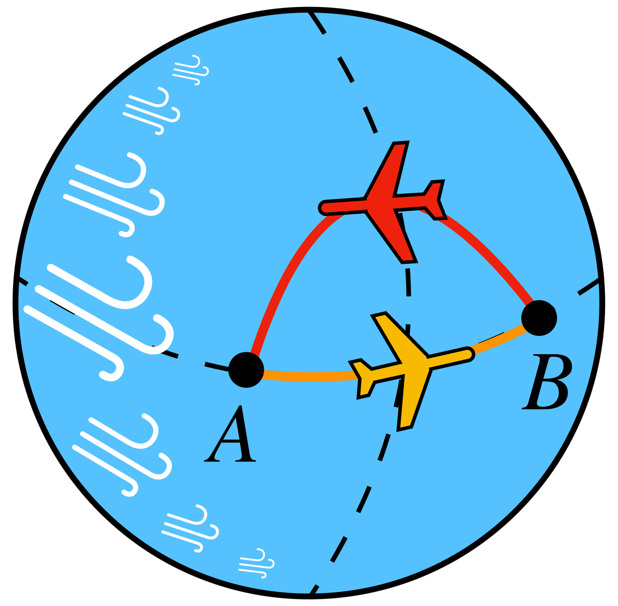
Finsler-Laplace-Beltrami Operators with Application to Shape Analysis

Simon Weber[1,2*], Thomas Dagès[3*], Maolin Gao[1,2], Daniel Cremers[1,2]

[1]Technical University of Munich [2]Munich Center for Machine Learning [3]Technion - Israel Institute of Technology

Summary

- From **Riemann** to **Finsler** manifolds
- **Heat diffusion** on Finsler manifolds
- **Theoretically motivated** anisotropic LBO: **Finsler-based LBO**
- **Shape correspondence** with Finsler-based approach



Geometry

RIEMANN

- **Riemann metric** (M)

$$\mathcal{R}_x(v) = \|v\|_{M(x)}$$

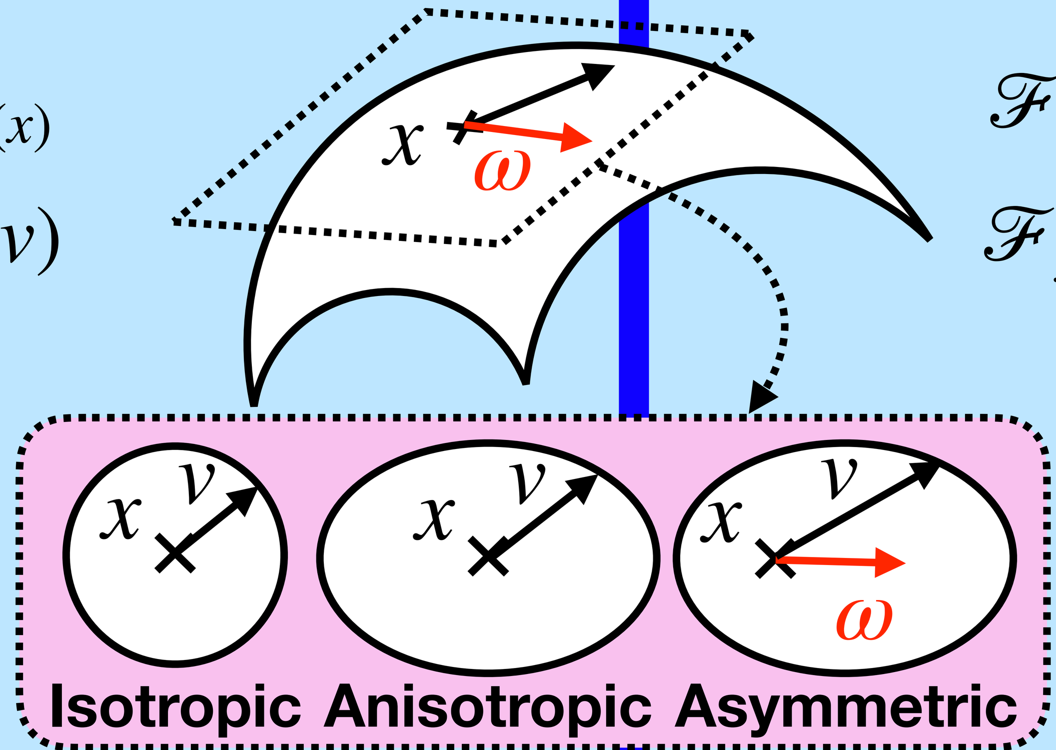
$$\mathcal{R}_x(v) = \mathcal{R}_x(-v)$$

FINSLER

- **Randers metric** (M, ω)

$$\mathcal{F}_x(v) = \|v\|_{M(x)} + \omega(x)^\top v$$

$$\mathcal{F}_x(v) \neq \mathcal{F}_x(-v)$$



RIEMANN

- **Traditional Heat Equation**

$$\partial_t u = \mathrm{div}\left(\nabla_X f\right)$$

- **Traditional LBO**

$$\Delta_{LBO} f = -\,\mathrm{div}_X\left(M^{-1}\nabla_X f\right)$$

$$\phi_k^{LBO} = \lambda_k \Delta_{LBO}\phi_k$$

FINSLER

- **Finsler Heat Equation**

$$\partial_t u = \mathrm{div}\left(\mathcal{F}_x^*(\nabla u)\nabla\mathcal{F}_x^*(\nabla u)\right)$$

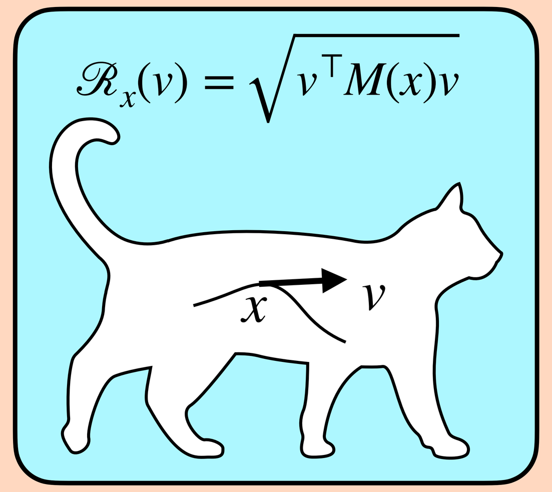
- **Finsler LBO**

$$\Delta_{FLBO} f = -\,\mathrm{div}_X\left(\left(M^* - w^* w^{*\top}\right)\nabla_X f\right)$$

$$\phi_k^{FLBO} = \lambda_k \Delta_{FLBO}\phi_k$$

Roadmap

Riemann metric



Heat equation

Laplace-Beltrami operator

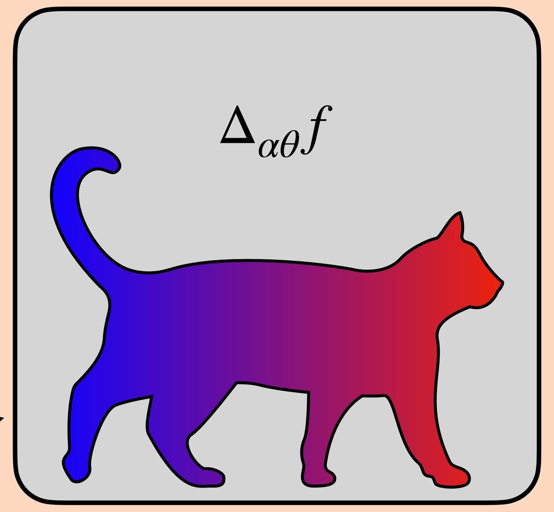
$$\Delta_X f = -\,\mathrm{div}_X(\nabla_X f)$$

Skew

Anisotropic diffusivity (heuristic)

$$D_{\alpha\theta}$$

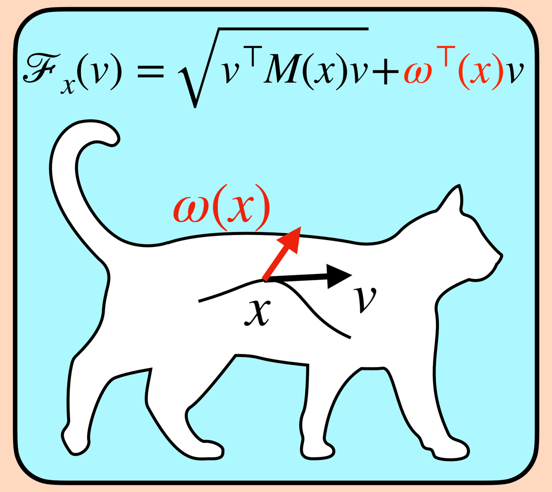
Heuristic ALBO



$$\Delta_D f = -\,\mathrm{div}_X\left(D\nabla_X f\right)$$

if $\omega = 0$

Finsler metric



Heat equation

(optional: further skew $D_{\alpha\theta}$)

Laplace-Finsler operator

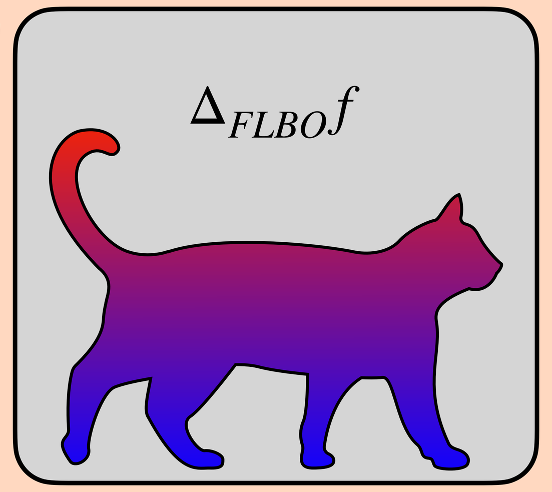
$$\Delta_{\mathcal{F}} f = -\,\mathrm{div}_X\left(\mathcal{F}_x^*(\nabla_X f)\nabla\mathcal{F}_x^*(\nabla_X f)\right)$$

Simplified special case

Anisotropic diffusivity (Finsler-based)

$$D_{\mathcal{F}_x^*} = M^* - \omega^*\omega^{*\top}$$

Finsler-based LBO (Ours)

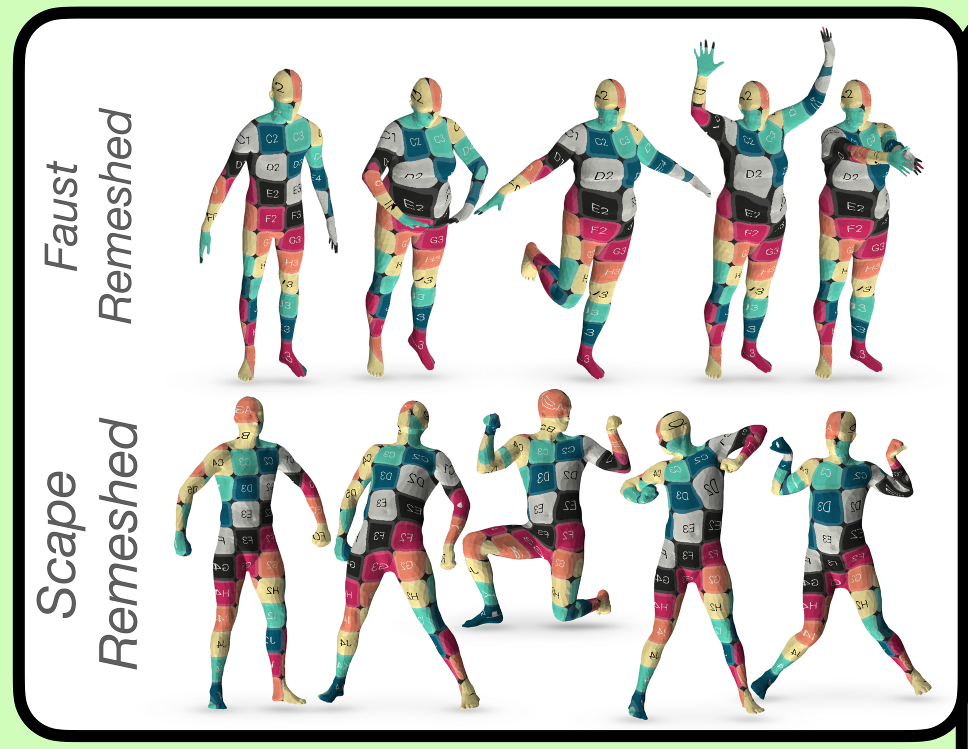


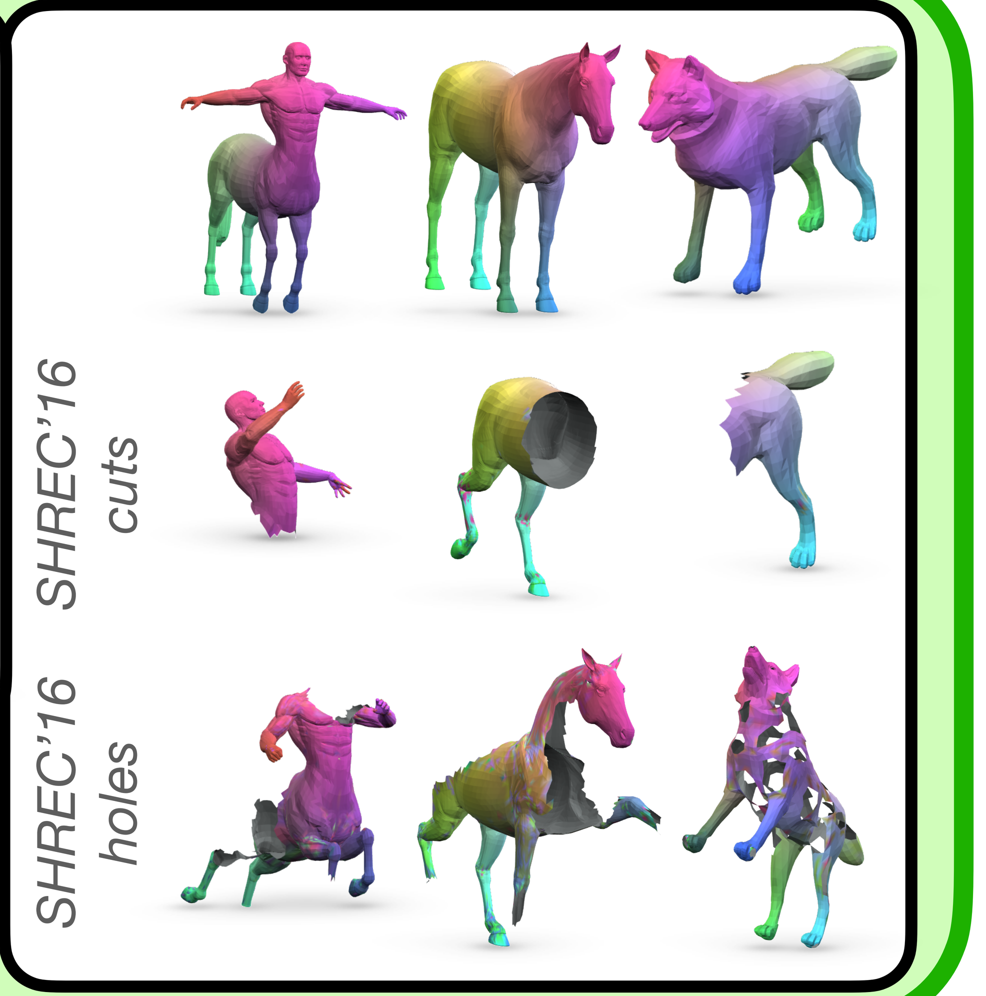
An Application: Shape Matching

- **FLBO** = a valuable alternative to traditional LBO
- **Convolution** in the spectral domain

$$(f * g)_\alpha(x) = \int_\theta (f * g)_{\alpha\theta}(x)\, d\theta$$

$$(f * g)_{\alpha\theta} = \sum \hat{f}(\lambda_{\alpha\theta,k})\hat{g}(\lambda_{\alpha\theta,k})\phi_{\alpha\theta,k}$$





- **Discretization**: generalization of the cotangent weight scheme
- **Shape correspondence** via FLBO CNN