Problem 3:

a) R: arbitary logical position
v: variable
t: term

$(\exists x_0.R)[t/v] = \exists\ x_0.R[t/v]$?

- We have to consider the variable x_0 in R
  - If R is x_0 free. This condition ensures that subtitution preserves the meaning of quantified expression.
  - If R is t free. It means that subtituing t for v in R does not introduce any new variable.

b) Ex: $(\exists x_0.R)[t/v] = \exists\ x_0.R[t/v]$ does not hold
R(x_0) = "x_0 % 2 == 0"
$v = x_0$
$t = x_0 + 1$

$(\exists x_0.R)[t/v] = (\exists x_0.\ (x_0 \% 2{=}{=}0))[t/v]$
-> $(\exists x_0.\ (x_0 \% 2{=}{=}0))[\ x_0 + 1/\ x_0]$
-> $(\exists x_0.\ (x_0{+}1 \% 2{=}{=}0))$
- there is an x_0 sothat x_0+1 % 2 == 0

$(\exists x_0.R)[t/v] \neq \exists x0.R[t/v]$
In this example the logical proposition doesn't hold because $((\exists x_0.R)[t/v]$ say that: "there's x_0 that x_0 % 2 == 0 where as $\exists\ x_0.R[t/v]$ say that also x_0+1%2==0. -> contradiction $\perp$.

c)Floyd's rule $\{P\ \}\ v := t\ \{\exists\ v_0.(P\ [v_0/v] \wedge v = t[v_0/v])\}$.

1. $\{P\}\ v := t\ \{Q[t/v]\}$ (Hoare's rule)
2. $\{P\}\ v := t\ \{Q[t/v] \wedge v = t\}$ (Conjunction introduction)
3. $\{P\}\ v := t\ \{\exists v_0.(Q[t/v] \wedge v = t[v_0/v])\}$ (Existential introduction)
4. $\{P\}\ v := t\ \{\exists v_0.(P[v_0/v] \wedge v = t[v_0/v])\}$ (Substitution)

Problem 4:

a) Determine under which circumstances the following program establishes 0 <= y < 100.

Refactor the code:

```
if (x < 34 && x == 2)
    y := x + 1;
else if (x < 34 && x != 2)
    y := 233;
else if (x >= 34 && x < 55)
    y:=21;
else if (x >= 34 && x >= 55)
    y:=144;
```

Conditions:

1. For any x in the range x==2, the value of y will be x + 1, which satisfies 0 <= y < 100.
2. For any x in the range x<2 or 2 < x < 34, the value of y will be 233, which does not satisfy the condition
3. For any x in the range 34 <= x < 55, the value of y will be 21, which satisfies 0 <= y < 100.
4. For any x in the range 55 <= x < 99, the value of y will be 144, which does not satisfy the condition.

b) if B1 then C1 elif B2 then C2 else C3

$$\frac{\{P \land B1\}\ C1\ \{Q\},\ \{P \land \neg B1 \land B2\}\ C2\ \{Q\},\ \{P \land \neg B1 \land \neg B2 \land B3\}\ C3\ \{Q\}}{\text{if } B1 \text{ then } C1 \text{ elif } B2 \text{ then } C2 \text{ else } C3}$$