Mathematics For Me

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Chapter 1

Linear Algebra

1.1 The Beginning, Systems of Linear Equations

When we first started learning about variables in middle school, one of the earliest forms of an equation we were introduced to may have looked something like this:

$$mx + b = c$$

Your first instinct is probably, "Oh look! It's the slope-intercept formula!" and you'd be right! The slope-intercept formula is an equation representing a line in 2D space (because you can move in the x and y direction).

As we move along and learn about more complex equations, we most likely encountered equations of higher dimensions (i.e., containing more variables). For example:

$$ax + +cz = d \text{ Plane in 3D space.} \tag{1.1}$$

$$ax_1 + by + cz + dx_2 = e \text{ Something in 4D space.} \tag{1.2}$$

$$a_1x_1 + a_2x_2 + ... + a_nx_n = d \; n\text{D space.} \tag{1.3}$$

As you might have guessed from the equations above, we can easily scale our **linear equations** to higher dimensions just by adding another **constant** multiplied by a new **variable**. You might have been thrown off by the number of terms introduced, so let's give them a proper definition!

- **Constant**: Any term that does not change. For example, in the equation $2x + 3$, 2 and 3 are constants because we know their value. We also know that 2 and 3 are definite values, meaning they do not change at all, regardless of what x is. In the equations provided above, a, b, c, a_1, a_2, etc are constants.

- **Variable**: Any value that will change. Typically, these are represented by an alphabetic letter (x, y, z, etc). We are usually trying to solve the equation to find the value for these variables. For example, we might be provided a question that asks us to solve for x in: 10 = 2x + 5. In this case, x is the variable we are solving for.
- **Linear Equation**: A simple equation that *does not involve any exponents or square roots of a variable.* Basically, if you see equations like: $x^2+x=0$ or $x*y+x=2$, then they are NOT linear equations. Equations like: $x+2=10$ or $x+y+z=10$ would be considered linear equations (because they do not involve any powers or square roots, which would alter the **linearity** of the equation, basically it will no longer be a straight line).

In any linear equation, the *constants cannot all be 0's.* "Why not?" well, because if they were, then the equation would just be 0 = some number (remember, 0 times anything is going to be 0)! That's not going to be a straight line, or even a point! However, the d and e (shown in equation 1.3) CAN be 0. Any linear equation where the other side of the = is 0 is called a **homogeneous linear equation**.

Generally, a **linear equation in n variables** (where n is the number of variables for the linear equation) can be expressed via the equation shown in 1.3 (the nD one)!

1.1.1 There are more of them now!

Just like how we can add more variables, we can also add more equations.

A finite set of linear equations is called a **systems of linear equations** or **linear system** (we will be calling them linear systems from now on for ease of typing). In a linear system, the variables may also be called **unknowns**.

The equation below is an example of a simple linear system in 2 variables (x_1 and y_1):

$$\begin{aligned} 4x_1 + 10y_1 &= 6 \\ 6x_1 - y_1 &= 10 \end{aligned} \tag{1.4}$$

Again, like the number of variables...we can scale the number of equations as much as we want!

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_3x_3 + ... + a_{1n}x_n &= b_1 \\ &. \\ &. \\ &. \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n &= b_m \end{aligned} \tag{1.5}$$

For a general linear system, we can say that a **solution** in n unknowns (x_1, x_2, ..., x_n) is a sequence of n numbers that, when substituted into unknowns x_1, x_2, ..., x_n, will make all linear equations in the linear system to be true.

Here is an example:

$$\begin{aligned} 2x - y &= 7 \\ 3x - 2y &= 10 \end{aligned} \tag{1.6}$$

Equation 1.6 has a solution: $x = 4$ and $y = 1$. We can also write the solution as coordinates too: (4, 1). More often times than not, we'll be writing the solution as coordinates because it is a lot easier to write out and visualize. We can think of the solutions to our linear systems as coordinates where all of the equations in the linear system intersects (cross).