SUBTITLE - Tommy O - Last edit: January 1, 2019

## > An introductory section

Plug the series expansion into the equation, collect powers of  $\epsilon$  and set to 0. Let A(T) and B(T) be constants of the slow time. Set coefficients of forcing terms to zero to avoid resonance. Find A(0) and A(0) using  $0 = x(0, \epsilon) = x_0(0, 0) + \epsilon x_1(0, 0) + \ldots$  This implies that all  $x_i(0, 0) = 0$ .

## $\triangleright$ Phase plane ( $\mathbb{R}^2$ )

- A good website for plotting phase planes is http://comp.uark.edu/~aeb019/pplane.html.
- Quantitative behavior with exact formulas is often unattainable, so we settle for qualitative behavior.
- Nulliclines are curves where either  $\dot{x} = 0$  or  $\dot{y} = 0$ .

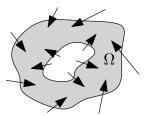
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- Ruling out closed orbits:
  - If  $\dot{\mathbf{x}} = -\nabla V$  (gradient system) there are no closed orbits.
  - If there exists a Liapunov function there is no closed orbit. A Liapunov function has:
    - i.  $V(\mathbf{x}) > 0$  for all  $\mathbf{x} \neq \mathbf{x}^*$  (pos.def).
    - ii.  $\dot{V}(\mathbf{x}) < 0$  for all  $\mathbf{x}$  (downhill flow)

– **Dulac's criterion** states that if  $\nabla \cdot (g\dot{\mathbf{x}})$  has one sign there are no closed orbits. This is from the div. theorem:

$$\iint \nabla \cdot \mathbf{F} \ dA = \oint \mathbf{F} \cdot \hat{\mathbf{n}} \ dr$$

- Finding closed orbits:
  - The **Poincaré-Bendixson theorem** states that if one can construct a trapping region then  $\Omega$  must have a limit cycle.



• Relaxation oscillations operate on two time scales; a slow buildup and a fast release.

## $\triangleright$ References

- Book number one.
- Book number two.