Program Verification Prof. Dr. H. Peter Gumm
Summer 2023

Exercise sheet 03

Deadline: May 13, 8:00 p.m.

This time, you should submit a pdf-file, named **ex03_*xyz*.pdf**, where *xyz* is your ilias user name. Clearly mark your solutions as *Problem 1(a)*, *Problem 1(b)*, *Problem 1(c)*, *Problem 2(a)* ,... . Before submitting your solutions, you should have checked them using Dafny, using a setup similar to the following:

```
87  method test()
88  {   var x:int,y:int, z:int  := *, *, *;
89
90      assume P ;      // the preconditionm
91        x := t;       // the body
92      assert Q ;      // this, or
93  }
```

You are **not required** to submit the Dafny solution - this serves only for your own reassurance.

Problem 1. Using Hoare's assignment rule $\{Q[t/v]\}v := t\{Q\}$, solve the following Hoare triples, where you are given $v := t$ and Q. Simplify as much as possible and check your answer using Dafny:

(a). $\{\ ?\ \}\, x := 2 * x + 1\, \{x \leq 100\}$

(b). $\{\ ?\ \}\, y := 2 * x + y\, \{2xy \leq y^2\}$

(c). $\{\ ?\ \}\, y := x + 1\, \{\exists x.(x^2 \leq y \leq (x+1)^2)\}$

(3 points)

Problem 2. Using Floyd's rule $\{P\}\ v := t\ \{\exists v_0.(P[v_0/v] \wedge v = t[v_0/v])\}$, calculate a postcondition in the following examples. Simplify as much as possible and show every step.

(a). $\{\ x \leq y\ \}\ x := 2 * x + 1\, \{\ ?\ \}$

(b). $\{\, 0 \leq x < 100\,\}\, x := 2 * x - 1\, \{\ ?\ \}$

(c). $\{\, \exists x.x < y\,\}\, y := x - 1\, \{\ ?\ \}$

(3 point)

Problem 3. Use the Hoare calculus, to find appropriate preconditions

(a). $\{\,?\,\}\ x := x + 1$ **;** $y := y + 1 \{x = y + 1\}$

(b). $\{\,?\,\}$ **if** $x < y$ **then** $x := x + 1$ else $y := y + 1 \{x = y + 1\}$

(c). $\{\,?\,\} x := y + 1$; **if** $x < y$ then $x := x + 1$ else $y := y + 1 \{x \neq y\}$

(3 points)

Problem 4. Consider the Hoare triple $\{\, P\,\}\, x := x^2 + 1\, \{Q\}$.

(a). Given $Q = \{|x| \leq 10\,\}$, find P, using Hoare's assignment rule.

(b). With the P that you found in part (a), use Floyd's rule to compute Q' so that $\{P\}x := x^2 + 1\{Q'\}$.

(c). Simplify and show that $Q \not\equiv Q'$, but $Q' \Rightarrow Q$.

(3 points)