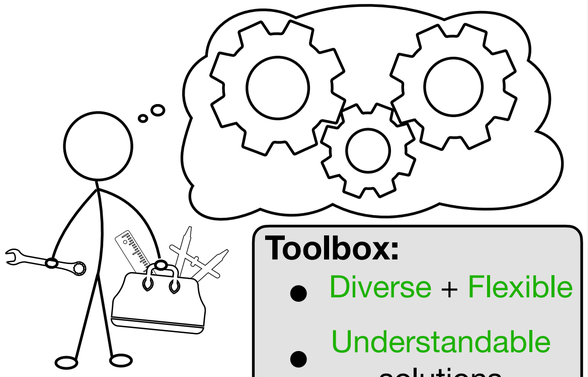
From Compass and Ruler to Convolutions and Nonlinearity: How a simple CNN solves an Elementary Geometric Task

Thomas Dagès, Michael Lindenbaum, Alfred M. Bruckstein
Technion - Israel Institute of Technology
The Henry and Marilyn Taub Faculty of Computer Science

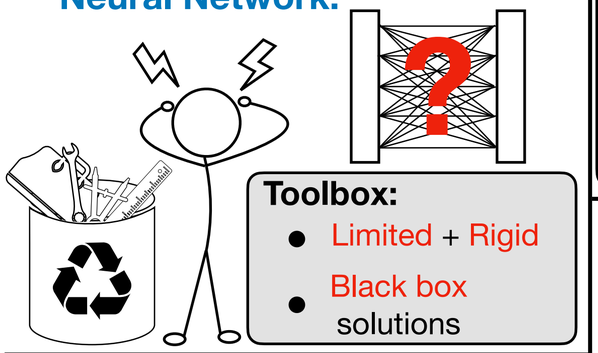
What Does a NN Learn?

Expert Engineer:



Toolbox:
- Diverse + Flexible
- Understandable solutions

Neural Network:



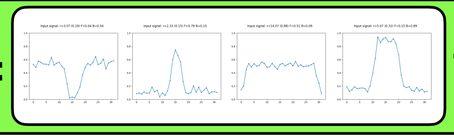
Toolbox:
- Limited + Rigid
- Black box solutions

Take-home Message

- Simple NN + Simple problem: $\Longrightarrow$ Still difficult to understand
- **Invariance** (non geometric): $\Longrightarrow$ From the weights (difficult)
- **Non linearity shape**: More plateaux $\Longrightarrow$ More thresholding $\Longrightarrow$ More invariance $\Longrightarrow$ More expressivity

1D Pulse Signals

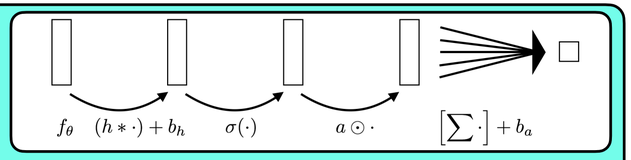
Data:



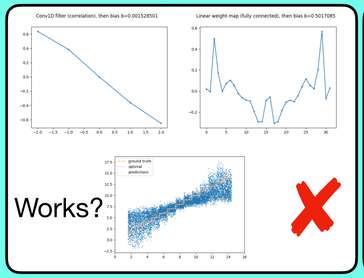
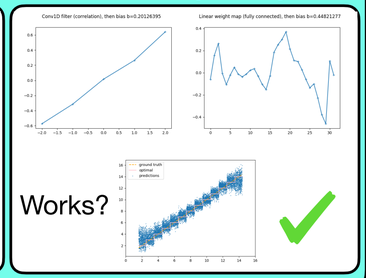
Task: (Half) Width Estimation

CNN:

Architecture:



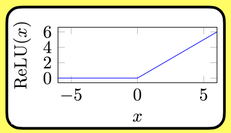
Learned weights: $\sigma = \mathrm{ReLU}$

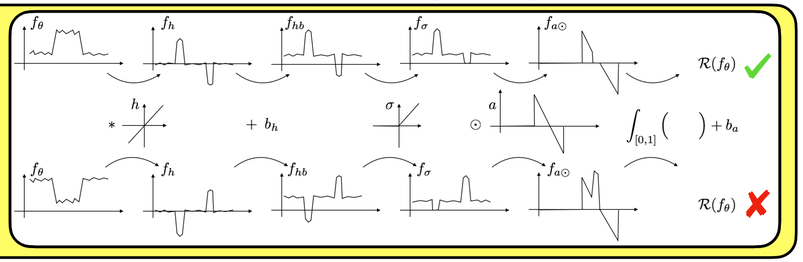
Both polarities	Positive polarity
	
Works? ✗	Works? ✓

Theory:

Positive polarity invariance: only one weight design possible!

$\sigma = \mathrm{ReLU}$

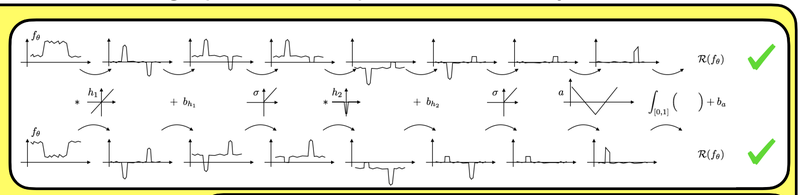




Problem: 2 thresholding operations required for intensity invariance

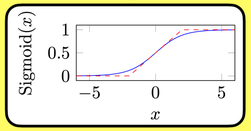
Changing architecture:

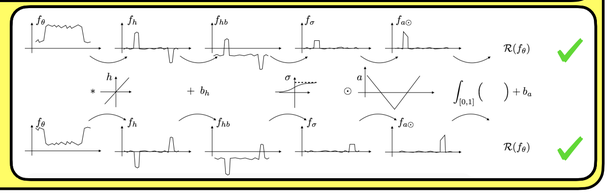
- More **depth**



- More **plateaux** in activation function

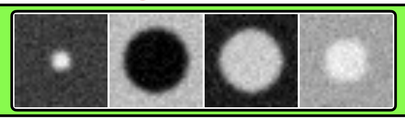
$\sigma = \mathrm{Sigmoid}$





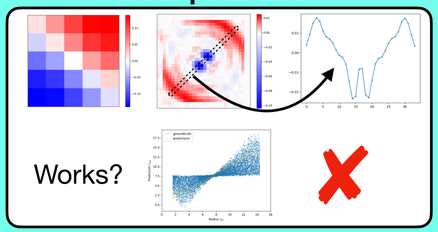
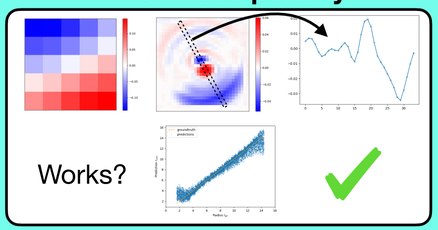
2D Disk Images

Data:



Task: Radius Estimation

CNN:

Both polarities	Positive polarity
	
Works? ✗	Works? ✓

"Perimeter" estimation of the radius

CHALLENGE - Multi-channels:

Both polarities	Positive polarity