MATH 4L03/6L03 Assignment #5

Due: Friday, November 20, 11:59pm.

Upload your solutions to the Avenue to Learn course website. Detailed instructions will be provided on the course website.

You may not use the Completeness Theorem in this assignment to show that deductions of various formulas exist. Other theorems, such as the Soundness Theorem and any meta-theorem can be used in your solutions, unless otherwise indicated.

1. Let L be the first order language that has a single 2 place relation symbol E. Let

$$\Gamma = \{\forall x E(x,x), \forall x \forall y \forall z\,(E(x,y) \to (E(y,z) \to E(z,x)))\}.$$

   Produce a deduction for $\Gamma \vdash \forall x \forall y\,(E(x,y) \to E(y,x))$. You may use any of the meta-theorems from Section 5.2 of the text.

   In the previous assignment, you were asked to show that a minor variant of Γ logically implies this sentence. Now you are asked to show that this sentence can be deduced from Γ.

2. Let L be the first order language with two 1-place relation symbols A and B. This question is related to a question from the previous homework assignment.

   (a) Show that there is a deduction for

$$\vdash \forall x(A(x) \to B(x)) \to (\forall x A(x) \to \forall x B(x)).$$

   (b) Show that there is a deduction for

$$\vdash \forall x(A(x) \to B(x)) \to (\exists x A(x) \to \exists x B(x)).$$

   (c) Is there a deduction for

$$\vdash [\forall x A(x) \to \forall x B(x)] \to [\forall x(A(x) \to B(x))]?$$

3. Let L be the first order language that has one 2-place relation symbol E and one 1-place function symbol g. Show that there is a deduction of

$$\vdash (\forall x E(g(x), x) \rightarrow \forall x \exists y E(y, x)).$$

   Compare this question to question #9 from the previous assignment.

4. Let G be the set of group axioms found on page 217 of the text (at the start of section 5.1). Show that there is a deduction of

$$G \vdash \forall x \forall y (((x + (-x)) + y) = y).$$

5. Find a first order language L and a sentence ϕ of L such that there is some infinite L-structure that satisfies ϕ but such that no finite L-structure satisfies ϕ.