h_{rb}c_b(t)+\sum_{bcd}\mathcal{V}_{rbcd}(c_b^\dagger c_d c_c)(t)$

TROVIAMO $E_{GS}=\langle GS|H|GS\rangle$ DA PROP $\langle GS|\hat O|GS\rangle\to E_{GS}=\langle GS|H_1|GS\rangle+\langle GS|H_2|GS\rangle$

$$\sum_r\langle GS|c_r^\dagger(t)i\hbar\partial_t c_r(t)|GS\rangle=\langle GS|H_1|GS\rangle+2\langle GS|H_2|GS\rangle=\lim_{t'\to t^+}\langle GS|\sum_r c_r^\dagger(t')i\hbar\partial_t c_r(t)|GS\rangle=\lim_{t'\to t^+}i\hbar\sum_r\langle GS|c_r^\dagger(t')c_r(t)|GS\rangle$$

$$=\lim i\hbar\partial_t\sum_r\langle GS|T c_r^\dagger(t')c_r(t)|GS\rangle=\lim_{t'\to t^+}(\pm)i\hbar\partial_t\,iG_{rr}(tt')$$

$$\Rightarrow E_{GS}=\frac12\langle GS|H_1|GS\rangle\pm\frac12\lim_{t'\to t^+}\sum_r i\hbar\partial_t\,iG_{rr}(tt')=\sum_{rr'}(\pm)\frac{i}{2}\langle r|h|r'\rangle G_{r'r}(tt')\pm\sum_r\frac{i}{2}\lim_{t'\to t^+}i\hbar\partial_t G_{rr}(tt')$$

EQ MOTO $iG_{ab}(tt')=\langle GS|T c_a(t)c_b^\dagger(t')|GS\rangle=\theta(t-t')\langle GS|c_a(t)c_b^\dagger(t')|GS\rangle\pm\theta(t'-t)\langle GS|c_b^\dagger(t')c_a(t)|GS\rangle$

→ DERIVO $i\partial_t G_{ab}(tt')=\delta(t-t')\langle c_a(t)c_b^\dagger(t')\mp c_b^\dagger(t')c_a(t)|GS\rangle+\theta(t-t')\langle\partial_t c_a(t)c_b^\dagger(t')\rangle\pm\theta(t'-t)\langle c_b^\dagger(t')\partial_t c_a(t)\rangle$

$$=\delta(t-t')\langle[c_a,c_b^\dagger]\rangle+\langle T\partial_t c_a(t)c_b^\dagger(t')\rangle=\delta_{ab}\delta(t-t')+\langle T\partial_t c_a(t)c_b^\dagger(t')\rangle$$

$$\Rightarrow -\hbar\partial_t G_{ab}(tt')=i\hbar\delta_{ab}\delta(t-t')+\sum_c h_{ac}\langle T\partial_t c_c(t)c_b^\dagger(t')\rangle+\sum_{b'c'd'}\mathcal{V}_{ab'c'd'}\langle T(c_{b'}^\dagger c_{d'}c_{c'})(t)c_b^\dagger(t')\rangle$$

$$\Rightarrow\boxed{\left[-\hbar\partial_t G_{ab}(tt')-i\sum_c h_{ac}G_{cb}(tt')\right]-\sum_{b'c'd'}\mathcal{V}_{ab'c'd'}\langle T(c_{b'}^\dagger c_{d'}c_{c'})(t)c_b^\dagger(t)\rangle=i\hbar\delta_{ab}\delta(t-t')}\quad (*)$$

IN ASSENZA DI OP A 2CORPI ⇒ EQ MOTO FG DI ORDINE SUP → FORMA NON CHIUSA = GERARCHIA MARTIN & SCHWINGER ⇒ TRONCO $\langle 4\psi\rangle=\langle 2\psi\rangle\times\langle 2\psi\rangle$ HF

$$i^2G_{abcd}(t_1t_2t_3t_4)=\langle GS|T c_a(t_1)c_b(t_2)c_d^\dagger(t_4)c_c^\dagger(t_3)|GS\rangle=\pm\langle GS|T c_b(t_2)c_a(t_1)c_d^\dagger(t_4)c_c^\dagger(t_3)|GS\rangle=\pm i^2G_{bacd}(t_2t_1t_3t_4)$$

→ ⊛ × $i\delta_{ac}$

$$T(c_{b'}^\dagger c_{d'}c_{c'})(t)c_b^\dagger(t')\equiv T c_{b'}^\dagger(t^{++})c_{d'}(t^+)c_{c'}(t)c_b^\dagger(t')\Rightarrow\boxed{\sum_c\left(i\hbar\delta_{ac}\partial_t-h_{ac}\right)G_{cb}(tt')+i\sum_{b'c'd'}\mathcal{V}_{ab'c'd'}\,i^2G_{d'c'bb'}(t^+tt't^{++})=\hbar\delta_{ab}\delta(t-t')}$$

⊕ 2 SALTI

→ CASO NON INTERAG ⇒ $\sum_c(i\hbar\delta_{ac}\partial_t-h_{ac})G^0_{cb}(tt')=\hbar\delta_{ab}\delta(t-t')$ → SP ω $\sum_c(\hbar\omega-h_{ac})G^0_{cb}(\omega)=\hbar\delta_{ab}\to(\hbar\omega\mathbb{1}-\hat h)G^0(\omega)=\hbar\mathbb{1}$

$\Rightarrow G^0(\omega)=\left(\omega-\frac{\hat h}{\hbar}\right)^{-1}$ RISOLVENTE ⇒ $\{|j\rangle\}$ BASE $h\Rightarrow G^0(\omega)=\sum_j\frac{|j\rangle\langle j|}{\omega-\varepsilon_j/\hbar}$ → SINGOLARITÀ SU Re ⇒ POLI IN $\mathbb{C}$

I #MODI ⇒ DEF #FG → IN BASE $\{|\underline{x}m\rangle\}$ OP h È LOC $\hat h=-\frac{\hbar^2}{2m}\nabla^2+U(\underline{x})\Rightarrow\left[\hbar\omega+\frac{\hbar^2}{2m}\nabla^2-U(\underline{x})\right]G_{mm'}(\underline{x}\underline{x}'\omega)=\hbar\delta(\underline{x}-\underline{x}')$

→ $\left[\hbar\omega+\frac{\hbar^2}{2m}\nabla^2-U(\underline{x})\right]f(\underline{x})=g(\underline{x})$ CON g SORG ASSEGNATA EQ DIF LIN NON OMOG ⇒ $f(\underline{x})=f_0(\underline{x})+\frac{1}{\hbar}\int d^3x'\,G^0(\underline{x}\underline{x}'\omega)g(\underline{x}')$

→ EQ DIF IN INDIC G^0 → SE NON SO U ⇒ ACCOPP A g E POI POSTULO/RISOLVO

TEOR. GELL-MANN & LOW

COLLEGA GS DI H A GS H_0 → ACCENSIONE ADIABATICA INTERAZIONE

$H = H_0 + gV$ → V A 2 CORPI ⇒ QUASI INTRATTABILE SENZA T. PERTURBAZ ⇒ INTERPOLO

DEF $H_\varepsilon = H_0 + g\, e^{-\varepsilon|t|} V = \begin{cases} H & |t|\to\infty \\ H_0 & t=0 \end{cases}$ $\quad \forall \varepsilon>0 \quad [H_t, H_{t'}] \neq 0$

$\Rightarrow U_\varepsilon(t,t')$ IN RAPPRES. INTERAZ. $U_\varepsilon(t,0) = e^{-\frac{i H_0 t}{\hbar}} U_{I\varepsilon}(t,0)$

→ TEOR SE $|\Phi_0\rangle$ EIGEN H_0 ⇒ $\lim_{\varepsilon\to 0^+} |\psi_{\varepsilon\pm}\rangle = \lim_{\varepsilon\to 0^+} \dfrac{U_{\varepsilon I}(0,\pm\infty)|\Phi_0\rangle}{\langle\Phi_0|U_{\varepsilon I}(0,\pm\infty)|\Phi_0\rangle} = |\psi_\pm\rangle$ EIGEN A H

SE $|\Phi_0\rangle$ NON DEG ⇒ COINCIDONO A MENO DI UNA FASE

NB NON GARANTISCE $GS^0 \to GS$ (POSSIB LEVEL CROSSING) MA NON AVVIENE SE CONSERVO LE SIMM

DIM $H_t = H_0 + g e^{+\varepsilon t} V \Rightarrow U_g(t,s)$ $s<t<0$ → $g = e^{\varepsilon\theta} \Rightarrow H_t = H_0 + e^{\varepsilon(t+\theta)} V \Rightarrow U_g(t,s) = U_1(t+\theta, s+\theta)$

$\Rightarrow \frac{\partial}{\partial g} \to \frac{\partial}{\partial\theta} \to \frac{\partial}{\partial g} U_g(t,s) = \frac{\partial\theta}{\partial g}\frac{\partial U_1}{\partial\theta}(t+\theta,s+\theta) = \frac{\partial\theta}{\partial g}\left[\frac{\partial U_1}{\partial t}(t+\theta,s+\theta) + \frac{\partial U_1}{\partial s}(t+\theta,s+\theta)\right] = \frac{\partial\theta}{\partial g}\left[\frac{\partial U_g}{\partial t}(t,s) + \frac{\partial U_g}{\partial s}(t,s)\right]$

$g = e^{\varepsilon\theta} \Rightarrow 1 = \frac{\partial\theta}{\partial g} g\varepsilon \Rightarrow \varepsilon g \frac{\partial}{\partial g} U_g(t,s) = \frac{H_t}{i\hbar} U_g(t,s) - U_g(t,s)\frac{H_s}{i\hbar}$ → $t=0 \Rightarrow H_{t=0} = H$

$\Rightarrow i\hbar\varepsilon g \frac{\partial}{\partial g} U(0,s) = H U(0,s) - U(0,s)[H_0 + g e^{\varepsilon s} V]$ ⟵ $U(\tau,0) = e^{-\frac{i}{\hbar}\tau H_0} U_I(\tau,0) = U^+(0,\tau)$

$i\hbar\varepsilon g \frac{\partial}{\partial g} U_I(0,s) = H U_I(0,s) - U_I(0,s) H_0 - g U_I(0,s) V_{H_0}(s) e^{\varepsilon s}$

$i\hbar\varepsilon g \frac{\partial}{\partial g} U_I(0,s)|\Phi_0\rangle = (H - E_0) U_I(0,s)|\Phi_0\rangle - g e^{\varepsilon s} U_I(0,s) V_{H_0}(s)|\Phi_0\rangle$

CONSID $\varepsilon<\infty \Rightarrow s \to -\infty \Rightarrow i\hbar g\varepsilon \frac{\partial}{\partial g} U_I(0,-\infty)|\Phi_0\rangle = (H-E_0) U_I(0,-\infty)|\Phi_0\rangle$ SE ELIMINO DIM $\varepsilon \to 0 \Rightarrow E_0$ AUTOVAL H

$\to \varepsilon i\hbar g \frac{\partial}{\partial g}\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle = \langle\Phi_0|(H-E_0)U_I(0,-\infty)|\Phi_0\rangle$

$$\Rightarrow \varepsilon i\hbar g \frac{\partial}{\partial g} \frac{U_I(0,-\infty)}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}|\Phi_0\rangle = \frac{(H-E_0)U_I(0,-\infty)}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}|\Phi_0\rangle - \frac{U_I(0,-\infty)}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}\frac{\langle\Phi_0|(H-E_0)U_I(0,-\infty)|\Phi_0\rangle}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}$$

$\downarrow \varepsilon \to 0$

$$0 = \frac{(H-E_0)U_I(0,-\infty)}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}|\Phi_0\rangle - \frac{U_I(0,-\infty)}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}\frac{\langle\Phi_0|(H-E_0)U_I(0,-\infty)|\Phi_0\rangle}{\langle\Phi_0|U_I(0,-\infty)|\Phi_0\rangle}$$

$\Rightarrow 0 = (H-E_0)|\psi_-\rangle - |\psi_-\rangle\langle\Phi_0|H-E_0|\psi_-\rangle$

X DIMOSTRARE U_I USO TEXP DYSON

$|\psi_\pm\rangle$ NON NORMALIZ, IDEM X $+\infty$ ∎

FORMULA DI RIDUZIONE

$H = H_0 + V$, $H|gs\rangle = E_{gs}|gs\rangle$, $H_0|\Phi_0\rangle = E_0|\Phi_0\rangle$

$\Rightarrow \forall \hat{O}$ VACO $\langle gs|T\hat{O}_{1H}(t_1)\ldots\hat{O}_{NH}(t_N)|gs\rangle = \dfrac{\langle\Phi_0|TS\,O_{1H_0}(t_1)\ldots O_{NH_0}(t_N)|\Phi_0\rangle}{\langle\Phi_0|S|\Phi_0\rangle}$

CALCOLABILE ALMENO PERTURBATIVAMENTE CON $S = U_I(+\infty,-\infty) = \mathrm{Texp}\frac{1}{i\hbar}\int_{-\infty}^{+\infty} dt\, V_{H_0}(t)$

DIM

$t_1>t_2>\ldots>t_N \Rightarrow \langle gs|O_{1H}(t_1)\ldots O_{NH}(t_N)|gs\rangle = \dfrac{\langle\psi_+|U_I^+(t_1,0)O_{1H_0}(t_1)U_I(t_1,t_2)O_{2H_0}(t_2)\ldots|\psi_-\rangle}{\langle\psi_+|\psi_-\rangle}$

$O_H(t) = U_I^+(t,0)O_{H_0}(t)U_I(t,0)$

$$= \frac{\dfrac{\langle\Phi_0|U_I^+(0+\infty)}{\langle\Phi_0|U_I(0+\infty)|\Phi_0\rangle^*}\, U_I(0\,t_1)O_1(t_1)U_I(t_1 t_2)O_2(t_2)\ldots\dfrac{U_I(0-\infty)|\Phi_0\rangle}{\langle\Phi_0|U_I(0-\infty)|\Phi_0\rangle}}{\dfrac{\langle\Phi_0|U_I^+(0+\infty)U_I(0-\infty)|\Phi_0\rangle}{\langle H\rangle^*\ \langle H\rangle}}$$

$$= \frac{\langle\Phi_0|U_I(\infty t_1)O_1(t_1)U_I(t_1,t_2)\ldots U_I(t_N-\infty)|\Phi_0\rangle}{\langle\Phi_0|U_I(+\infty,-\infty)|\Phi_0\rangle}$$

$$= \frac{\langle\Phi_0|TS\,O_1(t_1)\ldots O_N(t_N)|\Phi_0\rangle}{\langle\Phi_0|S|\Phi_0\rangle} \quad ∎$$

SE V A 2 CORPI TEXP HA # PARI DI cc^+ ⇒ ESATTO NO SGN

⇑

→ $T=1$ ⊕ $U_I(t_1,t_2) = \mathrm{Texp}\frac{1}{i\hbar}\int_{t_2}^{t_1} V_{H_0}(t')dt'$

X COMPLETARE

$iG_{mm'}(\underline{x}t\underline{x}'t') = \langle GS|T\psi_m(\underline{x}t)\psi^+_{m'}(\underline{x}'t')|GS\rangle = \dfrac{\langle\Phi_0|TS\,\psi_{mH_0}(\underline{x}t)\psi^+_{m'H_0}(\underline{x}'t')|\Phi_0\rangle}{\langle\Phi_0|S|\Phi_0\rangle}$

SOSTITUENDO A S NA VAGONATA DI V ⇒ UN BOTTO DI CORPI ⇒ TEOR DI WICK

PROPAGATORE LIBERO

$\left[\hbar\omega + \frac{\hbar^2}{2m}\nabla^2 - U(\underline{x})\right]G(\underline{x}\underline{x}'\omega) = \hbar\,\delta_3(\underline{x}-\underline{x}')$

$G_{mm'} = G\,\delta_{mm'}$

SE RISOLVIB X SINGOLA PART ✓ ⇒ G

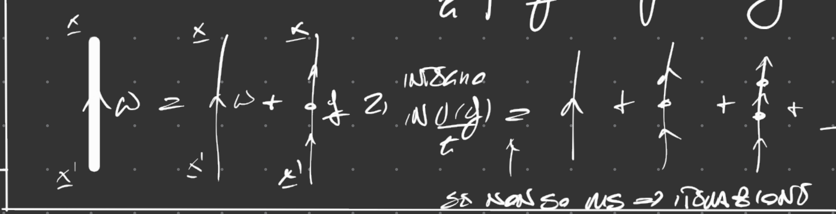
ALTRIMENTI PERTURBAZ ⇒ U IN SECONDO

$\left[\hbar\omega + \frac{\hbar^2}{2m}\nabla^2\right]G(\underline{x}\underline{x}'\omega) = \hbar\delta_3(\underline{x}-\underline{x}') + U(\underline{x})G(\underline{x}\underline{x}'\omega)$ EQ DIFF NON OMOG

G^0 = PARTIC LIBERA

$\Rightarrow G(\underline{x}\underline{x}'\omega) = G^0(\underline{x}\underline{x}'\omega) + \frac{1}{\hbar}\int d_3y\, G^0(\underline{x}y\omega)U(y)G(y\underline{x}'\omega)$ → ITERABILE

SE FACCIO AGIRE $\hbar\omega + \frac{\hbar^2\nabla^2}{2m}$ TORNO A EQ



BORN SERIES → SVIL PERTURB. USANDO INTERAZ U

TEOR DI WICK STATICO

$|GS\rangle$, base di op associati canonici $\alpha_k^\pm$ non autoag

$[\alpha_k^\pm,\alpha_j^\pm]_\pm=0$ $\quad [\alpha_k^-,\alpha_j^+]_\pm=\delta_{kj}$ t.c. $\alpha_k^-|GS\rangle=0$, $\langle GS|\alpha_k^+=0$ $\neq)$ uno creazione dell'altro

es $|F\rangle$ $\quad \alpha_k|F\rangle=0 \to \begin{cases} \text{crea } a^+_{km}|F\rangle=0 & |k|<k_F \\ \text{distrug } a_{km}|F\rangle & |k|>k_F \end{cases}$ sono una base ⇓ posso svilupp ogni op

$\langle F|\alpha_k^+=0 \to \begin{cases} \langle F|a_{km}=0 & |k|<k_F \\ \langle F|a^+_{km}=0 & |k|>k_F \end{cases}$

$$\psi_m(x)=\sum_k a_{km}\langle x|k\rangle=\sum_{|k|<k_F}\langle x|k\rangle a_{km}+\sum_{|k|>k_F}\langle x|k\rangle a_{km}=\psi_m^{(+)}(x)+\psi_m^{(-)}(x)$$

$\langle F|\psi_m^{(+)}(x)=0$ $\quad \psi_m^{(-)}|F\rangle=0$ → vale ∀ op A

$A=A^{(+)}+A^{(-)}$ → $[A_i^{(+)},A_j^{(+)}]=[A_i^{(-)},A_j^{(-)}]=0$ $\quad [A_i^{(-)},A_j^{(+)}]=c\mathbb{1}\neq0$

DEF [op ordinato normale]

$N[A_1^{(\pm)}\dots A_N^{(\pm)}]=(\pm)^\sigma A_{i_1}^{(+)}\dots A_{i_N}^{(-)}$ norm ordin (+ sx, − dx)

→ non univoco

$A_1A_2=(A_1^++A_1^-)(A_2^++A_2^-)=N[A_1A_2]+[A_1^-,A_2^+]$ ↗ c-numero

DEF [contraz] $A_1A_2=N[A_1A_2]+\overline{A_1A_2}$ ⇒ $\overline{A_1A_2}=[A_1^-,A_2^+]$

⇒ $\langle GS|A_1A_2|GS\rangle=\overline{A_1A_2}$ ⇒ $\langle N\rangle=0$

→ se $A_1\dots A_N$ ⇒ teor Wick

PROP [analoga T-ord] $N[A_1\dots A_N]=(\pm)^\sigma N[A_{\sigma_1}\dots A_{\sigma_N}]$

DEF [contraz dist n] $\overline{A(A_1\dots A_n)A'}=(A_1\dots A_n)\overline{AA'}(\pm)^n$

DIM x STEPS ① $A_0^-A_1^+\dots A_n^+=([A_0^-,A_1^+]_\pm)A_2^+\dots A_n^+\mp A_1^+A_0^-A_2^+\dots A_n^+=\overline{A_0A_1}A_2^+\dots A_n^+\pm A_1^+\underbrace{[A_0^-,A_2^+]_\pm}_{\overline{A_0A_2}}A_3^+\dots A_n^+ + A_1^+A_2^+A_0^-A_3^+\dots A_n^+$

$=\overline{A_0A_1}A_2^+\dots A_n^+ + \overline{A_0A_1A_2}\dots A_n^+ + \dots + \overline{A_0A_1^+\dots A_{n-1}^+A_n}+\underbrace{(\pm)^nA_1^+\dots A_n^+A_0^-}_{N[A_0A_1^+\dots A_n^+]}$

② x induz $A_0^-N[A_1\dots A_n]=N[A_0^-A_1\dots A_n]+\sum_{k=1}^n N[\overline{A_0\dots A_k}\dots A_n]$

③ banale $A_0N[A_1\dots A_n]=N[A_0\dots A_n]+\sum_k N[\overline{A_0\dots A_k}\dots A_n]$

TEOR

$$A_1\dots A_n=\underbrace{N[A_1\dots A_n]}_{N_{n0}}+\underbrace{\sum_{\sqcap}N[A_1\overline{\dots}A_n]}_{N_{n1}}+\underbrace{\sum_{\sqcap\sqcap}N[A_1\overline{\dots}A_n]}_{N_{n2}}+\dots+\sum_{\text{contraz tot}}N[\overline{A_1\dots A_n}]$$

n pari ⇒ c-num

n disp ⇒ a spari.

$$\Rightarrow \langle GS|A_1\dots A_n|GS\rangle=\begin{cases}0 & \text{n disp}\\ \sum_{\text{contraz tot}}N[\overline{A_1\dots A_n}] & \text{n pari}\end{cases}$$

DIM (x induz)

$A_1A_2=N[A_1A_2]\pm\overline{A_1A_2}$ → Hp $A_1\dots A_n=\sum_{k=0}^{[n/2]}N_{nk}$

$A_0A_1\dots A_n=A_0(N_{n0}+N_{n1}+\dots)$ ← $A_0N[A_1\dots A_n]=N[A_0\dots A_n]+\sum_k N[\overline{A_0\dots A_k}\dots A_n]$ $=N_{n+1,0}+N[\overline{A_0N_{n0}}]$

$=N_{n+1,0}+N[\overline{A_0N_{n0}}]+N[A_0N_{n1}]+N[\overline{A_0N_{n1}}]+\dots=N_{n+1,0}+N_{n+1,1}+\dots$

↪ $N[\overline{A_0N_{nk}}]+N[A_0N_{nk}]=N_{n+1,k+1}$

ESEMPIO

$$\langle F|\hat n_\uparrow(x)\hat n_\uparrow(x')|F\rangle=\langle F|\psi^+_\uparrow(x)\psi_\uparrow(x)\psi^+_\uparrow(x')\psi_\uparrow(x')|F\rangle=$$

$$=\langle F|\psi^+_\uparrow(x)\psi_\uparrow(x)|F\rangle\langle F|\psi^+_\uparrow(x')\psi_\uparrow(x')|F\rangle+\langle F|\psi^+_\uparrow(x)\psi_\uparrow(x')|F\rangle\langle F|\psi_\uparrow(x)\psi^+_\uparrow(x')|F\rangle$$

$$\Rightarrow\langle F|\hat n_\uparrow(x)\hat n_\uparrow(x')|F\rangle-\langle F|\hat n_\uparrow(x)|F\rangle\langle F|\hat n_\uparrow(x')|F\rangle=\langle F|\psi^+_\uparrow(x)\psi_\uparrow(x')|F\rangle\langle F|\psi_\uparrow(x)\psi^+_\uparrow(x')|F\rangle$$

non banale $\lim_{x\to x'}$ ⇒ esclusione Pauli

TEOR DI WICK ORDINATO

OSS $\langle GS|A_1\ldots A_n|GS\rangle = \begin{cases} 0 & \text{se } n \text{ DISP } \forall |GS\rangle \\ \sum \text{CONTRAZ TOT} & \text{se } n \text{ PARI} \end{cases}$ → CONTRAZ = FUNZ A 2 PT ⇒ FUNZ n PARTIC = $\prod$ FUNZ A 2 PT → CALCOLO ESATTO FUNZ LIBERE

INTERAZ A 2 CORPI

$$V = \frac{e^2}{2V}\sum_{k p q\neq 0}\sum_{m m'} a^\dagger_{q m} a^\dagger_{k m'} a_{k+q\, m'} a_{p-q\, m} \Rightarrow \langle F|V|F\rangle = \langle F|a^\dagger_{p m} a_{p-q\, m}|F\rangle\langle F|a^\dagger_{k m'} a_{k+q\, m'}|F\rangle - \langle F|a^\dagger_{q m} a_{k+q\, m'}|F\rangle\langle F|a^\dagger_{k m'} a_{p-q\, m}|F\rangle$$

$\neq 0 \Leftrightarrow q = 0$

x EVOLUZ + BASTANO → EVOLUZ TEMP NON DEVE MUTARE IL CARATT DI α^I_k

$H = \sum \hbar\omega_a c^\dagger_a c_a \longrightarrow c_a(t) = e^{-i\omega_a t} c_a \qquad c^\dagger_a(t) = e^{+i\omega_a t} c^\dagger_a$ ⇒ ANCORA ANN NORM

TEOR $|GS\rangle, \alpha^\pm_k, H$ → x TEOR WICK NEC CHE $\alpha^\pm_k(t) = f_k(t)\,\alpha^\pm_k \Rightarrow \alpha^-_k(t)|GS\rangle = 0,\ \langle GS|\alpha^+_k(t) = 0\ \forall t \Rightarrow [\alpha^-_j(t), \alpha^-_k(t')]_\pm = [\alpha^+_j(t), \alpha^+_k(t')]_\pm = 0 \qquad [\alpha^-_j(t), \alpha^+_k(t')]_\pm = \delta_{kj}\, g(t,t')\,\mathbb{1}$

$$\Rightarrow T A_1(t_1)\ldots A_n(t_n) = N[A_1(t_1)\ldots A_n(t_n)] + \sum N[A_1\ldots A_n] + N[A_1\ldots A_n] + \ldots$$

STEP 1 $T A_1(t_1)A_2(t_2) = \theta(t_1-t_2)A_1(t_1)A_2(t_2) \pm \theta(t_2-t_1)A_2(t_2)A_1(t_1) = \theta(t_1-t_2)\left(N[A_1(t_1)A_2(t_2)] + A_1(t_1)A_2(t_2)\right) \pm \theta(t_2-t_1)\left(N[A_2(t_2)A_1(t_1)] + A_2(t_2)A_1(t_1)\right)$

$= [\theta(t_1-t_2)+\theta(t_2-t_1)]N[A_1(t_1)A_2(t_2)] + \theta(t_1-t_2)A_1(t_1)A_2(t_2) + \theta(t_2-t_1)A_2(t_2)A_1(t_1) = N[A_1(t_1)A_2(t_2)] + A_1(t_1)A_2(t_2)$ ↳ comb c-NUM = c-NUM

$\Rightarrow \langle GS|T A_1(t_1)A_2(t_2)|GS\rangle = A_1(t_1)A_2(t_2)$ ⟶ OSS (PROP VALIDO SOLO x TEOR) $A_1(t_1)A_2(t_2) = \pm A_2(t_2)A_1(t_1)$

$\Rightarrow \langle GS|T\psi_{m_1}(1)\psi^\dagger_{m_2}(2)|GS\rangle = i\,G^0_{m_1 m_2}(12)$ ⟶ $\psi\psi = 0$ SE $|GS\rangle$ EIGEN DI $\hat N$; $\neq 0$ SE SBILANCIATO SE ROTTO SPECK $\Rightarrow \langle GS|T\psi_{m_1}\psi_{m_2}|GS\rangle = i\,F^0(12)$ CORRELATORE ANOMALO

STEP 2

x Hp INDUZ SU N $\quad T A_1(t_1)\ldots A_N(t_N) = \sum_{k=0}^{[N/2]} N_{Nk}$ ⟶ $N_{Nk} = \sum_{k \text{ CONTRAZ TOT}} N[\ldots]$ ⟶ x N+1 OP $T A_0(t_0)A_1(t_1)\ldots A_N(t_N) = (\pm)^\ell A_\ell\, T A_0\ldots \cancel{A_\ell}\ldots A_N$ PONGO PRIMA A_ℓ CORRISP A t max

$= (\pm)^\ell A_\ell\left(N[A_1\ldots\cancel{A_\ell}\ldots A_N] + \sum_m N[A_0\ldots\cancel{A_\ell}\ldots A_N] + \ldots\right)$

ORA OSS CONTRAZ INDIP DA t

$A_0 N[A_1\ldots A_n] = N[A_0\ldots A_n] + \sum_k N[A_0\ldots A_k\ldots A_n] \Rightarrow (\pm)^\ell N[A_\ell A_0\ldots\cancel{A_\ell}\ldots A_n] + (\pm)^\ell\sum_k N[A_\ell A_0\ldots A_k\ldots\cancel{A_\ell}\ldots A_n] + (\pm)^\ell\sum_m N[A_\ell A_0\ldots\cancel{A_\ell}\ldots A_n] + (\pm)^\ell\sum_k N[A_\ell A_0\ldots A_k\,\cancel{A_\ell}\ldots]$

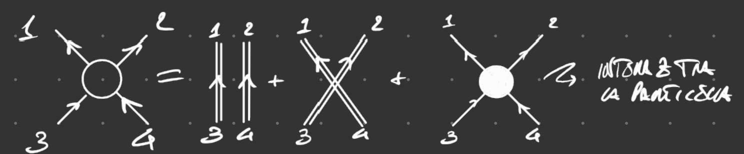
REINSERISCO ℓ A POSIZ NEL I TERM ⟶ $N[A_0\ldots A_n]$

II ⟶ $A_\ell A_k = \langle T A_\ell A_k\rangle$ ma A_ℓ HA t max ⇒ $T = \mathbb{1}$ ⟶ SPOSTO ⇒ ± DAVANTI ⇒ NO SEGN ⇒ $N[A_0\ldots A_n]$ x RES TERM, INSERISCO $A(A_1\ldots A_n)A' = (\pm)^n AA'(A_1\ldots A_n)$

$\Rightarrow N[A_0\ldots A_n] + N[A_0\ldots A_n] + \ldots$ ■

APPLICAZIONE $\quad i^2 G(1234) = \langle GS|T\psi(1)\psi(2)\psi^\dagger(4)\psi^\dagger(3)|GS\rangle = iG^0(13)\,iG^0(24) \pm iG^0(14)\,G^0(23)$ ⟶ INTER DI PARTIC NON INTERAG SE POSSO SCRIVERE BENE $\hat H$ ⇒ FG CHE FATTORIZZANO

+ IN GEN $\quad G(1234) = \underbrace{G(13)G(24) \pm G(14)G(23)}_{\text{PT NON CONNESSA}} + \underbrace{G(1234)}_{\text{PT CONNESSA}}$



DIAGRAMMI DI FEYNMAN

se $H = H_0 + V$ e vale Wick Tond $\Rightarrow$ calcolo il propag.

Ferm idue

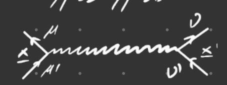
$$G_{mm'}(xx') = \frac{1}{i}\langle E|T\varphi_m(x)\varphi^+_{m'}(x')|E\rangle \overset{\downarrow}{=} \frac{1}{i}\frac{\langle E_0|TS\varphi_m(x)\varphi^+_{m'}(x')|E_0\rangle}{\langle E_0|S|E_0\rangle}$$

$$S = T\exp\frac{1}{i\hbar}\int_{\pm\infty} dt_1 V_{H_0}(t_1) = T\exp\frac{1}{2i\hbar}\int_{\pm\infty} dt_1\int d_3x_1 d_3x_1' \sum_{\mu\mu'\nu\nu'} \mathcal{V}_{\mu\mu'\nu\nu'}(x_1,x_1')\varphi^+_\mu(x_1t_1^{+++})\varphi^+_\nu(x_1't_1^{++})\varphi_{\nu'}(x_1't_1^+)\varphi_{\mu'}(x_1t_1)$$

2 pt. parziali $\neq$ ma svolti ciascuno x re stesso t_1 $\to$ Tond $t_1^{+++}\ t_1^{++}\ t_1^{+}$ $\to$ stesso tempo xk H_0 interaz. statica $\Rightarrow$ istantanea

$$= T\exp\frac{1}{2i\hbar}\int d_4x_1 d_4x_1' \sum_{\mu\mu'\nu\nu'} \mathcal{V}^0_{\mu\mu'\nu\nu'}(x_1x_1')\varphi^+_\mu(x_1^{++})\varphi^+_\nu(x_1'^{++})\varphi_{\nu'}(x_1'^{+})\varphi_{\mu'}(x_1)$$ con $\mathcal{V}^0_{\mu\mu'\nu\nu'} = \mathcal{V}_{\mu\mu'\nu\nu'}(xx')\,\delta(t-t')$

$$= T\exp\frac{1}{2i\hbar}\int d1\,d2 \sum_{\mu\mu'\nu\nu'} \mathcal{V}_{\mu\mu'\nu\nu'}(12)\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)$$



→ faccio sviluppo → num ord 0 $S = \mathbb{1} \Rightarrow G^0_{mm'}$

ord 1 $\frac{1}{2i\hbar}\int d1\,d2 \sum_{\text{spin}} \mathcal{V}^0_{\mu\mu'\nu\nu'}(12)\langle E_0|T\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)\varphi_m(x)\varphi^+_{m'}(x')|E_0\rangle$

Calcolo contraz tot x applicare T con Wick Tond

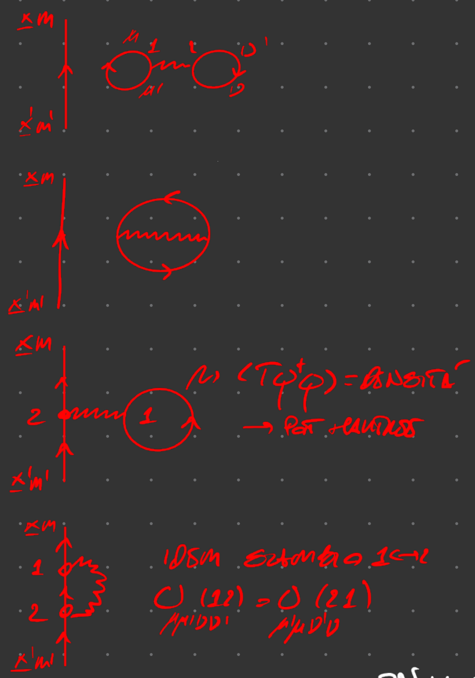
$\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)\varphi_m(x)\varphi^+_{m'}(x')$

$\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)\varphi_m(x)\varphi^+_{m'}(x')$

$\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)\varphi_m(x)\varphi^+_{m'}(x')$

stesso diag x scambio $1 \leftrightarrow 2$

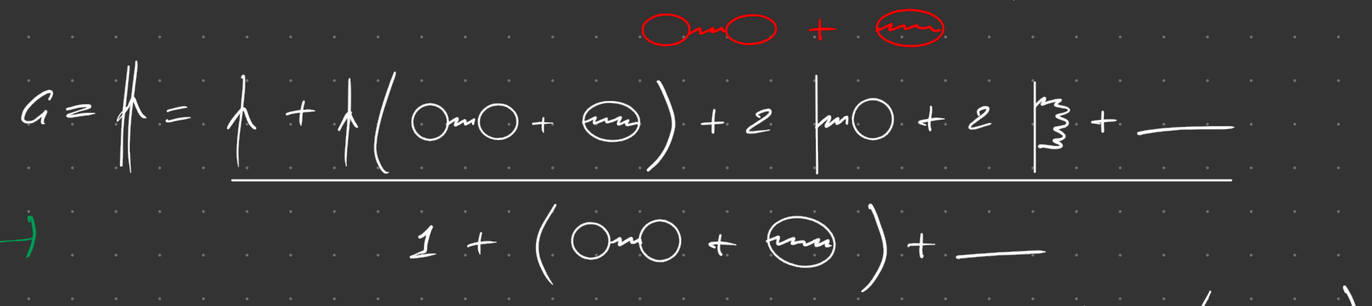
$\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)\varphi_m(x)\varphi^+_{m'}(x')$



→ tot 6 diag → 3! contraz tot → svilupp a ord N $\Rightarrow$ 2N+1 creat / 2N+1 distrutt $\Rightarrow$ $(2N+1)!$ contraz = $(2N+1)!$ diag

CANCELLAZIONE DIAGRAMMI DI VUOTO

$\langle E_0|S|E_0\rangle = 1 + \underbrace{\langle E_0|T\varphi^+_\mu(1)\varphi^+_\nu(2)\varphi_{\nu'}(2)\varphi_{\mu'}(1)|E_0\rangle}$ I° ord



Tr. fondam $\Rightarrow$ sviluppo denom come serie geom $\Rightarrow$ al num $1-(\ldots)$

$= \ldots \Rightarrow$ inutile calcolare i diag di vuoto

→ fattore 2 degli ordini cancellato da 1/2 al num $\Rightarrow$ solo diag

DIAG VUOTO $\longleftrightarrow$ CONTRAZ INTERNE = INTERAZ TRA PT MUTI

$$G_{mm'}(xx') = \frac{1}{i}\frac{\langle E_0|TS\varphi_m(x)\varphi_{m'}(x')|E_0\rangle}{\langle E_0|S|E_0\rangle} = \frac{1}{i\langle E_0|S|E_0\rangle}\sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\frac{1}{k!}\int d1\ldots dk\,\langle E_0|TV_{H_0}(t_1)\ldots V_{H_0}(t_k)\varphi_m(x)\varphi^+_{m'}(x')|E_0\rangle$$

$$= \sum_{k=0}^{\infty} G^{(k)}_{mm'}(xx')$$ $\Rightarrow$ come calcolare $G^{(k)}$ di serie perturbativa?

→ somma di diag di ord k con → k linee $\sim\mathcal{V}^0$
2k+1 → G^0 contraz tot di 4k+2 φ
2k pt sp-tempo

il denom cancella tutti i diag con fattori di vuoto

in gen $\dfrac{\langle E_0|TSO_1(t_1)\ldots O_N(t_N)|E_0\rangle}{\langle E_0|S|E_0\rangle} = \langle E_0|TSO_1(t_1)\ldots O_N(t_N)|E_0\rangle_*$

→ fattori di vuoto quando contraggo le V tra loro

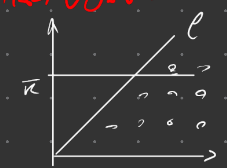
→ numeratore $\displaystyle\sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\frac{1}{k!}\int dt_1\ldots dt_k\,\langle E_0|TV_1\ldots V_k O_1(t_1)\ldots O_N(t_N)|E_0\rangle$

classifico contraz in * e non (1 o + V contratti tra loro comunque)

$$= \sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\frac{1}{k!}\int dt_1'\ldots dt_k' \sum_{\ell=0}^{k}\binom{k}{\ell}\langle E_0|TV_1\ldots V_\ell\ldots V_k\ldots A_1\ldots A_N|E_0\rangle$$

modi in cui l di si contraggono con se stessi — contratti tot — contratti tot senza diag vuoto

$$= \sum \underline{\qquad} \langle E_0|TV_1\ldots V_\ell|E_0\rangle\langle E_0|TV_{\ell+1}\ldots V_k A_1\ldots A_n|E_0\rangle$$



scambio somme → se fix $\ell \Rightarrow k: 0 \to \infty$

$$= \sum_{\ell=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{\ell!(k-\ell)!}\frac{1}{(i\hbar)^{\ell+k-\ell}}\int dt_1\ldots dt_\ell\langle E_0|TV_1\ldots V_\ell|E_0\rangle\int dt_{\ell+1}\ldots dt_k\langle E_0|V_{\ell+1}\ldots V_k A_1\ldots A_n|E_0\rangle_*$$

$$= \sum_{\ell=0}^{\infty}\frac{1}{\ell!}\frac{1}{(i\hbar)^\ell}\int dt_1\ldots dt_\ell\langle E_0|V_1\ldots V_\ell|E_0\rangle\sum_{k'=0}^{\infty}\frac{1}{k'!}\frac{1}{(i\hbar)^{k'}}\int dt_1\ldots dt_{k'}\langle E_0|TV_1\ldots V_{k'}A\ldots A|E_0\rangle_*$$

num fattorizzato $\Rightarrow$ I° fattore = $\langle E_0|S|E_0\rangle$ / II fatt = sviluppo *

REGOLE DI FEYNMAN

$G = \sum G^{(k)}$ $\qquad G^{(k)}$ diagrammi distinti senza fattori di vuoto
- → 2k pt interni
- → k $\mathcal{V}^0$ che uniscono pt int.
- → 2k+1 G^0 da contraz $TV_1\ldots V_k\varphi_x\varphi^+_{x'}$

- ∀ diag $G^{(k)}$ prefattore $\frac{1}{(i\hbar)^k}\frac{1}{i}\, i^{2k+1} = \left(\frac{1}{\hbar}\right)^k$
- prefattore $(\pm1)^{\#\text{loop}}$ $\Rightarrow$ conteggia le contraz a dist. $VV_1\ldots V_kV'$
- ∀ vertice ho $\int d\underline{x}_i\,dt_i \sum_{\sigma\sigma'}$
- $G^0(gtg't) \equiv G^0(gtg't^+)$
- si omettono i fatt. $\frac{1}{k!}$ → x simm scambio indici muti

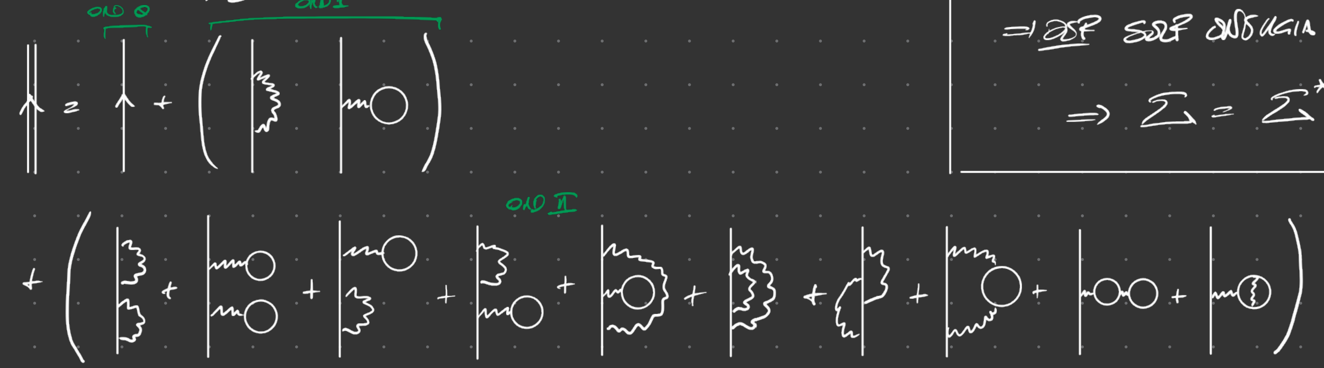
  $\frac{1}{2^k}$ → x simm scambio V in T prod

ESEMPIO DIAG FEYNMAN $\quad T\,\varphi_3^+\varphi_4^+\varphi_4\varphi_3\,\varphi_1^+\varphi_2^+\varphi_2\varphi_1\,\varphi_x\varphi_{x'}^+$

$V_{34} \qquad V_{12}$

SCAMBIO INTRA NS LA COPPIA $1/2^K$

SCAMBIO COPPIE (INVERTO i V) $\to K!$



ORD 0 ... ORD I ... ORD II

$$\overset{x\,m}{\underset{x'm'}{\mu'\,\nu \ \ \nu'}} = \frac{i}{\hbar}\sum_{\mu\mu'\nu\nu'}\int d1\,d2\; G^0_{m\mu}(x1)\,G^0_{\mu'\nu}(12^+)\,G^0_{\nu'm'}(2x')\,U_{\mu\mu'\nu\nu'}(12)$$

$\times$ GAS ELETTRONICO INTERAZ COULOMB

$$\Rightarrow G^0_{\mu\nu} = \delta_{\mu\nu}G^0 \quad U_{\mu\mu'\nu\nu'}(12) = \frac{4\pi e^2}{|x_1-x_2|}\,\delta(t_1-t_2)\,\delta_{\mu\mu'}\delta_{\nu\nu'} \quad \delta_{\mu\nu} G^0$$

$$= \frac{i}{\hbar}(-1)\sum_{SPIN}\int d1\,d2\;\delta_{m\mu}G^0(x1)\,\delta_{\mu'm'}G^0(1x')\,\delta_{\mu\mu'}\delta_{\nu\nu'}\,\frac{4\pi e^2}{|x_1-x_2|}\,\delta(t_1-t_2)\,G^0(22^+)$$

$$= -\frac{i}{\hbar}\sum_{\mu\nu}\int d1\,d2\;\delta_{m\mu}\delta_{\mu m'}\,G^0(\underline{x}t,\underline{x}_1t_1)\,G^0(\underline{x}_1t_1\underline{x}'t')\,\frac{4\pi e^2}{|x_1-x_2|}\,\frac{1}{i}\langle T\varphi_\nu(x_2t_1)\varphi^+_\nu(x_2t_1^+)\rangle$$

$$= \frac{1}{\hbar}\delta_{mm'}\int d_3x_1\,dt_1\;G^0(\underline{x}t\,\underline{x}_1t_1)\,G^0(\underline{x}_1t_1\,\underline{x}'t')\,U_H(\underline{x}_1) \qquad \Rightarrow U_H(\underline{x}) = \int d\underline{y}\,\hat n(\underline{y})\,v(\underline{x}\underline{y})$$

INTEG SPAZIALE COULOMB $\times$ $\hat n$ DENSITÀ SU $|E_0\rangle$ IMPERTURB = POT HARTREE

NO ESPR ANALITICA SOTTRARMAM INFINIS

$\to$ SE INVAR $\times$ TRASLAZ MOLTO SEMPLICE $\to$ SU K $\frac{1}{2}$ DELL'INTEG

SELF ENERGIA $\quad$ = + (Σ) $\Rightarrow$ SELF EN = SOMMA INSIEME PERTURBATIVO AL PROPAGATORE $\to$ FINE A 2 PTI

$\Sigma_{\mu\mu'}(xy) = \ldots + \ldots + \ldots + \_\_\_ \to$ 2 PTA DI COULOMB $\to$ LOCALI = 1 PTO SP TEMPO, BILOC = 2 PTI SP TEMPO

$\Sigma_{LOC} = \ldots + \ldots + \ldots + \ldots + \_\_\_ \to$ SENSIBILE AMMETTENDO G $\quad G^0 \to G^1$

ORD I $\qquad$ ORD II $\qquad$ ORD III

$\Rightarrow$ IN GEN $\quad \boxed{\Sigma_{LOC} = \ldots G} \to$ SE U^0 INDIP DA SPIN $\Rightarrow$

$$\Sigma_{LOC\,\mu\mu'} = \frac{\pm i}{\hbar}\delta(x-x')\sum_{\nu\nu'}\int dy\,U^0_{\mu\mu'\nu\nu'}(xy)\,G^0_{\nu\nu'}(xy) \quad (S+\ldots)$$

$$= \frac{\pm i}{\hbar}\delta(x-x')\,\delta_{\mu\mu'}(2S+1)\int dy\,v(xy)\,G(y t y t^+) \quad n(y)$$

$G^1 \to G^2$

$\times$ LA BILOCALE POSSO SPEZZARE ALCUNI DIAG IN 2 DIAG DI Σ ELIMINANDO G^0 $\to$ NON POSSO

$\Rightarrow$ DEF SELF ENERGIA PROPRIA = SOTTOSET DIAG DI Σ NON SEPARABILI RIMUOVENDO G^0

$$\Rightarrow \Sigma = \Sigma^* + \Sigma^*\Sigma^* + \Sigma^*\Sigma^*\Sigma^* + \_\_\_ = \Sigma^* + \Sigma^*\Sigma \text{ INDUTTIVA} \Rightarrow G = G^0 + G^0\Sigma G^0 = G^0 + G^0\Sigma^*G^0 + \ldots + \_\_\_$$

$\boxed{G = G^0 + G^0\Sigma^* G} \to$ EQ DYSON

CALCOLANDO Σ^* TROVO G

APPROX $\boxed{\Sigma^* = \ldots} \Rightarrow G^{APP} = \ldots + \ldots = \ldots + \ldots + \ldots + \_\_\_$

$$G(12) = G^0(12) + \int d3\,d4\; G^0(13)\,\Sigma^*(34)\,G(42)$$

SE ORA CONSIDERO $\boxed{\Sigma^* = \ldots} \Rightarrow \ldots = \ldots + \ldots = \ldots + \ldots + \ldots + \_\_\_$

CONSIDERO E SCAMBIO $\qquad$ # DIAG SONO PARTICOLARI CON L'ORDINE

EXPLICIT $\quad G_{\mu\nu}(xx') = G^0_{\mu\nu}(xx') + \sum_{\rho\sigma}\int dx_1dx_2\; G^0_{\mu\rho}(xx_1)\,\Sigma^*_{\rho\sigma}(x_1x_2)\,G^0_{\sigma\nu}(x_1x')$ $\quad$ CON $\Sigma^* = \Sigma_{LOC} + \Sigma^*_{XC} = \ldots + \ldots + \ldots + \ldots + \_\_\_$

POSSO VAL

$\Rightarrow$ RISOLVE PARZIALE $\times$ SE SOLO CONTRIB LOCALI $\to$ QUANDO SO' $\hat n$ ($\times$ SE DA DFT) $= U_H(x) \Rightarrow \Sigma^* = \Sigma_{LOC} \to$ POT HARTREE $\int dy\,v(xy)\,n(y) \to$ VALE $\times$ INTERAZ INDIP DA SPIN

$G^H = G^0 + G^0 \ldots G^H = \ldots + \ldots + \ldots + \ldots + \ldots + \_\_\_$

INVAR $\times$ TRASLAZ $\quad$ SE H E H_0 INVAR $\times$ TRASLAZ $\Rightarrow$ LO È $G \Rightarrow$ ANCHE $\Sigma \Rightarrow$ EQ DYSON IN SP K DIVENTA ALGEBRICA

$$G_{\mu\nu}(\underline{k}\omega) = G^0_{\mu\nu}(\underline{k}\omega) + \sum_{\rho\sigma}G^0_{\mu\rho}(\underline{k}\omega)\,\Sigma^*_{\rho\sigma}(\underline{k}\omega)\,G_{\sigma\nu}(\underline{k}\omega)$$

$\to$ SE I COMMUTATORI SONO DIAG NELLO SPIN E G^0 È GAS IDEALE $\Rightarrow$

$$G(\underline{k}\omega) = \frac{1}{\omega - \frac{\varepsilon_k}{\hbar} - \Sigma^*(\underline{k}\omega)}$$

HARTREE-FOCK $\quad$ PROPAGATORE DI 2 PARTIC CREATE A TEMPI $\pm$

$\to i^2 G_{abcd}(t_at_bt_ct_d) = \langle T c_a(t_a)c_b(t_b)c_d^+(t_d)c_c^+(t_c)\rangle$

$\times$ TEOR HO SIMM DI SCAMBIO

$$G_{abcd}(t_at_bt_ct_d) = \pm G_{bacd}(t_bt_at_ct_d) = \pm G_{abdc}(t_at_bt_dt_c) \to \text{LA FINE A 4 PTI APPARE IN EQ MOTO}$$

$\times$ INTERAZ INDIP DA SPIN $\quad [i\hbar\partial_t - h(\underline{x})]\,G_{mm'}(\underline{x}t\underline{x}'t') = \hbar\delta_{mm'}\delta(\underline{x}-\underline{x}')\delta(t-t') + i\sum_{m''}\int dy\,v(\underline{x}y)\,G_{mm''m'm''}(\underline{x}t\,\underline{y}t\,\underline{x}'t'\,\underline{y}t^{++})$

$\to$ I° EQ DI UNA GERARCHIA ∞ $\to$ MARTIN E SCHWINGER $\Rightarrow$ APPROX + SEMPLICE = TRONCARE QUI $\to$ BASATO SU DECOMPOSIZ ESATTA DETTA CLUSTER EXPANSION

FORMULA MENZIONE $\qquad$ * SOTTOINTEG.

CLUSTER EXPANSION $\quad G(1234) = G(13)G(24) \pm G(14)G(23) + G^{CON}(1234)$

DIM $\quad i^2G(1234) = \langle \Phi_0|T\varphi_1\varphi_2\varphi_4^+\varphi_3^+|\Phi_0\rangle_K = \sum_{K=0}^{\infty}\frac{1}{(i\hbar)^K}\frac{1}{K!}\int dt_1\_dt_K\langle\Phi_0|T V(t_1)\_V(t_K)\varphi_1\varphi_2\varphi_4^+\varphi_3^+|\Phi_0\rangle$

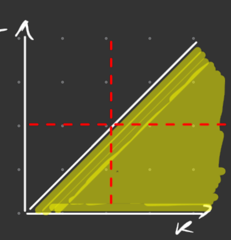
NEL CONSID LE CONTRAZ TOT ESCLUDERE DIAG DI VUOTO ETC

DOPO L COMB SI IMPONE $\qquad$ # SCELTE PERM

① CONTRAZ TOT FATTORIZZ $\quad \varphi_1\varphi_3^+$ CON L V, $\varphi_2\varphi_4^+$ CON $(K-L)$ V $\quad L = Q\_R \to$

$$\sum_{K=0}^{\infty}\frac{1}{(i\hbar)^K}\frac{1}{K!}\sum_{L=0}^{K}\binom{K}{L}\int dt_1\_dt_L\langle V(t_1)\_V(t_L)\varphi_1\varphi_3^+\rangle\int ds_1\_ds_{K-L}\langle T V(s_1)\_V(s_{K-L})\varphi_2\varphi_4^+\rangle$$

SCAMBIO SOMME



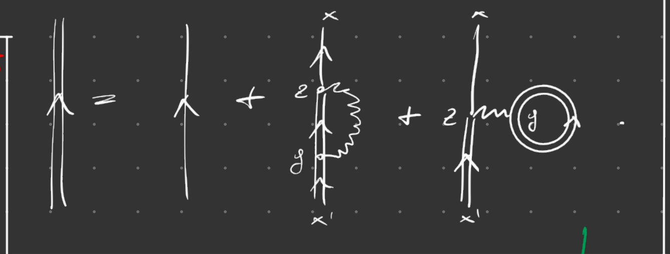
EQ DYSON X HF G^{HF} RISOLVE DYSON X SCELTA SPECIALE DI Σ^*

X G^0 PARTIC LIBERA $\left[i\hbar\partial_t - h(\underline{x})\right] G^0_{mm'} = \delta_{mm'}\,\hbar\,\delta_3(x-x')\,\delta(t-t')$

DA ⊛

X HF $G^{HF}_{mm'}(xx') = G^0_{mm'}(xx') + \frac{i}{\hbar}\sum_{m''m'''}\int d_4z\, G^0_{mm''}(xz)\int d_4y\, U^0(zy)\, G^{HF}_{m'''m''}(zy)\, G^{HF}_{m''m'}(yx')$

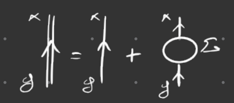
$-\frac{i}{\hbar}\sum_{m''m'''}\int d_4z\, G^0_{mm''}(xz)\, G^{HF}_{m''m'}(zx')\int d_4y\, U^0(zy)\, G^{HF}_{m''m'''}(yy^+)$



INTERAZ EFFETTIVA E POLARIZZAZIONE

IN SIS MB U^0 TRA 2 PARTIC "VESTITA"

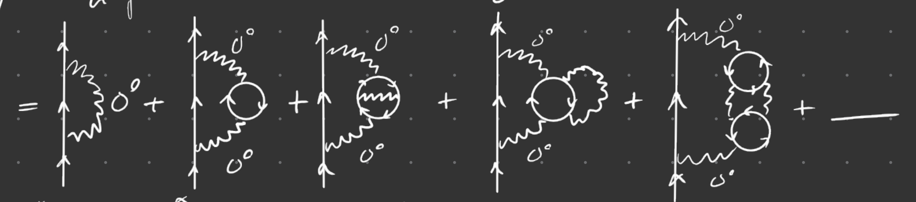
DA RISULTATI DI POL MB

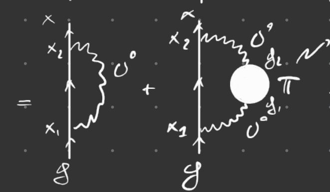


→ I ORDINE $\quad \wr = \frac{i}{\hbar}\int dx_1 dx_2\, G^0(xx_1)\, G^0(x_1x_2^+)\, G^0(x_2y)\, U^0(x_1x_2)$

SE ESCLUDO TUTTI I DIAG CON CONFIG FIX DI 3 PROPAG G^0 LA SOMMA DI QUESTI MI DEF L'INTERAZ EFF $U(x_1x_2)$

$U = \frac{i}{\hbar}\int dx_1 dx_2\, G^0(xx_1)\, G^0(x_1x_2)\, G^0(x_2y)\, U(x_1,x_2)$





POLARIZZAZIONE

$\longrightarrow U(x_1x_2) = U^0(x_1x_2) + \int dy_1 dy_2\, U^0(x_1y_1)\, \Pi(y_1y_2)\, U(y_2x_2)$

DEFINIAMO Π CONSID LE CONTRAZ DI G CHE MANTENGONO LE 3 G^0

DA $G = \langle E_0|TS\varphi_x\varphi_y^+|E_0\rangle_*$ ESPANDIAMO S E SPECIFICHIAMO LE 3 G^0 $\begin{smallmatrix} y\to x_2\\ x_2\to x_1\\ x_1\to x\end{smallmatrix}$ LEVANDOCI U^0 INIZ E LE 2 U^0 $\begin{smallmatrix} x_1\to y_1\\ x_2\to y_2\end{smallmatrix}$

↑ + ⌇

NO K! XKE' STO CONSID V PARTIC ⇒ NO SCAMBIO — NO EQUIV XKE' COMB

$\frac{1}{i}\sum_{k=2}^{\infty}\frac{1}{(i\hbar)^k}\int_{\pm\infty}\langle TV_1 \ldots V_k\varphi_x\varphi_y^+\rangle^c_* dt_1\ldots dt_k \overset{V_1}{=} \frac{1}{i}\sum_{k=2}^{\infty}\frac{1}{(i\hbar)^k}\int_{\pm\infty} dt_1\ldots dt_k\int dx_1dy_1\, U^0(x_1,y_1)\langle T\varphi^+_{x_1}\varphi^+_{y_1}\varphi_{y_1}\varphi_{x_1}V_2\ldots V_k\varphi_x\varphi^+_y\rangle^c_*$

$= \frac{1}{i}\sum_{k=2}^{\infty}\frac{1}{(i\hbar)^k}\int_{\pm\infty} dt_1\ldots dt_k\int dx_1dy_1\, U^0(x_1y_1)\, iG^0(xx_1)\langle T\varphi^+_{y_1}\varphi_{y_1}\varphi_{x_1}V_2\ldots V_k\varphi^+_y\rangle^c_* =$ $\quad\langle\ \rangle^c$ VOGLIO QUELLE CHE CONTRAGGO SOLO CON $x_1\,x_2$

$\overset{V_2}{=} \frac{1}{i}\sum_{k=2}^{\infty}\frac{1}{(i\hbar)^k}\int_{\pm\infty} dt_1\ldots dt_k\int dx_1dy_1dx_2dy_2\, U^0(x_1y_1)\, U^0(x_2y_2)\, iG^0(xx_1)\langle T\varphi^+_{y_1}\varphi_{y_1}\varphi_{x_1}\varphi^+_{x_2}\varphi^+_{y_2}\varphi_{y_2}\varphi_{x_2}\underbrace{V_3\ldots V_k}_{k-2}\varphi^+_y\rangle^c_*$

$= \frac{1}{i}\sum_{k=2}^{\infty}\frac{1}{(i\hbar)^k}\int_{\pm\infty} dt_1\ldots dt_k\int dx_1dy_1dx_2dy_2\, U^0(x_1y_1)\, U^0(x_2y_2)\, iG^0(xx_1)\, iG^0(x_1x_2)\, iG^0(x_2y)\langle TV_3\ldots V_k\varphi^+_{y_1}\varphi_{y_1}\varphi^+_{y_2}\varphi_{y_2}\rangle^c_*$

TRASLATO

$= \frac{1}{i}\frac{i^3}{(i\hbar)^2}\int dx_1dy_1dx_2dy_2\, G^0(xx_1)\, G^0(x_1x_2)\, G^0(x_2y)\, U^0(x_1y_1)\, U^0(x_2y_2)\sum_{k=0}^{\infty}\frac{1}{(i\hbar)^k}\int dt_1\ldots dt_k\langle TV(t_1)\ldots V(t_k)\varphi^+_{y_1}\varphi_{y_1}\varphi^+_{y_2}\varphi_{y_2}\rangle^c_*$

$= \frac{i}{\hbar}\int dx_1dx_2\, G^0(xx_1)\, G^0(x_1x_2)\, G^0(x_2y)\left[\frac{1}{i\hbar}\int dy_1dy_2\, U^0(x_1y_1)\, U^0(x_2y_2)\langle TS\varphi^+_{y_1}\varphi_{y_1}\varphi^+_{y_2}\varphi_{y_2}\rangle^c_*\right.$ USCIAMO INTERACTION PICTURE $\Rightarrow \langle E_0|TS\varphi^+\varphi\varphi^+\varphi|E_0\rangle^c_* = \langle E|T\varphi^+\varphi\varphi^+\varphi\rangle^c$ — LE CONTRAZ CHE NON SONO IN $\langle\ \rangle^c$ SONO QUELLE FATTORIZZABILI IN FUNZ DI y_1 E FUNZ DI y_2

$\Rightarrow \langle E|T\varphi^+_{y_1}\varphi_{y_1}\varphi^+_{y_2}\varphi_{y_2}\rangle^c = \langle E|T\,\delta(\varphi^+_{y_1}\varphi_{y_1})\,\delta(\varphi^+_{y_2}\varphi_{y_2})|E\rangle$ DOVE $\delta\hat{O} = \hat{O} - \langle E|\hat{O}|E\rangle\mathbb{1} \Rightarrow \langle E|T\delta A\delta B|E\rangle = \langle E|TAB|E\rangle - \langle A\times B\rangle \Rightarrow U_{\mu\mu'\sigma\sigma'}(x_1x_2) = U^0_{\mu\mu'\sigma\sigma'}(x_1x_2) + \sum_{\rho\rho'\sigma\sigma'}\int dy_1dy_2\, U^0_{\mu\mu'\rho\rho'}(x_1y_1)\,\Pi_{\rho\rho'\sigma\sigma'}(y_1y_2)\, U^0_{\sigma\sigma'\nu\nu'}(y_2x_2) \rightarrow \Pi_{\rho\rho'\sigma\sigma'}(xy) = \frac{1}{i\hbar}\langle E|T\delta[\varphi^+_{\rho}(y_1)\varphi_{\rho'}(y_1)]\,\delta[\varphi^+_{\sigma}(y_2)\varphi_{\sigma'}(y_2)]|E\rangle$

SOMMA DI TUTTI I DIAG TOPOLOGICAM DISTINTI CONNESSI IN CUI SI CREA PARTIC CON SPIN ρ E SI DISTRUGGE CON SPIN ρ' IN X E DOPO X Y, X TALE SIMM X SCAMBIO PARTIC

$\Rightarrow$ LA SIMM X SCAMBIO DI U^0 VIENE EREDITATA DA $U \Rightarrow U$ SIMM X SCAMBIO PARTIC, INOLTRE SE U^0 INDIP DA SPIN $U^0_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\delta_{\nu\nu'}U^0 \Rightarrow$ LO E' ANCHE U

$U(x_1x_2) = U^0(x_1x_2) + \int dy_1dy_2\, U^0(x_1y_1)\,\Pi(y_1y_2)\, U^0(x_2y_2)$ CON $\boxed{\Pi(xy) = \sum_{\mu\nu}\Pi_{\mu\mu\nu\nu}(xy) = \frac{1}{i\hbar}\langle E|T\delta n(x)\,\delta n(y)|E\rangle}$

$= \sum_{\ell=0}^{\infty}\frac{1}{(i\hbar)^{\ell}}\frac{1}{\ell!}\sum_{k=\ell}^{\infty}\frac{1}{(i\hbar)^{k-\ell}}\frac{1}{(k-\ell)!}[\ \ldots\ ] \overset{k'=k-\ell}{=} \sum_{\ell=0}^{\infty}\frac{1}{(i\hbar)^{\ell}}\frac{1}{\ell!}\int dt_1\ldots dt_\ell\langle TV(t_1)\ldots V(t_\ell)\varphi_1\varphi_3^+\rangle\sum_{k'=0}^{\infty}\frac{1}{(i\hbar)^{k'}}\frac{1}{k'!}\int ds_1\ldots ds_{k'}\langle TV(s_1)\ldots V(s_{k'})\varphi_2\varphi_4^+\rangle$

RIASSEMBLO TEXP

$= \langle E_0|TS\varphi_1\varphi_3^+|E_0\rangle\langle E_0|TS\varphi_2\varphi_4^+|E_0\rangle = iG(13)\, iG(24)$

② PRENDO 14 E 23 E FACCIO IL SALTO $\Rightarrow \pm iG(14)G(23)$ $\quad\Sigma\rightarrow$ DIAGRAMMI FATTORIZZATI

③ CONTRAGGO TUTTO CON TUTTO $\Rightarrow$ G CONNESSA

L'EQ DEL MOTO $\Rightarrow$ DIVENTA

$\left[i\hbar\partial_t - h(\underline{x})\right] G_{mm'}(\underline{x}t\underline{x}'t') = \hbar\delta_{mm'}\delta_3(\underline{x}-\underline{x}')\delta(t-t') + i\sum_{m''}\int dy\, v(\underline{x}y)\, G^c_{\substack{mm''\\m''m'}}(\underline{x}t\,yt^+\,yt^{++}\,\underline{x}'t')$

$+ i\sum_{m''}\int dy\, v(\underline{x}y)\left[G_{mm''}(\underline{x}t\,yt^+)\, G_{m''m'}(yt\,\underline{x}'t') - G_{mm'}(\underline{x}t\underline{x}'t')\, G_{m''m''}(yt\,yt^+)\right]$

SE TRASCURO LA PT CONNESSA $G^{con} = 0$

$\Rightarrow$ EQ CHIUSA X IL PROPAGATORE $\longrightarrow$ APPROSSIMAZ HF (IN TERMINI DEL PROPAGATORE)

EQ QUADRATICA IN G^{HF}

$\int v(xy)\, n^{HF}(y)\, dy = U_H(x)$

$\left[i\hbar\partial_t - h(\underline{x})\right] G^{HF}_{mm'}(\underline{x}t\underline{x}'t') = \hbar\delta_{mm'}\delta(\underline{x}-\underline{x}')\delta(t-t') + i\sum_{m''}\int dy\, v(\underline{x}y)\, G^{HF}_{mm''}(\underline{x}t\,yt^+)\, G^{HF}_{m''m'}(yt\,\underline{x}'t') - iG^{HF}_{mm'}(\underline{x}t\underline{x}'t')\sum_{m''}\int d_3y\, v(\underline{x}y)\, G^{HF}_{m''m''}(yt\,yt^+)$

$\Rightarrow \left[i\hbar\partial_t - h(\underline{x}) - U_H(\underline{x})\right] G^{HF}_{mm'}(\underline{x}t\underline{x}'t') = \hbar\delta_{mm'}\delta_3(\underline{x}-\underline{x}')\delta(t-t') + i\sum_{m''}\int d_3y\, v(\underline{x}y)\, G^{HF}_{m''m'}(yt\underline{x}'t')\, G^{HF}_{mm''}(\underline{x}t\,yt^+)$

IN SP ω

$\left[\hbar\omega - h(\underline{x}) - U_H(\underline{x})\right] G^{HF}_{mm'}(\underline{x}\underline{x}'\omega) = \hbar\delta_{mm'}\delta_3(\underline{x}-\underline{x}') + i\sum_{m''}\int d_3y\, v(\underline{x}y)\, G^{HF}_{m''m'}(y\underline{x}'\omega)\int\frac{d\omega'}{2\pi}G^{HF}_{mm''}(\underline{x}y\omega')\, e^{i\eta\omega'}$ ⊛

SE SUPP RAPPR SPETTRALE DI PARTIC INDIP $\longrightarrow G^{HF}_{mm'}(\underline{x}\underline{x}'\omega) = \sum_a\frac{u_a(\underline{x}m)\, u_a^*(\underline{x}'m')}{\omega-\omega_a+i\eta\,\mathrm{sgn}(\omega_a-\omega_F)}$ $\quad$ INCOGNITE: $\rightarrow u_a$ WF ORTONORM, $\omega_a\in\mathbb{R}$ FASE

INSERENDO LA RAPPR E MOLTIP X $u_a(\underline{x}'m')$ ⊕ $\int d_3x'\sum_{m'}$ $\Rightarrow$ X ORTOGONALITA' SI OTTIENE

$\left[\hbar\omega - h(\underline{x}) - U_H(\underline{x})\right]\frac{u_a(\underline{x}m)}{\omega-\omega_a\pm i\eta\,\mathrm{sgn}(\omega_a-\omega_F)} = \hbar u_a(\underline{x}m) + i\sum_{m''}\int d_3y\, v(\underline{x}y)\frac{u_a(ym'')}{\omega-\omega_a\pm i\eta\,\mathrm{sgn}(\omega_a-\omega_F)}\underbrace{\int\frac{d\omega'}{2\pi}G^{HF}_{mm''}(\underline{x}y\omega')\, e^{i\omega'\eta}}_{i\sum_b u_b(\underline{x}m)\, u_b^*(ym'')\,\theta(\omega_F-\omega_b)}$

CON U_H DA $n^{HF}(y) = \sum_a\sum_m|u_a(\underline{x}m)|^2\theta(\omega_F-\omega_a)$

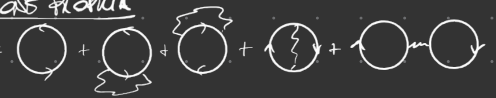
ORA PRENDO IL $\omega\to\omega_a \Rightarrow$ SIS EQ HF $\quad\left[h(\underline{x}) + U_H(\underline{x})\right] u_a(\underline{x}m) - \sum_{bm''}\theta(\omega_F-\omega_b)\, u_b(\underline{x}m)\int d_3y\, v(\underline{x}y)\, u_a(ym'')\, u_b^*(ym'') = \hbar\omega_a u_a(\underline{x}m)$

SE $u_{a\sigma}(\underline{x}m) = f_a(\underline{x})\,\chi_\sigma(m) \Rightarrow$ ORTOGONALITA' SPIN $\sum_{m''}\chi_\sigma(m'')\,\chi_{\sigma'}(m'') = \delta_{\sigma\sigma'}$

$\Rightarrow$ N EQ $\quad\left[h(\underline{x}) + U_H(\underline{x})\right] f_a(\underline{x}) - \sum_{\omega_b<\omega_F} f_b(\underline{x})\int d_3y\, v(\underline{x}y)\, f_a(y)\, f_b^*(y) = \hbar\omega_a f_a(\underline{x})$

POLARIZZAZIONE PROPRIA

DEF Π^* I DIAG NON

$\Pi(12) =$  → RIDUCIBILI LEVANDO U^0

$\Rightarrow \Pi = \Pi^* + \Pi^* U^0 (\Pi^* + \Pi^* U^0 (\Pi^* + \_\_)) = \Pi^* + \Pi^* U^0 \Pi = \Pi^* + \Pi^* U^0 \Pi$ EQ DYSON

EXPLICIT

$$\Pi_{\rho\rho'\sigma\sigma'}(12) = \Pi^*_{\rho\rho'\sigma\sigma'}(12) + \int d3\, d4 \sum_{\mu\mu'\nu\nu'} \Pi^*_{\rho\rho'\mu\mu'}(13)\, U^0_{\mu\mu'\nu\nu'}(34)\, \Pi_{\nu\nu'\sigma\sigma'}(42)$$

SE $U^0_{\mu\mu'\nu\nu'} = \delta_{\mu\mu'}\delta_{\nu\nu'} U^0 \Rightarrow \Pi(12) = \Pi^*(12) + \int d3\, d4\, \Pi^*(13)\, U^0(34)\, \Pi(42)$

EQ DYSON

$$U = U^0 + U^0 \Pi U^0 = U^0 + U^0(\Pi^* + \Pi^* U^0 \Pi) U^0 \Rightarrow \boxed{U = U^0 + U^0 \Pi^* U}$$

SE H INVAR X TRASLAZ $\Rightarrow$ ANCHE Π E Π^* (DIM Π X ES) $\Rightarrow$ ANCHE U

$$U(xy) = \int \frac{d_3k}{(2\pi)^3} \frac{d\omega}{2\pi}\, U(k\omega)\, e^{i\underline{k}(\underline{x}-\underline{y}) - i\omega(t_x - t_y)}$$

IDEM X Π^* E U^0

$$\Rightarrow U(k\omega) = U^0(k\omega) + U^0(k\omega)\, \Pi^*(k\omega)\, U(k\omega)$$

$$\Rightarrow U(k\omega) = \frac{U^0(k\omega)}{1 - U^0(k\omega)\Pi^*(k\omega)}$$

$$\Pi(k\omega) = \frac{\Pi^*(k\omega)}{1 - U^0(k\omega)\Pi^*(k\omega)}$$

X POT STATICO $U^0(xx') = V(\underline{x}-\underline{x}')\delta(t-t') \Rightarrow U^0(k\omega) = V(\underline{k})$

FUNZ DIELETTRICA CON

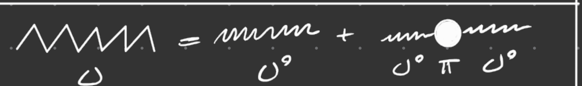
$$\Rightarrow U(k\omega) = \frac{V(k)}{\varepsilon(k\omega)}, \quad \Pi(k\omega) = \frac{\Pi^*(k\omega)}{\varepsilon(k\omega)} \quad \Rightarrow \varepsilon(k\omega) = 1 - V(k)\Pi^*(k\omega)$$

ANCHE SE U^0 STATICO U DIP DAL TEMPO X ε CHE DESCRIVE LA RISPOSTA DEL MEZZO

CASO COULOMB $\quad U^0(k\omega) = \dfrac{4\pi e^2}{k^2} \Rightarrow U(k\omega) = \dfrac{4\pi e^2}{k^2 - 4\pi e^2 \Pi^*(k\omega)}$

→ L'INTERAZ A LUNGO RANGE DI COULOMB È MODIF DALLO SCREENING DEL MEZZO POLARIZZATO

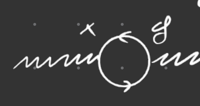
CALCOLO PARZIALE DI Π^0



$$\Pi(xy) = \frac{1}{i\hbar}\langle E| T\, \tilde{n}(x)\, \tilde{n}(y) |E\rangle$$

COME ASSEGNO DATO DALL'ESPRESS ANALITICA?

→ PREFATTORE $\frac{i}{\hbar}$ ∀ DIAG INTERNO

 $= \Pi^0(xy) = \frac{i}{\hbar}(-1)\, 2\, G^0(xy)\, G^0(yx)$

→ IN SP K

REGOLE DI FEYNMAN IN SP K

(OMETTO SPIN XKÈ INFLUENZATO DA TRANSFORMATA) $G = \sum G^{(\gamma)}$

$$G_\gamma(xx') = \left(\frac{i}{\hbar}\right)^k (-1)^\ell \sum_{\text{SPIN}} \int d_4z_1 \_\_ d_4z_{2k}\, \underbrace{U^0(z_i z_j) \_\_ U^0(z_i z_j)}_{k}\, \underbrace{G^0(xz) \_\_ G^0(zz) \_\_ G^0(zx')}_{2k+1}$$

γ = 2k-ESIMO ORD K

START $\qquad$ END

$V = \frac{1}{2}\int dx\, dy\, U^0 \psi^\dagger \psi^\dagger \psi \psi$

LE G^0 FORMANO 2 TIPI DI STRUTTURE

① START ⟶ END

② LOOP

SE HO INVAR X TRASLAZ SIA X G^0 CHE X INTERAZIONE

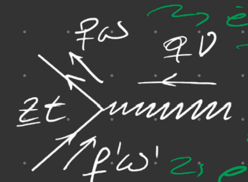
$$G^0(z_1 z_2) = \int \frac{d_3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, e^{i\underline{k}(\underline{z}_1 - \underline{z}_2) - i\omega(t_1 - t_2)}\, G^0(k\omega)$$

IN SP POSIZ HO CHIARAM START/END

$$U^0(z_1 z_2) = \int \frac{d_3q}{(2\pi)^3}\frac{d\omega}{2\pi}\, e^{iq(z_1 - z_2) - i\omega(t_1 - t_2)}\, U^0(q\omega)$$

IN SP K U^0 NON HA ORDINE NATURALE ⇒ DECIDO IO IL VERSO

INTEG DI EXP → δ



$$\longrightarrow \int d_3z\, e^{iz(p' + q - p)} \int dt\, e^{it(\omega - \nu - \omega')} = (2\pi)^3 \delta_3(p' + q - p)\, 2\pi\delta(\omega - \nu - \omega')$$

⇒ CONSERVAZ MOMENTO NEL VERTICE ⇒ INVAR X TRASLAZ INTORNO AL VERTICE

IDEM X FREQ ⇒ INVAR TRASLAZ TEMP

⇒ 2K δ X 2K INTEG ⇒ RIMANGONO K INTEG DELLE U E 1 TERMINE G

→ 2K δ CANCELLANO GLI INTEG LASCIANDO CONSERVATI I MOMENTI ∀ VERTICE

→ STESSE REGOLE FEYNMANN + FATTORE

$$(2\pi)^4 \left[\frac{1}{(2\pi)^4}\right]^{k + 2k + 1} \left[(2\pi)^4\right]^{2k} = \left[\frac{1}{(2\pi)^4}\right]^k$$

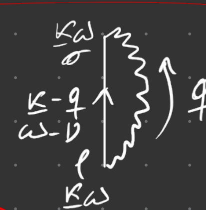
$\#U^0 \qquad \#G^0$

→ K QUADRIMOM DA INTEGRARE $\int \frac{d_4p}{(2\pi)^4}$

QUANDO TEMPI UGUALI $G(\underline{x}t\, \underline{y}t^+) \to G(k\omega)\, e^{i\omega\eta}$

ESEMPIO: DIAGRAMMA Σ DEL I ORD

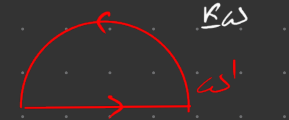
FIX q, ν ⇒ IL RESTO DA CONSERVAZ



$$\Sigma^*_{B\,\rho\sigma}(k\omega) = \left(\frac{i}{\hbar}\right) \sum_{\rho\sigma} \int \frac{d_3q\, d\nu}{(2\pi)^4}\, G^0_{\sigma\rho}(k - q, \omega - \nu)\, \delta_{\nu\sigma}\, \delta_{\rho\mu}\, U^0(q\nu)\, e^{i(\omega-\nu)\eta}$$

↳ INTERAZ ISTANTANEA

X COULOMB $U^0(q\nu) = \frac{4\pi e^2}{q^2}$

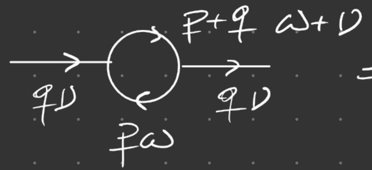


I PEZZO SOTTO → 0

II PEZZO SOPRA → RES

$$= \delta_{\mu\nu}\frac{i}{\hbar}\int \frac{d_3q}{(2\pi)^3}\frac{4\pi e^2}{q^2}\int \frac{d\omega'}{2\pi}\left[\frac{\theta(|\underline{k}-\underline{q}| - k_F)}{\omega' - \frac{\varepsilon_{k-q}}{\hbar} + i\eta} + \frac{\theta(k_F - |\underline{k}-\underline{q}|)}{\omega' - \frac{\varepsilon_{k-q}}{\hbar} - i\eta}\right] e^{i\omega'\eta}$$

$$= \delta_{\mu\nu}\frac{i}{\hbar}\int \frac{d_3q}{(2\pi)^3}\frac{4\pi e^2}{q^2}\frac{2\pi i}{2\pi}\,\theta(k_F - |\underline{k}-\underline{q}|) = \delta_{\nu\mu}\left(\frac{-1}{\hbar}\right)\int_{q' < k_F} \frac{d_3q'}{(2\pi)^3}\frac{4\pi e^2}{|\underline{k}-\underline{q}'|}$$



$$= \frac{i}{\hbar}(-1)2\int\frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\left[\frac{\theta(|p|-k_F)}{\omega-\frac{\varepsilon_p}{\hbar}+i\eta}+\frac{\theta(k_F-p)}{\omega-\frac{\varepsilon_p}{\hbar}-i\eta}\right]\left[\frac{\theta(|p+q|-k_F)}{\omega+\nu-\frac{\varepsilon_{p+q}}{\hbar}+i\eta}+\frac{\theta(k_F-|p+q|)}{\omega+\nu-\frac{\varepsilon_{p+q}}{\hbar}-i\eta}\right]=\Pi^0(q\nu)$$

integrando in $\omega \to \frac{1}{\omega^2}$ conv. con poli sotto e sopra → gli integ convergono ⇒ chiudo dove mi pare →

se ho 2 poli dallo stesso lato, integro dal lato opposto ⇒ 0

⇒ contrib solo integ con poli opposti → 2/4

$$\Pi^0(q\nu) = -\frac{2i}{\hbar}\int\frac{d^3p}{(2\pi)^3}\frac{2\pi i}{2\pi}\left[\frac{\theta(k_F-p)\theta(|p+q|-k_F)}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}+i\eta}+\frac{\theta(p-k_F)\theta(k_F-|p+q|)}{-\nu+\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}+i\eta}\right]$$

con i termini ho una particella dentro e una fuori FS → esplicito Re e Im

→ Plemelj-Sokhotski $\frac{1}{x-y\pm i\eta} = \frac{P}{x-y} \mp i\pi\delta(x-y)$ 2° vale se integro su una distrib

$$P = \mathrm{Re}\,\Pi^0(q\nu) = \frac{2}{\hbar}\int\frac{d^3p}{(2\pi)^3}\frac{\theta(k_F-p)\theta(|p+q|-k_F)-\theta(p-k_F)\theta(k_F-|p+q|)}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}$$

$$\mathrm{Im}\,\Pi^0(q\nu) = -\frac{2\pi}{\hbar}\int\frac{d^3p}{(2\pi)^3}\underbrace{\theta(k_F-p)\theta(|p+q|-k_F)\,\delta\left(\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}\right)}_{\nu>0}+\underbrace{\theta(p-k_F)\theta(k_F-|p+q|)\,\delta\left(\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}\right)}_{\nu<0}$$

da $\mathrm{Im}\,\Pi$ → ho una particella dentro e una fuori e la diff en è $\hbar\nu$

integrando la Re sapendo che

$$\theta(k_F-p)[1-\theta(k_F-|p+q|)]-[1-\theta(k_F-p)]\theta(k_F-|p+q|)=\theta(k_F-p)-\theta(k_F-|p+q|)$$

$$\mathrm{Re}\,\Pi^0(q\nu)=\frac{2}{\hbar}\int\frac{d^3p}{(2\pi)^3}\frac{\theta(k_F-p)-\theta(k_F-|p+q|)}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}} \underset{-p'=p+q \Rightarrow \text{cambio segno}}{=} \frac{2}{\hbar}\int\frac{d^3p}{(2\pi)^3}\theta(k_F-p)\left[\frac{1}{\nu-\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}-\frac{1}{\nu+\frac{\varepsilon_{p+q}-\varepsilon_p}{\hbar}}\right]$$

$\frac{1}{\hbar}\left[\frac{\hbar^2}{2m}\right](q^2+2qp)$ — è quadratica $\varepsilon_{p'}=\varepsilon_{-p'}$

[conto da finire x $\nu=0$ (caso statico)]

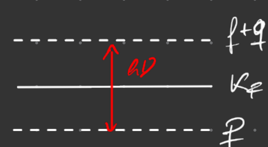
di Im → I° $\hbar$ $\begin{cases} p+q>k_F \\ p<k_F\end{cases} \Rightarrow \varepsilon_{p+q}-\varepsilon_p>0 \Rightarrow \nu>0$

II° $\hbar$ $\begin{cases} p+q<k_F \\ p>k_F\end{cases} \Rightarrow \varepsilon_{p+q}-\varepsilon_p<0 \Rightarrow \nu<0$

→ Dove ho $\mathrm{Im}\,\Pi = 0$?

→ dove non sono rispettati i vincoli

$\boxed{\nu>0}$



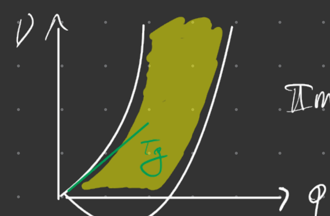
e ⇒ $\hbar\nu = \frac{\hbar^2}{2m}(q^2+2qp)$ →

posso maggiorare e minorare

$$\hbar\nu \le \frac{\hbar^2}{2m}(q^2+2|q||p|) \le \frac{\hbar^2}{2m}(q^2+2qk_F)$$

$\theta(k_F-p)$ ⇒ e sopra SF

$$\hbar\nu \ge \frac{\hbar^2}{2m}(q^2-2qp) \ge \frac{\hbar^2}{2m}(q^2-2qk_F)$$

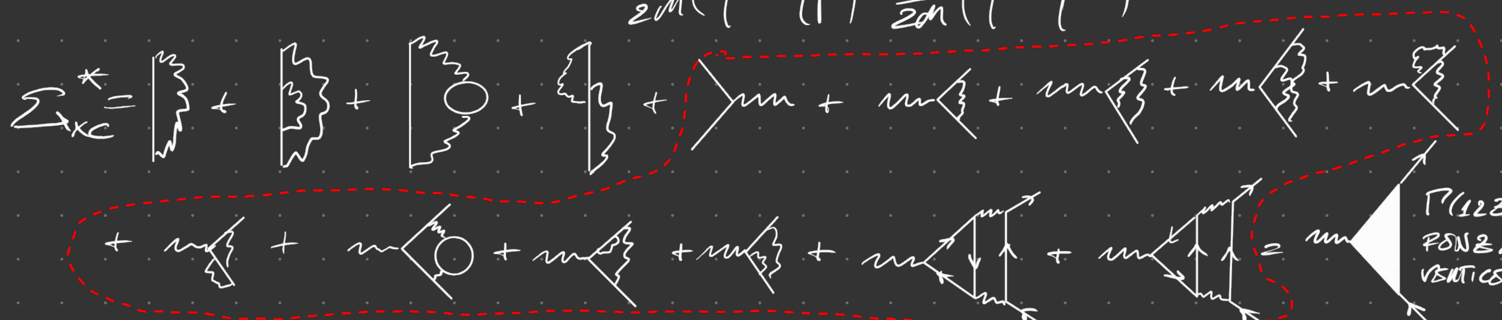


$\mathrm{Im}\,\Pi^0 = 0$

interessante andam di ν vicino q=0

$\hbar\nu = \frac{\hbar^2}{m}k_F q = 2\varepsilon_F q$

$\Sigma^*_{xc} =$

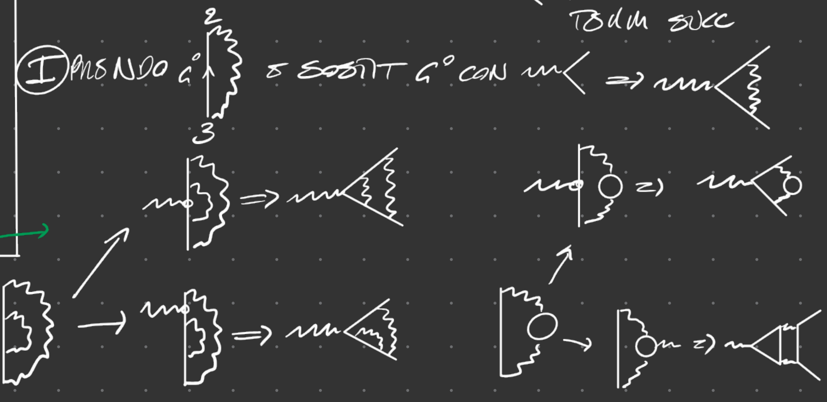


$\Gamma(123)$ funz di vertice

Funz di vertice

$\Gamma^0 =$ posso ricavare tutti succ

I prendo G^0 e sostit G^0 con

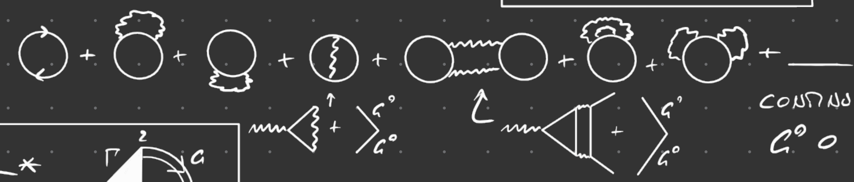


II

ecc. → da Σ ricavo diag di vertice

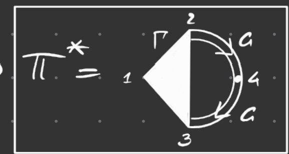
Eq di Hedin

- $\boxed{G = G^0 + G^0\Sigma^* G}$ Dyson I
- $\boxed{U = U^0 + U^0\Pi^* U}$ Dyson II
- $\Pi^* =$



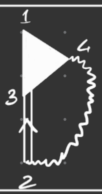
continuando a rivestire o le G^0 o i vertici o entrambi

⇒ $\Pi^* =$



rivestendo ext $\Pi^*(14) = \frac{i}{\hbar}(-1)\underset{\text{spin}}{2}\int d2d3\,\Gamma(123)G(42)G(34)$

- $\Sigma^* = \frac{i}{\hbar}$



$\Sigma^*(12) = \frac{i}{\hbar}\int d3d4\,\Gamma(413)G(32)U(42)$

- $\Gamma(123) = \Gamma^0(123) + \int d4d5\, G^0(41)\,G^0(15)\,\frac{\delta\Sigma^*(23)}{\delta G^0(45)}$

→ kernel di Bethe-Salpeter

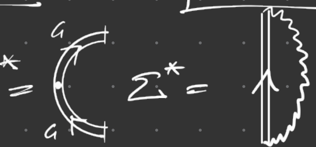
kernel creato levando G^0 e sostituendo con il

manco il papa se risolverebbe sta robaccia xké manco conosco Σ^*

⇒ ne uso solo alcuni

I approccio jihadista → <u>approx GW</u> in cui $\boxed{\Gamma=\Gamma^0}$

⇒ eq Hedin → 4 eq integrabili $\Pi^* =$ $\Sigma^* =$



$\Sigma^* = F(G^0(2\cdot)G^0(\cdot 3)G^0 \_\_ G^0)$ ⇒ $G^0 \to G^0+\delta G^0$ ⊕ $\Sigma \to \Sigma+\delta\Sigma$

⊕ $\frac{\delta G(ab)}{\delta G(cd)} = \delta_{ac}\delta_{bd}$ (sp-temp.)

poi prendo il funzionale e vario 1×1 le G^0 → $\delta\Sigma$ lin in δG

$$\delta\Sigma(23) = \int d4d5\,\frac{\delta\Sigma(23)}{\delta G(45)}\,\delta G(45)$$

in approx GW mentre il # di diag diverge fattorialmente ora la serie ha raggio di converg finito

RAPPR. DI LEHMAN DEL PROPAGATORE

$H(t) \to SP\omega \Rightarrow$ RAPPR LEHMAN

$$iG_{\mu\mu'}(x x') = \langle E_0^N | T \varphi_\mu(x) \varphi^+_{\mu'}(x') | E_0^N \rangle \quad \text{con } H|E_n^N\rangle = E_n^N |E_n^N\rangle$$

CONSERVA N PARTIC

⊕ ESPLICIT T ORD

$$= \theta(t-t') \langle E_0^N | \varphi_\mu(x) \varphi^+_{\mu'}(x') | E_0^N \rangle - \theta(t'-t) \langle E_0^N | \varphi^+_{\mu'}(x') \varphi_\mu(x) | E_0^N \rangle$$

φ^+ CREA PARTIC $\Rightarrow$ INSERISCO COMPLETEZZA SU N+1 PARTIC

$$= \sum_n \Big[\theta(t-t') e^{-i/\hbar (E_n^{N+1} - E_0^N)(t-t')} \langle E_0^N | \varphi_\mu(x) | E_n^{N+1} \rangle \langle E_n^{N+1} | \varphi^+_{\mu'}(x') | E_0^N \rangle - \theta(t'-t) e^{i/\hbar (E_n^{N-1} - E_0^N)(t'-t)} \langle E_0^N | \varphi^+_{\mu'}(x') | E_n^{N-1} \rangle \langle E_n^{N-1} | \varphi_\mu(x) | E_0^N \rangle \Big]$$

ORA CHE IL TEMPO È ESPLICIT $\Rightarrow$ SP ω

$$= i \sum_n \left[\int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t') - i/\hbar (E_n^{N+1} - E_0^N)(t-t')}}{\omega + i\eta} \langle \varphi_\mu \rangle \langle \varphi^+_{\mu'} \rangle - \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t') + i/\hbar (E_n^{N+1} - E_0^N)(t-t')}}{?} \langle \varphi^+_{\mu'} \rangle \langle \varphi_\mu \rangle \right]$$

MANGIO IN ω IL CONTRIB ΔE ⊕ MANGIO i A DX E SX

$$\Rightarrow G_{\mu\mu'}(xx') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \sum_n \left[\frac{\langle E_0^N | \varphi_\mu(x) | E_n^{N+1} \rangle \langle E_n^{N+1} | \varphi^+_{\mu'}(x') | E_0^N \rangle}{\omega - \frac{E_n^{N+1} - E_0^N}{\hbar} + i\eta} + \frac{\langle E_0^N | \varphi^+_{\mu'}(x') | E_n^{N-1} \rangle \langle E_n^{N-1} | \varphi_\mu(x) | E_0^N \rangle}{\omega - \frac{E_n^{N-1} - E_0^N}{\hbar} - i\eta} \right]$$

$\Rightarrow G_{\mu\mu'}(xx') = \sum_n [\ -\ ]$ POSSO ASSORBIRE EN ED EP $+$ XKÈ LE SOMME SONO $\neq$ E SHIFTO TUTTO A UN NUMERO

ORA $E_n^{N+1} - E_0^N = E_n^{N+1} \pm E_0 - E_0^N$ ⊕ $\varepsilon_n^{N+1} \equiv E_n^{N+1} - E_0^{N+1} \geq 0$ x DEF <u>EN ECCITAZ</u> E $\mu = E_0^{N+1} - E_0^N$ <u>POT CHIMICO</u>

PER $N \to \infty$ INDIP DA N MA DA ρ DENSITÀ

IDEM x N-1 $E_n^{N-1} - E_0^N = \underbrace{E_n^{N-1} - E_0^{N-1}}_{\varepsilon_n^{N-1}} + \underbrace{E_0^{N-1} - E_0^N}_{-\mu}$

$$\Rightarrow G_{\mu\mu'}(xx') = \sum_n \frac{\langle E_0^N | \varphi_\mu(x) | E_n^{N+1} \rangle \langle E_n^{N+1} | \varphi^+_{\mu'}(x') | E_0^N \rangle}{\omega - \frac{\varepsilon_n^{N+1}}{\hbar} - \frac{\mu}{\hbar} + i\eta} + \sum_n \frac{\langle E_0^N | \varphi^+_{\mu'}(x') | E_n^{N-1} \rangle \langle E_n^{N-1} | \varphi_\mu(x) | E_0^N \rangle}{\omega + \frac{\varepsilon_n^{N-1}}{\hbar} - \frac{\mu}{\hbar} - i\eta}$$

SOMMO TRA STATI DISCRETI $\Rightarrow$ POLI VICINISSIMI IN PT Im

FACENDO ∞ TD I POLI SI CONDENSANO ACQUISENDO PT Im FINITA MA MANTENGONO IL SGN

I PT $\to$ SOPRA

I PT $\to$ SOTTO

$\times\times\times\times\times \ \mu/\hbar \ \times\times\times\times\times\times$

INVAR x TRASLAZ $[H, \vec{p}] = 0 \Rightarrow |E_n(\kappa) k\rangle \Rightarrow \sum_n \longleftrightarrow \sum_{nk}$ AZIONE TRASLAZ

$$e^{i/\hbar\, \vec{a}\cdot\vec{p}} \varphi_\mu(x) e^{-i/\hbar\, \vec{a}\cdot\vec{p}} = \varphi_\mu(x - a) \Rightarrow \varphi_\mu(x) = e^{-i/\hbar\, x p} \varphi_\mu(0) e^{i/\hbar\, x p} \Rightarrow$$

SCARICO TUTTO SUGLI AUTOSTATI

POSSO SUPP CHE GS SIA $k = 0$ (PRENDO IN BARICENTRO)

$$G_{\mu\mu'}(x x' \omega) = \sum_{nk} \frac{\langle E_0^N 0 | e^{-\frac{i}{\hbar} xp} \varphi_\mu(0) e^{\frac{i}{\hbar} xp} | E_n^{N+1} k \rangle \langle E_n^{N+1} k | e^{-\frac{i}{\hbar} x'p} \varphi^+_{\mu'}(0) e^{\frac{i}{\hbar} x'p} | E_0^N 0 \rangle}{\omega - \frac{\varepsilon_n^{N+1}}{\hbar} - \frac{\mu}{\hbar} + i\eta} + \sum_{nk} \frac{\langle E_0^N 0 | e^{-\frac{i}{\hbar} x'p} \varphi^+_{\mu'}(0) e^{\frac{i}{\hbar} x'p} | E_n^{N-1}(-k), -k \rangle \langle E_n^{N-1}(-k), -k | e^{-\frac{i}{\hbar} xp} \varphi_\mu(0) e^{\frac{i}{\hbar} xp} | E_0^N \rangle}{\omega + \frac{\varepsilon_n^{N-1}(k)}{\hbar} - \frac{\mu}{\hbar} - i\eta}$$

$\sum_k = \sum_{-k}$

$$= \sum_{nk} e^{ik(x-x')} \left[\frac{\langle E_0^N 0 | \varphi_\mu(0) | E_n^{N+1} \rangle \langle E_n^{N+1} | \varphi^+_{\mu'}(0) | E_0^N \rangle}{\omega - \frac{\varepsilon_n^{N+1}}{\hbar} - \frac{\mu}{\hbar} + i\eta} + \frac{\langle E_0^N 0 | \varphi^+_{\mu'}(0) | E_n^{N-1} \rangle \langle E_n^{N-1} | \varphi_\mu(0) | E_0^N \rangle}{\omega + \frac{\varepsilon_n^{N-1}}{\hbar} - \frac{\mu}{\hbar} - i\eta} \right]$$

SE SCELGO $\mu = \mu'$

S FUNZ SPETTRALE ≥ 0 XKÈ CE DI NORMA POS

$$\Rightarrow G_{\mu\mu}(k\omega) = V \sum_n \left[\frac{|\langle E_0^N 0 | \varphi_\mu(0) | E_n^{N+1}(k) k \rangle|^2}{\omega + \frac{\varepsilon_n^{N+1}(k)}{\hbar} - \frac{\mu}{\hbar} + i\eta} + \frac{|\langle E_0^N | \varphi^+_\mu(0) | E_n^{N-1}(-k), -k \rangle|^2}{\omega - \frac{\varepsilon_n^{N-1}(-k)}{\hbar} - \frac{\mu}{\hbar} - i\eta} \right] \Rightarrow \sum_\mu G_{\mu\mu}(k\omega) = \int d\omega' \frac{A(k\omega')}{\omega - \omega' + i\eta\, \mathrm{sgn}(\omega' - \frac{\mu}{\hbar})}$$

$$A(k\omega) = V \sum_{n\mu} |\langle\ |\ \rangle|^2 \delta\left(\omega' - \frac{\varepsilon_n^{N+1}(k)}{\hbar} - \frac{\mu}{\hbar}\right) + |\langle\ |\ \rangle|^2 \delta\left(\omega' + \frac{\varepsilon_n^{N-1}(-k)}{\hbar} - \frac{\mu}{\hbar}\right)$$

$\omega' > \mu/\hbar$ $\quad$ $\omega' < \mu/\hbar$ $\Rightarrow$ CONTA IL TERM IN BASE AL SGN

VEDIAMO CHE $G(k\omega) \xrightarrow{|\omega| \to \infty} \frac{1}{\omega} \Rightarrow \int d\omega' A(k\omega') = 1$

$(\int \delta = 0$ ALTRIMENTI$)$

TEORIA DELLA RISPOSTA LINEARE

H INFAME A PIACERE $\to$ PERTURBO CON $V(t) = 0$ PER $t < 0$

$\to$ SUPP DI ESSERE $|\varphi(0)\rangle = |GS\rangle$ DI $H \Rightarrow$ SE $t > 0$ $|\varphi(t)\rangle = U(t,0)|\varphi(0)\rangle = e^{-i/\hbar Ht} U_I(t,0)|GS\rangle$

SE CONSID O_L CHE HA PROPRIA EVOLUZ TEMP INDIP

$\langle\varphi(t)|O_L|\varphi(t)\rangle = \langle GS| U_I^+(t,0) e^{i/\hbar Ht} O_L e^{-i/\hbar Ht} U_I(t,0)|GS\rangle = \langle GS| U_I^+(t,0) O_{H} U_I(t,0)|GS\rangle$ CON EVOLUZ HEISENBERG DI O_L SECONDO H

I 2 EVOLUZIONI DI HEISENBERG SCALANO SU $|GS\rangle$

SUPP DI POTER SVILUP TEXP AL I ORD

$$U_I(t,0) = T\exp \frac{1}{i\hbar} \int_0^t dt' V_H(t') \Rightarrow \langle GS| \left(1 - \frac{1}{i\hbar} \int_0^t dt' V_H(t') + \dots\right) O_{H} \left(1 + \frac{1}{i\hbar} \int_0^t dt' V_H(t') \dots\right) |GS\rangle = \langle\varphi(t)|O|\varphi(t)\rangle - \langle GS|O_H|GS\rangle$$

$$= \frac{1}{i\hbar} \int_0^t dt' \langle GS| [O_H(t), V_H(t')] |GS\rangle$$

$$\Rightarrow \langle\varphi(t)|O_L|\varphi(t)\rangle - \langle GS|O_L|GS\rangle = \frac{1}{i\hbar} \int_{\pm\infty} dt' \theta(t-t') \langle GS| [O_H(t), V_H(t')] |GS\rangle \quad \text{LINEARE IN } V$$

<u>PERTURBAZIONE EXT</u>

$H_t = H + \int d_3x\, n(x) \varphi(xt)$ DIP TEMP SOLO DA CAMPO EXT $\quad \varphi(xt) = 0$ SE $t < 0$

VADO A MIS $\hat{n} \to \langle\varphi(t)|\hat{n}(x)|\varphi(t)\rangle - \langle GS|\hat{n}(x)|GS\rangle = \frac{1}{i\hbar} \int_{\pm\infty} dt' \int d_3x'\, \theta(t-t') \langle GS| [\hat{n}_H(xt), n_H(x't')] |GS\rangle \varphi(x't')$

VARIAZ DENSITÀ IN xt $\quad \delta\hat{n}(xt) = \frac{1}{i\hbar} \int_{\pm\infty} dt' d_3x'\, D^{RET}(xt\, x't') \varphi(x't')$

$\hookrightarrow$ OGGETTO DI H

$$iD^{RET}(xx') = \theta(t-t') \langle GS| [n_H(x), n_H(x')] |GS\rangle$$

VARIAZ SE INDOTTA DA FUNZ EXT È UN IN QUESTA DA UNA RISPOSTA IN CON NON LOC

θ CONSERVA CAUSALITÀ $(t' < t) \Rightarrow$ FUNZ DI RISPOSTA RITARDATA $D^R(t,t')$ DIP DA $t - t'$

<u>H INVAR x TRASLAZ SP-TEMPO</u> $\Rightarrow D^R(xt\, x't')$ DIP DA $x - x'$ E $t - t'$ $\to$ DIM CON TRASLAZ OP DI CAMPO

$[\vec{p}, H] = 0 \Rightarrow$ RIMUOVO EVOLUZ TEMP E AGISCO SU $|GS\rangle \Rightarrow$ DIP DA $x - x' \Rightarrow$ INDIETRO $\to$ CONVOLUZ SE SP-TEMPO $\Rightarrow$ SP FOURIER $\Rightarrow$ SPARISCE INTEGRALE

$$\delta n(xt) = \int \frac{d_3k}{(2\pi)^3} \frac{d\omega}{2\pi} e^{+ikx - i\omega t} \delta n(k\omega) \Rightarrow \boxed{\delta n(k\omega) = \frac{1}{\hbar} D^{RET}(k\omega) \varphi(k\omega)}$$

IN SP $k\omega$ I VALORI MOD RISP IN MANIERA LIN ATTRAVERSO I COEFF $D(k\omega)/\hbar$

$\to$ <u>SUSCETTIBILITÀ GENERALE</u>

<u>x ES</u> $\quad \vec{j} = \sigma\vec{E}$ E $\vec{D} = \varepsilon\vec{E}$ IDENTITÀ VALIDA FREQ x FREQ

<u>DEF</u> $iD^T(xt\, x't') = \langle GS| T\, \delta n_H(xt)\, \delta n_H(x't') |GS\rangle = i\hbar\, \Pi(xt\, x't')$ GIÀ VISTO x PARTICELLE x INVAR TRASLAZ SP-TEMPO $\to D^T(k\omega)$

RAPPR LEHMAN x δA E δB $\quad$ Ⓘ $iC^R_{AB}(t,t') = \langle GS| [A_H(t), B_H(t')] |GS\rangle \theta(t-t') \to$ SVILUPPO

Ⓘ $iC^T_{AB}(tt') = \langle GS| T\, \delta A_H(t)\, \delta B_H(t') |GS\rangle = \theta(t-t') \langle GS| \delta A_H(t) \delta B_H(t') |GS\rangle - \theta(t'-t) \langle GS| \delta B_H(t') \delta A_H(t) |GS\rangle$

SVILUPPO I $\quad$ CON $|E_0\rangle = |GS\rangle$, $\{|E_n\rangle\}$ SONO DI H IN SSP N PARTIC $\to$ SUPP A,B DEFINITE SU HN

$\Rightarrow$ # PARI DI C, C^+

$$iC^R_{AB}(tt') = \sum_n \left(e^{-\frac{i}{\hbar}(E_n - E_0)(t-t')} A_{0n} B_{n0} - e^{\frac{i}{\hbar}(E_n - E_0)(t-t')} B_{0n} A_{n0} \right) \theta(t-t')$$

$n = 0$ SI CANCELLA ⊕ SP ω E SHIFTO $(E_n - E_0)/\hbar$

$$= i \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \left[\sum_n \frac{A_{0n} B_{n0}}{\omega - \frac{E_n - E_0}{\hbar} + i\eta} - \frac{B_{0n} A_{n0}}{\omega + \frac{E_n - E_0}{\hbar} + i\eta} \right]$$

$$\Rightarrow C^R_{AB}(\omega) = \sum_n \frac{A_{0n} B_{n0}}{\omega - \frac{E_n - E_0}{\hbar} + i\eta} - \frac{B_{0n} A_{n0}}{\omega + \frac{E_n - E_0}{\hbar} + i\eta} = \int_{\pm\infty} d\omega' \frac{C_{AB}(\omega')}{\omega - \omega' + i\eta} \quad \text{FUNZ SPETTRALE}$$

SCREENING impurezza in 0 di carica Ze in HEG $\Rightarrow H_{ext} = -Ze^2 \int d_3x \frac{n(x)}{|x|}$ → perturb. indip da t accesa a $t=-\infty$

$$\delta n(\underline{x}t) = \frac{1}{\hbar}\int dt' d_3\underline{x}' \, D^R(\underline{x}t\underline{x}'t')\,\varphi(\underline{x}') \Rightarrow \varphi(\underline{x}) = -\frac{Ze^2}{|\underline{x}|}$$

D^R invar x traslaz xké lo è HEG ⇒ sp $\underline{k}\omega$

$$\delta n(\underline{k}\omega) = \frac{1}{\hbar}D(\underline{k}\omega)\varphi(\underline{k}\omega) \Rightarrow \varphi(\underline{k}\omega) = -Ze^2\frac{4\pi}{k^2}2\pi\delta(\omega)$$

da azimut $\frac{1}{ikr}\left(e^{ikr}-e^{-ikr}\right) = \int_{-1}^{1} d\xi\, e^{ikr\xi}$ $\quad k \to -k$

$$\delta n(\underline{x}t) = \int \frac{d_3k}{(2\pi)^3} e^{i\underline{k}\underline{x}} \frac{1}{\hbar} D^R(\underline{k}0)(-Ze^2)\frac{4\pi}{k^2} = -Ze^2\frac{4\pi}{(2\pi)^3}\int_0^\infty k^2dk \frac{1}{\hbar}D^R(\underline{k}0)\frac{2\pi}{k^2}\int d\theta \sin\theta\, e^{ikx\cos\theta} = -\frac{Ze^2}{i4\pi^2 r}\int_0^\infty kdk\frac{4\pi}{k^2}\frac{1}{\hbar}D^R(k0)\left(e^{ikr}-e^{-ikr}\right)$$

$$= -\frac{Ze^2}{i4\pi^2 r}\int_{\pm\infty} kdk\frac{4\pi}{k^2}\frac{1}{\hbar}D^R(k0)e^{ikr}$$

Dyson

sappiamo che $D^T(xx') = \hbar\Pi(xx')$ ⇒ col calcolatore $D^R(xx')$ ⇒ $D^R(xx') = \hbar\Pi^R(xx') \overset{k}{\Rightarrow} D^R(\underline{k}\omega) = \hbar\Pi^R(\underline{k}\omega)$ e so che $\Pi(\underline{k}\omega) = \frac{\Pi^*(\underline{k}\omega)}{1-v(k)\Pi^*(\underline{k}\omega)}$

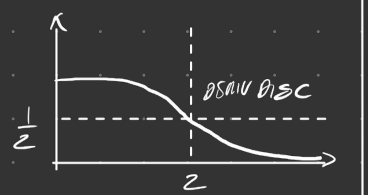
⇒ facile dim che se $\Pi^{*R} = \mathrm{Re}\Pi^* + i\,\mathrm{Im}\Pi^*\,\mathrm{sgn}(\omega) \Rightarrow \Pi^R = \frac{\Pi^{*R}}{1-v\Pi^{*R}} \Rightarrow \frac{D^R(\underline{k}0)}{\hbar} = \Pi^R(\underline{k}0) = \frac{\Pi^{*R}(\underline{k}0)}{1-\frac{4\pi e^2}{k^2}\Pi^{*R}(\underline{k}0)}$

$$\Rightarrow \delta n(r) = -\frac{Z}{i4\pi^2 r}\int_{\pm\infty} kdk \frac{\frac{4\pi e^2}{k^2}\Pi^{*R}(\underline{k}0)}{1-\frac{4\pi e^2}{k^2}\Pi^*(\underline{k}0)} e^{ikr}$$

→ non so Π^{*R} ma solo che $\Pi^{*R} = \mathrm{Re}\Pi^*(\underline{k}0)$ ($\not\exists$ Im nell') ⇒ approx $\Pi^* \simeq \Pi^0$

$\nexists$ calcolo esatto → vado a $r\to\infty \Rightarrow e^{ikr}$ oscilla un botto ⇒ integr. va a 0 velocem. ⇒ domina zona $k\to0$ in integ.

$$\Rightarrow \text{in } \boxed{k\to0} \quad \Pi^{*R}(\underline{k}0) \simeq \Pi^0(\underline{k}0) = -\frac{mk_F}{4\pi^2\hbar^2}g\left(\frac{k}{k_F}\right) \Rightarrow g(x) = \frac{1}{2} - \frac{1}{2x}\left(1-\frac{x^2}{4}\right)\lg\left|\frac{2-x}{2+x}\right|$$



g nell'intorno da integrando ⇒ $\boxed{g(0)=1}$ → approx Thomas-Fermi $\boxed{g(x)\simeq1}$

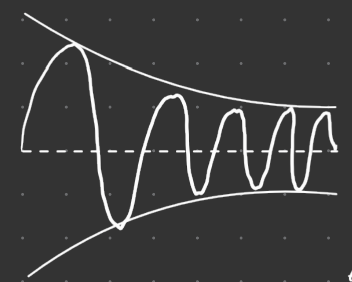
$$\delta n(r) = -\frac{Z}{4\pi^2 ir}\int_{\pm\infty} kdk \frac{4\pi e^2\left(\frac{-mk_F}{4\pi^2\hbar^2}\right)}{k^2 + \frac{mk_F 4\pi e^2}{4\pi^2\hbar^2}}$$

$$k_{TF} = \frac{mk_F e^2}{\pi\hbar^2} = \frac{k_F}{a_0} \quad \left(a_0 = \frac{\hbar^2}{me^2}\right)$$

$\frac{k_{TF}}{k_F} \simeq 1 \Rightarrow$ non così ≠ da densità materia cond → $\delta n(r) = -\frac{Z}{4\pi^2 ir}\int kdk \frac{k_{TF}^2}{k^2+k_{TF}^2}e^{ikr} = \frac{e^{-r}}{r}k_{TF}^2\frac{Z}{4}$ (2 poli $\pm ik_{TF}$) andamento Thomas Fermi

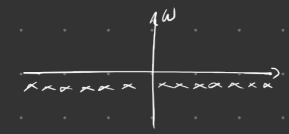
in realtà essendo nel caso g ⇒ fase Friedel

$$\Rightarrow \delta n(r) \sim Z\frac{1}{r^3}\cos(2k_F r) \quad \text{zone a densità pos e neg}$$



charge esaltata oltre dalla schermatura → oscillaz di Friedel a $T>0$ scompaiono

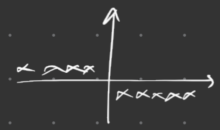
$$C_{AB}(\omega) = \sum_n A_{0n}B_{n0}\,\delta\left(\omega - \underbrace{\frac{E_n^N - E_0^N}{\hbar}}_{>0}\right) - B_{0n}A_{n0}\,\delta\left(\omega + \frac{E_n - E_0}{\hbar}\right)$$



funz. rit ha poli neg ⇒ analitica in piano sup

funz tond x ss → ognuna ma 2θ

$$C^T_{AB}(\omega) = \int_{\pm\infty} d\omega \frac{C_{AB}(\omega')}{\omega - \omega' + i\eta\,\mathrm{sgn}(\omega)}$$



se $\boxed{A = \hat{n}(\underline{x})}$ e $\boxed{B = \hat{n}(\underline{x}')}$ ⇒ elem matr autostato di H

$$\varphi(\underline{x}) = e^{-\frac{i}{\hbar}\underline{x}\underline{p}}\varphi(\underline{0})e^{+\frac{i}{\hbar}\underline{x}\underline{p}} \Rightarrow \hat{n}(\underline{x}) = e^{-\frac{i}{\hbar}\underline{x}\underline{p}}n(\underline{0})e^{+\frac{i}{\hbar}\underline{x}\underline{p}}$$

in base $|E_n(k)\underline{k}\rangle \to \langle E_0 0|n(\underline{x})|E_0 0\rangle =$ svil di Fourier

$$iD^R(\underline{k}\omega) = \int_{\pm\infty} d\omega' \frac{D(\underline{k}\omega')}{\omega - \omega' + i\eta} \qquad D^T(\underline{k}\omega) = \int d\omega' \frac{D(\underline{k}\omega')}{\omega - \omega' + i\eta\,\mathrm{sgn}(\omega)}$$

facendo conto $D(\underline{k}\omega) \in \mathbb{R}$

uso $\frac{1}{\omega - \omega' \pm i\eta} = \frac{P}{\omega - \omega'} \mp i\pi\delta(\omega - \omega') \Rightarrow D^R(\underline{k}\omega) = ⨍ d\omega' \frac{D(\underline{k}\omega')}{\omega - \omega'} - i\pi D(\underline{k}\omega)$

$$D^T(\underline{k}\omega) = ⨍ d\omega' \frac{D(\underline{k}\omega')}{\omega - \omega'} - i\pi\,\mathrm{sgn}(\omega)D(\underline{k}\omega)$$

$\to D(\underline{k}\omega) \in \mathbb{R}$

ho che $\mathrm{Re}\,D^R(\underline{k}\omega) = \mathrm{Re}\,D^T(\underline{k}\omega)$ e $\mathrm{Im}\,D^R(\underline{k}\omega) = \mathrm{Im}\,D^T(\underline{k}\omega)\,\mathrm{sgn}(\omega)$

OSCILLAZIONI DI PLASMA e⁻ sparati su sup

⇒ e⁻ riflessi di coi alcuni perdono en multiplo di una certa quantità

→ idem se colpisce metallo ⇒ e⁻ perde x attivaz di oscillaz collettive nel metallo

se li illumino con en $\begin{cases} < \hbar\omega_{pl} \Rightarrow \text{riflessi} \\ > \hbar\omega_{pl} \Rightarrow \text{assorbiti} \end{cases}$

$\omega_{pl} = \frac{4\pi e^2}{m}n \to \hbar\omega_{pl} \sim 10\,eV > E_F$

↳ indip da $\hbar$ ⇒ fenomeno universale x sys cl

$$\delta n(\underline{x}t) = \int \frac{d_3k}{(2\pi)^3}\frac{d\omega}{2\pi} e^{i\underline{k}\underline{x} - \omega t}\,\Pi^R(\underline{k}\omega)\,\varphi(\underline{k}\omega)$$

dove la risposta è maggiore? se Π^R ha poli vicino a Re → polo ∈ Re ⇒ integ. esplode, ogni funz rit ha poli neg → $\Pi(\underline{k}\omega) = \frac{\Pi^*}{1-v\Pi^*}$ ⇒ poli = zeri funz dielettrica $\varepsilon(\underline{k}\omega) = 1 - v(k)\Pi^*(\underline{k}\omega)$ ⇒ $\varepsilon^R(\underline{k}\omega) = 1 - v(k)\Pi^{*R}(\underline{k}\omega)$

poli $0 = 1 - v(\underline{k})\Pi^*(\underline{k}\omega)$ → fix $\underline{k}$ ⇒ $\omega(\underline{k}) = \underbrace{\omega_1(\underline{k})}_{\mathrm{Re}} + i\underbrace{\omega_2(\underline{k})}_{\mathrm{Im}}$ → rit ⇒ $\omega_2 < 0$ → $0 = 1 - v(\underline{k})\Pi^{*R}(\underline{k}, \omega_1 + i\omega_2)$ → impongo ω_2 piccolo ⇒ separo pt Re e Im e sviluppo

idem x pt Im ma esemplifica qua

$$1 - v(\underline{k})\left[\mathrm{Re}\Pi^{*R}(\underline{k}\omega_1) + i\,\mathrm{Im}\Pi^{*R}(\underline{k}\omega_1) + i\omega_2\frac{\partial}{\partial\omega}\mathrm{Re}\Pi^{*R}(\underline{k}\omega)\Big|_{\omega=\omega_1} + \dots\right]$$

→ pt Re $0 = 1 - v(\underline{k})\mathrm{Re}\Pi^{*R}(\underline{k}\omega_1)$ (+ pt Im x $\omega_2 \to 0$) pt Im $0 = -v(\underline{k})\,\mathrm{Im}\Pi^{*R}(\underline{k}\omega_1) + \omega_2\frac{\partial}{\partial\omega}\mathrm{Re}\Pi^{*R}(\underline{k}\omega)\Big|_{\omega_1} v(\underline{k})$ → da qui ω_2

$\Omega = 1 - \varepsilon(k)\,\mathrm{Re}\,\Pi^{*R}(k\omega_1(k))$

$\omega_2(k) = \dfrac{\mathrm{Im}\,\Pi^{*R}(k\omega_2(k))}{\frac{\partial}{\partial\omega}\mathrm{Re}\,\Pi^{*R}(k\omega)\big|_{\omega_1(k)}}$

TROVATO IL POLO MASSIMO Π^{R}

$\Rightarrow \Pi^R(k\omega) = \dfrac{\overbrace{Z(k)}^{\text{RESIDUO}}}{\omega - [\omega_1(k) + i\omega_2(k)]} + \Pi^{REG}(k\omega) \longrightarrow$ CONTRIBUTO POLO $\delta n^{polo}(x t) = \displaystyle\int \frac{d^3k}{(2\pi)^3} Z(k) e^{-ikx} \int_{\pm\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - \omega_1 - i\omega_2}\varphi(k\omega)$

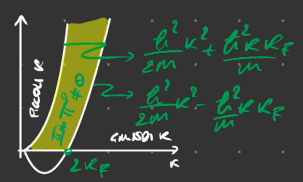
$t > 0 \rightarrow$ CHIUDO SOTTO

$-\frac{2\pi i}{2\pi} e^{-i(\omega_1 + i\omega_2)t}\varphi(k\omega)$

$\rightarrow \omega_2 < 0 \Rightarrow$ TRICAM È INCLUSO TEMPO DI DECADIMENTO

CASO COULOMB

APPROX $\Pi^{*R} = \Pi^{0R} \longrightarrow$ x VEDERE A LUNGO MOTO COLLETTIVO $k\omega$ 2 REGIONI IN CUI $\mathrm{Im}\,\Pi^0 = 0$



PATOLOGICO XKÉ 33 POLO E $\mathrm{Re} \Rightarrow$ INTEG ESPLODE $\longrightarrow \frac{1}{\varepsilon}$ COMUNQUE $\Rightarrow$ LO USO x BUONA APPROX ω_1 $\longrightarrow$ IN ZONA K GRANDE $\Rightarrow$ LUNGH PICCOLE MA VOGLIO GUARDARE SOPRA SCALA RETICOLARE

$\Rightarrow$ ZONA K PICCOLO $\rightarrow \omega$ GRANDI $\longrightarrow 1 = \frac{4\pi e^2}{k^2}\Pi^R(k\omega) \longrightarrow 1 = \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\displaystyle\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\left[\frac{1}{\omega - \frac{\varepsilon^0_{q}}{\hbar} - \frac{\varepsilon_{q+k}}{\hbar}} - \frac{1}{\omega - \frac{\varepsilon^0_q}{\hbar} + \frac{\varepsilon_{q+k}}{\hbar}}\right] = \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\frac{2\left(\frac{\varepsilon^0_{q+k} - \varepsilon^0_q}{\hbar}\right)}{\omega^2 - \left(\frac{\varepsilon^0_{q+k} - \varepsilon^0_q}{\hbar}\right)^2}$

$\hookrightarrow \omega = \omega_1(k)$ SOMMO E SOTT. $= 2\left(\frac{\hbar k^2}{2m} + \frac{\hbar k q}{m}\right)^2 \cdot q \cdot k$

Θ XKÉ DISP IN q

$= \frac{4\pi e^2}{k^2}\frac{2}{\hbar}\displaystyle\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\,2\left(\frac{\hbar k^2}{2m} + \frac{\hbar k q}{m}\right)\left[1 + \left(\frac{\hbar k^2}{2m} + \frac{\hbar k q}{m}\right)^2 + \ldots\right]$ QUINDI $\Theta \Rightarrow \omega_p \oplus$ CONSIDERAZIONI

$\Rightarrow 1 = \frac{4\pi e^2}{k^2\omega^2}\frac{4}{\hbar}\displaystyle\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\left[\left(\frac{\hbar k^2}{2m} + \frac{\hbar k q}{m}\right) + \frac{1}{\omega^2}\left(\frac{\hbar k^2}{2m} + \frac{\hbar k q}{m}\right)^3 + \ldots\right] \longrightarrow$ MIRACOLO COULOMB $= k^2$ NEI TERM SI CANCELLA CON FATTORE $\longrightarrow$ ALTRIM XSS POT COST PICCOLO

$\varepsilon(k) = \varepsilon(0) + \ldots \Rightarrow \omega^2 = \text{cost}\, k^2$

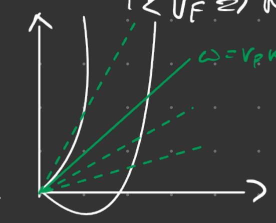
$\Rightarrow$ LEGGE LIN $=$ ONDE

INTERSEZ. $\begin{cases} > v_F \Rightarrow \text{ONDE} \\ < v_F \Rightarrow \text{NON LO VEDO} \end{cases}$ (He_3 SUPERFL.)

$1 = \frac{4\pi e^2}{\omega^2}\frac{4}{\hbar}\frac{\hbar}{2m}\displaystyle\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q) + \frac{4\pi e^2}{k^2\omega^2}\frac{\hbar}{\hbar\omega^2}\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\left[\left(\frac{\hbar k^2}{2m}\right)^3 + 3\left(\frac{\hbar k^2}{2m}\right)^2\cancel{\frac{\hbar k q}{m}} + 3\left(\frac{\hbar k^2}{2m}\right)\left(\frac{\hbar k q}{m}\right)^2 + \cancel{\left(\frac{\hbar k q}{m}\right)^3}\right] + \ldots$

K PICCOLO, $\oplus$ POT GRANDE $\Rightarrow$ TRASCURABILE

$\Rightarrow 1 = \frac{4\pi e^2}{m\omega^2}n + \frac{4\pi e^2}{k^2\omega^4}\frac{4}{\hbar}\displaystyle\int\frac{d^3q}{(2\pi)^3}\theta(k_F - q)\,3\left(\frac{\hbar k^2}{2m}\right)\left(\frac{\hbar}{m}\right)^2 k_i k_j q_i q_j$



TRASF CANONICHE

CONSERVANO I COMMUTATORI FONDAM

$x' = O^\dagger x O \quad p' = O^\dagger p O \quad s' = O^\dagger s O$

$\rightarrow [x_i, p_j] = [x'_i, p'_j] = i\hbar\delta_{ij} \qquad [s_i, s_j] = [s'_i, s'_j] = -i\hbar\varepsilon_{ijk}s_k$

ES $[x, s$ INVAR$]$

$x' = x, \; s' = s, \quad p'_i\varphi = [p_i - i\hbar(O^\dagger\partial_i O)]\varphi$

x AVERE INVAR LOC INTRODUCO $\pi_i = p_i - A_i$ DERIVATA COVARIANTE

$\pi'_i = O^\dagger\pi_i O = p_i - i\hbar O^\dagger\partial_i O - A'_i$

$= \pi_i + A_i - i\hbar O^\dagger\partial_i O - A'_i \longrightarrow$ IMPONGO $\pi_i = \pi'_i$ x INVAR LOC

$\Rightarrow A'_i = A_i - i\hbar(O^\dagger\partial_i O)$

$[\pi_i, \pi_j] = [p_i - A_i, p_j - A_j] = -i\hbar\,\partial_i A_j + i\hbar\,\partial_j A_i + [A_i, A_j]$

INVAR x FASE GLOBALE $\longrightarrow$ SIMM $U(1)$ $\quad \varphi' = e^{i\alpha}\varphi$

SE PASSO A SIMM $U(1)_{loc}$ $\quad \varphi'(x) = e^{i\alpha(x)}\varphi(x)$ INCON LA FISICA NON È INVAR

XKÉ LO SIA DEVO PASSARE DA p_i A π_i

$O \in U(1)_{loc}$ È $O = e^{i\alpha(x)} \Rightarrow A'_i(x) = A_i - \mathrm{grad}_i\,\alpha(x)$

SUPPONIAMO ORA LA TEORIA MI INTERESSA SIA INVAR x UNITARIE $\varphi'(x) = O\varphi(x)$, $O \in SU(N)$

$\rightarrow$ IMPONENDO INVAR LOC $A'_i = A_i - i\hbar O^\dagger_x \partial O_x \longrightarrow$ MATR $\in SU(N)$ CAMBIA ORA x HERMITIANA $\oplus$ TR $= 0$

LIVELLI DI LANDAU

FASE SINGOLE PARTIC $h = \frac{p^2}{2m} \longrightarrow$ e IN CAMPO EM $\frac{1}{2m}(\vec p + \frac{e}{c}\vec A)^2 = \frac{1}{2}m v^2 \rightarrow v = \dot x = \frac{p + \frac{e}{c}A}{m}$

FISSIAMO $\vec B \parallel \hat z \longrightarrow$ LIBERTÀ DI SCELTA $A \longrightarrow A = (A_x, A_y, 0)^T$

$B = \begin{vmatrix} \hat i & \hat j & \hat k \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix} \Rightarrow B_z = \partial_x A_y - \partial_y A_x \Rightarrow$ INDIP DA z

$[v_x, v_y] = \frac{1}{m^2}\left[p_x + \frac{e}{c}A_x, p_y + \frac{e}{c}A_y\right] = \frac{1}{m^2}\left(-i\hbar\,\partial_x A_y + i\hbar\,\partial_y A_x\right) = -\frac{eB_z}{m^2 c}i\hbar$ COMMUTATORE GAUGE INVAR $\rightarrow B$ INDIP DA GAUGE

SE B_z UNIF A MENO DI FATTORI NUMERICI DIVENTANO $[\;]$ CANONICHE

DEF $\hat P = v_x\left(\sqrt{\frac{eB_z}{m^2 c}\hbar}\right)^{-1}, \quad \hat Q = v_y\left(\sqrt{\frac{eB_z}{m^2 c}\hbar}\right)^{-1} \longrightarrow [P, Q] = -i\hbar \Rightarrow$ HAMILTONIANA $\frac{1}{2}m(v_x^2 + v_y^2) + \frac{p_z^2}{2m} = \frac{1}{2}m\frac{eB\hbar}{m^2 c}(P^2 + Q^2) + \frac{p_z}{2m} = \overbrace{\hbar\omega_c}^{\text{FREQ DI CICLOTRONE}}\left(\frac{P^2 + Q^2}{2}\right) + \frac{p_z^2}{2m}$

$\longrightarrow$ TERMINE CINETICO IN Z $\oplus$ OSCILLATORE ARM EIGEN. $n + \frac{1}{2}$

$\Rightarrow$ SPETTRO OSCILLATORE TRASLATO IN B

$\longrightarrow$ LIVELLI DI LANDAU

PARTICELLE IN BOX $\longrightarrow$ CONSERVARE $p_x\,p_y\,p_z \longrightarrow$ A FORTIORI DEG. XKÉ COSA SI GDL UNIFORM p_x E p_y

FACENDO CONTI LANDAU SI VEDE CHE $\mathcal{J}$ MOLTO SCELTE DI A x STESSA FISICA $\longrightarrow$ 2 ESEMPI ① GAUGE LANDAU $\vec A_L = \begin{pmatrix} 0 \\ xB \\ 0 \end{pmatrix} \rightarrow$ PORTA A FORMA DI HERMITE $\;\mathcal{Z}$ OSC CA

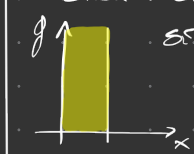
② GAUGE SIMMETRICO $\vec A_S = \begin{pmatrix} -\frac{yB}{2} \\ \frac{xB}{2} \\ 0 \end{pmatrix} \rightarrow$ NATURALE XKÉ COORD CILINDRICHE

$h = \frac{1}{2m}\left(p_x + \frac{e}{c}A_x\right)^2 + \left(p_y + \frac{e}{c}A_y\right)^2 = \frac{1}{2m}\left[p_x^2 + \left(p_y + \frac{e}{c}Bx\right)^2\right] \Rightarrow p_y$ COST MOTO $\Rightarrow$ x AUTOVAL $h\varphi = E\varphi \rightarrow \varphi(x, y) = e^{ik_y y}a(x) \Rightarrow \frac{1}{2m}\left[p_x^2 + \left(\hbar k_y + \frac{eB}{c}x\right)^2\right]a = Ea$ OSCILLATORE TRASLATO

SI VEDE LUNGH CARATT FOND $[\hbar k] = \left[\frac{eB}{c}\right] \quad \left(k = \frac{1}{L}\right) \Rightarrow L^2 = \frac{\hbar c}{eB} \longrightarrow$ RISCALO $\xi = \frac{x}{L}$

$\Rightarrow$ EQ AUTOVAL INS $\frac{1}{2m}\left(-\frac{\hbar^2}{L^2}\frac{d^2}{d\xi^2} + \frac{\hbar^2}{L^2}(\xi + k_y L)^2\right)a(\xi) = E a(\xi) \Rightarrow -\frac{a''}{2} + (\xi + kL)^2\frac{a}{2} = \underbrace{\left(\frac{mL^2}{\hbar^2}\right)}_{\frac{m}{\hbar^2}\frac{\hbar c}{eB}} a E = \frac{E}{\hbar\omega_c}a$ DEVO CALCOLARE ETA MONNEZZA

$\rightarrow$ STIMA POSSIBILE: È NON È OVUNQUE MA CONFINATO XSS IN UNA BARRIERA $\rightarrow$ GEOM STRISCIA

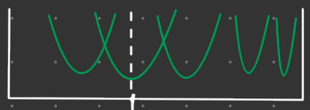


SE SIAMO LONTANO DAI BORDI $\Rightarrow$ SOL $e^{-\frac{1}{2L^2}(x + kL^2)^2}H_n\left(\frac{x + kL^2}{L}\right)$

CENTRATE IN $-kL^2$ CON OSCILLAZ SU SCALA L E AUTOVAL $\hbar\omega_c\left(n + \frac{1}{2}\right)$

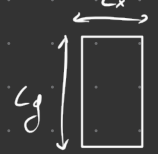
$L = \frac{2{,}56\ nm}{\sqrt{B}}$ ↳ TESLA $\longrightarrow$ SE LA MICRO SCALA DECADONO AUTOFUNZ $\Rightarrow$ A DIST L DAL BORDO NON NE SENTO L'EFFETTO

⇒ NEL BULK LIV LANDAU EQUISPAZIATI → OSCILLATORE ARM → AVVICINANDOMI AL BORDO SENTO LA PARETE



+ MI AVVICINO AL BORDO + SI INNALZA LA PARABOLA → CLASSICAM AL CENTRO CORRENTE NULLA, AI BORDI UNA SALE L'ALTRA SCENDE

→ QUANTO SONO DEGENERI I LIV DI LANDAU NEL BULK?



$$\frac{L_x L_y B}{\Phi_0}$$

SCRIVO HAM DIP DA A —

MI LIBERO DI $\underline{x}_i$ USANDO δ

$$H_A=\sum_{i=1}^{N}\frac{1}{2m}\left(\underline{p}_i+\frac{e}{c}\underline{A}(\underline{x}_i,t)\right)^2+U(\underline{x}_1 \dots \underline{x}_N)\ \text{I QUANT} =\sum\frac{\underline{p}_i^2}{2m}+\frac{e}{2mc}\sum\left(\underline{p}_i\underline{A}(\underline{x}_i t)+\underline{A}(\underline{x}_i t)\underline{p}_i\right)+\frac{e^2}{2mc^2}\sum\underline{A}(x_i t)+U(x_1 \dots x_N)=H_{A=0}+\int d_3x\,\frac{e}{2mc}\sum\left[\underline{p}_i\delta_3(\underline{x}-\underline{x}_i)+\delta_3(\underline{x}-\underline{x}_i)\underline{p}_i\right]\underline{A}(\underline{x}t)+\frac{e^2}{2mc^2}\int d_3\underline{x}\sum\delta_3(\underline{x}-\underline{x}_i)A^2(\underline{x}t)$$

$$=H_{A=0}+\frac{e}{c}\int d_3\underline{x}\ \underline{j}(\underline{x})\underline{A}(\underline{x}t)+\frac{e^2}{2mc^2}\int d_3\underline{x}\ A^2(\underline{x}t)\hat{n}(\underline{x})$$

→ IN EQ CONTINUITÀ × PROB POSIZ EMERGE QUESTA CORRENTE DI PROB IN ASSENZA DI $\vec{A}$ → NB $\underline{v}\neq \underline{p}/m$

$\underline{j}$ NON È LA CORRENTE CHE OSS × CAMPO MAGN

$$n_{H_A}(\underline{x}t)=e^{\frac{i}{\hbar}H_A t}\,\hat{n}(\underline{x})\,e^{-\frac{i}{\hbar}H_A t}\longrightarrow -\frac{i}{\hbar}\frac{\partial}{\partial t}n_{H_A}(\underline{x}t)=e^{\frac{i}{\hbar}H_A t}\left[H_A,n(\underline{x})\right]e^{-\frac{i}{\hbar}H_A t}$$

↳ IN GEN DIP DA t XKÈ A DIP DA t

$[\hat{n},U]=0$ ⇒ PT CINETICA NON COMMUTA → DEF $\hat{\rho}(\underline{x}t)=-e\hat{n}(\underline{x}t)$ DENSITÀ DI CARICA

→ CONSERVAZ CARICA ⇒ EQ CONTINUITÀ $\frac{1}{c}\frac{\partial}{\partial t}\hat{\rho}(\underline{x}t)=-\mathrm{div}\,\underline{J}(\underline{x}t)$ CON $\underline{J}=-e\underline{j}-\frac{e^2\underline{A}}{mc^2}\hat{n}(\underline{x})=\rho\,\underline{v}$ CORRENTE CONSERVATA CHE MIS IN PRESENZA DI CAMPO

↳ × NOI $\underline{v}=\frac{\underline{p}+\frac{e}{c}\underline{A}}{m}$ ⇒ EQ SOPRA

RISPOSTA LINEARE

ACCENDO $\underline{A}$ A UN CERTO TEMPO E PRIMA SONO IN $|GS\rangle$ DI $H|_{A=0}$ (DIP DA t ⇒ A DIP DA t)

$$\langle\psi(t)|J_e(\underline{x}t)|\psi(t)\rangle=\langle GS|J_e(\underline{x}t)|GS\rangle+\frac{1}{i\hbar}\int dt'\,\theta(t-t')\langle GS|\left[J_e(\underline{x}t),V_{H_{A=0}}(t')\right]|GS\rangle$$

$$\underline{J}=e\underline{j}-\frac{e^2}{mc^2}\hat{n}\underline{A}(\underline{x}t)\ \ =-\frac{e^2}{mc^2}n_{GS}(\underline{x})A_e(\underline{x}t)+\frac{1}{i\hbar}\frac{e}{c}\int dt'd_3x'\,\theta(t-t')\langle GS|\left[J_e(\underline{x}t),j_m(\underline{x}'t')\right]|GS\rangle A_m(\underline{x}'t')$$

OMETTO L'ALTRO TERM XKÈ QUADRATICO NELLA PERTURB (ANCHE SE CONTRIB A $\underline{j}$)

NEL GS $\underline{j}$ HA $\langle\underline{j}\rangle=0$ MA IL II TERM NO

→ VOGLIO ROBA LIN IN $\underline{A}$ ⇒ NEL COMMUTATORE USO SOLO $-ej_e$

$$J_e(\underline{x}t)=-\frac{e}{mc^2}n_{GS}(\underline{x})A_e(\underline{x}t)-\frac{e^2}{\hbar}\int dt'd_3\underline{x}'\,D^{RET}_{em}(\underline{x}t\,\underline{x}'t')A_m(\underline{x}'t')$$

$iD^{RET}_{em}(xx')=\theta(t-t')\langle GS|\left[j_e(x),j_m(x')\right]|GS\rangle$ PARENTE DELLA FORMULA → [DENSITÀ, DENSITÀ]

SE FACCIO $\sum_e\partial_e D^{RET}_{em}=-\frac{\partial}{\partial t}D^{RET}_{0m}(xx')$

↳ COMP 0 VETT QUADRIDIM ⇒ DENSITÀ

SE FACCIO TORD $iD^{T}_{em}(xx')=\langle GS|T\{j_e(x)j_m(x')\}|GS\rangle$ → II MODULO

$$J_e(\underline{x}t)=\int dt'd_3\underline{x}'\left[-\frac{e^2}{mc}n(\underline{x}')\delta_{em}\delta_3(\underline{x}-\underline{x}')\delta(t-t')-\frac{e^2}{\hbar}D^{RET}_{em}(\underline{x}t\,\underline{x}'t')\right]A_m(\underline{x}'t')$$

A NOI INTERESSA CAMPO $\underline{E}$ → $\frac{1}{c}\frac{\partial A_e}{\partial t}=E_e\xrightarrow{\text{SPW}}\frac{i\omega}{c}A_e(\underline{x}\omega)=E_e(\underline{x}\omega)$

POSSO RISCRIVERE $\underline{J}$ LIN IN $\underline{E}$ ⇒ TENSORE CONDUTTIVITÀ

$$J_e(\underline{x}\omega)=\int d_3\underline{x}'\,\sigma_{em}(\underline{x}\,\underline{x}'\omega)E_m(\underline{x}'\omega)$$

LEGGE DI OHM (NO INVARIANZE × ORA) CON $\sigma_{em}(\underline{x}\underline{x}'\omega)=\frac{e^2}{2m\omega}n(\underline{x})\delta_{em}\delta_3(\underline{x}-\underline{x}')+\frac{e^2}{\hbar i\omega}D^{RET}_{em}(\underline{x}\underline{x}'\omega)$

SE INVAR × TRASLAZ $J_e(\underline{k}\omega)=\sigma_{em}(\underline{k}\omega)E_m(\underline{k}\omega)$ → D^{R}_{em} × LEGGE DI OHM SERVE → MODELLO CONTIN DI SCATTERING

SE MANDO ω→0 (CORRENTE COST) → SI CANCELLA SINGOLARITÀ IN σ_{em} ⇒ MODELLO DI SCATTERING FUNZIONA

LEGAME D=εE

$$H_t=H-e\int d_3\underline{x}\,n(\underline{x})\int d_3\underline{x}'\frac{\rho_{EXT}(\underline{x}'t')}{|\underline{x}-\underline{x}'|}$$

⇒ INTERAZ DENSITÀ È INTERNA - EXT ⇒ TR RISPOSTA LINEARE

$$\delta n(\underline{x}t)=\frac{1}{\hbar}\int d_3\underline{x}'dt'\,D^{RET}(\underline{x}t\,\underline{x}'t')(-e)\int d_3\underline{x}''\frac{\rho_{EXT}(\underline{x}''t'')}{|\underline{x}'-\underline{x}''|}\,\varphi(\underline{x}'t')$$

VARIAZ DENS × CARICA INDOTTA

CORRELATORE RITARD. DENSITÀ-DENSITÀ

$$\rho_{ind}(\underline{x}t)=\frac{1}{\hbar}\int d_3\underline{x}'dt'\,D^{RET}(\underline{x}t\,\underline{x}'t')\int d_3\underline{x}''dt''\frac{e^2}{|\underline{x}'-\underline{x}''|}\delta(t'-t'')\rho_{EXT}(\underline{x}''t'')=\int d_4x'\,\Pi^{R}(xx')\int d_4x''\,U^0(x'x'')\rho_{EXT}(x'')$$

→ EQ MAXWELL $\nabla\underline{E}=4\pi(\rho_{EXT}+\rho_{ind})$

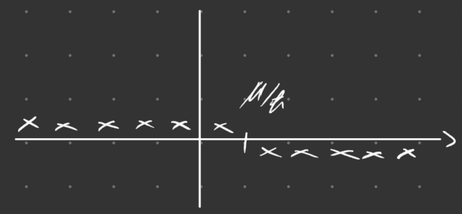
SE SP R (INVAR TEMP-SPAZ) ⇒ $i\underline{k}\underline{E}(\underline{k}\omega)=4\pi\left[1+\Pi^R(\underline{k}\omega)U(\underline{k})\right]\rho_{EXT}(\underline{k}\omega)=\frac{4\pi\,\rho_{EXT}(\underline{k}\omega)}{\varepsilon^R(\underline{k}\omega)}$ con $\Pi^{R}_{\varepsilon}=\frac{\Pi^{*R}}{1-U(\underline{k})\Pi^{*R}}$

→ SE METTO A SYS CON $i\underline{k}\underline{D}=4\pi\rho_{EXT}$ ⇒ HO ROBA SE COMP ∥ $\underline{k}$ → SUPP ISOTROPIA ⇒ $\underline{D}\parallel\underline{k}$ ⇒ $\underline{D}=\varepsilon^R\underline{E}$

→ ε CALCOLABILE SOLO CON Π^{*} XKÈ ∄ NOTA ROBA ANALITICA

QUASI PARTICELLE

SVILUPPO LEHMANN PROPAGATORE $G_{\mu\mu'}(\underline{x}\underline{x}'\omega) = \sum_n \frac{\langle E_0^N|\psi_\mu(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\psi^\dagger_{\mu'}(\underline{x}')|E_0^N\rangle}{\omega - \frac{1}{\hbar}(\mu+\varepsilon_n^{N+1})+i\eta} + \frac{\langle E_0^N|\psi^\dagger_{\mu'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\psi_\mu(\underline{x})|E_0^N\rangle}{\omega-\frac{1}{\hbar}(\mu-\varepsilon_n^{N-1})-i\eta}$



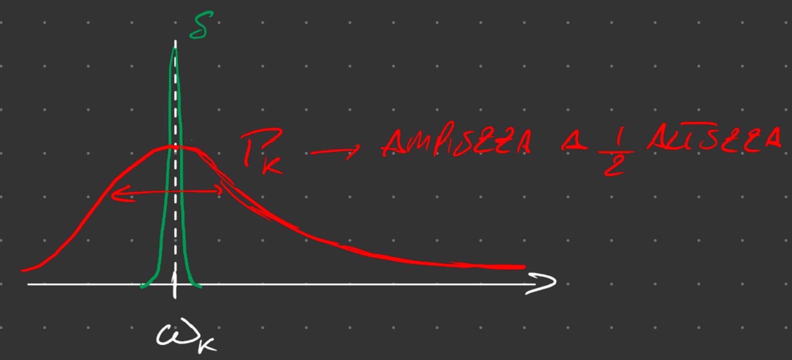
SE $\omega \to \pm\infty \Rightarrow G = \frac{1}{\omega}\sum_n \langle \underbrace{|\rangle\langle|}_{1}\rangle + \langle\underbrace{|\rangle\langle|}_{1}\rangle$ + CORREZIONI

$= \frac{1}{\omega}\langle E_0^N|\{\psi_\mu(\underline{x}),\psi^\dagger_{\mu'}(\underline{x}')\}|E_0^N\rangle = \frac{1}{\omega}\delta_{\mu\mu'}\delta_3(\underline{x}-\underline{x}')$ → SE ANCHE CONSID INVARIANZA X TRASLAZ E INDIP SPIN $G_{\mu\mu'}(\underline{k}\omega) = \delta_{\mu\mu'}G(\underline{k}\omega)$

$\Rightarrow G(\underline{k}\omega) = \int d\omega' \frac{A(\underline{k}\omega')}{\omega-\omega'+i\eta\,\mathrm{sgn}(\omega-\frac{\mu}{\hbar})}$ → $\begin{cases} A(\underline{k}\omega)\ge 0 \\ \int_{\pm\infty} d\omega\, A(\underline{k}\omega) = 1\end{cases}$ → SEMBRA SOVRAPP CON UNA MIS DI PROB DI $\frac{1}{\text{PROPAG PARTIC LIBERA}}$

CASO ELEMENTARE [FREE PARTICLE]

$A^0(\underline{k}\omega) = \delta\left(\omega - \frac{\varepsilon_0(\underline{k})}{\hbar}\right)$ ⇒ δ CENTRATA SU LEGGE DISPERS X FREE PARTICLE ⇒ PROPAG LIBERO



SE HO INTERAZ ⇒ SI ALLARGA δ → APPROX LORENTZIANA

$A(\underline{k}\omega) = \frac{\Gamma_k}{\pi}\frac{1}{(\omega-\omega_k)^2+\Gamma_k^2} \xrightarrow[\Gamma_k\to 0]{} A^0$ $\Rightarrow G(\underline{k}\omega) = \frac{1}{\omega-\omega_k+i\Gamma_k\,\mathrm{sgn}(\omega-\frac{\mu}{\hbar})}$

$n(\underline{k})$ = # OCCUPAZ STATO k → CALCOLAB DA ORDER ε_g

→ APPROX DISASTROSA: CASO È HA MOMENTI FINITI MENTRE QUI DECAY TROPPO LENTO X AMMETTERLI

$G(\underline{k}\omega) = \fint_{\pm\infty} d\omega' \frac{A(\underline{k}\omega')}{\omega-\omega'} - i\pi A(\underline{k}\omega)\,\mathrm{sgn}(\omega-\frac{\mu}{\hbar})$

Im G CAMBIA sgn IN $\frac{\mu}{\hbar}$ $\begin{cases} <0 & \omega > \mu/\hbar \\ >0 & \omega<\mu/\hbar \end{cases}$ → 0 IN $\omega = \mu/\hbar$

CON $G(\underline{k}\omega) = \frac{1}{\omega-\omega_k+i\Gamma_k\,\mathrm{sgn}(\omega-\mu/\hbar)}$ → 1 POLO ⇒ COME QUANDO FACCIO TRASF FOURIER E ANDO PIANO

⇒ 1 ESP COME NOTE

→ TRASF DI LORENTZ

∃ ALTRA RAPPR DI G → $G(\underline{k}\omega) = \frac{1}{\omega-\frac{\varepsilon_0^k}{\hbar}-\Sigma^*(\underline{k}\omega)}$ EQ DYSON → SEPARO x ω IN PT Re E Im $= \frac{1}{(\omega-\frac{\varepsilon_0^k}{\hbar}-\mathrm{Re}\,\Sigma^*)-i\,\mathrm{Im}\,\Sigma^*}$

$\Rightarrow \mathrm{Im}\,G(\underline{k}\omega) = \frac{\mathrm{Im}\,\Sigma^*(\underline{k}\omega)}{[\omega-\frac{\varepsilon_0^k}{\hbar}-\mathrm{Re}\,\Sigma^*]^2+\mathrm{Im}\,\Sigma^{*2}}$ → 0 IN $\omega = \frac{\mu}{\hbar}$ $\forall \underline{k}$ ⇒ DENOM PUNTI NSACO ⇒ IN SE $\underline{k}$ È SUP FERMI $\underline{k}$ TC $\frac{\mu}{\hbar}-\frac{\varepsilon_0(\underline{k})}{\hbar}-\Sigma^*(\underline{k}\frac{\mu}{\hbar})=0$

Re xkè $\mathrm{Im}\,\Sigma^*(\underline{k}\frac{\mu}{\hbar})=0$

TEOR [LUTTINGER-WARD] IL VOL RACCHIUSO IN SUP FERMI È PARI ALLA DENSITÀ (x DIV DI SPIN, x NOI ½) $\int\frac{d\underline{k}}{(2\pi)^3}\,\theta(G(\underline{k},\frac{\mu}{\hbar})) = \hat{n}$

CONTINUITÀ se HEG e accendo Coulomb ⇒ sup HEG → sf Fermi (interaz nulla)   il vol $\frac{4}{3}\pi k_F^3$ rimane invar accendendo interaz

→ con Coulomb sys ancora isotropo ma ∃ interaz che deformano la sup

$G(\underline{k}\omega) = \frac{1}{\omega - \frac{\varepsilon^0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega)}$ una quasipartic deve essere un polo semplice del propag → $\omega - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega) = 0$ con $\omega(\underline{k}) = \underbrace{\omega_1(\underline{k})}_{Re} + i\underbrace{\omega_2(\underline{k})}_{Im}$

⇒ G funz di l'andament $G(\underline{k}\omega) = \frac{z(\underline{k})}{\omega - \omega_{polo}(\underline{k})} + G^{reg}(\underline{k}\omega)$
↳ f analitica

↳ assomiglia a quella di una particella ma con p. Im finita ω_2 ⇒ roba che decade

è come se da funz spettrale avessimo estratto δ davanti a $z(k) \le 1$ ⇒ se $z(k)$ di ordine 1 ⇒ contrib importante

$\omega_1 + i\omega_2 - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega_1 + i\omega_2) = 0$ $\omega_2 = (\text{vita partic})^{-1}$ ⇒ lo voglio piccolo → $0 = \omega_1 + i\omega_2 - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega) - i\omega_2 \frac{\partial\Sigma^*}{\partial\omega}\Big|_{\omega_1} + \dots$

p. Re $0 = \omega_1(\underline{k}) - \frac{\varepsilon_0(\underline{k})}{\hbar} - \mathrm{Re}\,\Sigma^*(\underline{k}\,\omega_1(\underline{k}))$

p. Im $0 = \omega_2(\underline{k}) - \mathrm{Im}\,\Sigma^*(\underline{k}\,\omega_1(\underline{k})) - \omega_2(\underline{k})\frac{\partial\,\mathrm{Re}\,\Sigma^*}{\partial\omega}\Big|_{\omega=\omega_1}$ ⇒ $\omega_2 = \frac{\mathrm{Im}\,\Sigma^*(\underline{k}\,\omega_1(\underline{k}))}{1 - \frac{\partial}{\partial\omega}\mathrm{Re}\,\Sigma^*(\underline{k}\omega)\Big|_{\omega_1(\underline{k})}}$ $z, 0$ dove $\mathrm{Im}\,\Sigma^* = 0$ ⇒ su sup Fermi

$z(\underline{k}) = \lim_{\omega\to\omega_{polo}} G(\underline{k}\omega)(\omega-\omega_{polo}) = \lim_{\omega\to\omega_{polo}} \frac{\omega-\omega_{polo}}{\omega - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}\omega)}$

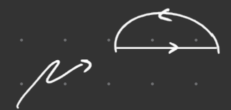
ω_2 trasc → $\simeq \lim_{\omega\to\omega_1} \frac{\omega-\omega_1}{(\omega-\omega_1)\left[1 - \frac{\partial\,\mathrm{Re}\,\Sigma^*}{\partial\omega}\Big|_{\omega_1}\right]} = \frac{1}{1 - \frac{\partial\,\mathrm{Re}\,\Sigma^*}{\partial\omega}\Big|_{\omega_1}}$ ⇒ $\omega_2 = z(\underline{k})\,\mathrm{Im}\,\Sigma^*(\underline{k}\omega)$

Quasi partic = poli semplici di G con peso $z(\underline{k})$ di ordine 1 e vivono in regione di k vicino sup di Fermi (ω_2 piccolo)

→ in caso di QP la sup di Fermi è discontinuità del # di occupaz (come sf Fermi lo è x # occupaz 1 in 0 out)

→ il salto vale esattam z

Teor. di Migdal   sup di Fermi è sup di disc x # occupaz $n_{\underline{k}} = \langle GS|a^\dagger_{\underline{k}\mu} a_{\underline{k}\mu}|GS\rangle$



→ uso formula vera a prescindere da QP $n(\underline{k}) = \sum_\mu \langle GS|a^\dagger_{\underline{k}\mu} a_{\underline{k}\mu}|GS\rangle = \int_{\pm\infty} \frac{d\omega}{2\pi i}\, G(\underline{k}\omega)\, e^{i\omega\eta} = \int_{\pm\infty} \frac{d\omega}{2\pi i}\frac{z(\underline{k})}{\omega-\omega_1-i\omega_2}\, e^{i\omega\eta} + n^{reg}(\underline{k}) = n^{reg}(\underline{k}) + z(\underline{k})\,\theta\left(\frac{\mu}{\hbar} - \omega_1(\underline{k})\right)$

⇒ il polo QP da p. Re che sia ± p. Im a cui contrib 0

z, salto in $k = k_F$ (sf Fermi) $\frac{\mu}{\hbar} - \varepsilon_0(\underline{k}) - \Sigma^*(\underline{k}\frac{\mu}{\hbar}) = 0$ sup Fermi → caso isotropico $k_F = (3\pi^2 n)^{1/3}$ raggio sf → le QP vivono a lungo vicino sf Fermi ⇒ chiamiamo signif ω_1 e ω_2 ripassando in t

$G(\underline{k}t) = \int_{\pm\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\left(\frac{z(\underline{k})}{\omega-\omega_p} + G^{reg}(\underline{k}\omega)\right)$ $t > 0$ ⇒  $= \underbrace{\frac{-2\pi i}{2\pi}\, e^{-it(\omega_1(k)+i\omega_2(k))}\, z(\underline{k})}_{\text{p. di QP}} + \dots$

interpreto ω_1 effettivam come ecc a disposizione → $\varepsilon(\underline{k}) = \hbar\omega(\underline{k})$ x analogia evoluz temp libera

SE CHIUDO SOTTO $\Rightarrow$ POLO SOLO IN $\omega_2 < 0 \Rightarrow \omega_1 > \frac{\mu}{\hbar}$ HO 0 SE $\omega_1(k) < \mu/\hbar$ $\rightarrow$ SMORZ MOLTO LENTO DOVUTO A $e^{\omega_2 t}$ SE $\omega_1 > \mu/\hbar$

$\rightarrow$ QUASI PARTIC = MANIFESTAZ COLLETTIVA DOVUTA AL MIND ESTRONS $\rightarrow$ STESSA CARICA DI e^- MA $\neq$ MASSA E RISENTE DELLA SELF ENERGIA

LE PARTIC SONO INDISTINGUIB $\Rightarrow$ EFF COLLETTIVO $\rightarrow$ PROPAGAZ DI TUTTO L'INSIEME PUÒ MANIF IN EVIDENZA UN CONTRIB DI POLO $\Rightarrow$ QP

SE SONO VICINO A SUP FERMI $\Rightarrow$ SVIL ATTORNO k_F $\times$ QP $\omega_1(\underline{k}) = \omega_1(k_F) + \left.\frac{\partial \omega_1}{\partial k}\right|_{k_F}(\underline{k}-k_F) + \_\_$ $\times$ FREE PARTICLES $\omega^0(\underline{k}) = \omega^0(k_F) + \frac{\hbar k_F}{m}(k-k_F) + \_\_$ $\nearrow \frac{\hbar k^2}{2m}$

$\Rightarrow$ $\times$ ANALOGIA $\frac{\partial \omega_1}{\partial k} = \frac{\hbar k_F}{m^*}$ $\rightsquigarrow$ MASSA EFFETTIVA $\rightarrow$ SE NON FOSSIMO IN SF $\Rightarrow m^* \neq \forall$ DIR $\longrightarrow$ $\omega_1(\underline{k}) - \frac{\varepsilon_0(\underline{k})}{\hbar} - \Sigma^*(\underline{k}, \omega_1(\underline{k})) = 0$ (CONDIZ DI POLO)

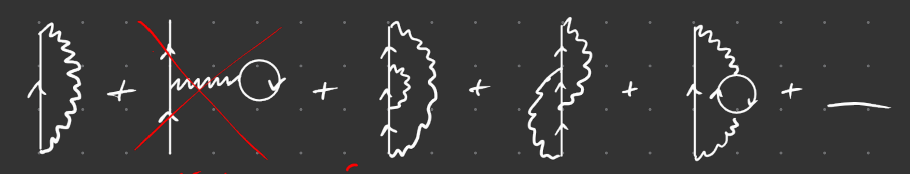
$\rightarrow$ SVILUPPO $\omega_1(k_F) + \frac{\hbar k_F}{m^*} - \frac{\varepsilon_0(k_F)}{\hbar} - \frac{\hbar}{m} k_F(k-k_F) - \Sigma^*(k_F, \omega_1(k_F)) - \left.\frac{\partial \Sigma^*}{\partial k}\right|_{k_F}(k-k_F) - \frac{\partial \Sigma^*}{\partial \omega}\left.\frac{\partial \omega_1}{\partial k}\right|_{k_F}(k-k_F) + \_\_ = 0$
$\hookrightarrow$ TERM II gr IN k_F CHE TRASC

$\rightarrow$ $\times$ DEF DI SUP FERMI IL TERM DI ORD 0 È NULLO $\rightarrow \frac{\mu}{\hbar} - \frac{\mu_0}{\hbar} - \Sigma^*(k_F, \frac{\mu}{\hbar}) = 0 \Rightarrow \frac{\hbar k_F}{m^*} - \frac{\hbar k_F}{m} - \left.\frac{\partial \Sigma^*}{\partial k}\right|_{k_F}(k_F, \omega_1(k_F)) - \frac{\partial \Sigma^*}{\partial \omega}\frac{\hbar k_F}{m^*} = 0$ $\rightsquigarrow$ DA QUI RICAVO m^* IN TERM DI Σ^*

$\rightarrow$ L'INTERAZ COULOMB NON MODIFICA MOLTO m^* RISP m_e $\longrightarrow$ CAMBIO DRASTICO $\times$ INTERAZ e^--FONONI (50-100%)

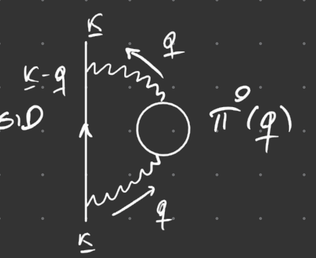
<u>SOTTOLINEA Z</u>

$\times$ COULOMB IN HEG $\Sigma^*(k) =$



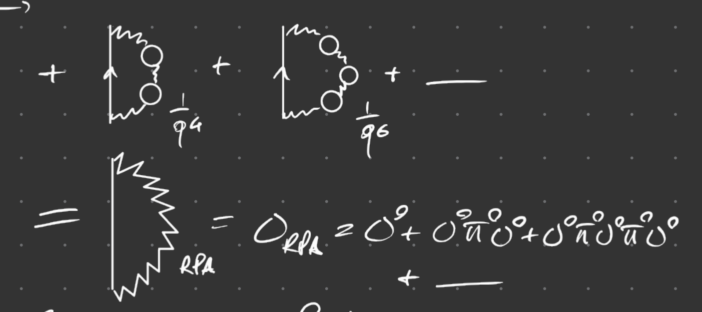
NON C'È IN HEG XKÉ HA $q=0$ CHE SI CANCELLA

$\rightsquigarrow$ CONSID



$\pi^0(q)$ $\rightsquigarrow$ BRUTTA DIVERG $\frac{1}{q^2}$ $\rightarrow$

PASSANDO A PERT SH E FACENDO PERTURBAZ PRENDO DIAG SEMPRE PEGGIORI IN ORD SUCC COME CORREZIONE

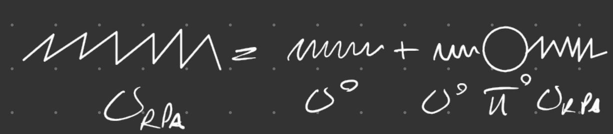


SOMMO SOLO BOLLE

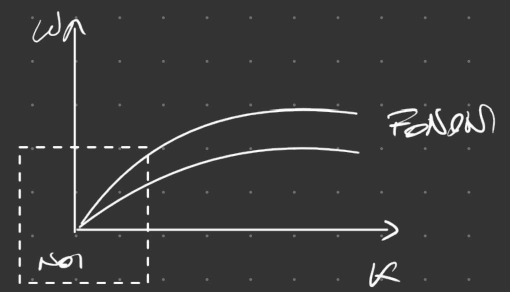
$\Rightarrow U_{RPA}(q) = \frac{4\pi e^2}{q^2 - 4\pi e^2 \pi^0(q)}$ $\longrightarrow$ LA DIV DIMINUENDA $\frac{1}{q^2}$ È ACCOMPAGNATA DA π^0

$\rightarrow$ SCHERMAGGIO CHE FA SCOMPARIRE DIVERGENZA log $\Rightarrow$ TEO PERTURBATIVA AL II ORDINE

$\Rightarrow$

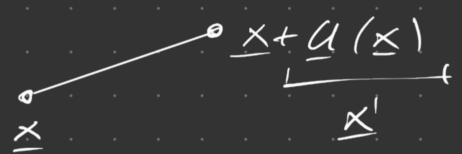


BASI TEO ELASTICITÀ   LA CARICA UNIF VERRÀ TRATTATA COME MEZZO ELASTICO OMOG E ISOTROPO INTERESSANDOCI SE K PICCOLI ($\Rightarrow$ SOPRA PASSO RETICOLARE)



PRIMA CARATT I MEZZI CONTINUI $\rightarrow$ POI GENERALIZ LEGGE HOOK $\rightarrow$ VOGLIO TENSORE STRESS PROP A GRADIENTI DI DEFORMAZIONE

MEZZO CONTINUO $\rightarrow$ APPLICO DEFORMAZ CON $u(\underline{x})$ PICCOLO E PICCOLI GRADIENTI (VOGLIAMO TUTTO LIN)



$\Rightarrow$ DESCRIVO COME CAMBIANO I RAPPORTI TRA PTI VICINI COL TENSORE METRICO CHE A RIPOSO È UNA δ

$$ds'^2 = dx'^i dx'_i = \frac{\partial x'^i}{\partial x_j} dx^j \frac{\partial x'_i}{\partial x_k} dx^k$$

$$\equiv g_{jk}(\underline{x}) dx^j dx^k \rightsquigarrow g_{jk}(\underline{x}) = \frac{\partial x'^i}{\partial x^j}\frac{\partial x'_i}{\partial x_k}$$

NEL NOSTRO CASO $g_{jk}(\underline{x}) = \left(\delta_{ij} + \frac{\partial u_i}{\partial x^j}\right)\left(\delta_{ki} \frac{\partial u_i}{\partial x^k}\right) = \delta_{jk} + \left(\frac{\partial u_k}{\partial x^j} + \frac{\partial u_j}{\partial x^k}\right)$

$\equiv \delta_{jk} + 2\partial_{jk}$ $\rightsquigarrow$ STRAIN TENSOR (DEFORMAZ)

[$M \geq 0 \Leftrightarrow \exists R$ TC
$M = R^T R$
$\Rightarrow$ È SIMMETRICA] $\leftarrow$ MATR NON NEG XKÉ PROD DI 2 MATR UNA TRASPOSTA DELL'ALTRA

→ come variano i volumi? $d_3\underline{x}' = \left|\det \frac{\partial x'}{\partial x}\right| d_3\underline{x} = \sqrt{g(x)}\, d_3\underline{x}$ ⇒ $\sqrt{g} = \sqrt{\det(\mathbb{1}+2D)}$ con D essere piccolo risp $\mathbb{1}$ ⇒ $\det(\mathbb{1}+\varepsilon M) = \mathbb{1}+\varepsilon\,\mathrm{Tr}M + O(\varepsilon^2)$

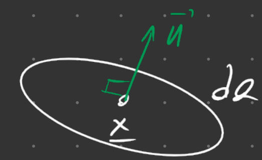
$g \equiv \det g_{ij}$ $\qquad = \sqrt{1+2\,\mathrm{Tr}D + \dots} \simeq 1+\mathrm{Tr}D = 1+\mathrm{div}\,\underline{u}$

⇒ $\delta V = d_3\underline{x}' - d_3\underline{x} = \mathrm{div}\,\underline{u}\, V$ ⇒ $\mathrm{div}\,\underline{u} = \frac{\delta V}{V}$ → $\mathrm{div}\,\underline{u}$ si calcola in un pto attorno a cui si centra il vol

→ cambia anche densità → no conservaz uniformità $\rho_0 d_3x = \left(\rho_0 + \delta\rho(\underline{x})\right) d_3x' = \left[\rho_0 + \delta\rho(\underline{x})\right](1+\mathrm{div}\,\underline{u})\, d_3x$ ⇒ $\delta\rho(\underline{x}) = -\rho_0\, \mathrm{div}\,\underline{u}$

così applicando deformazione il mezzo reagisce con forze interne

→ come si descrivono in un mezzo continuo? tensore di stress Cauchy $\sigma_{ij}(\underline{x})$



$$\boxed{df_i = \sigma_{ij}(\underline{x})\, n^j da}$$

comp i-esima forza su da
→ anche senza la deform si è nuovam all'equilibrio

x fluidi ideali e forza di taglio $\sigma_{ij} = -p\,\delta_{ij}$ ⇒ $\underset{\text{(traccia)}}{\sigma^k_{\ k}} = -3p$

in gen se? $p = -\frac{1}{3}\sigma^k_{\ k}$

preso V delim da sup S in mezzo continuo ⇒ tiriam vi agiscono forze short range

$$F_i = \oint_S da\, \sigma_{ij} n^j \underset{\text{flusso}}{=} \int d_3\underline{x}\, \partial^j \sigma_{ij}$$

↳ diverg σ = forza x unità di vol

⇒ posso scrivere legge di Newton x questa forza x unità di vol
→ consid densità imperturb xké già accelerazione è piccola
⇒ al I ord (gradienti di u tutti piccoli)

$$\rho_0\, \ddot{u}_j = \partial^i \sigma_{ij}$$

x scrivere σ_{ij} mi specializzo a mezzi elastici

I relaz generale $\sigma_{ij}(\underline{x}) = C^{kl}_{ij}(\underline{x})\, D_{kl}(\underline{x})$ $\qquad$ simm in kl e ij (xké σ_{ij} simm)

↳ prop elastiche che variano di pto in pto

se il mezzo ha prop omogenee ⇒ indip da $\underline{x}$

se ha simm tc $ij \leftrightarrow kl$ (simm x scambio) ⇒ conservaz en $\qquad$ inoltre σ_{ij} simm x conservaz mom ang

x mezzi elastici

⇒ $\sigma_{ij}(\underline{x}) = \lambda\, \delta_{ij} D^k_{\ k}(\underline{x}) + 2\mu D_{ij}(\underline{x})$

entrano le 2 quantità che si trasf in modo indip sotto rotaz → $D^k_{\ k}$ e D_{ij}

λ, μ cost di Lamé (dim di pressione)

sostituendo in eq moto

$$\rho_0\, \ddot{u}_j = \lambda\, \partial_j D^k_{\ k} + 2\mu\, \partial^i D_{ij} = \lambda\, \partial_j \partial_k u_k + \mu\, \partial^i\left(\frac{\partial u_j}{\partial x^i} + \frac{\partial u_i}{\partial x^j}\right) = (\lambda+\mu)\,\partial_j\partial_k u_k + \mu\, \nabla^2 u_j$$

⇒ vettorialmente $\rho_0\, \ddot{\underline{u}} = (\lambda+\mu)\,\mathrm{grad}(\mathrm{div}\,\underline{u}) + \mu\, \nabla^2 \underline{u}$

da questa eq di ordine 2 → ① divergenza $\rho_0 \frac{\partial^2}{\partial t^2}\mathrm{div}\,\underline{u} = (\lambda+2\mu)\,\nabla^2 \mathrm{div}\,\underline{u}$ → eq descrivente onde di vol ≠

→ ② rotore $\rho_0 \frac{\partial^2}{\partial t^2}\mathrm{rot}\,\underline{u} = \mu\, \nabla^2 \mathrm{rot}\,\underline{u}$

① VEL LONGITUD $v_L = \sqrt{\frac{\lambda+2\mu}{\rho_0}}$ $\Rightarrow$ $\left(\frac{1}{v_L^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) \mathrm{div}\,\underline{u} = 0$ $\Rightarrow$ $\mathrm{div}\,\underline{u}$ CAMPO SCALARE CHE DESCRIVE LA VARIAZ DI VOL $\longrightarrow$ ONDE DI PRESSIONE LONGITUD

v_L = VEL SUONO NEL MEZZO

② $v_T = \sqrt{\frac{\mu}{\rho_0}}$ ONDE TRASVERSALI $\rightarrow$ $(\mathrm{rot}\,\underline{u})_i = \varepsilon_{ijk}\partial_j u_k \Rightarrow \perp \underline{u}$ ONDE + PICCOLE DELLE LONGIT INFATTI (ES $\lambda=0$ COMUNQUE) $v_T \leq \sqrt{2}\, v_L$

I VARI MODI SONO LEGATI A VAL DI λ, μ

<u>COMODITÀ</u>

Al	$v_L \simeq 6100\ \frac{m}{s}$	
H_2O	1400	+ LEGGERO IL MAT + VELOCE È $\longrightarrow$ x H LIQUIDO HO $v_{max} = 3600$ m/s
DIAMANTE	1200	

x APPROFONDIRE STA CAPPONATA $\rightarrow$ <u>SP K</u> (TRATTAZ SIMILE A CAMPO em) $u_j(\underline{x}t) = \sum_{\underline{k}} u_j(\underline{k}t)\frac{e^{i\underline{k}\underline{x}}}{\sqrt{V}}$ $\Rightarrow$ $u_j(\underline{k}t) = \frac{k_j}{k} k^l a_l(\underline{k}t) + \left(\delta_{jl} - \frac{k_j k_l}{k^2}\right) u^l$

PROIEZ LONGITUDINALE — PROIEZ TRASVERSALE

TEOR CLEBSCH = CAMPO VETT SEMPRE SCOMPONIB IN 2 PT $\rightarrow$ 1 A ROTORE 0 $\longrightarrow$ $\mathrm{div} = k \Rightarrow$ ANNULLA PT $\perp \underline{k}$

$\rightarrow$ DIV 0 — $\mathrm{rot} \Rightarrow$ ANNULLA PT $/\!/\ \underline{k}$

IN SP K $\rho_0\frac{\partial^2}{\partial t^2} u_j(\underline{k}t) = -\mu k^2 u_j(\underline{k}t) - (\lambda+\mu)k_j k^l u_l(\underline{k}t)$ $\rightarrow$ APPLICO I PROIEZIONI LONG E TRASV $\longrightarrow$ $\begin{cases}\rho_0 \ddot{u}_j^L = -(\lambda+2\mu)k^2 u_j^L \\ \rho_0\ddot{u}_j^T = -\mu k^2 u_j^T\end{cases}$ $\Rightarrow$ EQ ONDE DISACCOPPIATE

$\Rightarrow$ SOL GENERALE $\vec{u}(\underline{x}t) = \frac{1}{\sqrt{V}}\sum_{\underline{k}\lambda} \vec{e}_\lambda(\underline{k})\left[e^{i\underline{k}\underline{x}-i\omega_\lambda t} d_{\underline{k}\lambda} - e^{-i\underline{k}\underline{x}+i\omega_\lambda t} d^*_{\underline{k}\lambda}\right]\frac{1}{i}$ $\Rightarrow$ $\vec{e}_3(\underline{k}) = \frac{\vec{k}}{k}$ DIR LONG; $\vec{e}_{1/2}(\underline{k}) \perp \underline{k}$ DIR TRASV

(λ $\Rightarrow$ POLARIZZAZ (DIR))

d, d^* AMPIEZZE ONDE, $\omega_\lambda = v_\lambda \underline{k}$ RELAZ DISPERSIONE (DIP DA CONDIZ INIZ) $\begin{cases} v_3 = v_L \\ v_{1/2} = v_T\end{cases}$

SI PUÒ DIM CHE EQ TRASV/LONG $\Rightarrow$ CONSERVAZ EN LONG/TRASV $\begin{cases}\frac{\partial^2 u^L}{\partial t^2} = v_L^2\nabla^2 u^L & \text{con } \mathrm{rot}\,\underline{u}^L = 0 \\ \frac{\partial^2 u^T}{\partial t^2} = v_T^2\nabla^2 \underline{u}^T & \text{con } \mathrm{div}\,\underline{u}^T = 0\end{cases}$ $\Rightarrow$ EN TOT $E = \frac{1}{2}\rho_0\int d_3x\left[\left(\frac{\partial u_L}{\partial t}\right)^2 + v_L^2(\mathrm{div}\,\underline{u}^L)^2 + \left(\frac{\partial u^T}{\partial t}\right)^2 + v_T^2(\mathrm{rot}\,u^T)^2\right]$ SI CONSERVA $\longrightarrow$ $\rho_0\left(\frac{\partial u}{\partial t}\right)^2$ DENSITÀ DI EN CINETICA INTEG SU VOL

IN $t=0$ $E = \sum_{\underline{k}\lambda}\rho_0\omega_\lambda^2(\underline{k})\left[d^*_{\underline{k}\lambda} d_{\underline{k}\lambda} + cc\right]$

ADESSO FACCIO UN CAZZO XKÉ SONO Re MA QUANDO QUANTIZZO È UN TROIAIO

$\rightarrow$ CALCOLIAMO IL # DI OSCILL CON FREQ < DI FREQ FIX ω (O MODI NORMALI $\underline{k}\lambda$) $N(\omega) = \sum_{\underline{k}\lambda}\theta(\omega - \omega_\lambda(\underline{k})) = \sum_{\underline{k}}\theta(\omega-\omega_3(\underline{k})) + 2\sum_{\underline{k}\lambda}\theta(\omega - \omega_\lambda(\underline{k})) = V\int\frac{d_3\underline{k}}{(2\pi)^3}\left[\theta(\omega - v_L k) + 2\theta(\omega - v_T k)\right] = \frac{V}{(2\pi)^3}\frac{4}{3}\pi\left[\left(\frac{\omega}{v_L}\right)^3 + 2\left(\frac{\omega}{v_T}\right)^3\right]$

(FREQ ≠ x ≠ POLARIZ $\Rightarrow$ SEPARO; $v_L k$; $v_T k$)

$\Rightarrow N(\omega) = \frac{V}{6\pi^2}\omega^3\left[\frac{1}{v_L^3} + \frac{2}{v_T^3}\right]$ SE HO MEZZO DI N IONI OSCILLANTI NON HO ∞ MODI NORM MA 3N OSCILLATORI $\Rightarrow$ CAMPO ω_D DEBYE TC $3N = N(\omega_D)$ COME CUTOFF $\Rightarrow$ IN E SI INSERISCE FATTORE $\theta(\omega_D - \omega(\underline{k}))$