Cheat Sheet SUBJECT

SUBTITLE – Tommy O – Last edit: April 11, 2017

LOGO

▷ An introductory section

Lorem ipsum. This is some text without meaning. Plug the series expansion into the equation, collect powers of ϵ and set to 0. Let $A(T)$ and $B(T)$ be constants of the slow time. Set coefficients of forcing terms to zero to avoid resonance. Find $A(0)$ and $A(0)$ using $0 = x(0,\epsilon) = x_0(0,0) + \epsilon x_1(0,0) + \dots$. This implies that all $x_i(0,0) = 0$.

▷ Phase plane ($\mathbb{R}^2$)

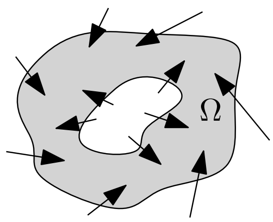
- A good website for plotting phase planes is `http://comp.uark.edu/~aeb019/pplane.html`.
- Quantitative behavior with exact formulas is often unattainable, so we settle for qualitative behavior.
- **Nulliclines** are curves where either $\dot{x} = 0$ or $\dot{y} = 0$.
- Linearizing around $f(\mathbf{x}) = 0$ gives correct information unless $\mathrm{Re}(\lambda_i) = 0$ for some i in the Jacobian matrix of the linearization. If $\mathrm{Re}(\lambda_i) = 0$ the point is "fragile"—linearizing does not always give the correct answer.
- The **basin of attraction** for a point $\mathbf{x}$ is the subset of the phase plane which sends all trajectories to $\mathbf{x}$.
- A **conserved quantity** is a real valued, nonzero, continuous function $E(\mathbf{x})$ such that $\dot{E}(\mathbf{x}) = 0$. The quantity $E(\mathbf{x})$ is conserved along trajectories.
  - A conservative system cannot have any attracting fixed points.
  - If a point $\mathbf{x}$ is a local minimum for $E(\mathbf{x})$, then $\mathbf{x}$ is a (perhaps non-linear) center.
- A system is **reversible** if it's invariant under the a of variables $\mathbf{x} \mapsto R(\mathbf{x})$, $t \mapsto -t$.
  - One common mapping is $y \mapsto -y$, $t \mapsto t$. (Reflection over x-axis.)

▷ Limit cycles(closed orbits)

- **Ruling out closed orbits**:
  - If $\dot{\mathbf{x}} = -\nabla V$ (**gradient system**) there are no closed orbits.
  - If there exists a **Liapunov function** there is no closed orbit. A Liapunov function has:
    1. $V(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{x}^*$(pos.def).
    2. $\dot{V}(\mathbf{x}) < 0$ for all $\mathbf{x}$ (downhill flow)
  - **Dulac's criterion** states that if $\nabla \cdot (g\dot{\mathbf{x}})$ has one sign there are no closed orbits. This is from the div. theorem:

$$\iint \nabla \cdot \mathbf{F}\, dA = \oint \mathbf{F} \cdot \hat{\mathbf{n}}\, dr$$

- **Finding closed orbits**:
  - The **Poincaré-Bendixson theorem** states that if one can construct a trapping region then Ω must have a limit cycle.



- **Relaxation oscillations** operate on two time scales; a slow buildup and a fast release.

▷ References

- Book number one.
- Book number two.