Problem 01:

a) $\{?\}\ x = 2*x + 1\ \{x \leq 100\} \Rightarrow \{Q[2x+1/x]\}\ x = 2x+1\ \{x \leq 100\}$

$P \equiv \{2*x + 1 \leq 100\}\ x = 2*x + 1\ \{x \leq 100\}$

$\{2*x \leq 99\}\ x = 2*x + 1\ \{x \leq 100\}$

$\{x \leq 49{,}5\}\ x = 2*x + 1\ \{x \leq 100\}$

b) $\{?\}\ y := 2*x + y\ \{2xy \leq y^2\}$

$P \equiv \{2x(2x + y) \leq (2*x + y)^2\}$

$\{4x^2 + 2xy \leq 4x^2 + 4xy + y^2\}$

$\{4x^2 \leq 4x^2 + 2xy + y^2\}$

$\{0 \leq 2xy + y^2\}$

$\{-2xy \leq y^2\}$

$\{(2x \geq y \ \&\& \ y \geq 0) \ \| \ (2x \leq y \ \&\& \ y \leq 0)\}$

c) $\{?\}\ y = x + 1\ \{\exists x.\ (x^2 \leq y \leq (x+1)^2)\}$

$P \equiv \{\exists x.\ x^2 \leq x + 1 \leq (x+1)^2\} \Rightarrow$ True sei $0 \leq x \leq 1$

Problem 02 Floyd's Rule

a) $\{x \leq y\}\ x = 2x + 1\ \{Q\}$

$Q = \{\exists x_0.\ x_0 \leq y \ \wedge \ x = 2x_0 + 1\}$

$\{\exists x_0.\ 2x_0 + 1 \leq 2y + 1 \ \wedge \ x = 2x_0 + 1\}$

$\{x \leq 2y + 1 \quad \exists x_0.\ x = 2x_0 + 1\}$

$\{x \leq 2y + 1\}$

b) $\{0 < x < 100\}\ x := 2x - 1\ \{Q\}$

$Q := \{\exists x_0.\ 0 < x_0 < 100 \ \wedge \ x = 2x_0 - 1\}$

$\{\exists x_0.\ -1 < 2x_0 - 1 < 199 \ \wedge \ x = 2x_0 - 1\}$

$\{-1 < x < 199 \ \wedge \ \exists x_0.\ x = 2x_0 - 1\}$

$\{-1 < x < 199\}$

c) $\{\exists x.\ x < y\}\ y = x - 1\ \{Q\}$

$Q := \{\exists y_0\ .\ (x < y_0) \wedge (y = x - 1 < y_0)\}$

$\{\exists y_0\ .\ (x < y_0) \wedge (y = x - 1 < x < y_0)\}$

$\{y < x\}$

Problem 03

a) $\underbrace{\{?\}}_{P}\ x = x + 1;\ y = y + 1\ \underbrace{\{x = y + 1\}}_{R}$

$\{Q\}\ y = y + 1\ \{\underbrace{x = y + 1}_{R}\} \Rightarrow \{Q\}\ y = y + 1\ \{\underbrace{y = x - 1}_{R}\}$

$\Rightarrow Q := \{y + 1 = x - 1\}$

$\{P\}\ x = x + 1\ \{\underbrace{y + 1 = x - 1}_{Q}\}$

$\{P\}\ x = x + 1\ \{y + 2 = x\}$

$\Rightarrow P := \{y + 2 = x + 1\}$

$\{y = x - 1\}$

b) $\underbrace{\{?\}}_{P}$ if $x < y$ then $x = x + 1$ else $y := y + 1$ $\{x = y + 1\}$

Linke Seite

$\vdash \{P \wedge (x < y)\}\ x = x + 1\ \{x = y + 1\}$

$\vdash (x + 1 = y + 1) \wedge (x < y)\ x = x + 1\ \{x = y + 1\}$

Rechte Seite

$\vdash \{P \wedge x \geq y\}\ y := y + 1\ \{x = y + 1\}$

$\vdash \{P \wedge x \geq y\}\ y := y + 1\ \{y = x - 1\}$

$\vdash \{y + 1 = x - 1 \wedge x \geq y\}\ y := y + 1\ \{y = x - 1\}$

c) $\{?\}\ x = y + 1\ ;$ if $(x < y)$ then $x = x + 1$ else $y = y + 1$ $\{x \neq y\}$

Linke_Seite $\{P \wedge (x < y)\}\ x := x + 1\ \{x \neq y\}$

Rechte_Seite $\{P \wedge (x \geq y)\}\ y := y + 1\ \{x \neq y\}$

LS: $\{P[x+1/x]\}\ x = y+1$ if $(x<y)$ then $x = x+1$ else $y = y+1$ $\{x \neq y\}$

da $x = y+1 \Rightarrow x > y \Rightarrow x < y$ false

$\Rightarrow \{P \wedge (x \geq y)\}\ y := y+1\ \{x \neq y\}$

$\{x \neq y+1 \wedge x \geq y\}\ y := y+1\ \{x \neq y\}$

$\rightarrow P := \{x \neq y+1\}$

Problem 04 $\{P\}\ x := x^2 + 1\ \{Q\}$

a) $Q = \{|x| \leq 10\}$

$\{P\}\ x := x^2+1\ \{-10 \leq x \leq 10\}$

$P := \{-10 \leq x^2+1 \leq 10\}$

$\{-11 \leq x^2 \leq 9\}$

$\{0 \leq x^2 \leq 9\}$

$\{-3 \leq x \leq 3\}$

b) Floyd's rule $\{P\}$ (aus a) $x := x^2+1\ \{Q'\}$

$\{P\} \equiv \{-3 \leq x \leq 3\}$
$\equiv \{0 \leq x^2 \leq 9\}$
$\equiv \{1 \leq x^2+1 \leq 10\}$

$\rightarrow \{1 \leq x^2+1 \leq 10\}\ x := x^2+1\ \{Q'\}$

$\{1 \leq x^2+1 \leq 10\}\ x := x^2+1\ \{1 \leq x \leq 10\}$

$Q' \equiv \{1 \leq x \leq 10\}$

c) $Q' \Rightarrow Q$ zeigen

$Q = \{-10 \leq x \leq 10\}$

$Q' = \{1 \leq x \leq 10\}$

for $x \in [-10, 1)$ is Q' false and Q true

$\Rightarrow Q' \Rightarrow Q$