Cheat Sheet SUBJECT

SUBTITLE – Tommy O – Last edit: March 4, 2018

Logo

▷ An introductory section

Plug the series expansion into the equation, collect powers of ϵ and set to 0. Let $A(T)$ and $B(T)$ be constants of the slow time. Set coefficients of forcing terms to zero to avoid resonance. Find $A(0)$ and $A(0)$ using $0 = x(0, \epsilon) = x_0(0,0) + \epsilon x_1(0,0) + \dots$. This implies that all $x_i(0,0) = 0$.

▷ Phase plane ($\mathbb{R}^2$)

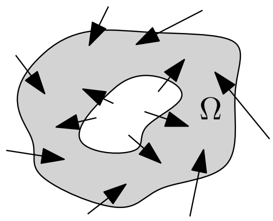
- A good website for plotting phase planes is `http://comp.uark.edu/~aeb019/pplane.html`.
- Quantitative behavior with exact formulas is often unattainable, so we settle for qualitative behavior.
- **Nulliclines** are curves where either $\dot{x} = 0$ or $\dot{y} = 0$.

▷ Limit cycles(closed orbits)

- **Ruling out closed orbits**:
  - If $\dot{\mathbf{x}} = -\nabla V$ (**gradient system**) there are no closed orbits.
  - If there exists a **Liapunov function** there is no closed orbit. A Liapunov function has:
    1. $V(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{x}^*$(pos.def).
    2. $\dot{V}(\mathbf{x}) < 0$ for all $\mathbf{x}$ (downhill flow)
  - **Dulac's criterion** states that if $\nabla \cdot (g\dot{\mathbf{x}})$ has one sign there are no closed orbits. This is from the div. theorem:

$$\iint \nabla \cdot \mathbf{F}\; dA = \oint \mathbf{F} \cdot \hat{\mathbf{n}}\; dr$$

- **Finding closed orbits**:
  - The **Poincaré-Bendixson theorem** states that if one can construct a trapping region then Ω must have a limit cycle.



- **Relaxation oscillations** operate on two time scales; a slow buildup and a fast release.

▷ References

- Book number one.
- Book number two.