Math 4A3/6A3

Kritik Assignment # 4

Creation Phase. Solve the following problems. Either write very neatly on paper, and scan the solution with your phone (use the Dropbox app rather than just a photo) and upload to your Kritik account. Or use an iPad, or use Latex, and upload a PDF file produced that way. This step is due by **10:00pm on Sunday November 1**.

Let (M, d) be any metric space.

(a) Assume that K_n, $n \in \mathbb{N}$, are each nonempty *compact* sets in M, which are nested: $K_n \subseteq K_{n-1}$. Show that $\bigcap_{n=1}^{\infty} K_n \neq \emptyset$.

(b) Let $K \subseteq M$ be *compact*, and $f : K \to K$ a continuous function which maps K to itself. Define a sequence of sets A_n, $n \in \mathbb{N}$ by:

$$A_1 = f(K), \quad A_n = f(A_{n-1}),\ \forall n \in \mathbb{N}.$$

Prove that the set $A = \bigcap_{n=1}^{\infty} A_n$ is nonempty, compact, and *invariant* in the sense that $f(A) = A$ (as sets.)

Evaluation Phase. After the due date you will receive 5 submissions of solutions to the above problems, which you are to evaluate using the criteria: Use of Notation; Use of Definitions; Logical Order; Clear and Concise Writing. The ideal is what you would expect in a textbook: a complete and perfectly written proof which explains everything and contains no irrelevant information. The written form of the solution counts just as much as the mathematical content. For this assignment you will receive a solution which you may use to help you in the evaluation phase. Remember that there may be more than one correct and clear method for proving any statement in mathematics.

The evaluate step is due by **10:00pm on Friday November 6.**

Important note: Whatever your opinion of your classmates' solutions, please make your comments polite and respectful. There is no call for sarcastic, abusive, or offensive language when making your criticisms.

The final step, your evaluation of each evaluator, is due by **10:00pm on Sunday November 8.**