

Math 4A3/6A3

Kritik Assignment # 2

Creation Phase. Solve the following problems. Either write very neatly on paper, and scan the solution with your phone (use the Dropbox app rather than just a photo) and upload to your Kritik account. This step is due by **10:00pm on Tuesday September 22**.

We define ℓ^1 to be the class of all real sequences, $x = (x(1), x(2), x(3), \dots)$, $x(i) \in \mathbb{R}$, $\forall i \in \mathbb{N}$, which are absolutely summable,

$$\|x\|_1 = \sum_{i=1}^{\infty} |x(i)| < \infty.$$

- (a) Show that if $x, y \in \ell^1$ then $x + y \in \ell^1$ and $\|x + y\|_1 \leq \|x\|_1 + \|y\|_1$.
(b) For any fixed $x = (x(1), x(2), x(3), \dots) \in \ell^1$, define the sequence

$$x^n = (x(1), x(2), \dots, x(n), 0, 0, \dots), \quad n \in \mathbb{N}, \quad (1)$$

that is, each x^n is the element of ℓ^1 which coincides with x for the first n coordinates, and $x^n(i) = 0$ for all $i > n$.

Show that $x^n \xrightarrow{n \rightarrow \infty} x$ in the metric space ℓ^1 .

- (c) Now assume $x \in \ell^\infty$, with $\|x\|_\infty = \sup_i |x(i)| < \infty$. Define the sequence x^n as in (1). Give an example of a particular $x \in \ell^\infty$ for which $x^n \not\rightarrow x$ in the metric space ℓ^∞ .

Evaluation Phase. After the due date you will receive 5 submissions of solutions to the above problems, which you are to evaluate using the criteria: Use of Notation; Use of Definitions; Logical Order; Clear and Concise Writing. The ideal is what you would expect in a textbook: a complete and perfectly written proof which explains everything and contains no irrelevant information. The written form of the solution counts just as much as the mathematical content. For this assignment you will receive a solution which you may use to help you in the evaluation phase. Remember that there may be more than one correct and clear method for proving any statement in mathematics.

The evaluate step is due by **10:00pm on Saturday September 26**.

Important note: Whatever your opinion of your classmates' solutions, please make your comments polite and respectful. There is no call for sarcastic, abusive, or offensive language when making your criticisms.

The final step, your evaluation of each evaluator, is due by **10:00pm on Monday September 28**.