Assignment 8

Monday, October 17, 2022   11:01 PM

1.

a. $P(\text{toothache})$

$0.108 + 0.012 + 0.016 + 0.064$

$= \boxed{0.20}$

b. $\vec{P}(\text{cavity})$

$= \langle P(\text{cavity}), P(\neg\text{cavity}) \rangle$

$= \langle 0.108 + 0.012 + 0.072 + 0.008, 0.016 + 0.064 + 0.144 + 0.576 \rangle$

$= \boxed{\langle 0.20, 0.80 \rangle}$

c. $\vec{P}(\text{Toothache} \mid \text{cavity})$

$= \alpha \langle P(\text{toothache} \mid \text{cavity}), P(\neg\text{toothache} \mid \text{cavity}) \rangle$

$= \alpha \langle 0.108 + 0.012, 0.072 + 0.008 \rangle$

$= \alpha \langle 0.12, 0.08 \rangle \qquad \alpha = \dfrac{1}{0.12 + 0.08}$

$= \boxed{\langle 0.60, 0.40 \rangle} \qquad 0.2\,\alpha = 1$

$\alpha = 5$

2.

a. $P(b, i, \neg m, g, j)$

$= P(b) P(i \mid b, \neg m) P(\neg m) P(g \mid b, i, \neg m) P(j \mid g)$

$= 0.9 \times 0.5 \times 0.8 \times 0.8 \times 0.8$

$\boxed{0.23}$

$= 0.9 \times 0.5 \times \ldots$

$= \boxed{0.23}$

b. $\vec{P}(J \mid b, i, m)$

$= \alpha \vec{P}(J, b, i, m) = \alpha \sum_{g'} \vec{P}(J, b, i, m, g')$

$= \alpha \sum_{g'} \vec{P}(J \mid g') P(b) P(i \mid b, m) P(m) \vec{P}(g' \mid b, i, m)$

$= \alpha \sum_{g'} \vec{P}(J \mid g') \vec{P}(g' \mid b, i, m)$

$$= \alpha \left(\begin{array}{l} P(j) \left(\begin{array}{l} P(j \mid g) P(g \mid b, i, m) \\ + P(j \mid \neg g) P(\neg g \mid b, i, m) \end{array} \right) \\ \\ P(\neg j) \left(\begin{array}{l} P(\neg j \mid g) P(g \mid b, i, m) \\ + P(\neg j \mid \neg g) P(\neg g \mid b, i, m) \end{array} \right) \end{array} \right.$$

$= \alpha \langle 0.8 \times 0.9 + 0.1 \times 0.1,\ 0.2 \times 0.9 + 0.9 \times 0.1 \rangle$

$= \alpha \langle .73,\ 0.27 \rangle$

$= \boxed{\langle 0.73, 0.27 \rangle}$

$\alpha = 1 / (0.73 + 0.27)$

$1\alpha = 1$

$\alpha = 1$

c. $\vec{P}(J \mid \neg b, \neg i, m)$

$$= \alpha \vec{P}(J, \neg b, \neg i, m) = \alpha \sum_{g'} \vec{P}(J, \neg b, \neg i, m, g')$$

$$= \alpha \sum_{g'} \vec{P}(J|g')\, P(\neg b)\, P(\neg i|\neg b, m)\, P(m)\, \vec{P}(g'|\neg b, \neg i, m)$$

$$= \alpha \sum_{g'} \vec{P}(J|g')\, P(g'|\neg b, \neg i, m)$$

$$= \alpha \left(\begin{array}{ll} P(j) & \left(\begin{array}{l} P(j|g)\,P(g|\neg b, \neg i, m) \\ + P(j|\neg g)\,P(\neg g|\neg b, \neg i, m) \end{array} \right. \\ P(\neg j) & \left(\begin{array}{l} P(\neg j|g)\,P(g|\neg b, \neg i, m) \\ + P(\neg j|\neg g)\,P(\neg g)\neg b, \neg i, m) \end{array} \right. \end{array} \right.$$

$$= \alpha \langle 0.8 \times 0 + 0.1 \times 1,\ 0.2 \times 0 + 0.9 \times 1 \rangle$$

$$= \alpha \langle 0.1, 0.9 \rangle$$

$$= \boxed{\langle 0.1, 0.9 \rangle}$$

$$\alpha = 1/0.1+0.9$$

$$1\alpha = 1$$

$$\alpha = 1$$