Sunny Side Up

Can you recreate the surface of the Sun in a frying pan?

Group 17:

Cezara Cornea, Thomas Curson Smith, Pat Dizon, Giorgia Riley, Aaron Stace, Noah Wessel

Scan QR code to see animation of modelled convection currents!!

MANCHESTER 1824

The University of Manchester

Abstract

This project investigates chaotic turbulent flow on the surface of the Sun, and how well it can be recreated using olive oil in a frying pan. This was done using a computational simulation of Rayleigh-Benard convection flow, modelled from a Boussinesq approximation of the Navier-Stokes equation. Using the Rayleigh number of olive oil calculated via thermodynamic theory, a simulation was ran to demonstrate the convection patterns in heated olive oil. The granulation patterns of this convection were shown to replicate those on the surface of the sun.

Introduction

Convection, defined by the movement of liquid cells in a fluid, is a vital part of modern physics. The fluid dynamics of convection have applications everywhere, from explaining the movement of plasma on the surface of the sun in astrophysics, to the temperature gradient of your pan that you use to cook dinner. This project investigates the formation and behaviour of convection cells of thin films under thermal gradients, and answers the burning question on everyone's mind; **how close can we get to recreating the convection cells on the surface of the sun**, **turning our understanding of this process sunny side up?** The Navier-Stokes and diffusion equations were used to model convection in heated fluid layers, and a simulation was produced showing the creation and time evolution of convection cells. From this simulation, an investigation was done into the cells' dependance on key physical parameters, such as the Rayleigh number, kinematic viscosity, and thermal diffusivity.

Theory

Plasma motion in the Sun's outer layer is driven by **Rayleigh-Benard convection**, a type of natural convection not driven by external forces. A cycle of hot plasma rising & cooler plasma sinking drives the convection through the production of a temperature gradient. An important parameter related to this model is the **Rayleigh number** [1], which is the ratio of convection to conduction happening in a fluid; the higher the Rayleigh number the stronger the convection.

$$Ra = \frac{\alpha g \Delta T d^3}{\nu \kappa}$$

ΔT temperature difference
α thermal expansion coefficient
d depth
g gravitational field strength
κ thermal diffusivity
ν kinematic viscosity

Different regimes

Ra < 1708 = purely conductive
Ra > 1708 = laminar convection
Ra >> 1708 = turbulent flow

The Rayleigh number for the convection zone of the Sun is Ra ~ 4.7 × 10^23. This indicates highly turbulent flow shown in solar granulation patterns. The governing equations for Rayleigh-Benard convection are:

Navier-Stokes equation [2] - with a buoyancy force term in the z-direction due to the temperature gradient

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{u} + g\alpha(T - T_0)\hat{\mathbf{z}}$$

ρ_0 = density p = pressure $\mathbf{u}$ = velocity T_0 = reference temperature

Heat equation [3]

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \kappa\nabla^2 T$$

We performed a **Boussinesq approximation**[4] to the Navier-Stokes equation which treats the density as constant throughout the liquid, with density differences only affecting the buoyancy term. Another key parameter is the **vorticity**, given by the curl of the velocity of fluid flow **u** at any given time, which helps us visualize the rotation of the oil. The equations were dimensionalized using free-fall time-scaling of characteristic time to make them more computationally manageable, and to allow us to visualize the time evolution of our system.

Constants & starting parameters for olive oil in a typical pan [5]:

α	$7\times10^{-4}\ K$
g	$9.81\ ms^{-2}$
κ	$7.99\times10^{-8}\ m^2\ s^{-1}$
ν	$24.1\times10^{-6}\ m^2 s^{-1}$
d	2 cm
ΔT	10 K

A reference Rayleigh number was calculated for olive oil in a pan by approximating a temperature gradient of 10K and a oil layer depth of 2cm, giving an answer of **Ra (olive oil) ~ 3× 10^5.** This value being much greater than 1708 indicates that **we can comfortably be in a regime for turbulent flow, meaning we can try to recreate the surface of the Sun in a pan of olive oil**. However it is many orders of magnitude below the Rayleigh number of the Sun's convective zone which will lead to less chaotic patterns being observed, or perhaps a different time-evolution of the system.

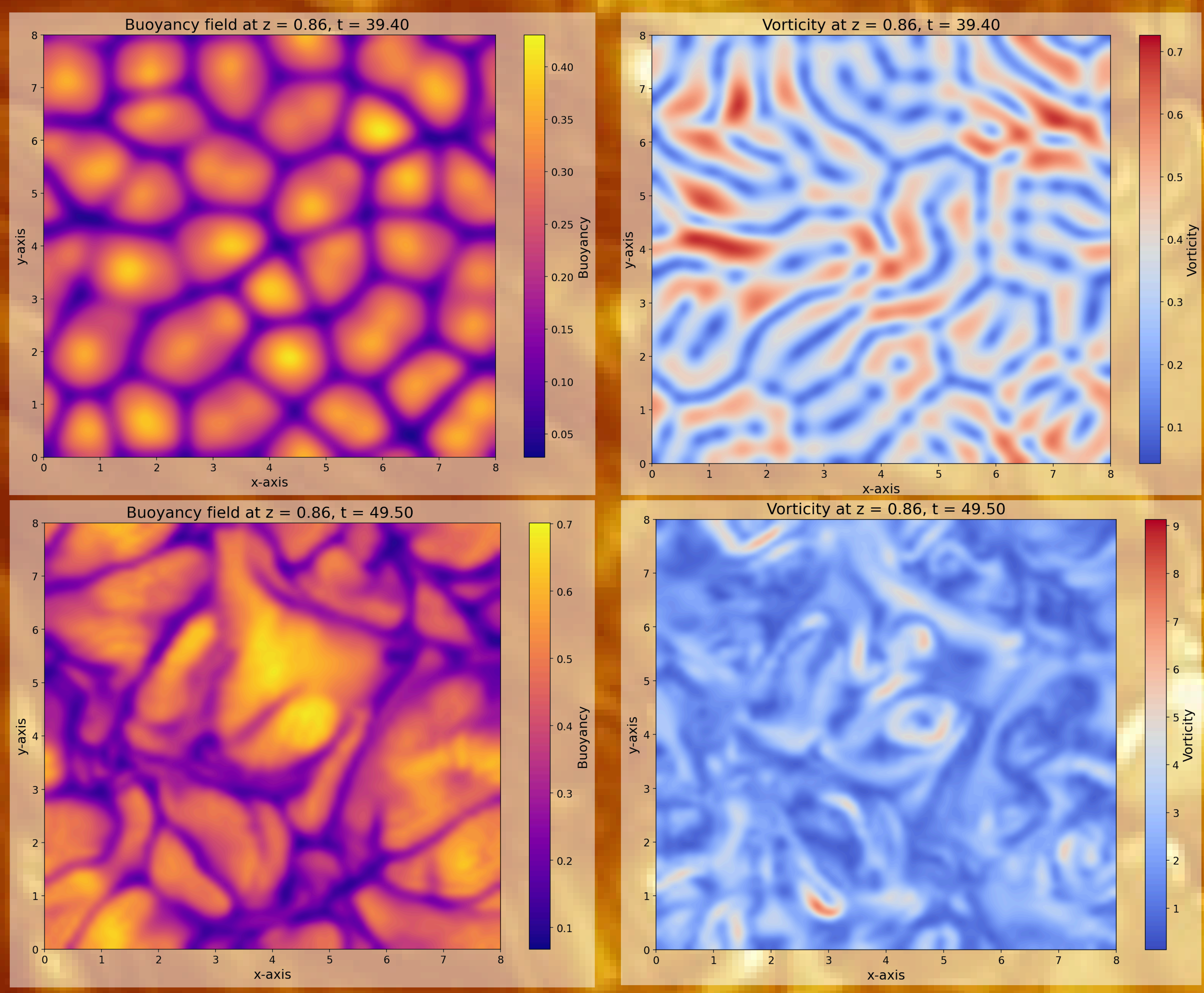
Results

Computational Method

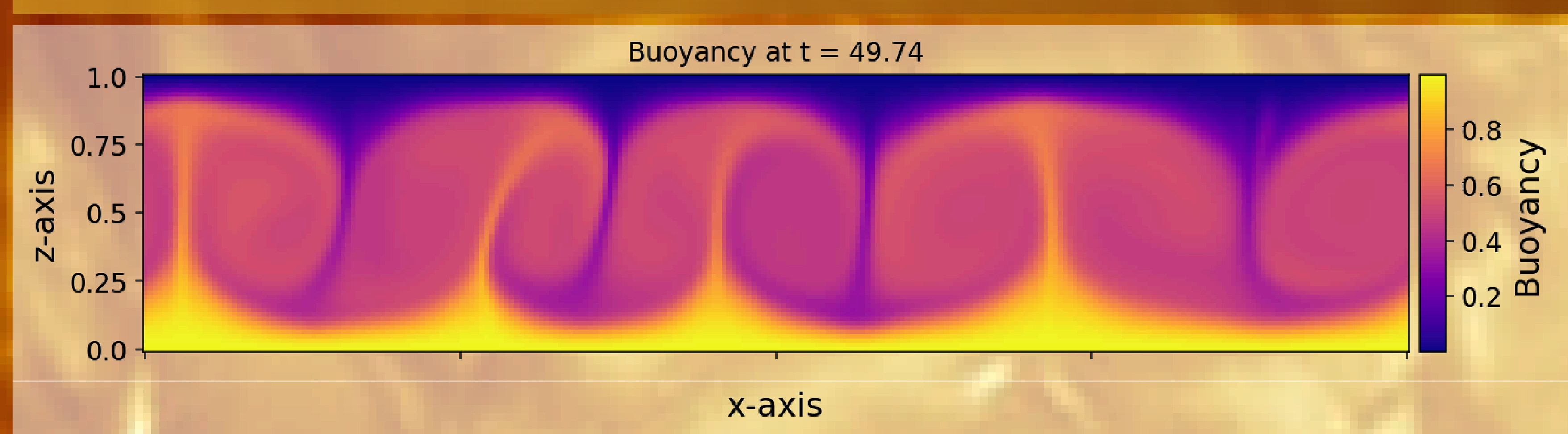
The code uses open-source package "Dedalus" [6] to numerically solve our partial differential equations, and to simulate the associated fluid fields. We can't always find an algebraic solution, or family of solutions, to systems of PDEs like for our convection. However we can approximate using a series of orthogonal functions; Dedalus assumes Chebyshev functions. One of our boundary conditions is horizontal periodicity; the left and right edges of our plots should be the same. So the Fourier series is an easier basis to work with in the x-direction, as they will already be periodic. For our other boundary conditions we considered our choice of pressure gauge, incompressibility, and setting the speeds of fluid at the very top and bottom of our pan to be 0. Dedalus uses the tau method, which adds extra degrees of freedom to each equation to adhere to these extra conditions. These degrees of freedom come from perturbing each equation by the previous polynomial.

We varied the buoyancy force i.e the Rayleigh number, felt by the olive oil, and solved our Navier-Stokes and heat equation to observe the changing behaviour of the convection patterns obtained.

In the figures below we show the convection patterns created in the oil along the x and y axes, cut along a z-axis cross-section near the top of the fluid layer (a layer 86% of the way up the oil depth). **The first row represents the simulation at Ra = 2 x 10 ^4, and the second one at Ra = 2 x 10^6. The left and right columns showcase the buoyancy and vorticity fields respectively.**



Below we show a "side-view" of the layer of fluid convection patterns in our oil for **Ra = 2 x 10^4**, where we can see the clear shape of the plumes formed.



Discussion

We can observe the **clearest sun-spot like patterns formed at a Rayleigh number of 2 x 10^4. At Ra = 2 x 10^6 there is more turbulence and irregularity in the cells.** This confirms that our initial olive oil parameters are in between the regime of highly chaotic patterns and clear sun-spot like behaviour. We need to be closer to the 10^4 order of magnitude in Ra to observe the best sun-spot like periodicity and clarity, which could be done by using a thinner layer of olive oil, or an oil with greater thermal diffusivity and/or kinematic viscosity values than those from our inital parameters.

Conclusion

The convective patterns produced in a thin layer of oil in a frying pan were successfully modelled computationally, using dimensionalised equations of Rayleigh-Bernard convection. **Using a Rayleigh number of 2x10^4 produces very similar, defined patterns of buoyancy and vorticity, characteristic of those seen on the Sun,** with parameters easy to recreate in your home.

References

[1] Incropera, F.P., DeWitt, D.P., Bergman et. al (2011) "*Fundamentals of Heat and Mass Transfer*". 7th Edition, Ch. 9, NJ: Wiley
[2] Bejan, A., (2013) "*Convection Heat Transfer - Natural convection in single-phase fluids*" 4th Edition, pp. 177–230.
[3] Blundell, SJ, & Blundell, KM (2009), "*Concepts in Thermal Physics*" Ch. 10, Oxford University Press, Incorporated, Oxford.
[4] 1. Kleinstreuer C. (1997) "Engineering Fluid Dynamics: An Interdisciplinary Systems Approach-Derivations and Transformations of the Conservation Equations". Cambridge University Press; pp. 39-112
[5] The Engineering ToolBox [online] Available at: https://www.engineeringtoolbox.com/ [Accessed: 1st April 2025].
[6] KJ Burns, GM Vasil, JS Oishi, D Lecoanet, BP Brown. Dedalus: A flexible framework for numerical simulations with spectral methods. DOI: 10.1103/PhysRevResearch.2.023068