ESERCIZI TEORIA DEI SISTEMI A MOLTI CORPI 1

TOMMASO DI LUCIANO

ESERCIZIO 6

DIM CHE LE FUNZIONI DI GREEN PER SISTEMI CON HAMILTONIANE $\hat{H}$ E $\lambda\hat{H}$ SONO COLLEGATE DA

$$G_\lambda(\underline{x}\,\underline{x}'\,\omega) = \frac{1}{\lambda}\, G\left(\underline{x}\,\underline{x}'\,\frac{\omega}{\lambda}\right)$$

$$H|GS\rangle = E_0^N |GS\rangle$$

$$G_{mm'}(\underline{x}t, \underline{x}'t') = \langle GS| T\, \psi_m(\underline{x}t)\, \psi^+_{m'}(\underline{x}'t')|GS\rangle$$

$$= \theta(t-t')\, \langle GS|\psi_m(\underline{x}t)\, \psi^+_{m'}(\underline{x}'t')\,|GS\rangle \pm$$

$$\pm\, \theta(t'-t)\langle GS|\psi^+_{m'}(\underline{x}'t')\,\psi_m(\underline{x}t)|GS\rangle$$

CON $\psi_m(\underline{x}t) = e^{+\frac{i}{\hbar}Ht}\,\psi_m(\underline{x})\, e^{-\frac{i}{\hbar}Ht}$

$$\Rightarrow G_{mm'}(\underline{x}t\,\underline{x}'t') = \theta(t-t')\, e^{\frac{i}{\hbar}E_0^N(t-t')} \langle GS|\psi_m(\underline{x})\, e^{-\frac{i}{\hbar}H(t-t')}\, \psi^+_{m'}(\underline{x}')|GS\rangle$$

$$\pm\,\theta(t'-t)\, e^{-\frac{i}{\hbar}E_0^N(t-t')} \langle GS|\psi^+_{m'}(\underline{x}')\, e^{+\frac{i}{\hbar}H(t-t')}\,\psi_m(\underline{x})|GS\rangle$$

INSERISCO IDENTITÀ $\sum_n |E_n^{\tilde{N}}\rangle\langle E_n^{\tilde{N}}| = \mathbb{1}$ CON $\{|E_n^{\tilde{N}}\rangle\}$ AUTOSTATI DI $\hat{H}$ AD $\tilde{N}$ PARTICELLE

$$G_{mm'}(\underline{x}t\,\underline{x}'t') = \theta(t-t')\, e^{\frac{i}{\hbar}E_0^N(t-t')} \langle GS|\psi_m(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\, e^{-\frac{i}{\hbar}H(t-t')}\,\psi^+_{m'}(\underline{x}')|E_0^N\rangle$$

$$\pm\,\theta(t'-t)\, e^{-\frac{i}{\hbar}E_0^N(t-t')} \langle GS|\psi^+_{m'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\, e^{+\frac{i}{\hbar}H(t-t')}\,\psi_m(\underline{x})|GS\rangle$$

SOMMA SU n SOTTOINTESA

$$= \theta(t-t')\, e^{\frac{i}{\hbar}\mathcal{E}_0^N (t-t')} \langle GS|\psi_m(\underline{x})|\mathcal{E}_n^{N+1}\rangle\langle\mathcal{E}_n^{N+1}|\, e^{-\frac{i}{\hbar}H(t-t')}\, \psi^+_{m'}(\underline{x}')|\mathcal{E}_0^N\rangle \longrightarrow e^{-\frac{i}{\hbar}\mathcal{E}_n^{N+1}(t-t')}$$

$$\pm\, \theta(t'-t)\, e^{-\frac{i}{\hbar}\mathcal{E}_0^N (t-t')} \langle GS|\psi^+_{m'}(\underline{x}')|\mathcal{E}_n^{N-1}\rangle\langle\mathcal{E}_n^{N-1}|\, e^{+\frac{i}{\hbar}H(t-t')}\, \psi_m(\underline{x})|GS\rangle \hookrightarrow e^{+\frac{i}{\hbar}\mathcal{E}_n^{N-1}(t-t')}$$

$$= \theta(t-t')\, e^{\frac{i}{\hbar}(\mathcal{E}_0^N - \mathcal{E}_n^{N+1})(t-t')} \langle GS|\psi_m(\underline{x})|\mathcal{E}_n^{N+1}\rangle\langle\mathcal{E}_n^{N+1}|\psi^+_{m'}(\underline{x}')|GS\rangle$$

$$\pm\, \theta(t'-t)\, e^{\frac{i}{\hbar}(\mathcal{E}_n^{N-1} - \mathcal{E}_0^N)(t-t')} \langle GS|\psi^+_{m'}(\underline{x}')|\mathcal{E}_n^{N-1}\rangle\langle\mathcal{E}_n^{N-1}|\psi_m(\underline{x})|GS\rangle$$

$$\theta(t-t') = \int_{-\infty}^{+\infty} \frac{e^{-i\omega(t-t')}}{\omega + i\eta}\frac{d\omega}{2\pi} \qquad \theta(t'-t) = \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{+i\omega(t-t')}}{\omega + i\eta}$$

$$\Rightarrow G(\underline{x}\,t\,\underline{x}'t') = \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\left(\omega + \frac{\mathcal{E}_n^{N+1}-\mathcal{E}_0^N}{\hbar}\right)(t-t')}}{\omega + i\eta} \langle GS|\psi_m(\underline{x})|\mathcal{E}_n^{N+1}\rangle\langle\mathcal{E}_n^{N+1}|\psi^+_{m'}(\underline{x}')|GS\rangle$$

$$\pm \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{+i\left(\omega + \frac{\mathcal{E}_n^{N-1}-\mathcal{E}_0^N}{\hbar}\right)(t-t')}}{\omega + i\eta} \langle GS|\psi^+_{m'}(\underline{x}')|\mathcal{E}_n^{N-1}\rangle\langle\mathcal{E}_n^{N-1}|\psi_m(\underline{x})|GS\rangle$$

TRASLO L'INTEGRALE $\longrightarrow$ $\underline{\text{I}^a}$ $\quad \omega \longrightarrow \omega + \dfrac{\mathcal{E}_n^{N+1} - \mathcal{E}_0^N}{\hbar} = \omega'$

$\underline{\text{II}^a}$ $\quad \omega \longrightarrow \omega + \dfrac{\mathcal{E}_n^{N-1} - \mathcal{E}_0^N}{\hbar} = \omega'$

E RINOMINO ω' COME ω

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega - (E_n^{N+1} - E_0^N)/\hbar + i\eta} \langle GS | \psi_m(\underline{x}) | E_n^{N+1} \rangle \langle E_n^{N+1} | \psi^+_{m'}(\underline{x}') | GS \rangle \pm$$

$$\pm \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{+i\omega(t-t')}}{\omega - \left(\frac{E_n^{N-1} - E_0^N}{\hbar}\right) + i\eta} \langle GS | \psi^+_{m'}(\underline{x}') | E_n^{N-1} \rangle \langle E_n^{N-1} | \psi_m(\underline{x}) | GS \rangle$$

NELLA II PT $\omega \to -\omega = \omega'$

$$\Rightarrow \int_{-\infty}^{+\infty} d\omega\, f(\omega) = -\int_{+\infty}^{-\infty} d\omega'\, f(-\omega') = \int_{-\infty}^{+\infty} d\omega'\, f(\omega')$$

POI DI NUOVO RINOMINO ω' COME ω

$$\text{II PT} \Rightarrow \pm \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{-\omega - \left(\frac{E_n^{N-1} - E_0^N}{\hbar}\right) + i\eta} \langle GS | \psi^+_{m'}(\underline{x}') | E_n^{N-1} \rangle \langle E_n^{N-1} | \psi_m(\underline{x}) | GS \rangle$$

$$\Rightarrow G_{mm'}(\underline{x} t\, \underline{x}' t') = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[\frac{\langle GS | \psi_m(\underline{x}) | E_n^{N+1} \rangle \langle E_n^{N+1} | \psi^+_{m'}(\underline{x}') | GS \rangle}{\omega - \left(\frac{E_n^{N+1} - E_0^N}{\hbar}\right) + i\eta} \mp \right.$$

$$\left. \mp \frac{\langle GS | \psi^+_{m'}(\underline{x}') | E_n^{N-1} \rangle \langle E_n^{N-1} | \psi_m(\underline{x}) | GS \rangle}{\omega + \frac{E_n^{N-1} - E_0^N}{\hbar} - i\eta} \right] e^{-i\omega(t-t')}$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} G_{mm'}(\underline{x}\, \underline{x}'\, \omega)$$

$$\Rightarrow G_{mm'}(\underline{x}\,\underline{x}'\omega) = \sum_n \frac{\langle GS|\psi_m(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\psi^+_{m'}(\underline{x}')|GS\rangle}{\omega - \frac{(E_n^{N+1}-E_0^N)}{\hbar} + i\eta} \mp$$

$$\mp \sum_n \frac{\langle GS|\psi^+_{m'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\psi_m(\underline{x})|GS\rangle}{\omega + \frac{E_n^{N-1}-E_0^N}{\hbar} - i\eta}$$

A QUESTO PUNTO LE 2 HAMILTONIANE POSSIEDONO GLI STESSI AUTOVETTORI (SONO NORMALIZZATI)
→ LA DIFFERENZA STA NEGLI AUTOVALORI

$$\hat{H} \longrightarrow H|E_n^N\rangle = E_n^N|E_n^N\rangle$$

$$\lambda\hat{H} \rightarrow \lambda H|E_n^N\rangle = \lambda E_n^N|E_n^N\rangle$$

$$\Rightarrow G_\lambda(\underline{x}\,\underline{x}'\omega) = \sum_n \frac{\langle GS|\psi_m(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\psi^+_{m'}(\underline{x}')|GS\rangle}{\omega - \lambda\frac{(E_n^{N+1}-E_0^N)}{\hbar} + i\eta} \mp$$

$$\mp \sum_n \frac{\langle GS|\psi^+_{m'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\psi_m(\underline{x})|GS\rangle}{\omega + \lambda\left(\frac{E_n^{N-1}-E_0^N}{\hbar}\right) - i\eta}$$

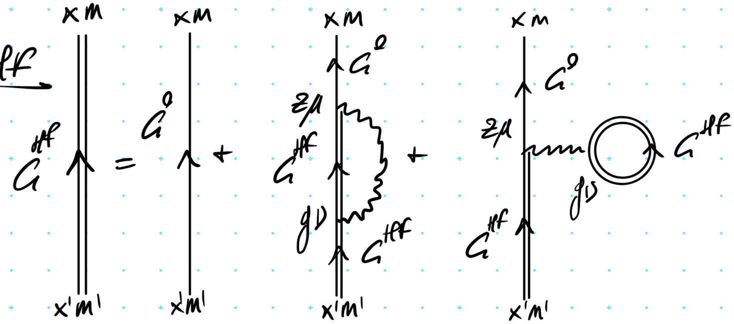
$$= \left[\sum_n \frac{\langle GS|\psi_m(\underline{x})|E_n^{N+1}\rangle\langle E_n^{N+1}|\psi^+_{m'}(\underline{x}')|GS\rangle}{\frac{\omega}{\lambda} - \frac{(E_n^{N+1}-E_0^N)}{\hbar} + i\eta} \mp \sum_n \frac{\langle GS|\psi^+_{m'}(\underline{x}')|E_n^{N-1}\rangle\langle E_n^{N-1}|\psi_m(\underline{x})|GS\rangle}{\frac{\omega}{\lambda} + \left(\frac{E_n^{N-1}-E_0^N}{\hbar}\right) - i\eta}\right]\frac{1}{\lambda}$$

$$= \frac{1}{\lambda} G\left(\underline{x}\ \underline{x}' \frac{\omega}{\lambda}\right) \qquad \square$$

ESERCIZIO HARTREE-FOCK 1

SCRIVERE LA SERIE ENERGIA PROPRIA DI HF E DIM CHE È INDIPENDENTE DAL TEMPO

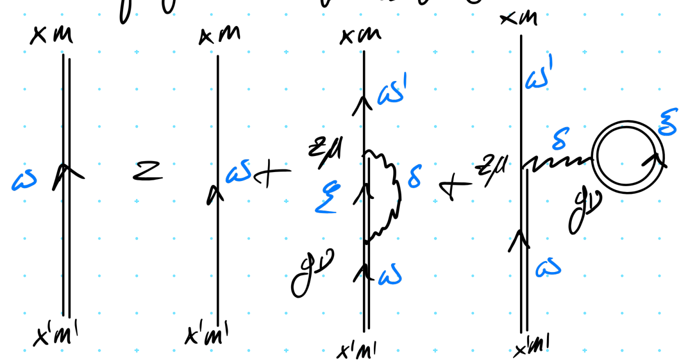
EQUAZIONE DI DYSON x HF →



$$G^{HF}_{mm'}(\underline{x}t\underline{x}'t') = G^{0}_{mm'}(\underline{x}t\underline{x}'t') + \frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,dt''\, G^{0}_{m\mu}(\underline{x}t\underline{z}t'') \int d_3\underline{y}\,dt'''\, \mathcal{V}(\underline{z}\underline{y})\, G^{HF}_{\mu\nu}(\underline{z}t''\underline{y}t''')\, G^{HF}_{\nu m'}(\underline{y}t'''\underline{x}'t')$$

$$-\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,dt''\, G^{0}_{m\mu}(\underline{x}t\underline{z}t'')\, G^{HF}_{\mu m'}(\underline{z}t''\underline{x}'t') \int d_3\underline{y}\,dt'''\, \mathcal{V}(\underline{z}\underline{y})\, G^{HF}_{\nu\nu}(\underline{y}t'''\underline{y}t'''^{+})$$

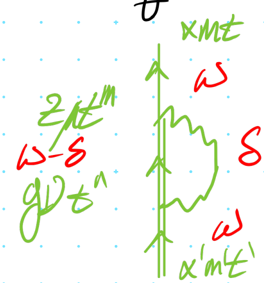
→ PASSANDO IN SP ω



PT BILOCALE

$$\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,dt'' \int\frac{d\omega'}{2\pi}\, G^{0}_{m\mu}(\underline{x}\underline{z}\omega')\, e^{-i\omega'(t-t'')} \int d_3\underline{y}\,dt'''\, G^{HF}_{\mu\nu}(\underline{z}\underline{y}\xi)\, e^{-i\xi(t''-t''')} \int\frac{d\delta}{2\pi}\, e^{-i\delta(t''-t''')}\, \mathcal{V}(\underline{z}\underline{y})$$

$$\int\frac{d\omega}{2\pi}\, G^{HF}_{\nu m'}(\underline{y}\underline{x}'\omega)\, e^{-i\omega(t'''-t')}$$



RACCOLGO I TERMINI IN t''' E t'' DA CUI OTTENGO

$$(2\pi)^2\,\delta(\omega-\omega')\,\delta(\xi+\delta-\omega)$$

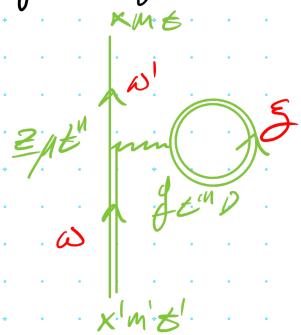
$$= \frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,\frac{d\omega}{2\pi}\frac{d\delta}{2\pi}\,G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3\underline{y}\,G^{Hf}_{\mu\nu}(\underline{z}\,\underline{y}\,\omega-\delta)\,v(\underline{z}\,\underline{y})\,G^{Hf}_{\nu m'}(\underline{y}\,\underline{x}'\omega)\,e^{-i\omega(t-t')}$$

$$= \frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,\frac{d\omega}{2\pi}\,G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3\underline{y}\,\frac{d\delta}{2\pi}\,G^{Hf}_{\mu\nu}(\underline{z}\,\underline{y}\,\omega-\delta)\,v(\underline{z}\,\underline{y})\,G^{Hf}_{\nu m'}(\underline{y}\,\underline{x}'\omega)\,e^{-i\omega(t-t')}$$

PT LOCALE

$$-\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,dt''\int\frac{d\omega'}{2\pi}\,G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega')\,e^{-i\omega'(t-t'')}\int\frac{d\omega}{2\pi}\,G^{Hf}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\,e^{-i\omega(t''-t')}\int d_3\underline{y}\,dt'''\,v(\underline{z}\,\underline{y})$$

$$\int\frac{d\xi}{2\pi}\,G^{Hf}_{\nu\nu}(\underline{y}\,\underline{y}\,\xi)\,e^{-i\xi\eta}$$



COME PRIMA INTEGRO IN t'' E t'''

$$= -\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\int\frac{d\omega'}{2\pi}\,G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega')\int\frac{d\omega}{2\pi}\,G^{Hf}_{\mu m'}(\underline{z}\,\underline{x}'\omega)$$

$$e^{-i\omega t}\,e^{+i\omega t'}\left(\underbrace{\int dt''\,e^{-i(\omega-\omega')t''}}_{=2\pi\,\delta(\omega-\omega')}\right)\int d_3\underline{y}\,v(\underline{z}\,\underline{y})\int\frac{d\xi}{2\pi}\,G^{Hf}_{\nu\nu}(\underline{y}\,\underline{y}\,\xi)\,e^{-i\xi\eta}$$

$$= -\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\int\frac{d\omega}{2\pi}\,G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\,G^{Hf}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\,e^{-i\omega(t-t')}$$

$$\int d_3\underline{y}\,v(\underline{z}\,\underline{y})\int\frac{d\xi}{2\pi}\,G^{Hf}_{\nu\nu}(\underline{y}\,\underline{y}\,\xi)\,e^{-i\xi\eta}$$

RICOMBINANDO

$$G^{HF}_{mm'}(\underline{x}t\,\underline{x}'t') = \int \frac{d\omega}{2\pi}\, G^{0}_{mm'}(\underline{x}\,\underline{x}'\omega)\, e^{-i\omega(t-t')} + \frac{i}{\hbar}\sum_{\mu\nu}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3y \frac{d\delta}{2\pi} G^{HF}_{\mu\nu}(\underline{z}\,\underline{y}\,\omega-\delta)\, v(\underline{z}\,\underline{y})\, G^{HF}_{\nu m'}(\underline{y}\,\underline{x}'\omega) e^{-i\omega(t-t')}$$

$$-\frac{i}{\hbar}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\, G^{HF}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\int d_3y \frac{d\delta}{2\pi} G_{\nu\nu}(\underline{y}\,\underline{y}\,\delta) e^{-i\delta\eta}\, 2v(\underline{z}\,\underline{y})\, e^{-i\omega(t-t')}$$

SOSTITUISCO LA RAPPRESENTAZIONE SPETTRALE DI SINGOLA PARTICELLA

$$\boxed{G^{HF}_{\mu\nu}(\underline{x}\,\underline{y}\,\omega) = \sum_{\lambda} \frac{u_\lambda(\underline{x}m)\, u^*_\lambda(\underline{x}'m')}{\omega-\omega_\lambda+i\eta\, \mathrm{sgn}(\omega_\lambda-\omega_F)}}$$

$$= \int \frac{d\omega}{2\pi} G^{0}_{mm'}(\underline{x}\,\underline{x}'\omega) e^{-i\omega(t-t')} + \frac{i}{\hbar}\sum_{\mu\nu}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3y \frac{d\delta}{2\pi} \frac{\sum_\lambda u_\lambda(\underline{z}\mu)\, u^*_\lambda(\underline{y}\nu)}{\omega-\delta-\omega_\lambda+i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)}\, v(\underline{z}\,\underline{y})\, G^{HF}_{\nu m'}(\underline{y}\,\underline{x}'\omega) e^{-i\omega(t-t')}$$

$$-\frac{i}{\hbar}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\, G^{HF}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\int d_3y \frac{d\delta}{2\pi} \frac{\sum_\lambda u_\lambda(\underline{y}\nu)\, u^*_\lambda(\underline{y}\nu)}{\delta-\omega_\lambda+i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)} e^{-i\delta\eta}\, 2v(\underline{z}\,\underline{y})\, e^{-i\omega(t-t')}$$

$$= \int \frac{d\omega}{2\pi} G^{0}_{mm'}(\underline{x}\,\underline{x}'\omega) e^{-i\omega(t-t')} + \frac{i}{\hbar}\sum_{\mu\nu}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3y\, G^{HF}_{\nu m'}(\underline{y}\,\underline{x}'\omega)\, 2v(\underline{z}\,\underline{y})\, e^{-i\omega(t-t')}$$

$$\sum_\lambda u_\lambda(\underline{z}\mu)\, u^*_\lambda(\underline{y}\nu)\, \underbrace{\int \frac{d\delta}{2\pi}\frac{1}{\omega-\delta-\omega_\lambda+i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)}}_{\frac{2\pi i}{2\pi}}$$

POLO ESPLICITO

$$\Rightarrow \omega-\omega_\lambda-i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)$$

$$-\frac{i}{\hbar}\int d_3z \frac{d\omega}{2\pi} G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\, G^{HF}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\, e^{-i\omega(t-t')}\int d_3y\, v(\underline{z}\,\underline{y}) \sum_\lambda |u_\lambda(\underline{y}\nu)|^2 \underbrace{\int \frac{d\delta}{2\pi}\frac{e^{-i\delta\eta}}{\delta-\omega_\lambda+i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)}}_{\frac{2\pi i}{2\pi}\exp\left(-i\omega_\lambda\eta+\eta\,\mathrm{sgn}(\,)\right)}$$

POLO ESPLICITO

$$\omega_\lambda-i\eta\,\mathrm{sgn}(\omega_\lambda-\omega_F)$$

$$= G^{0}_{mm'}(\underline{x}t\,\underline{x}'t') + \frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\,\frac{d\omega}{2\pi}\, G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\int d_3\underline{y}\, G^{Hf}_{\nu m'}(\underline{y}\,\underline{x}'\omega)\, e^{-i\omega(t-t')}\, \underbrace{2S(\underline{z}\,\underline{y})_i \sum_a u_a(\underline{z}\mu)\,u_a^{*}(\underline{y}\nu)}_{\Sigma^{*Hf}_{bil\,\mu\nu}(\underline{z}\,\underline{y})}$$

$$-\frac{i}{\hbar}\int d_3\underline{z}\,\frac{d\omega}{2\pi}\, G^{0}_{m\mu}(\underline{x}\,\underline{z}\,\omega)\, G^{Hf}_{\mu m'}(\underline{z}\,\underline{x}'\omega)\, e^{-i\omega(t-t')} \int d_3\underline{y}\, \delta S(\underline{z}\,\underline{y})_i \sum_a |u_a(\underline{y}\nu)|^2$$

SE ORA RIASSORBO GLI INTEGRALI IN ω

$$G^{Hf}_{mm'}(\underline{x}t\,\underline{x}'t') = G^{0}_{mm'}(\underline{x}t\,\underline{x}'t')$$

$$+\frac{i}{\hbar}\sum_{\mu\nu}\int d_3\underline{z}\, dt''\, d_3\underline{y}\; G^{0}_{m\mu}(\underline{x}t\,\underline{z}t'')\, G_{\nu}(\underline{y}t''\,\underline{x}'t')\; \Sigma^{*Hf}_{bil\,\mu\nu}(\underline{z}\,\underline{y})$$

$$-\frac{i}{\hbar}\sum_{\mu}\int d_3\underline{z}\, dt''\; G^{0}_{m\mu}(\underline{x}t\,\underline{z}t'')\, G^{Hf}_{\mu m'}(\underline{z}t''\,\underline{x}'t')\; \Sigma^{*Hf}_{loc\,\mu\mu}(\underline{z})$$

$$\Sigma^{*Hf}_{bil\,\mu\nu}(\underline{z}\,\underline{y}) = -\frac{1}{\hbar}\,\delta S(\underline{z}\,\underline{y}) \sum_a u_a(\underline{z}\mu)\,u_a^{*}(\underline{y}\nu)$$

$$\Sigma^{*Hf}_{loc\,\mu\mu}(\underline{z}) = \frac{1}{\hbar}\sum_{\nu}\int d_3\underline{y}\; \delta S(\underline{z}\,\underline{y}) \sum_a |u_a(\underline{y}\nu)|^2$$

ESERCIZIO QUASI PARTICELLE 5

Σ^* PER KEG CON POTENZIALE DI COULOMB SCHERMATO IN APPROSSIMAZ DI THOMAS-FERMI è

$$\Sigma^*(k) = -\frac{1}{\hbar}\int \frac{d^3q}{(2\pi)^3}\,\frac{4\pi e^2}{|\underline{k}-\underline{q}|^2+k_{TF}^2}\,\theta(k_F-q) \longrightarrow \text{INDIP DA } \omega$$

$$= -\frac{\hbar k_F^2}{m}\,\frac{4}{3\pi^2}\,r_s^{3/2}\left\{1+\frac{1+Q^2}{4x}\lg\frac{(1+x)^2+Q^2}{(1-x)^2+Q^2}-\left(Q+\frac{x}{2Q}\right)\left[\operatorname{arctg}\frac{x+1}{Q}-\operatorname{arctg}\frac{x-1}{Q}\right]\right\}$$

CON $x \equiv \frac{k}{k_F}$ $\quad Q \equiv \frac{k_{TF}}{k_F} = \left(\frac{16}{3\pi^2}\right)^{1/3}\sqrt{r_s} \simeq 0{,}8145\sqrt{r_s}$

$\rightarrow$ VALUTIAMO m^*

X AVERE QP $\Rightarrow$ DEVO ESSERE VICINO FERMI SURFACE

$$\omega_1(\underline{k}) = \mu - \hbar\Sigma^*(\underline{k}\,\omega_1(\underline{k}))$$

SVILUPPANDO VICINO A k_F

$$\omega_1(k) = \underbrace{\mu - \hbar\Sigma^*(k_F)}_{\omega_1(k_F)} - \left.\frac{\partial\Sigma^*}{\partial k}\right|_{k_F}(k-k_F) - \left.\frac{\partial\Sigma^*}{\partial\omega}\frac{\partial\omega}{\partial k}\right|_{k_F}(k-k_F)$$

$\rightarrow$ NEL NOSTRO CASO $\frac{\partial\Sigma^*}{\partial\omega}=0$

$$\frac{\partial\Sigma^*}{\partial k} = \left.\frac{\partial\Sigma^*}{\partial x}\frac{\partial x}{\partial k}\right|_{k_F} = \frac{1}{k_F}\left.\frac{\partial\Sigma}{\partial x}\right|_{x=1}$$

MA $\quad \omega_1(k) = \omega_1(k_F) + \frac{\hbar k_F}{m^*}(k-k_F)$

$$\Rightarrow \frac{\hbar k_F}{m^*} = -\left.\frac{\partial\Sigma^*}{\partial k}\right|_{k_F}$$

$$\Rightarrow \frac{\hbar k_F}{m^*} = -\frac{\partial \Sigma^*}{\partial k}\Big|_{k_F} = -\frac{1}{k_F}\frac{\partial \Sigma^*}{\partial x}\Big|_{x=1}$$

$$= \frac{\hbar k_F}{m}\frac{4}{3\pi^2} r_s^{3/2}\left\{\frac{\partial}{\partial x}\left[\frac{1+Q^2}{4x}\lg\frac{(1+x)^2+Q^2}{(1-x)^2+Q^2}\right] - \frac{\partial}{\partial x}\left[\left(Q+\frac{x}{2Q}\right)\left(\arctg\frac{x+1}{Q} - \arctg\frac{x-1}{Q}\right)\right]\right\}\Big|_{x=1}$$

$$= \frac{\hbar k_F}{m}\frac{4}{3\pi^2} r_s^{3/2}\left\{\left[-\frac{1+Q^2}{4x^2}\lg\frac{(1+x)^2+Q^2}{(1-x)^2+Q^2} + \frac{1+Q^2}{4x}\left(\frac{2(1+x)}{(1+x)^2+Q^2}+\frac{2(1-x)}{(1-x)^2+Q^2}\right)\right] - \frac{1}{2Q}\left(\arctg\frac{x+1}{Q}-\arctg\frac{x-1}{Q}\right)\right.$$

$$\left. - \frac{1}{Q}\left(Q+\frac{x}{2Q}\right)\left(\frac{1}{1+\left(\frac{x+1}{Q}\right)^2} - \frac{1}{1+\left(\frac{x-1}{Q}\right)^2}\right)\right\}\Big|_{x=1}$$

$$= \frac{\hbar k_F}{m}\frac{4}{3\pi^2} r_s^{3/2}\left\{\frac{1+Q^2}{4}\left(\frac{4}{4+Q^2} - \lg\frac{4+Q^2}{Q^2}\right) - \frac{1}{2Q}\arctg\frac{2}{Q} - \frac{2Q^2+1}{Q^2}\frac{1}{Q^2+2}\right\}$$

$$\Rightarrow \quad \frac{\hbar k_F}{m^*} = \frac{\hbar k_F}{m}\frac{4}{3\pi^2} r_s^{3/2}\left\{\frac{1+Q^2}{4}\left(\frac{4}{4+Q^2} - \lg\frac{4+Q^2}{Q^2}\right) - \frac{1}{2Q}\arctg\frac{2}{Q} - \frac{2Q^2+1}{Q^2}\frac{1}{Q^2+2}\right\}$$

$$\boxed{m^* = m\,\frac{3\pi^2}{4} r_s^{-\frac{3}{2}}\left[\frac{1+Q^2}{4}\left(\frac{4}{4+Q^2} - \lg\frac{4+Q^2}{Q^2}\right) - \frac{1}{2Q}\arctg\frac{2}{Q} - \frac{2Q^2+1}{Q^2}\frac{1}{Q^2+2}\right]^{-1}}$$

con $Q = \frac{k_{TF}}{k_F} = \frac{1}{Q_0} = k_F r_s\left(\frac{4}{9\pi}\right)^{1/3} \longleftarrow Q_0 k_F = \left(\frac{9\pi}{4}\right)^{1/3}\frac{1}{r_s}$