EquiPy: A Python Package implementing Sequential Fairness with Optimal Transport

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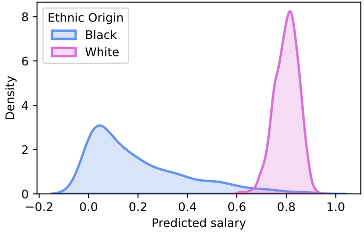
Introduction: Algorithmic Fairness with Multiple Sensitive Attributes

Discrimination in Predictive Models

Consider a Machine Learning (ML) model f, its salary predictions on test set $\hat{Y}$ and one sensitive attribute to which we have access in our dataset, ethnic origin (White/Black).



Potential source of discrimination

1. **Statistical bias** in the data: reproduction of past injustices, under-represented minority in an unbalanced data set,
2. **Explanatory variables** of the model: proxy variables (correlation between a sensitive attribute and other explanatory variables),
3. **Intentional bias**: bias can be the result of deliberate choices, which can be benevolent or malicious.

Legislation

- AI Act (Europe, 2024) aims to ban or limit AI systems in production that present an "**unacceptable level of risk**."
- **Motor insurance** regulation (Zebra, 2022).

	United States									Canada				
	CA	HI	GA	NC	NY	MA	PA	FL	TX	AL	ON	NB	NL	QC
Gender	x	x	•	x	•	x	x	•	•	•	•	x	x	•
Age	x	x	•	x*	•	x	•	•	•	•*	•	x	x	•
Driving experience	•	x	•	•	•	•	•	•	•	•	•	•	•	•
Credit history	x	x	•	•	•	x	•*	•	•	x*	x	•*	x	•
Education	x	x	x	x	x	x	•	•	•	•	•	•	•	•
Profession	x	x	x	•	x	x	•	•	•	•	•	•	•	•
Employment	x	x	x	•	x	x	•	•	•	•	•	•	•	•
Family	•	x	•	•	•	x	•	•	•	•	•	•	•	•
Housing	x	x	•	•	•	x	•	•	•	x	x	•	•	•
Address/ZIP code	•	•	•	•	•	•	•	•	•	x	x	•	•	•

• Permitted attribute ✖ Prohibited attribute * with conditions

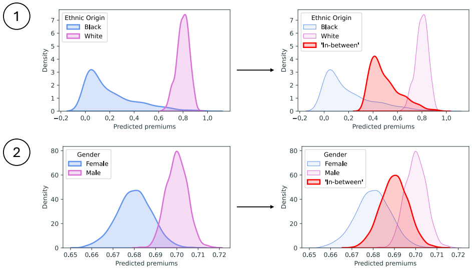
Proxy variables (Upton and Cook, 2014) Simply **eliminating the sensitive attributes** from models does not guarantee fair premiums (Feller et al., 2016).

Single Sensitive Attribute (SSA) Multiple **mitigation** approaches exist (Chzhen et al., 2020; Gouic et al., 2020; Hardt et al., 2016).

Multiple Sensitive Attributes (MSA) EquiPy: an approach to **evaluate and mitigate unfairness** in model predictions.

Objective

Consider an **insurance pricing model** f, its predicted premiums $\hat{Y}$ and two sensitive attributes, ethnic origin A_1 (White/Black) and gender A_2 (Male/Female).



We avoid selecting a **reference category** (White/Black and Male/Female) because:

- if "Black" and "Female" are the references, the total premiums would fall short of the planned amount needed to cover claims,
- if "White" and "Male" are the references, the premiums would exceed the planned amount, leading to higher costs for the insureds.

Intersectional Fairness

MSA $\rightarrow$ Single sensitive attribute (SSA), by intersection:

Female & White	Female & Black
Male & White	Male & Black

Sequential Fairness (Hu et al., 2024)

$$\hat{Y} \longrightarrow \hat{Y} \text{ fair for } A_1 \longrightarrow \hat{Y} \text{ fair for } A_1, A_2$$

- **Interpretability** accross MSA,
- Easily adding sensitive attributes (SA) to meet changing **regulatory demands**.

Paper: **Sequential Fairness**

Python package: **EquiPy**

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Unfairness Evaluation with Optimal Transport

Notations

- $\boldsymbol{X} \in \mathcal{X}$: 'non-sensitive' features,
- $\boldsymbol{A} = (A_1, \cdots A_r) \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_r$: r sensitive features,
- $\hat{Y}$: response variable (continuous or score from a binary classifier)
- f: predictive model on $(\boldsymbol{X}, \boldsymbol{A})$, with f^* the optimal Bayes estimator $\mathbb{E}[Y|\boldsymbol{X}, \boldsymbol{A}]$,
- ν_f: distribution of $f(\boldsymbol{X}, \boldsymbol{A})$ with cumulative distribution function F_f and quantile function Q_f,
- $\nu_{f|a_i}$: conditional distribution of $f(\boldsymbol{X}, \boldsymbol{A})|A_i = a_i$ with $F_{f|a_i}$ and $Q_{f|a_i}$,
- $\mathcal{R}(f) = \mathbb{E}[(Y - f(\boldsymbol{X}, \boldsymbol{A}))^2]$: risk metric.

Demographic Parity for Group Fairness

Demographic Parity requires that the predictions made by a model be **independent** of a specific sensitive attribute A (such as race, gender, or age).

Strong Demographic Parity $\forall\, a_i, a_i' \in \mathcal{A}_i,\ \nu_{f|a_i} = \nu_{f|a_i'}$ or $\text{distance}(\nu_{f|a_i}, \nu_{f|a_i'}) = 0$.

1. f is strongly fair regarding **a single sensitive attribute** (SSA) A_i, if and only if:

$$\mathcal{U}_i(f) = \max_{a_i \in \mathcal{A}_i} \text{distance}(\nu_f, \nu_{f|a_i}) = 0$$

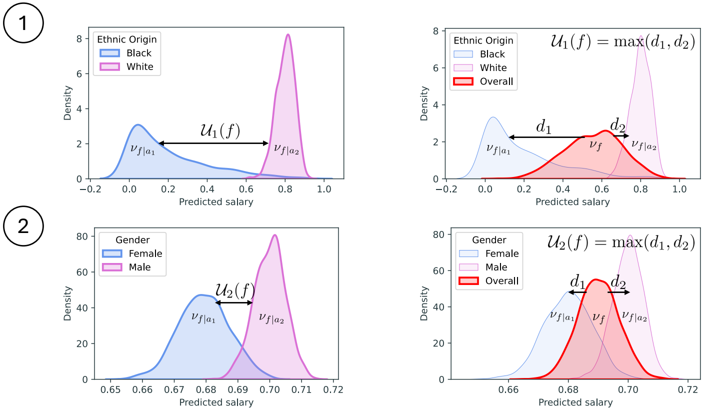
2. f is strongly fair regarding **MSA**, if and only if:

$$\mathcal{U}(f) = \mathcal{U}_1(f) + \cdots + \mathcal{U}_r(f) = 0$$

$\rightarrow$ **Wasserstein distance** from Optimal Transport (OT) theory is employed to compute the distance between distributions.

Example

Strong Demographic Parity for MSA: ethnic origin (A_1) and gender (A_2).



$$\mathcal{U}(f) = \mathcal{U}_1(f) + \mathcal{U}_2(f)$$

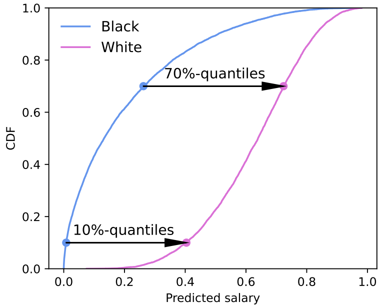
Optimal Transport (OT)

The objective of OT is to minimize the overall cost of moving one mass distribution (ν_A) onto another one (ν_B). We are searching for the most efficient mapping T to move mass between ν_A and ν_B, s.t. $\nu_B = T_{\#}\nu_A$, by solving (Monge, 1781)

$$\inf_{T_{\#}\nu_A = \nu_B} \int_{\mathcal{A}} c(x, T(x))\, d\nu_A(x)$$

For some strictly convex "cost" c, such as quadratic cost, and univariate distributions ν_A and ν_B, the **optimal transport map** T^* is (Santambrogio, 2015)

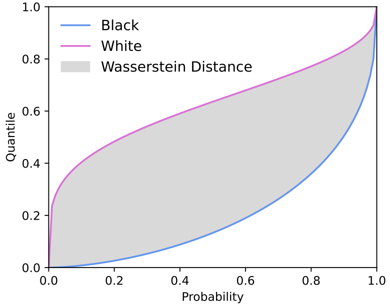
$$T^* = Q_B \circ F_A$$



Optimal Transport and Wasserstein distance

For univariate distributions ν_A and ν_B,
p-**Wasserstein distance** ($p \geq 1$) corresponds to the value of the minimum "cost" required to transform ν_A into ν_B (Wasserstein, 1969):

$$\mathcal{W}_p(\nu_A, \nu_B) = \left(\int_{u \in [0,1]} |Q_A(u) - Q_B(u)|^p \, du \right)^{1/p}$$



→ **Fairness criterion**: $\mathcal{U}_i(f) = \max_{a_i \in \mathcal{A}_i} \mathcal{W}_1(\nu_f, \nu_{f|a_i})$.

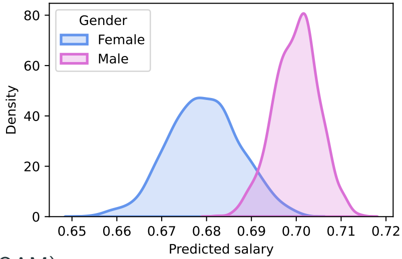
Unfairness Mitigation

Mitigation Methods

Objective: Transform model predictions $f(\boldsymbol{X}, \boldsymbol{A}) \in \mathbb{R}$ into fair ones $f_B(\boldsymbol{X}, \boldsymbol{A})$, while **preserving good performance** $\mathcal{R}(f)$.

- Pre-processing: transform multivariate distribution of $\boldsymbol{X}$,
- In-processing: add a "fairness" penalty in the objective function,

- **Post-processing**: transform univariate distribution of $\hat{Y} = f(\boldsymbol{X}, \boldsymbol{A})$.



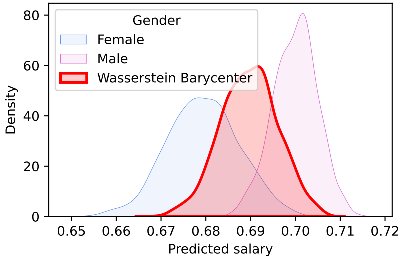
Unfairness Mitigation

EquiPy Mitigation Approach

Wasserstein Barycenter

The **Wasserstein Barycenter** finds a representative distribution that lies between K given distributions $(\nu_1, \ldots, \nu_K)$, and weights $(w_1, \ldots, w_K) \in \mathbb{R}_+^K$. The $\mathcal{W}_2$-Barycenter is the minimizer:

$$\mathrm{Bar}\{(w_k, \nu_k)_{k=1}^K\} = \underset{\nu}{\mathrm{argmin}} \sum_{k=1}^{K} w_k \cdot \mathcal{W}_2^2(\nu_k, \nu)$$



Constructing f_B with Wasserstein barycenter, Gouic et al. (2020) prove $f_B = \mathrm{arginf}_f\{\mathcal{R}(f) : \mathcal{U}(f) = 0\}$.

Achieving Fairness via Optimal Transport

Single Sensitive Attribute $(r = 1)$ (Chzhen et al., 2020)

$\forall\ (\mathbf{x}, a_1) \in \mathcal{X} \times \mathcal{A}_1$,

$$\nu_{f_B} = \mu_{\mathcal{A}_1}(\nu_{f^*}) = \inf_f \sum_{a_1 \in \mathcal{A}_1} \mathbb{P}(A_1 = a_1) \cdot \mathcal{W}_2^2\left(\nu_{f^*|a_1}, \nu_f\right)$$

$$f_B(\mathbf{x}, a_1) = \left(\sum_{a_1' \in \mathcal{A}_1} \mathbb{P}(A_1 = a_1') Q_{f^*|a_1'} \right) \circ F_{f^*|a_1}(f^*(\mathbf{x}, a_1))$$

$\rightarrow$ **EquiPy**: This approach is implemented in the function `FairWasserstein` of `fairness` module.

Example

Consider ML model predictions $\hat{y} = \hat{f}(\boldsymbol{x}, a_1)$ where $a_1 \in \mathcal{A}_1$ corresponds to the observations of the SSA, A_1: ethnic origin (White/Black).

Mitigation approach

$$\hat{f}_{B_1}(\boldsymbol{x}, a_1 = \text{White}) = \mathbb{P}[A_1 = \text{White}] \cdot \hat{f}(\boldsymbol{x}, a_1 = \text{White}) \\ + \mathbb{P}[A_1 = \text{Black}] \cdot Q_{\text{Black}} \circ F_{\text{White}}\big(\hat{f}(\boldsymbol{x}, a_1 = \text{White})\big)$$

$$\hat{f}_{B_1}(\boldsymbol{x}, a_1 = \text{Black}) = \mathbb{P}[A_1 = \text{Black}] \cdot \hat{f}(\boldsymbol{x}, a_1 = \text{Black}) \\ + \mathbb{P}[A_1 = \text{White}] \cdot Q_{\text{White}} \circ F_{\text{Black}}\big(\hat{f}(\boldsymbol{x}, a_1 = \text{Black})\big)$$

Multiple Sensitive Attributes $(r \geq 1)$ (Hu et al., 2024) $\forall\ (\mathbf{x}, \mathbf{a}) \in \mathcal{X} \times \mathcal{A}_{1:r}$,

$$f_B(\mathbf{x}, \mathbf{a}) := f_{B_1} \circ f_{B_2} \circ \cdots \circ f_{B_r}(\mathbf{x}, \mathbf{a})$$

$$f_{B_i} \circ f_{B_j}(\mathbf{x}, \mathbf{a}) = \left(\sum_{a_i' \in \mathcal{A}_i} \mathbb{P}(A_i = a_i') Q_{f_{B_j}|a_i'} \right) \circ F_{f_{B_j}|a_i}(f_{B_j|a_i}(\mathbf{x}, \mathbf{a})) \;,$$

with the i-th component of $\mathbf{a}$ denoted a_i.

Hu et al. (2024) prove the **associativity** of Wasserstein barycenters:
$\mu_{\mathcal{A}_1} \circ \mu_{\mathcal{A}_2}(\nu_{f^*}) = \mu_{\mathcal{A}_2} \circ \mu_{\mathcal{A}_2}(\nu_{f^*})$.

Fairness mitigation remains **unaffected by the order** of $A_{1:r}$.

$\rightarrow$ **EquiPy**: This approach is implemented in the function `MultiWasserstein` of `fairness` module.

Example (1/2)

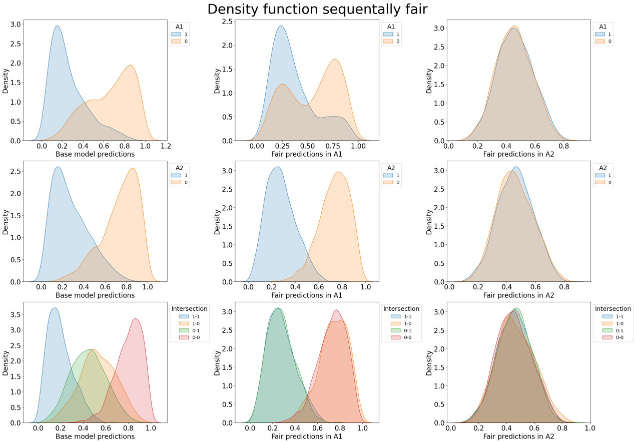
Consider transformed model predictions fair regarding ethnic origin $\hat{f}_{B_1|A_2=a_2}(\boldsymbol{x}, \boldsymbol{a})$ where $\boldsymbol{a} = (a_1, a_2) \in \mathcal{A}_1 \times \mathcal{A}_2$ corresponds to the observations of the MSA, A_1 and A_2: ethnic origin and gender (Male/Female).

Mitigation approach

$$\begin{aligned}\hat{f}_{B_2}(\boldsymbol{x}, a_1, a_2 = \text{Male}) = \mathbb{P}[A_2 = \text{Male}] \cdot \hat{f}_{B_1|A_2=a_2}(\boldsymbol{x}, a_1, a_2 = \text{Male}) \\ + \mathbb{P}[A_2 = \text{Female}] \cdot Q_{\text{Female}} \circ F_{\text{Male}}\left(\hat{f}_{B_1|A_2=a_2}(\boldsymbol{x}, a_1, a_2 = \text{Male})\right)\end{aligned}$$

$$\begin{aligned}\hat{f}_{B_2}(\boldsymbol{x}, a_1, a_2 = \text{Female}) = \mathbb{P}[A_1 = \text{Female}] \cdot \hat{f}_{B_1|A_2=a_2}(\boldsymbol{x}, a_1, a_2 = \text{Female}) \\ + \mathbb{P}[A_2 = \text{Male}] \cdot Q_{\text{Male}} \circ F_{\text{Female}}\left(\hat{f}_{B_1|A_2=a_2}(\boldsymbol{x}, a_1, a_2 = \text{Female})\right)\end{aligned}$$

Example (2/2)



Density function sequentally fair
Density
Base model predictions
Fair predictions in A1
Fair predictions in A2
A1
A2
Intersection
1-1
1-0
0-1
0-0

Illustrative Example

Life Insurance dataset

- Public SEER dataset: `https://seer.cancer.gov`,
- Prediction of **one-year mortality** of US individuals with melanoma skin cancer,
  $\rightarrow$ Utilizing the methodology presented in Sauce et al. (2023), we convert the dataset into survival data, by accounting for **exposure** over a given time interval.
- Sample size $n = 547,878$ from 2004 to 2018,
- Explanatory variables: 16 features describing patient characteristics (age, gender male/female, ethnic origin) and cancer attributes (tumor size, extent).

$\rightarrow$ **MSA** framework: use of the function `MultiWasserstein`.

Model fitting

1. Split the data into train and test sets,
2. Fit **Logistic Regression*** f,
3. Apply f on the test set to obtain $\hat{y}_{\text{test}}$.

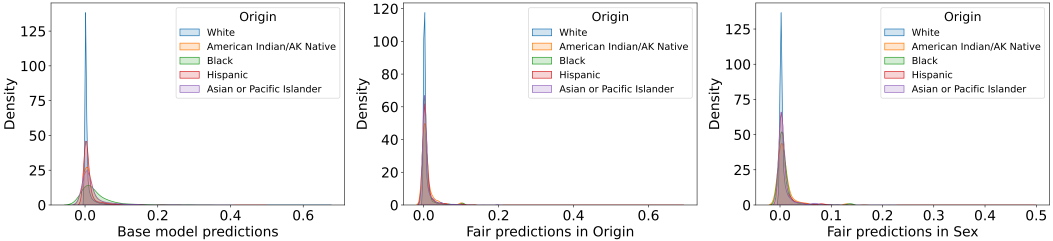
We consider different model fitting scenarios, in which we include or exclude sensitive attributes as explanatory variables:

Ethnic origin	Gender	AUC	Unfairness
No	No	0.87	0.22
Yes	**Yes**	0.87	0.27

***Model-agnosticity** of EquiPy: f can be any ML model.

Transforming predictions

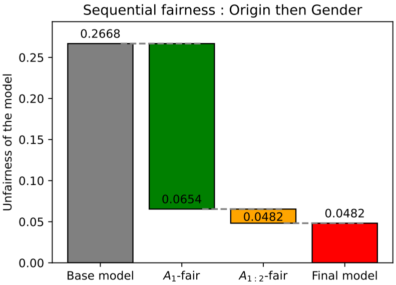
1. Split the test data into calibration and test sets,
2. Specify an order to sequentially correct: A_1 corresponds to ethnic origin and A_2 corresponds to gender,
3. Fit and transform your test predictions using `MultiWasserstein` from `fairness` module.

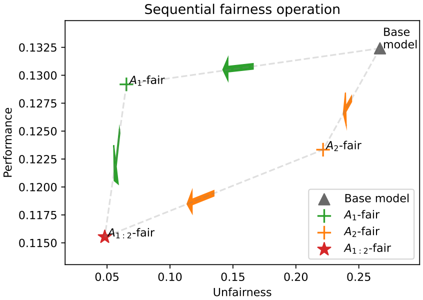


Visualizations

Unfairness and **metric** calculations with `graphs` module:

- `fair_waterfall_plot`: sequential gain in fairness for the order A_1 then A_2,
- `fair_multiple_arrow_plot`: fairness-performance relationship for all potential pathways.





Additional results: Approximate fairness

When correcting biases related to gender, we reduce fairness regarding origin:

Fairness step	Unfairness in origin	Unfairness in gender
Base model	**0.2371**	0.0297
Origin	**0.0345**	0.0309
Origin & Gender	**0.0469**	0.0013

We can **prioritize fairness accross attributes** by specifying $\epsilon = [0, 0.5]$ corresponding to exact fairness in A_1 and 0.5-approximate fairness in A_2.

$$f_B = 0.5 \cdot (f_{B_2} \circ f_{B_1}) + 0.5 \cdot f_{B_1}$$

Wrap up

- The novel approach of **Sequential Fairness**, introduced in Hu et al. (2024), allows to mitigate unfairness regarding **Multiple Sensitive Attributes**.
- The Python package **EquiPy** implements the Sequential Fairness approach and is applicable to **any continuous Machine Learning predictions** (Machado et al., 2025).

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Documentation **EquiPy**

Appendix

References

E. Chzhen, C. Denis, M. Hebiri, L. Oneto, and M. Pontil. Fair Regression with Wasserstein Barycenters. In *Advances in Neural Information Processing Systems*, 2020.

A. Feller, E. Pierson, S. Corbett-Davies, and S. Goel. A computer program used for bail and sentencing decisions was labeled biased against blacks. It's actually not that clear. *The Washington Post*, October 2016.

T. L. Gouic, J.-M. Loubes, and P. Rigollet. Projection to fairness in statistical learning, 2020.

M. Hardt, E. Price, E. Price, and N. Srebro. Equality of opportunity in supervised learning. In D. Lee, M. Sugiyama, U. Luxburg, I. Guyon, and R. Garnett, editors, *Advances in Neural Information Processing Systems*, volume 29. Curran Associates, Inc., 2016. URL `https://proceedings.neurips.cc/paper_files/paper/2016/file/9d2682367c3935defcb1f9e247a97c0d-Paper.pdf`.

F. Hu, P. Ratz, and A. Charpentier. A sequentially fair mechanism for multiple sensitive attributes. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(11):12502–12510, Mar. 2024. doi: 10.1609/aaai.v38i11.29143. URL `https://ojs.aaai.org/index.php/AAAI/article/view/29143`.

A. F. Machado, S. Grondin, P. Ratz, A. Charpentier, and F. Hu. Equipy: Sequential fairness using optimal transport in python, 2025. URL https://arxiv.org/abs/2503.09866.

G. Monge. Mémoire sur la théorie des déblais et des remblais. *Histoire de l'Académie Royale des Sciences de Paris*, 1781.

F. Santambrogio. *Optimal Transport for Applied Mathematicians: Calculus of Variations, PDEs, and Modeling*. Progress in Nonlinear Differential Equations and Their Applications. Springer International Publishing, 2015. ISBN 9783319208282. URL https://books.google.ca/books?id=UOHHCgAAQBAJ.

M. Sauce, A. Chancel, and A. Ly. AI and Ethics in Insurance: a new solution to mitigate proxy discrimination in risk modeling. *arXiv*, 2307.13616, 2023.

G. Upton and I. Cook. *A Dictionary of Statistics 3e*. Oxford Paperback Reference. OUP Oxford, 2014. ISBN 9780199679188. URL https://books.google.ca/books?id=4WygAwAAQBAJ.

Wasserstein. Markov processes over denumerable products of spaces, describing large systems of automata. *Probl. Peredachi Inf.*, 5, 1969.

T. Zebra. Car insurance rating factors by state. https://www.thezebra.com/, 2022.