Assignment 1 **MATH 4NA** **11 January 2021**

This assignment is due in class on Monday 25 January.

Instructions:

- Carefully read and answer **all** parts of each question. Four significant figures are sufficient for numerical answers.
- Include printouts of **all** `matlab` m-files and scripts you wrote to generate your results.
- Include and fully **label** all graphs: label axes, add a title or figure caption and indicate the corresponding question.
- Submit as a **single pdf file** on the Avenue dropbox by the due date.

Review exercises NOT TO BE HANDED IN:

1. Review the `matlab` tutorials available on the course website.
2. What is the smallest positive integer which *cannot* be represented exactly in `ieee` arithmetic?
3. Which of the following statements are True or False?
   - Using higher-precision arithmetic will make an ill-conditioned problem better conditioned.
   - The choice of algorithm for solving a problem has no effect on the propagated data error.
   - A stable algorithm applied to a well-conditioned problem necessarily produces an accurate solution.
   - Floating point numbers are uniformly distributed throughout their range.
4. In a floating point system with base β, precision p and rounding to nearest, what is the maximum relative error in representing any nonzero real number within the range of the system.

Exercises TO BE HANDED IN:

[6] 1. Let $f(x)$ be a polynomial of degree $\leq N$. Let $P_N(x)$ be the Lagrange polynomial of degree $\leq N$ based on the $N+1$ nodes $x_0, x_1, \ldots, x_N$. Show that $f(x) = P_N(x)$ for all x. [Hint: Show that the error term $E_N(x)$ is identically zero.]

2. Let $f(x) = x^x$.

[6] (a) Find the quadratic Lagrange polynomial $P_2(x)$ using the nodes $x_0 = 1, x_1 = 1.25, x_2 = 1.5$.

[4] (b) Use the polynomial from (a) to estimate the average value of $f(x)$ over the interval [1, 1.5].

[4] (c) Estimate the error in approximating $f(x)$ with $P_2(x)$.

3. (a) Write a `matlab` program to interpolate

$$\begin{array}{rcl} x & = & [\ -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\ ], \\ y & = & [\ -0.1923 \quad -0.2353 \quad -0.30 \quad -0.40 \quad -0.50 \quad 0.00 \quad 0.50 \quad 0.40 \quad 0.30 \quad 0.2353 \quad 0.1923\ ]. \end{array}$$

[4] (b) Calculate error at the interpolated values at $x = -4.5, -3.5, \ldots, 3.5, 4.5$ compared with the exact function $f(x) = x/(x^2+1)$. Are these errors reasonable?

[6] (c) Plot the exact function, the interpolating polynomial and the node points on the same graph. Where is the error the largest? Comment.

[8] 4. (a) Find the clamped cubic spline that passes through the points $\{(x_k, f(x_k))\}_{k=0}^{3}$, on the graph of $f(x) = x + 2/x$, using the nodes $x_0 = 1/2, x_1 = 1, x_2 = 3/2, x_3 = 2$. Use the first derivative boundary conditions $S'(x_0) = f'(x_0)$ and $S'(x_3) = f'(x_3)$. Graph f and the clamped cubic spline interpolant on the same coordinate system. [You should use program `csfit.m` from Avenue, or simply compute the spline by hand.]

[8] (b) Find the natural cubic spline that passes through the points $\{(x_k, f(x_k))\}_{k=0}^{3}$, on the graph of $f(x) = x + 2/x$, using the nodes $x_0 = 1/2, x_1 = 1, x_2 = 3/2, x_3 = 2$. Use the free boundary conditions $S''(x_0) = 0$ and $S''(x_3) = 0$. Graph f and the natural cubic spline interpolant on the same coordinate system.

[4] (c) Comment on any differences between the results you observed in (a) and (b).

[Total: 50]