Ejercicio 1. Sea $\Sigma = \{a, b\}$ un alfabeto. Hallar:

$$\Sigma^0, \quad \Sigma^1, \quad \Sigma^2, \quad \Sigma^*, \quad \Sigma^+, \quad |\Sigma|, \quad |\Sigma^0|$$

($|A|$ indica la cantidad de elementos de A).

$\Sigma^0 = \{\lambda\}$ $\quad \Sigma^1 = \{a, b\}$ $\quad \Sigma^2 = \{aa, ab, ba, bb\}$

$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa \ldots\}$ $\quad |\Sigma| = 2$

$\Sigma^+ = \{a, b, aa, ab, ba, bb, aaa \ldots\}$ $\quad |\Sigma^0| = 1$

Ejercicio 2. Decidir si, dado $\Sigma = \{a, b\}$, vale:

$$\lambda \in \Sigma, \quad \lambda \subseteq \Sigma, \quad \lambda \in \Sigma^+, \quad \lambda \in \Sigma^*, \quad \Sigma^0 = \lambda, \quad \Sigma^0 = \{\lambda\}$$

✗ ✗ ✗ ✓ ✗ ✓

Ejercicio 3. Sea $\alpha = abb$ una cadena. Calcular:

$$\alpha^0, \quad \alpha^1, \quad \alpha^2, \quad \alpha^3, \quad \prod_{k=0,\ldots,3} \alpha^k = \alpha^0.\alpha^1.\alpha^2.\alpha^3, \quad \alpha^r$$

$\alpha^0 = \lambda$ $\quad \alpha^1 = abb$ $\quad \alpha^2 = abb\,abb$ $\quad \alpha^3 = abb\,abb\,abb$

$\prod_{k=0,..3} \alpha^k = \alpha^0.\alpha^1.\alpha^2.\alpha^3 = \lambda.abb\,abb\,abb\,abb\,abb\,abb$

$\alpha^R = bba$

Ejercicio 4. Sean las cadenas $\alpha = abb$ y $\beta = acb$. Calcular:

$$\alpha\beta, \quad (\alpha\beta)^r, \quad \beta^r, \quad \beta^r\alpha^r, \quad \lambda\alpha, \quad \lambda\beta, \quad \alpha\lambda\beta, \quad \alpha^2\lambda^3\beta^2$$

$\alpha.\beta = abbacb$ $\quad (\alpha.\beta)^R = bcabba$ $\quad \beta^R = bca$

$\beta^R.\alpha^R = bcabba$ $\quad \lambda\alpha = abb$ $\quad \lambda\beta = acb$ $\quad \alpha\lambda\beta = abbacb$ $\quad \alpha^2\lambda^3\beta^2 = abbabb\,acbacb$

Ejercicio 5. Dado un alfabeto Σ, sean $x, y \in \Sigma$ y $\alpha, \beta \in \Sigma^*$. Demostrar que:

a. $|x.(y.\alpha)| = 2 + |\alpha|$
b. $|\alpha^r| = |\alpha|$
c. $|\alpha x\beta| = |x\alpha\beta|$
d. $|\alpha.\alpha| = 2|\alpha|$
e. $(\alpha.\beta)^r = \beta^r.\alpha^r$
f. $(\alpha^r)^r = \alpha$
g. $(\alpha^r)^n = (\alpha^n)^r$

($|\alpha|$ indica la longitud de la cadena α).

a) $|x.(y.\alpha)| \overset{LON}{=} 1 + |y.\alpha| \overset{LON}{=} 2 + |\alpha|$

b) Inducción en α

Caso Base $\alpha = \lambda \rightarrow |\lambda^R| = |\lambda|$

$\overset{REV}{=} |\lambda|$ ✓

Caso inductivo $\alpha = (x.\beta) \rightarrow |(x.\beta)^R| = |x.\beta|$ $\quad$ Hi $|\beta^R| = |\beta|$

$|(x.\beta)^R| \overset{REV}{=} |\beta^R.x| \overset{L1}{=} 1 + |\beta^R| \overset{Hi}{=} 1 + |\beta| \overset{LON}{=} |x.\beta|$

Lema 1 $|\alpha.x| = 1 + |\alpha|$

CB $\rightarrow |\lambda.x| = 1 + |\lambda|$ $\quad$ Ci $\quad |y.\beta.x| = 1 + |y.\beta|$ $\quad$ Hi: $|\beta.x| = 1 + |\beta|$

$|x.\lambda| = |x| = 1 + 0$ $\quad$ LON $= 1 + |\beta.x| \overset{LON}{=} 1 + 1 + |\beta|$

$= 1 + |\lambda|$ $\quad$ Hi $= 1 + 1 + |\beta|$ ✓

c) $|\alpha.x.\beta| = |x\alpha.\beta|$

$\overset{ASO}{=} |\alpha.(x.\beta)|$ $\quad \overset{ASO}{=} |x.(\alpha.\beta)|$

L2 $= |\alpha| + |x.\beta|$ $\quad \overset{L2}{=} |x| + |\alpha.\beta|$

L2 $= |\alpha| + |x| + |\beta|$ $\quad \overset{L2}{\underset{conm}{+}}= |\alpha| + |x| + |\beta|$ ✓

Lema 2 $\alpha, \beta \in \Sigma^*$ qvq $|\alpha.\beta| = |\alpha| + |\beta|$

Caso Base $\alpha = \lambda \rightarrow |\lambda.\beta| = |\lambda| + |\beta|$

$\overset{Neutro}{=} |\beta| \overset{LON}{=} 0 + |\beta|$ ✓

Caso inductivo: $\alpha = x.\gamma \rightarrow |x.\gamma.\beta| = |x.\gamma| + |\beta|$ $\quad$ Hi $|\gamma.\beta| = |\gamma| + |\beta|$

$\overset{LON}{=} 1 + |\gamma.\beta|$ $\quad \overset{LON}{=} 1 + |\gamma| + |\beta|$

$\overset{Hi}{=} 1 + |\gamma| + |\beta|$ ✓

d) $|\alpha.\alpha| = 2|\alpha|$

Caso Base $\alpha = \lambda \rightarrow |\lambda.\lambda| = 2|\lambda|$

$0 = 0$

Caso inductivo $\alpha = x.\beta$ $\quad |x.\beta.x.\beta| = 2|x.\beta|$ $\quad$ Hi $= |\beta\beta| = 2|\beta|$

L2 $= |x.\beta| + |x.\beta|$

$= 2 \cdot (|x.\beta|)$ ✓

e) $(\alpha.\beta)^R = \beta^R.\alpha^R$

Caso Base $\alpha = \lambda$ $\quad (\lambda.\beta)^R = \beta^R.\lambda^R$

$(\beta)^R = \beta^R.\lambda = \beta^R$ ✓

Caso inductivo $\alpha = x.\gamma$ $\quad (x.\gamma.\beta)^R = \beta^R.(x.\gamma)^R$ $\quad$ Hi: $(\gamma.\beta)^R = \beta^R.\gamma^R$

$\overset{REV}{=} (\gamma.\beta)^R.x$ $\quad \overset{REV}{=} \beta^R.\gamma^R.x$ ✓

Hi $= \beta^R.\gamma^R.x$

f) $(\alpha^R)^R = \alpha$

Caso Base $\alpha = \lambda \rightarrow (\lambda^R)^R = \lambda$

$= \lambda^R = \lambda$ ✓

Caso inductivo $\alpha = x.\beta \rightarrow ((x.\beta)^R)^R = x.\beta$ $\quad$ Hi: $(\beta^R)^R = \beta$

$\overset{REV}{=} (\beta^R.x)^R \overset{L3}{=} x.\beta^{R^R} \overset{Hi}{=} x.\beta$ ✓

Lema 3 $(\alpha.x)^R = x.\alpha^R$ $\quad x \in \Sigma$ $\alpha \in \Sigma^*$

Caso Base $\alpha = \lambda \rightarrow (\lambda.x)^R = x.\lambda^R$

$x.\lambda = x = x$ ✓

Caso inductivo $\alpha = y.\beta \rightarrow (y.\beta.x)^R = x.(y.\beta)^R$ $\quad$ Hi $(\beta.x)^R = x.\beta^R$

$\overset{REV}{=} (\beta.x)^R.y$ $\quad \overset{REV}{=} x.\beta^R.y$ ✓

$\overset{Hi}{=} x.\beta^R.y$

g) $(\alpha^R)^m = (\alpha^m)^R$

Caso Base $m = 0 \rightarrow (\alpha^R)^0 = (\alpha^0)^R$

$\lambda = \lambda^R = \lambda$ ✓

Caso inductivo $m = m+1 \rightarrow (\alpha^R)^{m+1} = (\alpha^{m+1})^R$ $\quad$ Hi: $(\alpha^R)^m = (\alpha^m)^R$

Pot $= \alpha^R.(\alpha^R)^m$ $\quad \overset{Pot}{=} (\alpha.\alpha^m)^R$

Hi $= \alpha^R.(\alpha^m)^R$ $\quad \overset{e)}{=} \alpha^R.(\alpha^m)^R$ ✓

Ejercicio 6. Dar ejemplos de cadenas que pertenezcan a los siguientes lenguajes:

a. $\mathcal{L} = \{a^n b^n \mid n \geq 0\}$
b. $\mathcal{L} = \{a^n b^n \mid n \geq 1\}$
c. $\mathcal{L} = \{a^n b^m \mid n \geq 1 \wedge m \geq 1\}$
d. $\mathcal{L} = \{a^n b^m \mid n \geq 1 \wedge m \geq 0\}$
e. $\mathcal{L} = \{a^n (ac)^p (bab)^q \mid n \geq 0 \wedge q = p + 2 \wedge p \geq 1\}$
f. $\mathcal{L} = \{a, b\}^3 \cap \Lambda$
g. $\mathcal{L} = \left\{\alpha\alpha^r \mid \alpha \in \{a, b\}^+\right\}$
h. $\mathcal{L} = \left\{\alpha \in \{a, b\}^+ \mid \alpha = \alpha^r\right\}$

a) $\lambda \in \mathcal{L}a$, $(ab) \in \mathcal{L}a$, $(aabb) \in \mathcal{L}a$

b) Los mismos que $\mathcal{L}a$ pero sin λ

c) $(aabb), (aaaab), (abb) \in \mathcal{L}c$

d) $(a), (aab), (aaaa), (aabbb) \in \mathcal{L}d$

e) $(ac\,bab\,bab\,bab), (aaa\,ac\,ac\,(bab)^4) \in \mathcal{L}e$

f) $\mathcal{L}f = \emptyset \quad (\{a,b\}^3 = \{aaa, aab, abb, \dots\}) \cap \{\lambda\} = \emptyset$

g) $(aa), (abbaa\,aabba), (bbaabb) \in \mathcal{L}g$

h) $(aaa), (aba), (aabaabbabaa) \in \mathcal{L}h$.

Ejercicio 7. Definir por comprensión los siguientes lenguajes:

a. $\mathcal{L}_1 = \{ab, aabb, aaabbb, \dots\}$
b. $\mathcal{L}_2 = \{aab, aaaabb, aaaaaabbb, \dots\}$
c. $\mathcal{L}_3 = \{aaabccc, aaaabcccc, aaaaabccccc, \dots\}$

(donde el «crecimiento» en la cantidad de cada símbolo es lineal en cada caso).

$\mathcal{L}_1 = \{a^n . b^n / n \geq 1\}$ $\quad$ $\mathcal{L}_3 = \{a^n . b . c^n / n \geq 3\}$

$\mathcal{L}_2 = \{a^n . b^m / m \geq 1 \wedge n = 2 \cdot m\}$

Ejercicio 8. Dados $\mathcal{L}_1 = \{a, bc\}$, $\mathcal{L}_2 = \{aaa, bc\}$, y siendo $\Lambda = \{\lambda\}$, calcular:

a. $\mathcal{L}_1 \cup \mathcal{L}_2$	*d.* $\mathcal{L}_1.(\mathcal{L}_2)^0$	*g.* $(\mathcal{L}_1.\mathcal{L}_2)^+$	*j.* $\mathcal{L}_1 \emptyset \mathcal{L}_2$
b. $\mathcal{L}_1 \cap \mathcal{L}_2$	*e.* $\mathcal{L}_1.(\mathcal{L}_2)^2$	*h.* $(\mathcal{L}_1.\mathcal{L}_2)^*$	*k.* $(\mathcal{L}_1)^r$
c. $\mathcal{L}_1.\mathcal{L}_2$	*f.* $\mathcal{L}_1.(\mathcal{L}_2)^+$	*i.* $\mathcal{L}_1.\Lambda.\mathcal{L}_2$	*l.* $(\mathcal{L}_1.\mathcal{L}_2)^r$

a) $\mathcal{L}_1 \cup \mathcal{L}_2 = \{a, bc, aaa\}$ $\quad$ b) $\mathcal{L}_1 \cap \mathcal{L}_2 = \{bc\}$

c) $\mathcal{L}_1 . \mathcal{L}_2 = \{a.aaa, a.bc, bc.aaa, bc.bc\}$

d) $\mathcal{L}_1 . (\mathcal{L}_2)^0 = \mathcal{L}_1 . \Lambda = \mathcal{L}_1$

e) $\mathcal{L}_1 (\mathcal{L}_2)^2 = \mathcal{L}_1 . \mathcal{L}_2 . \mathcal{L}_2 = \{aaaa.aaa, aaaa.bc, abc.aaa, abcbc, bcaaa.aaa, bcaaabc, bcbcaaa, bcbcbc\}$

f) $\mathcal{L}_1 . (\mathcal{L}_2)^+ = \mathcal{L}_1 . \{aaa, bc, aaaaaa, aaabc, bcaaa, bcbc \dots\}$

$= \{a.aaa, bc.aaa, abc, bcbc, aaaaaaa, bc\,aaaaaa, a.aaabc, bcaaabc, a.bcaaa, bcbcaaa, abcbc, bcbcbc, \dots\}$

g) $(\mathcal{L}_1 . \mathcal{L}_2)^+ = \{\overset{A}{a.aaa}, \overset{B}{a.bc}, \overset{C}{bc.aaa}, \overset{D}{bc.bc}\}^+ = \dots \{A, B, C, D, AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD \dots, AAA, AAB, AAC \dots\}$

h) $(\mathcal{L}_1 . \mathcal{L}_2)^* = \{\lambda, A, B, C, D, AA, AB, \dots\}$

i) $\mathcal{L}_1 . \Lambda . \mathcal{L}_2 = (\mathcal{L}_1 . \Lambda) . \mathcal{L}_2 = (\mathcal{L}_1 . \{\lambda\}) . \mathcal{L}_2 = \{a.\lambda, bc\lambda\} . \mathcal{L}_2 = \{aaaa, abc, bcaaa, bcbc\}$

j) $\mathcal{L}_1 \emptyset \mathcal{L}_2 = \emptyset . \mathcal{L}_1 = \emptyset$ (Absorvente)

k) $(\mathcal{L}_1)^R = \{a, cb\}$

l) $(\mathcal{L}_1 . \mathcal{L}_2)^R = \{aaaa, abc, bcaaa, bcbc\}^R = \{aaaa, cba, aaacb, cbcb\}$

Ejercicio 9. Determinar el complemento de los siguientes lenguajes, considerando los alfabetos indicados en cada caso.

a. $\mathcal{L} = \Lambda$ para $\Sigma = \{a, b\}$
b. $\mathcal{L} = \{\lambda, a\}$ para $\Sigma = \{a\}$ y $\Sigma = \{a, b\}$
c. $\mathcal{L} = \{b\alpha \mid \alpha \in \{a, b\}^*\}$ para $\Sigma = \{a, b\}$
d. $\mathcal{L} = \{a^{2n} \mid n \geq 0\}$ para $\Sigma = \{a\}$ y $\Sigma = \{a, b\}$
e. $\mathcal{L} = \{\alpha_1 b \alpha_2 \mid \alpha_1, \alpha_2 \in \{a, b\}^* \wedge |\alpha_1| > |\alpha_2|\}$ para $\Sigma = \{a, b\}$

a) $\mathcal{L}^c = \Sigma^+$

b) $\mathcal{L}^c = \Sigma^+ - \{a\}$ y $\mathcal{L}^c = \Sigma^+ - \{a\}$

c) $\mathcal{L}^c = \{\alpha \in \Sigma^* / \alpha = \lambda \vee (\alpha = a.\beta \wedge \beta \in \Sigma^*)\}$

d) $\mathcal{L}^c = \{a^n / n \bmod 2 \neq 0\}$ (impares) y $\mathcal{L}^c = \{a^n / n \bmod 2 \neq 0\} \cup \{\alpha \in \Sigma^* / |\alpha|_B \geq 1\}$

e) $\mathcal{L}^c = \{\alpha_1 b \alpha_2 / \alpha_1, \alpha_2 \in \{a,b\}^* \wedge |\alpha_1| \leq |\alpha_2|\} \cup \{\alpha_1 . a . \alpha_2 / \alpha_1, \alpha_2 \in \{a,b\}^* \wedge |\alpha_1| > (|\alpha_2| + 1)\}$

Ejercicio 10. Sea $\mathcal{L}$, $\mathcal{L}_1$, $\mathcal{L}_2$ lenguajes cualesquiera. Determinar si las siguientes afirmaciones son verdaderas o falsas. Si son verdaderas, demostrarlas. Si no, dar un contraejemplo.

a. $\mathcal{L}^+ \subseteq \mathcal{L}^+$.

b. $\mathcal{L}^+ \subsetneq \mathcal{L}^*$

c. $\mathcal{L}^n\mathcal{L}^m = \mathcal{L}^{n+m}$ para todo $n, m \geq 0$

d. $\mathcal{L}^n \subseteq \mathcal{L}^{n+1}$ para todo $n \geq 0$

e. $\mathcal{L}_1 \subseteq \mathcal{L}_2, n \geq 0 \Longrightarrow (\mathcal{L}_1)^n \subseteq (\mathcal{L}_2)^n$

f. $\mathcal{L}_1 \subseteq \mathcal{L}_2 \Longrightarrow (\mathcal{L}_1)^* \subseteq (\mathcal{L}_2)^*$

g. $(\mathcal{L}^*)^* = \mathcal{L}^*$

h. $(\mathcal{L}^+)^+ = \mathcal{L}^*$

i. $(\mathcal{L}^+)^* = \mathcal{L}^*$

j. $(\mathcal{L}_1 \cup \mathcal{L}_2)^* = (\mathcal{L}_1)^* \cup (\mathcal{L}_2)^*$

k. $(\mathcal{L}_1 \cap \mathcal{L}_2)^* = (\mathcal{L}_1)^* \cap (\mathcal{L}_2)^*$

l. $(\mathcal{L}^2)^* = \mathcal{L}^*$

m. $(\mathcal{L} \cup \mathcal{L}_2)^* = \mathcal{L}^*$

n. $(\mathcal{L}^n)^r = (\mathcal{L}^r)^n$ para todo $n \geq 0$

ñ. $(\mathcal{L}^*)^r = (\mathcal{L}^r)^*$

a) Son iguales

b) Contraej: $\mathcal{L}^+ = \{\lambda, a\} = \mathcal{L}^*$ / son iguales si $\lambda \in \mathcal{L}$ /

$\mathcal{L}^+ \subseteq \mathcal{L}^*$ ✓

c) $\mathcal{L}^n\mathcal{L}^m = \mathcal{L}^{n+m} \quad \forall n, m \geq 0$

Caso Base $n=0 \to \mathcal{L}^0\mathcal{L}^m = \mathcal{L}^m$

$\Lambda \cdot \mathcal{L}^m = \mathcal{L}^m$ ✓

Caso inductivo $n = \bar{n}+1$ $\mathcal{L}^{\bar{n}+1}\mathcal{L}^m = \mathcal{L}^{\bar{n}+1+m}$, Hi: $\mathcal{L}^{\bar{n}} \cdot \mathcal{L}^m = \mathcal{L}^{\bar{n}+m}$

$\mathcal{L}^{\bar{n}+1} \cdot \mathcal{L}^m \overset{Pot}{=} \mathcal{L}\mathcal{L}^{\bar{n}} \cdot \mathcal{L}^m \overset{Hi}{=} \mathcal{L} \cdot \mathcal{L}^{\bar{n}+m} \overset{Pot}{=} \mathcal{L}^{\bar{n}+1+m}$ ✓

d) $\mathcal{L}^n \subseteq \mathcal{L}^{n+1} \quad \forall n \geq 0$

Contraejemplo: $\mathcal{L} = \{a, b\}$ $n=0$ $\mathcal{L}^0 = \{\lambda\} \not\subseteq \mathcal{L}^1 = \{a, b\}$, o $n=2$ $\mathcal{L}^1 = \{a, b\} \not\subseteq \mathcal{L}^2 = \{aa\ ab\ ba\ bb\}$

e) $\mathcal{L}_1 \subseteq \mathcal{L}_2$, $n \geq 0 \to (\mathcal{L}_1)^n \subseteq (\mathcal{L}_2)^n$

Caso Base $n=0 \to \mathcal{L}_1^0 \subseteq \mathcal{L}_2^0 = \{\lambda\} \subseteq \{\lambda\}$ ✓

Caso inductivo $n = n+1 \to \mathcal{L}_1^{n+1} \subseteq \mathcal{L}_2^{n+1}$, Hi: $\mathcal{L}_1^n \subseteq \mathcal{L}_2^n$ $a \subseteq B \to a \cup C \subseteq B \cup C$

$\mathcal{L}_1^{n+1} = \mathcal{L}_1 \cdot \mathcal{L}_1^n \subseteq \mathcal{L}_1 \cdot \mathcal{L}_2^n \subseteq \mathcal{L}_2 \cdot \mathcal{L}_2^n = \mathcal{L}_2^{n+1}$

f) $\mathcal{L}_1 \subseteq \mathcal{L}_2 \to (\mathcal{L}_1)^* \subseteq (\mathcal{L}_2)^*$

$\bigcup_{i=0}^{\infty} \mathcal{L}_1^i \subseteq \bigcup_{k=0}^{\infty} \mathcal{L}_2^k$ $\mathcal{L}_1^i = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \ldots$ $\forall i\ \alpha_i \in \mathcal{L}_1$ y $\mathcal{L}_1 \subseteq \mathcal{L}_2$ ∴ $\alpha_i \in \mathcal{L}_2$ ∴

$\forall i \quad \mathcal{L}_1^i \subseteq \mathcal{L}_2^*$

g) $(\mathcal{L}^*)^* = \mathcal{L}^*$

$\subseteq$ $w \in (\mathcal{L}^*)^* \to w = \beta_1 \ldots \beta_n$ / $\forall i, 1 \leq i \leq n$. $\beta_i \in \mathcal{L}^*$. Cada $\beta_i \in \mathcal{L}^{m_i}$ para algun $m_i \geq 0$.

Luego $\alpha \in \mathcal{L}^{m_1} \ldots \mathcal{L}^{m_n} = \mathcal{L}^{m_1 + \ldots + m_n} \subseteq \mathcal{L}^*$

$\supseteq$ $\mathcal{L}^* \subseteq (\mathcal{L}^*)^*$. $\mathcal{L}^* = M$ qvq $M \subseteq M^*$ y vale por def.

h) $(\mathcal{L}^+)^+ = \mathcal{L}^*$ Es falso $\mathcal{L} = \{a, aa, \ldots\}$ $\mathcal{L}^+ = \{a, aa \ldots\} = \mathcal{L}^{++}$

pero $\mathcal{L}^* = \{\{\lambda\}, a, aa, \ldots\}$

i) $(\mathcal{L}^+)^* = \mathcal{L}^*$

$(\mathcal{L}^+)^* = \left(\bigcup_{n \geq 1} \mathcal{L}^n\right)^* = \bigcup_{m \geq 0} \left(\bigcup_{n \geq 1} \mathcal{L}^n\right)^m$ qvq

i) $(\mathcal{L}^+)^* = \mathcal{L}^*$

$\subseteq)$ $\mathcal{L}^+ \subseteq \mathcal{L}^*$ y Por F, $\mathcal{L}^{+*} \subseteq (\mathcal{L}^*)^*$ $\quad \mathcal{L}^{**} = \mathcal{L}^*$ ✓

$\supseteq)$ $\mathcal{L} \subseteq \mathcal{L}^+$ y Por F, $\mathcal{L}^* \subseteq \mathcal{L}^{+*}$ ✓

j) $(\mathcal{L}_1 \cup \mathcal{L}_2)^* = \mathcal{L}_1^* \cup \mathcal{L}_2^*$ Contraej:

$\mathcal{L}_1 = \{a\}$ $\quad \mathcal{L}_2 = \{b\}$ $\quad \mathcal{L}_1 \cup \mathcal{L}_2 = \{a, b\}$ $\quad \mathcal{L}_1^* = a, aa, aaa, \lambda \ldots$

$(\mathcal{L}_1 \cup \mathcal{L}_2)^* = \{\Lambda, a, b, aa, ab, bb, ba \ldots\}$ $\quad \mathcal{L}_2^* = b, bb, bbb, \lambda \ldots$

Luego $ab \notin \mathcal{L}_1^* \cup \mathcal{L}_2^*$

Jo que pienso que sea así:

(ab) Tiene que estar en $\mathcal{L}_1^*$ o $\mathcal{L}_2^*$

k) $(\mathcal{L}_1 \cap \mathcal{L}_2)^* = \mathcal{L}_1^* \cap \mathcal{L}_2^*$

$\mathcal{L}_1 = \{a\}$ $\quad \mathcal{L}_2 = \{aa\}$

$\mathcal{L}_1 \cap \mathcal{L}_2 = \emptyset$, $(\mathcal{L}_1 \cap \mathcal{L}_2)^* = \Lambda$, $\mathcal{L}_1^* = \{a, aa, aaa \ldots\}$ $\mathcal{L}_2^* = \{aa, aaaa, aaaaaa, \ldots\}$ $\mathcal{L}_1^* \cap \mathcal{L}_2^* = \{aa, aaaa, \ldots\}$

Son dif.

l) $(\mathcal{L}^2)^* = \mathcal{L}^*$ Contraej: $\mathcal{L} = \{a\}$ $\quad \mathcal{L}^2 = \{aa\}$ $\quad \mathcal{L}^{2*} = \{\Lambda, aa, aaaa, \ldots\}$ $\quad \mathcal{L}^* = \{\Lambda, a, aa, aaa, \ldots\}$

m) $(\mathcal{L}_1 \cup \mathcal{L}_2)^* = \mathcal{L}_1^*$ Contraej: $\mathcal{L}_1 = \{a\}$ $\quad \mathcal{L}_2 = \{b\}$ $\quad \mathcal{L}_1 \cup \mathcal{L}_2 = \{a, b\}$ $\quad \mathcal{L}_1 \cup \mathcal{L}_2^* = \{\Lambda, a, b, aa, ab, \ldots$ $ba, bb, \ldots\}$

$\neq$

$\mathcal{L}_1^* = \{\Lambda, a, aa, aaa\}$

n) $(\mathcal{L}^n)^R = (\mathcal{L}^R)^n$ $\quad \forall n \geq 0$

Caso Base $n = 0$

$(\mathcal{L}^0)^R = (\mathcal{L}^R)^n$

$\{\lambda\}^R = \{\lambda\}$ ✓

Caso inductivo $\quad Hi\ (\mathcal{L}^n)^R = (\mathcal{L}^R)^n$

QVQ $(\mathcal{L}^{n+1})^R = (\mathcal{L}^R)^{n+1}$

$(\mathcal{L}.\mathcal{L}^n)^R$

$P1 \Rightarrow \mathcal{L}^{nR}.\mathcal{L}^R$

$H1 \Rightarrow \mathcal{L}^{R^n}.\mathcal{L}^R$

$pot \Rightarrow (\mathcal{L}^R)^{n+1}$ ✓

$P1 = (\mathcal{L}_1 \mathcal{L}_2)^R = \mathcal{L}_2^R \mathcal{L}_1^R$

ñ) $(\mathcal{L}^*)^R = (\mathcal{L}^R)^*$ $\quad (\mathcal{L}^*)^R = \left(\bigcup_{n \geq 0} L^n\right)^R = \bigcup_{n \geq 0} (L^n)^R \overset{n}{=} \bigcup_{n \geq 0} (L^R)^n = (L^R)^*$

Ejercicio 11. Siendo:

- $\mathrm{Sub}(\mathcal{L})$: subcadenas del lenguaje $\mathcal{L}$.
- $\mathrm{Ini}(\mathcal{L})$: subcadenas iniciales (prefijos) del lenguaje $\mathcal{L}$.
- $\mathrm{Fin}(\mathcal{L})$: subcadenas finales (sufijos) del lenguaje $\mathcal{L}$.

Demostrar que, si $\mathcal{L}_1$ y $\mathcal{L}_2$ son lenguajes:

a. $\mathrm{Fin}(\mathrm{Fin}(\mathcal{L}_1)) = \mathrm{Fin}(\mathcal{L}_1)$
b. $\mathrm{Sub}(\mathrm{Sub}(\mathcal{L}_1)) = \mathrm{Sub}(\mathcal{L}_1)$
c. $\mathrm{Fin}(\mathcal{L}_1\mathcal{L}_2) = \mathrm{Fin}(\mathcal{L}_2) \cup \mathrm{Fin}(\mathcal{L}_1)\mathcal{L}_2$
d. $\mathrm{Ini}(\mathcal{L}_1 \cup \mathcal{L}_2) = \mathrm{Ini}(\mathcal{L}_1) \cup \mathrm{Ini}(\mathcal{L}_2)$
e. $\mathrm{Fin}(\mathcal{L}_1 \cup \mathcal{L}_2) = \mathrm{Fin}(\mathcal{L}_1) \cup \mathrm{Fin}(\mathcal{L}_2)$
f. $\mathrm{Ini}(\mathrm{Sub}(\mathcal{L}_1)) = \mathrm{Sub}(\mathrm{Ini}(\mathcal{L}_1)) = \mathrm{Fin}(\mathrm{Sub}(\mathcal{L}_1)) = \mathrm{Sub}(\mathrm{Fin}(\mathcal{L}_1)) = \mathrm{Sub}(\mathcal{L}_1)$

a) $Fin(L_1) = \{\alpha \in \Sigma^* \mid \exists \beta \in \Sigma^* / \beta.\alpha \in \mathcal{L}\}$

$\therefore L_1 \subseteq Fin\, L_1$. ya que si $\beta = \lambda$, $\alpha \in L_1$.

$\subseteq$ si $x \in Fin(Fin(\mathcal{L}_1))$, $z.y.x \in \mathcal{L}_1$, $z, y \in \Sigma^*$.

$y.x \in Fin(z.y.x)$

quería
ver, en un
lo de abajo

b) $Sub(Sub(L_1)) = Sub(L_1)$

$\subseteq$ si $x \in Sub(Sub(L_1))$, $\exists a, b / axb \in Sub(L_1)$. $\exists u, v / u(axb)v \in L_1$, osea $(ua)x(bv)$, $\therefore x \in Sub(L_1)$

$\supseteq$ si $x \in Sub(L_1)$, $\exists a, b / (axb) \in L_1$. Si tomo $u, v = \lambda$, $u.x.v = \underbrace{uaxbv}_{Sub(Sub(x))}$, y $Sub(L_1) \subseteq Sub(Sub(L_1))$

c) $Fin(L_1 L_2) = Fin(L_2) \cup Fin(L_1) L_2$

(o sufijo de L_2 o de L_1 seguido de L_2)

$\subseteq$ $x \in Fin(L_1 L_2)$, $\exists a / ax \in L_1 L_2$, si $ax = st$ con $s \in L_1$ y $t \in L_2$

si $|x| \leq |t| \rightarrow x \in Fin(L_2)$

si $|x| > |t| \rightarrow x \in Fin(L_1) L_2$

$\supseteq$ $x \in Fin(L_2)$, $x \in (L_1 L_2)$. ya que $ax \in L_2$ por lo primero, entonces $L_1 ax \in L_2$

$x \in Fin(L_1) L_2$, $a y L_2$ con $ay \in L_1$.

D) $ini(L_1 \cup L_2) = Ini(L_1) \cup Ini(L_2)$
e) $Fin(L_1 \cup L_2) = Fin(L_1) \cup Fin(L_2)$

osea es lo mismo ir elemento x elemento del conj en la unión que analizar luego.

f) $Ini(Sub(L_1)) = Sub(ini(L_1)) = Fin(Sub(L_1)) = Sub(Fin(L_1)) = Sub(L_1)$